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A quantitative experiment on the fountain effect in superfluid helium

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Abstract

Superfluid helium, a state of matter existing at low temperatures, shows many remarkable properties. One example is the so called fountain effect, where a heater can produce a jet of helium. This converts heat into mechanical motion; a machine with no moving parts, but working only below 2 K. Allen and Jones first demonstrated the effect in 1938, but their work was basically qualitative. We now present data of a quantitative version of the experiment. We have measured the heat supplied, the temperature and the height of the jet produced. We also develop equations, based on the two-fluid model of superfluid helium, that give a satisfactory fit to the data. The experiment has been performed by advanced undergraduate students in our home institution, and illustrates in a vivid way some of the striking properties of the superfluid state.

Keywords: superfluid helium 4, undergraduate experiment, fountain effect, thermodynamics

(Some figures may appear in colour only in the online journal)

Liquid helium is a substance which has unique physical properties. The behavior of the liquid at a temperature below 2.17 K shows non classical features. This state is known as the superfluid phase, and the transition temperature (at 2.17 K) has been named the λ-point because of the shape of the specific heat curve at the transition [1–4].

In a laboratory with some expertise on liquid helium, it is not too hard to display some of the unusual properties of superfluid helium, and in this paper we give an example of a relatively simple experiment that has been performed by advanced undergraduates in our home institution. We have made a quantitative measurement of one of the striking

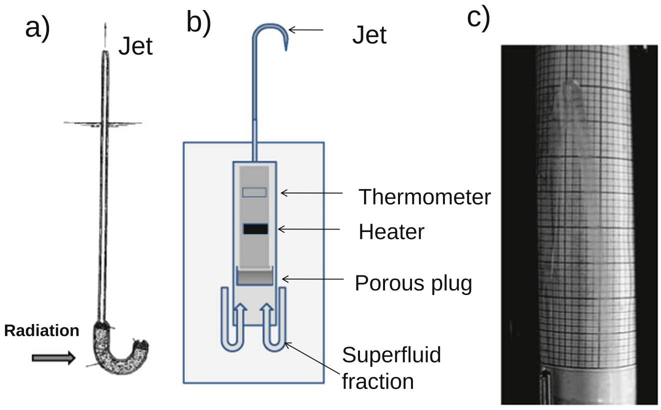

Figure 1. (a) Apparatus used by Allen and Jones in 1938 [5]. The wide and narrow tubes are made of glass and the device is submerged in superfluid helium. (b) Sketch of our setup for producing the jet. The heater increases the temperature inside a region, typically a few millidegrees, forcing superfluid into the volume. The helium level is below the narrow tube and we can measure the height of the jet to estimate the velocity. The interior cross section of the narrow tube is 1.013 mm^2 and that of the wide tube 113 mm^2. Construction is stainless steel with air tight double walls in the thick section. They have been sealed in air at room temperature. At 4 K a cryogenic vacuum is produced between the walls, helping with the thermal insulation inside the tube. (c) Photograph of the jet, seen through the glass windows of the Dewar.

manifestations of the superfluid phase, the so called fountain effect, which can be observed in helium below the λ point.

Many textbooks, among them the classical books by Wilks [1], Atkins [2] Lane [3] or Tilley and Tilley [4] mention the fountain effect, but without giving a quantitative treatment of this subject. In our laboratory we have introduced students to low temperature measurements using the fountain effect. The results of the experiments and a simple quantitative analysis are given in the present paper.

The first description of the fountain effect was given in 1938 by Allen and Jones [5]. In this classic experiment, a jet of liquid was produced when the beam of a flashlight hits a simple apparatus such as that shown in figure 1(a). The setup consists of a glass tube with a narrow and a wide section in series, with the wide section filled with powder of micron size. The beam from the flashlight hits the the powder, supplying heat and raising its temperature slightly. We can understand the origin of the jet using the two-fluid model of liquid helium [1–4].

The model postulates that superfluid helium can be described by two different, interpenetrating and almost non-interacting components. Perhaps, as Landau and Lifshitz [6] point out, it is better to speak of two modes of motion of the liquid, rather than two components, since these two parts do not exist separate of each other and cannot be obtained in isolation. They are related to the excitations and ground state in the description due to Landau [7]. The subject has many subtleties, and a thorough description is beyond the aim of this paper, so that keeping in mind this warning, we use the simplified phenomenological model below. For a more advanced discussion of the physics involved, the full account by Landau [7] or the slightly different approach by Feynman [8, 9] can be consulted. Books on liquid helium [1–4] also treat the problem in some detail and also apply the two-fluid concept to a variety of experimental scenarios. However, the special case considered here is described but not treated quantitatively in these texts.

The two-fluid model is widely used because it gives a very good phenomenological description. It considers a superfluid fraction ρ_s with no viscosity and no entropy, and a normal fraction ρ_n with finite entropy and viscosity. The ratio ρ_s/ρ is unity at $T = 0$ (all superfluid) and zero above 2.17 K (the lamda point), where the whole fluid behaves classically. Here $\rho = \rho_s + \rho_n$ is the total density of the liquid. The ratios ρ_s/ρ and ρ_n/ρ depend only on temperature, and this is important in understanding the origin of the jet.

Because the proportion of both components is strongly temperature dependent there is a strong correlation between concentration and temperature, so that diffusion occurs strongly whenever a temperature difference is established. This is different from convection or classical temperature diffusion because there is a unique connection here between concentration and temperature. There is only one atomic species in liquid helium so that the components considered are not the same as in ordinary liquid mixtures. In the Landau picture [1, 7] the two fluids refer to occupation of the excited states (normal fluid) and ground state (superfluid) of the quantum mechanical many body system. Occupation numbers are basically defined by the mean energy of the system (i.e. the temperature) so that both components can be 'created' or 'destroyed' by changing only the temperature. Conversely one could say that the concentration *defines* the temperature of the mixture. This creates a special type of difussion that is due to the quantum mechanical nature of the superfluid state. Also, as the superfluid fraction has no viscosity, it can very quickly re-establish equilibrium in concentration and therefore in temperature. For this is reason no boiling is observed in superfluid helium and temperature gradients are virtually absent, except in special geometries such as the one we are considering here.

When radiation from the flashlight heats the powder in the wide part of the tube, the hotter region round the powder becomes reduced in superfluid component. To re-establish equilibrium, superfluid tends to diffuse into the hot region and the normal fraction to diffuse out. However, the powder is so tightly packed that the viscosity of the normal component prevents its movement. It is therefore blocked from leaking out towards the cold region. In this way a net flow is established. Only the superfluid, inviscid, component can move into the cavity. When the heat is great enough, forcing superfluid at a high enough rate, some liquid emerges from the narrow part of the tube in the form of a jet or fountain.

The experiment has been repeated many times, and movies of the fountain effect can be seen in the internet [10]. It is mentioned in most texts on superfluid physics [1–4] but a quantitative measurements or equations are not available for the simple setup described here, although systems of related geometries have been studied experimentally and theoretically in the literature [11–14].

In the following, we show the results of a simple but quantitative experiment, in which we have measured the heat input, the temperature and the height of the jet. The data can be analyzed within the usual two-fluid model [1–4], and good agreement is found. However, some subtleties arise and we discuss these in the following.

1. Experimental details

The apparatus producing the jet is very similar to that used by Allen and Jones [5] in the first description of the fountain effect, but we have used an electrical resistance as a heater instead of the flashlight of the original experiment. We have also placed a thermometer inside to monitor the temperature of the liquid. The setup consists of a wide and a narrow tube in series, placed vertically on the cryostat, as is shown in figure 1(b). The bottom of the wide section is blocked by a plug of compacted fine ferric oxide powder (jeweler's rouge), and there is an electrical heater and a carbon resistance thermometer in the interior. The thermometer has been calibrated against the vapor pressure of the helium. The power supplied to the heater can be obtained by measuring the voltage and current using a four wire setup. The height of the jet is measured by sight, comparing it against a scale. For this purpose a wide slit has been left uncovered in the silvering of the glass Dewars used. A picture of the jet, photographed through the glass windows is seen in figure 1(c).

The apparatus is placed inside a glass Dewar and the helium is pumped, while regulating the pressure with a home built rubber diaphragm device. The area of the narrow channel is 1.013 mm^2 and that of the wide tube 113 mm^2. Since the section of the narrow channel is not exactly cylindrical it was measured with more precision by using a microscope image of the mouth of the tube and integrating the area of this image.

2. Analysis within the two-fluid model

The heater dissipates a power W, which raises the temperature inside. This causes the superfluid to diffuse towards the hot region, while the normal fraction is blocked by the packed powder. The superfluid then is heated up to a temperature T but as more superfluid enters the tube, the liquid in it is pushed out through the narrow tube, where it is ejected as a jet. Both components flow together in this jet in the proportion given by the temperature T, which is typically around 15 millidegrees above that of the main helium bath T_B.

Because the temperature difference $T - T_B$ produces a change in the ratio ρ_s/ρ_n as was explained above, the superfluid component difusses towards the warmer region, where its concentration is lower, through the powder and into the tube. The incoming helium must raise its temperature from essentially zero, to a temperature T, so that the proportion of normal and superfluid fractions arrives to the equilibrium ratio at the temperature T. The incoming superfluid must therefore absorb a heat

$$H = \int_0^T C(T')\mathrm{d}T' \tag{1}$$

per unit mass, where $C(T)$ is the heat capacity of the helium. This results in a net mass flow, which then exits the tube as a jet.

The mass flow $\mathrm{d}m/\mathrm{d}t$ in the narrow part of the tube, is given by

$$\frac{\mathrm{d}m}{\mathrm{d}t} = v\rho A, \tag{2}$$

where v is the velocity, ρ the density of helium and A the cross section of the narrow tube.

In a steady state, the power W supplied to the heater is dissipated as heat and if we neglect other losses this balances the heat that must be supplied to the superfluid entering the tube to raise its temperature from zero to T,

$$W = \frac{\mathrm{d}m}{\mathrm{d}t}H = v\rho A \int_0^T C(T')\mathrm{d}T'. \tag{3}$$

In the right hand side we have inserted equations (1) and (2). Here the losses we have neglected include heat going through the walls, and other irreversible effects such as turbulence or the heat that could be exchanged as the superfluid flows between the narrow channels in the powder.

The velocity of the jet is therefore controlled by the power supplied to the heater so that from equation (3)

$$v = \left(\frac{W}{\int_0^T C(T') \mathrm{d}T' \rho A} \right). \tag{4}$$

We think that it may be instructive to compare the above result to a second approach which actually was our first try at quantifying the problem. On reflection, we naively used formulas that are widely used in the literature, but have problems with our actual experimental situation. In the discussion section we will compare the experimental results with both approaches, to see which one captures better the underlying physics of the problem.

The classic book of Landau and Lifshitz [6] provides a way of calculating the mass flow due to a heat input in superfluid helium which is generally employed for obtaining the velocity in situations where a heater produces counterflow of both components [15–17]. Equation (137.1) in Landau and Lifshitz [6] gives, for a heat input q a normal component velocity v_n

$$q = \rho T s v_\mathrm{n}, \tag{5}$$

where s is the entropy per unit mass. We have at first adapted this result to our situation, by assuming that in our setup the power supplied is again producing a mass flow. Because the normal velocity is blocked, the relation $\rho_\mathrm{n} v_\mathrm{n} + \rho_\mathrm{s} v_\mathrm{s} = 0$ assumed by Landau and Lifshitz [6] is not valid, and instead under our conditions we measure a unique velocity v corresponding to both components moving together. However, one could assume that the flow of superfluid through the powder would take over the whole flow calculated for the blocked normal fraction. Integrating q over the heater we get a power W and integrating v over the cross section A we obtain

$$W = \rho T s v A \tag{6}$$

and therefore

$$v = \left(\frac{W}{T s \rho A} \right). \tag{7}$$

In the following we present an experimental test of equations (4) and (7).

3. Experimental Results

3.1. Transients

There is a transient for the temperature measured inside the tube as the heater is switched on.

In figure 2 we show the temperature T read by the thermometer as a function of time. The change of T appears in two steps, there is a plateau that lasts a brief time, from a few seconds to less than a second in the two instances shown. Afterward, a new plateau is observed, with a slight upwards drift. The height of the jet also has a transient, being higher in the first seconds. It is possible that the true stationary state takes a few seconds to be established, before the incoming superfluid comes to be thoroughly mixed with the liquid close to the heater. As we have not studied this behavior in detail we will not discuss it further. In the following we concentrate on the steady state of the system, where the flow reaches a stationary state. This is closer to our assumption of a steady state for the equations as well as making better

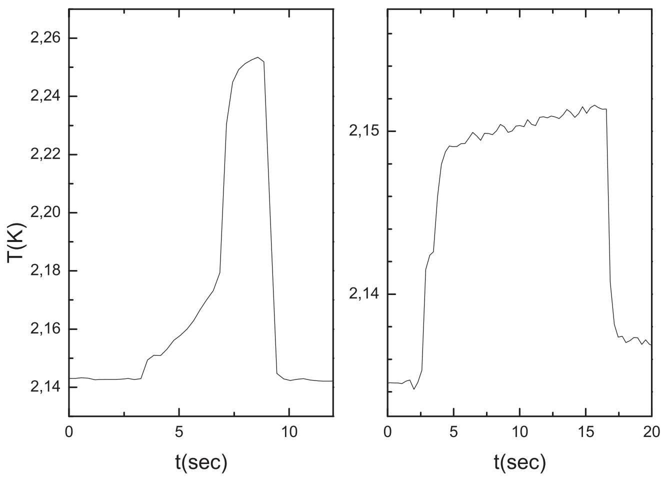

Figure 2. Transient response of the temperature inside the wide tube as heat is applied. The temperature rises as soon as the heater is switched on, there is a plateau which can last about a second (right panel) or a few seconds (left panel) and then a steady state is reached. The power applied is greater for the data shown on the left.

measurements of the height of the jet. These need more than a few seconds to be estimated accurately by sight.

3.2. Steady state

Once the stationary state was reached, we measured the height h of the jet by sight and calculated the velocity of the jet by applying conservation of energy, assuming that the kinetic energy $1/2\rho v^2$ equals the potential energy of the jet ρgh, with g the acceleration due to gravity. In this way $v = \sqrt{2gh}$. The power supplied to the resistance (W) was calculated from the current and voltage measured across the resistance of the heater.

In figure 3 we present a graph of the velocity plotted against the power supplied to the heater, for different values of the bath temperature T_B. The linear behavior is quite clear, except at high power, where the temperature T inside the tube has become higher than the bath temperature T_B. In fact, the slight increase in temperature above T_B necessary for producing the fountain is of the order of a few tens of a millidegree, as was shown in figure 2. The temperature difference $T - T_B$ increases with the applied power W as is to be expected, so the constant temperature which we have assumed in deriving the equations of the velocity starts to break down eventually. Turbulence effects inside the wide tube could also be present, as reported in experiments of Murakami *et al* [13, 14].

We have fitted the linear part of the curves shown in figure 3 and have compared the linear coefficient with that expected from equations (4) and (7). We show the results in figure 4. The experimental points have a spread in temperature which is indicated by the horizontal bars in the graph. This is a measure of the difference in $T - T_B$ produced when W is increased. The slopes are calculated for a given value of T_B assuming that the change in T is small enough when the observed curves is a straight line, as happens over most of the range in figure 3. The spread in temperature is seen to be greater at the lower temperatures measured.

In deriving the equations we have ignored irreversible effects. Losses of heat through the walls of the tube, heat given off to the superfluid as it goes through the packed powder or

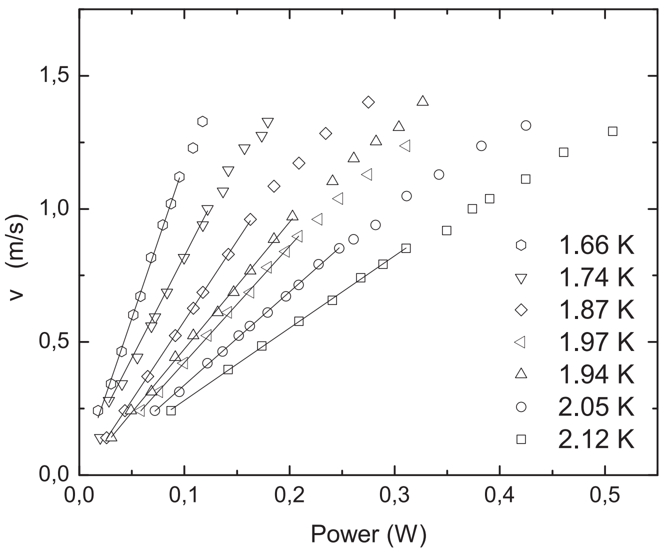

Figure 3. Velocity of the jet plotted against the power supplied to the heater for different temperatures T_B of the bath, indicated on the graph. The linear trend is clear, but it fails at higher heater power. The lines are least square fits to the points inside the linear range.

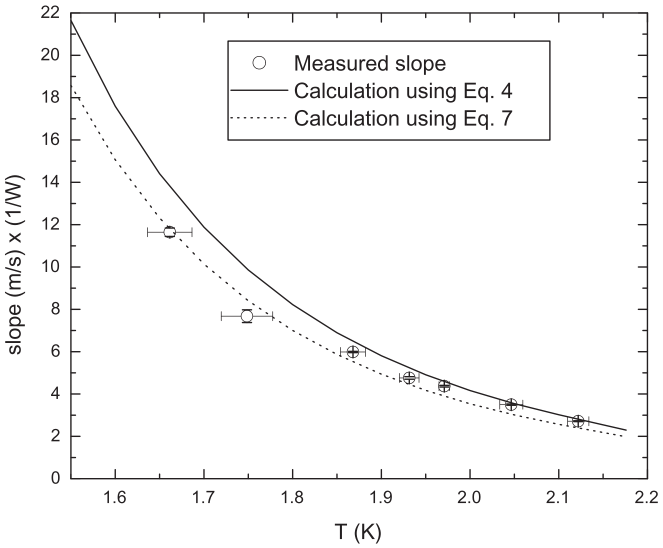

Figure 4. Measured slopes (full circles) and calculated values using equation (4) (upper curve) and equation (7) (lower curve) as a function of the average temperature inside the tube T. The bars are the spread in the temperature T recorded by the thermometer. It can be seen that most of the measured slopes fall between both calculated curves. It could be expected that experimental points would fall below the estimated values, because the calculations assume no extra energy dissipation mechanisms. These are present in the experiment and will reduce the kinetic energy available and therefore the slope of the curves.

dissipation of energy of the heater through any other channel will decrease the mechanical energy of the jet. All these effects will produce a lower slope in the experimental data of figure 3. Because of this, by conservation of energy, the experimental slopes should fall below the calculated values, which assume no losses.

It can be seen that the experimental slopes fall in general above the prediction of equation (7) and below that of equation (4). Only the two points at lower temperature are below the predictions of both equations, in this case the spread in the temperatures seen by the thermometer is greater and turbulence effects could be higher.

Because of the discrepancy observed it may be instructive to analyze with more care the assumptions made in the two ways from which we have calculated the expected velocity of the jet.

4. Discussion

Mathematically, the difference in the terms of the equations are the factors $\int_0^T C(T')\mathrm{d}T'$ in equation (4) which is replaced by sT in equation (7). The entropy is calculated from the expression

$$s = \int_0^T \frac{C(T')}{T'}\mathrm{d}T'. \tag{8}$$

In physics terms, and carrying out the calculations with the T dependence of the specific heat, obtained from the measurements reported in standard tables [18], it turns out that

$$sT = \int_0^T \frac{C(T')}{T'}\mathrm{d}T' \cdot T > \int_0^T C(T')\mathrm{d}T'. \tag{9}$$

Since the factors are in the denominator of both equations the slope is smaller in the case of equation (7).

Physically equation (7) postulates that the heater must supply an entropy sT per unit mass, the entropy of the helium at the temperature T. On the other hand, equation (4) makes the assumption that the superfluid must absorb a heat which takes it from zero temperature to T. Thus, the gradual increase in entropy implied by equation (4) absorbs less heat for the same mass flow than the entropy change assumed to occur at uniform T implied by equation (7). This in turn is reflected in more energy being imparted as kinetic energy to the jet which acquires a higher velocity than expected from equation (7).

Equation (4) takes into account that a 'superfluid filter' is present, in the form of the packed powder and so the liquid entering is heated from zero to T. On the other hand equation (7) has been derived for a continuous volume of liquid helium at temperature T. The presence of the 'filter' therefore influences the way heat is absorbed. The physical reason for sT being larger than the integral $\int_0^T C(T')\mathrm{d}T'$ comes from the fact that a given amount of heat is more effective in raising the temperature when the temperature is lower, as the specific heat is much lower at low temperatures. We believe this is the explanation for favoring equation (4) over (7), because our naive first assumption did not take into account the presence of the superfluid leak that 'filters' the superfluid fraction.

The discussion in terms of components should not be taken too literally. More properly speaking, in the Landau interpretation, one should say that the presence of the packed powder only allows the movement of the collective mode corresponding to the ground state of the helium atoms. However, the Landau description, as happens with quantum mechanical concepts, is intuitively harder to grasp and in general the quasi classical two-fluid description is adequate in most cases. A full treatment of the subtleties of the subject is beyond the scope of this article, but it may perhaps stimulate further discussion among interested students and teachers.

The difference between the velocities calculated is not too great however. In average it is around 14% in the temperature range explored here. The precision of the velocity measurements in many experiments of counterflow or flow through plugs are perhaps not enough to distinguish between the two equations used in this work. However, in situations where a superfluid leak is involved, it would seem that the velocity calculated with equation (4) gives a better estimate, while in counterflow equation (7) may reflect better the experimental situation.

In conclusion we have presented an experimental, quantitative version of the fountain effect of Allen and Misener, which highlights some of the features and subtleties of the two-fluid model of superfluid helium. The experiment is relatively simple and has been performed by advanced undergraduate students, but it is best attempted in an institution where the necessary expertise in low temperature techniques is available. In particular the pressure/temperature regulation and the safe pumping of the helium, although standard in many low temperature laboratories, are the main difficulty that needs to be taken into account.

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