

### Current-voltage characteristics in collective pinning

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The critical-current curves of amorphous samples which exhibit collective flux pinning have been analyzed, and it was found that the nonlinear part can be fitted by an expression of the form  $V(I) \sim [(I - I_K)/I_K]^\zeta$ . This form, which could be due to critical behavior, is consistent with an interpretation of the unpinning of vortices in the collective regime as a phase transition between the static flux-line lattice (FLL) and the FLL in steady-state movement as has been proposed by Fisher.

There has been steady progress in the understanding of critical currents in type-II superconductors, but several aspects of this fundamental and practical problem still remain unsolved. Full quantitative agreement between theory and experiment is more the exception than the norm and there are usually several unknown factors which make the comparison difficult, e.g., one does not know in general the number or the type of defects responsible for the pinning or the actual way in which the forces from individual pinning centers should be added.

In a typical experiment the current-voltage ( $I$ - $V$ ) characteristics are measured at a fixed magnetic field  $H$ . Normally three regimes are observed (Fig. 1):

If the current  $I < I_c$ , where  $I_c$  is the critical current, the voltage  $V = 0$ . There is no power dissipation because the pinning force  $F_p$  is greater than the Lorentz force  $F_L = \mathbf{J} \times \mathbf{B}$  and the vortices do not move.

If  $I \gg I_c$  the voltage is linear with the current,  $V = C(I - I_c^*)$ , and the vortices move viscously at uniform velocity. This is usually known as the "flux-flow" state. The proportionality constant  $C$  is a function of the superconducting properties of the material and does

not depend at all on the pinning centers which determine  $I_c$ .

In general,  $I_c^* \neq I_c$  because there is a third zone where the voltage is not linear and so the extrapolation of the flux-flow region to  $V = 0$  does not coincide with  $I_c$ .

Recently Fisher<sup>1</sup> has suggested the possibility that the static to flux-flow change in regime can be treated as a phase transition and so the nonlinear regime could be a manifestation of critical behavior.

He considers the unpinning of vortices in the collective regime as a phase transition between the static flux-line lattice (FLL) and the FLL in a steady-state motion, with the mean velocity of the FLL as the order parameter. In this case the voltage measured close to the transition would have the form

$$V(I) \sim [(I - I_K)/I_K]^\zeta \tag{1}$$

and the critical exponent would be given by

$$\zeta = \lim_{I \rightarrow I_K} \frac{\ln[V(I)]}{\ln[(I - I_K)/I_K]} \tag{2}$$

Measurements performed in our laboratory<sup>2,3</sup> have shown that in the amorphous superconductors  $Zr_{70}Cu_{30}$  and  $Zr_{75}Rh_{25}$  the two-dimensional (2D) collective flux-pinning theory of Larkin and Ovchinnikov<sup>4</sup> quantitatively adjusts the  $F_p$  data. The measured pinning forces, as in other amorphous materials, are among the lowest measured in the literature, indicating a great degree of homogeneity and making our samples suitable for studying the proposed phase transition. In the present paper results of the nonlinear regime of the  $I$ - $V$  curves for samples which show 2D collective pinning will be analyzed.

To check Eq. (1) we plotted  $V^{1/\zeta}$  against  $I$  and the value of  $\zeta$  was changed until the best straight line, corresponding to the highest correlation coefficient when fitted by least squares, was obtained. The extrapolation of that line to  $V = 0$  is then  $I_K$ . Data of samples with different heat treatments and different materials were analyzed in this way. Typical results of such fits are shown in Fig. 2.

In Fig. 3, we plot  $\ln(V)$  against  $\ln[(I - I_K)/I_K]$ . The flux-flow regime and the nonlinear part are seen as straight lines of different slope, the slope of the nonlinear part being  $\zeta$  and that of the linear part being close to 1.<sup>5</sup>

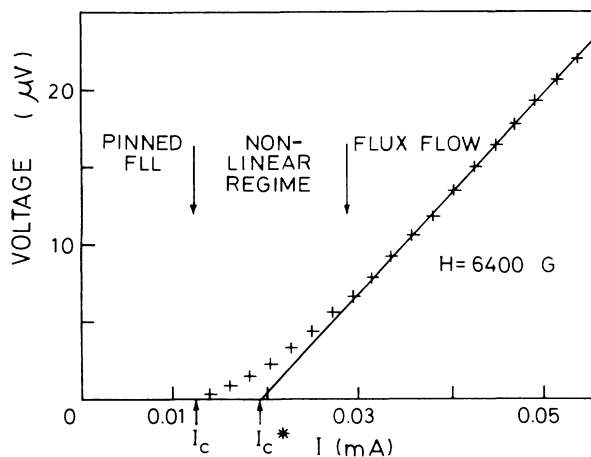


FIG. 1. The three regimes in the  $I$ - $V$  characteristic in flux pinning. The experimental points correspond to a  $Zr_{70}Cu_{30}$  alloy that has been annealed for 2 h at 210°C.

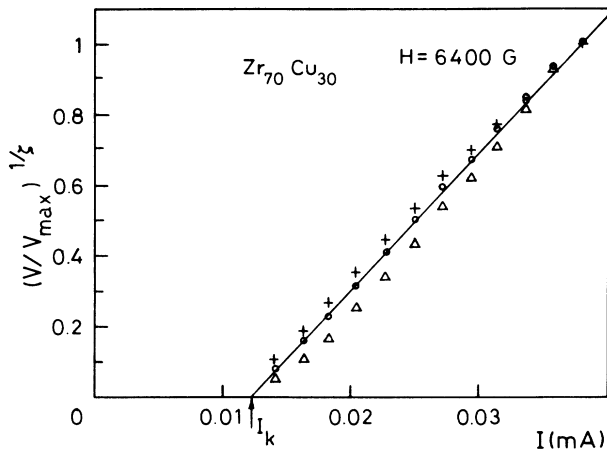


FIG. 2. Voltage raised to  $1/\zeta$  against current. Plus signs denote  $\zeta=1.6$ , circles denote  $\zeta=1.43$ , and triangles denote  $\zeta=1.2$ . Experimental points are the same as those of Fig. 1 and the voltage is normalized with respect to the highest voltage shown.

From Fig. 3 it can be seen that the region over which Eq. (1) is valid is rather wide, as predicted by Fisher<sup>1</sup> for the critical region, so that the logarithmic plot gives a straight line up to values of  $I - I_K$  which are about the same magnitude as  $I_K$ . This makes it possible to define  $\zeta$  using the experimental points over a greater span, although strictly speaking  $\zeta$  should be defined through Eq. (2); that is, through the measurements very close to  $I_K$ . As eliminating the points of higher  $I$  from the least-squares fit used to define  $\zeta$  only changes its value within the experimental dispersion, we consider  $\zeta$  to be a well-defined quantity.

The values of  $\zeta$  obtained in this way are shown in Fig. 4 as a function of reduced magnetic field, for data taken at different measuring temperatures and two different materials. It can be seen that  $\zeta$  remains remarkably con-

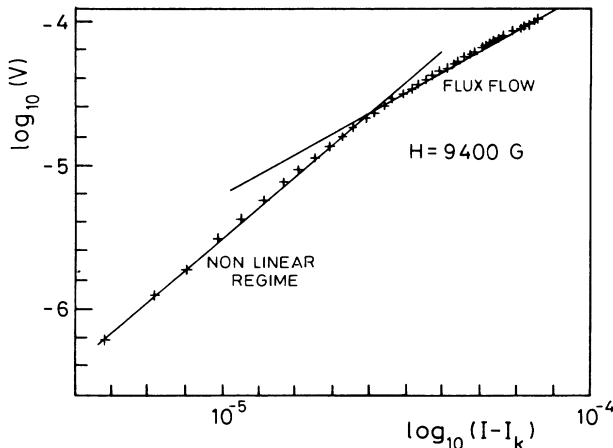


FIG. 3. Logarithmic plot of  $V$  against  $I - I_K$ . Same sample as in previous figures but at different magnetic field.

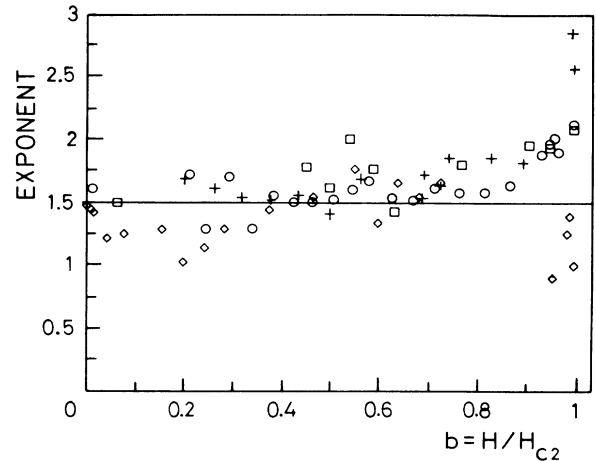


FIG. 4. Values of the exponent calculated from Eq. (1) as a function of reduced field  $b \equiv H/H_{c2}$  for different samples. Plus signs represent  $Zr_{70}Cu_{30}$  annealed for 20 h at  $150^\circ C$  and a further 11 h at  $215^\circ C$  measured at  $t \equiv T/T_c = 0.72$ ; circles are for same sample measured at  $t = 0.59$ , squares are for  $Zr_{75}Cu_{25}$  annealed for 380 h at  $230^\circ C$  measured at  $t = 0.67$ , and diamonds for same as preceding sample but at  $t = 0.82$ .

stant although the pinning force changes from  $22\,800\text{ N/m}^3$  for  $Zr_{75}Rh_{25}$  at  $b=0.9$  and  $T=2.7\text{ K}$ , to  $4000\text{ N/m}^3$  for  $Zr_{70}Cu_{30}$  at  $b=0.3$  and  $T=1.39\text{ K}$ . The value  $\frac{3}{2}$ , which is what mean-field theory<sup>1</sup> predicts for  $\zeta$ , is shown as a reference. The only thing the samples have in common is that they exhibit two-dimensional collective pinning.

Both conditions seem to be necessary to obtain well-defined, temperature- and field-independent exponents. We have studied data for samples which do not show collective pinning at all, such as the sputtered amorphous  $Zr_{70}Cu_{30}$  measured by Frank.<sup>6</sup> The values of  $\zeta$  calculated from these data are not well defined in the sense that excluding the highest-current points from the  $V^{1/\zeta}$ -versus- $I$  plots changes the best-fit exponent (this "pruning," as stated above, does not affect the exponents for the samples represented in Fig. 4). We obtain values above 6, which are strongly temperature and field dependent. We have also tried to extract exponents from measurements on samples which showed a non-2D collective behavior, as evidenced by pinning-force densities which are strongly decreasing functions of applied field. In a previous publication<sup>7</sup> we have interpreted such behavior as three-dimensional collective pinning. For these cases the exponents which can be found are again not well defined, and although they do not depend on temperature, they vary strongly with applied field. As such behavior could be due to a narrower critical region or size effects in the 3D regime, these cases merit further study. Accordingly, in this paper we report results on samples which show clear 2D collective pinning only.

The nonlinear part of the  $I$ - $V$  curves has also been interpreted as a result of a distribution of critical currents in the sample<sup>8</sup> so that different regions reach the flux-flow regime at different current densities, each region

contributing in turn to the voltage observed. In this model the voltage increases as more and more vortices start to move, until all are moving and the linear regime is reached. The model proposed has a distribution of critical currents along the sample  $l(I_c)$  such that

$$V(I) = (\rho/A) \int_{I_K}^I l(I_c)(I - I_c) dI_c, \quad (3)$$

where  $\rho$  is the resistivity in the flux-flow regime and  $A$  the cross section of the sample.

Jones *et al.*<sup>8</sup> have observed that since  $d^2V/dI^2 \propto l(I_c)$ , if  $l(I_c)$  is approximately constant a parabola should fit the nonlinear regime. Integrating (3) with  $l(I_c) = \text{const}$ ,

$$V(I) = A(I - I_K)^2. \quad (4)$$

This is equivalent to using Eq. (1) with  $\zeta = 2$  and eliminating one free parameter, but this form obviously does not fit our data. If one wishes to propose a more complex distribution of currents and still retain a parabolic fit, one could use the general form of the parabola,

$$V(I) = a + bI + cI^2. \quad (5)$$

Both Eqs. (1) and (5) have three adjustable parameters [ $I_K$ ,  $\zeta$ , and  $K$  in (1) and  $a$ ,  $b$ , and  $c$  in (5)], and both can be made to fit the data with similar correlation coefficients, but in our view Eq. (1) gives a more satisfactory explanation from a physical point of view. The fact that  $\zeta$  remains constant appears to be significant, while the coefficients  $a$ ,  $b$ , and  $c$  obtained when adjusting our data with Eq. (5) show no particular trend. Also, since Eq. (5) has a nonzero slope at  $I_c$ , the form of Eq. (3) means that the distribution of currents has a singularity at  $I = I_c$ . Furthermore, it can be seen from the experimental data that the nonlinear part of the curves spans an interval which is roughly the same as the value of the critical current  $I_c$ . Such a broad distribution of critical currents in the same sample would produce a greater dispersion in the values of  $I_c$  measured for different parts of the same amorphous ribbon than is observed in exper-

iments. The overall homogeneity in glasses which is responsible for their low critical current and for the almost universal behavior of their pinning centers<sup>9</sup> also would make it unlikely that the nonlinear region is due to inhomogeneities in the sample. Although other authors report 2D collective flux pinning,<sup>10,11</sup> due to lack of detail in the published  $I$ - $V$  curves we were unable to test Eqs. (1) and (5) against their data. It would be interesting to see if their results give a similar value of  $\zeta$  when Eq. (1) is used.

If one uses Eq. (1) and the procedure outlined here to obtain the current  $I_K$ , one has a way of defining critical current which is free from the arbitrary definition of the threshold voltage and the separation of the voltage contacts in the sample. Comparing the values of  $I_K$  against those of  $I_c$  obtained with the usual definition ( $I_c$  is the current at which  $V$  equals  $1 \mu\text{V}$  with contacts approximately 1 cm apart) it is seen that the values are reasonably close to each other and that the shapes of the  $Fp$ -versus- $H$  curves is the same in both cases.

In conclusion, although it would be necessary to perform other experiments such as measuring the noise<sup>12</sup> generated by the unpinning of vortices to have unambiguous evidence that  $\zeta$  is a critical exponent, the measurements presented here certainly support the hypothesis.

Expression (1) is appropriate for extrapolating  $I_K$  and defining a critical current in absolute terms, independently of its physical meaning.

Theoretical models seeking to explain the  $V(I)$  curves in collective pinning should also take into account the fact that  $\zeta$  is independent of field, temperature, and sample in the 2D regime.

#### ACKNOWLEDGMENTS

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<sup>1</sup>D. S. Fisher, Phys. Rev. B **31**, 1396 (1985).

<sup>2</sup>E. J. Osquiguil, V. L. P. Frank, and F. de la Cruz, Solid State Commun. **55**, 227 (1985).

<sup>3</sup>J. Luzuriaga, C. D'Ovidio, and F. de la Cruz, Solid State Commun. **57**, 753 (1986).

<sup>4</sup>A. I. Larkin and Yu. N. Ovchinnikov J. Low Temp. Phys. **34**, 409 (1979).

<sup>5</sup>If  $I_K = I_c^*$  the slope would be exactly 1. Since the difference is small compared to  $I - I_K$  the deviations from a straight line cannot be observed in this plot.

<sup>6</sup>V. L. P. Frank, Ph.D. thesis, Universidad Nacional de Cuyo, 1985 (unpublished).

<sup>7</sup>E. N. Martinez, V. L. P. Frank, E. J. Osquiguil, and F. de la Cruz, Solid State Commun. **60**, 151 (1986).

<sup>8</sup>R. G. Jones, E. H. Rhoderick, and A. C. Rose-Innes, Phys. Lett. **24A**, 318 (1967); O. V. Magradze, L. V. Matyushkina, and V. A. Shukman, J. Low Temp. Phys. **55**, 475 (1984).

<sup>9</sup>J. Luzuriaga, Phys. Rev. B **35**, 3625 (1987).

<sup>10</sup>P. H. Kes and C. C. Tsuei, Phys. Rev. B **28**, 5126 (1983).

<sup>11</sup>S. Yoshizumi, W. L. Carter, and T. H. Geballe, J. Non-Cryst. Solids **61&62**, 589 (1984).

<sup>12</sup>G. Mozurkewich and G. Gruner, Phys. Rev. Lett. **51**, 2206 (1983).