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T^5 law and Matthiessen's rule

O J Bressan, A E Ridner and F de la Cruz

Centro Atómico Bariloche, Comisión Nacional de Energía Atómica, Instituto de Física "Dr J A Balseiro", Universidad Nacional de Cuyo, San Carlos de Bariloche (RN), Argentina

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Abstract. Precise electrical resistivity measurements on very dilute non magnetic indium alloys show no doubt in a T^5 dependence on temperature between 1–4 K and strong deviations from Matthiessen rule. Measurements of thermal resistivity show that the extrapolation method to obtain the data to $H = 0$ is correct. Comparison of the results with recent theories is discussed.

1. Introduction

Recently a renewed interest in the low temperature behaviour of the electrical conductivity of metals has been shown. Substantial deviations from the Bloch–Gruneisen T^5 law are obtained either theoretically (Ehrlich 1970, Kaveh and Wiser 1972, Lawrence and Wilkins 1972, Bass 1972, Bergman *et al* 1974, Dosdale and Morgan 1974) or experimentally (see eg Salvadori *et al* 1973, and references therein). These deviations are explained by a more realistic treatment of the electrical resistivity when different electron–phonon scattering mechanisms are considered. If these processes are allowed for polyvalent metals, not only departures from the T^5 law are found but also deviations from the Matthiessen rule (DMR) should be observed.

Indium is one of the metals that has been reported (Garland and Bowers 1968) and used (Ehrlich 1970, Lawrence and Wilkins 1972) to show this behaviour. Nevertheless in a previous work Bressan *et al* (1970) have shown that if a careful experimental study of the electrical resistivity is done, no deviations from the T^5 law are found. In this paper we want to insist on this point with more experimental evidence of what we feel is the behaviour of the electrical resistance of indium at low temperatures.

It is important to remark that indium is a superconductor below 3·4 K; consequently to obtain the resistivity at low temperatures it is necessary to extrapolate the magnetoresistance to $H = 0$. It is possible to make important errors if the extrapolation procedure is not correct. We claim that a very careful study of the magnetoresistance is required in order to be able to extrapolate correctly. We have shown (Bressan *et al* 1970) that the Kohler rule is not obeyed and that if it is used as a universal curve for the analysis of the results at $H = 0$, it gives a misleading interpretation.

Table 1. Impurity ratio defined as $r = R(293)/R(0)$. Values for the coefficients ρ_0 and β, according to equation (1), for $x = 5{\cdot}0$.

Sample	Purity	Resistivity ratio $(\rho(293\ \mathrm{K})/\rho(0\mathrm{K}))$	ρ_0 $(10^{-11}\ \Omega\mathrm{cm})$	β $(10^{-11}\ \Omega 3\ \mathrm{K}^{-5})$
A	99·9999% pure indium from Consolidated Mining & Smelting Co Ltd, Canada.	100000	8·86	0·023
B	Sample A + 0·23 ppm of Sn by thermal neutron activation.	73000	12·45	0·027
C	Sample B + 0·23 ppm of Sn by thermal neutron activation.	57000	15·91	0·030

2. Method and results

We report new results for two samples with different resistivity ratios obtained using the same impurification method as the one described before (Bressan *et al* 1970). We obtained for these samples the same universal magnetic behaviour within the experimental error, which was about 2%. Once the universal function is found the resistivity at zero magnetic field, for temperatures below the critical, is a parameter to be determined by fitting the points obtained at a field greater than the critical. This method allows for the determination of the resistivity with an accuracy of 2%. Details of how the extrapolation is done and results showing how an incorrect procedure could lead to important mistakes in the interpretation of the data at zero field can be seen in Bressan *et al* (1970). The results and characteristics of the three samples are shown in table 1 and in figure 1.

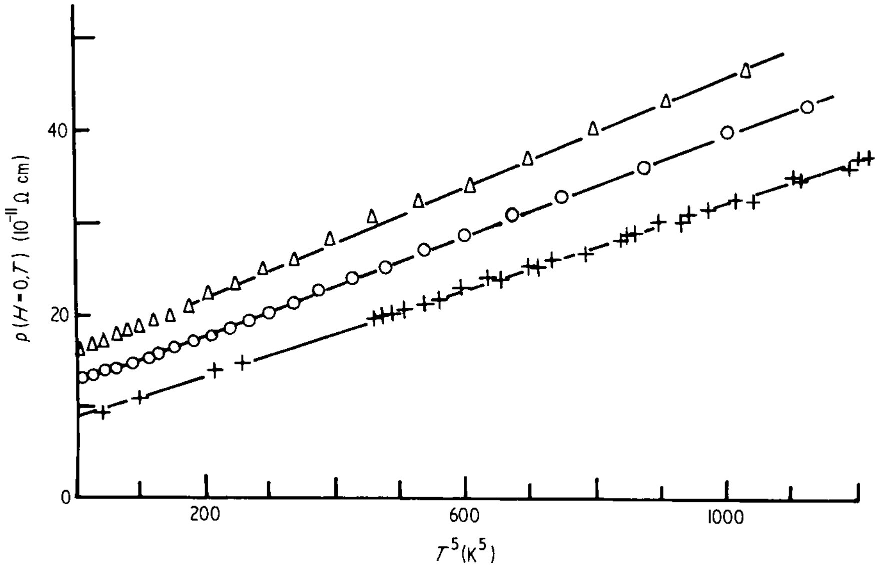

Figure 1. Temperature dependence of three samples with different impurity concentrations as indicated in table 1. Sample A, +; sample B, ○; sample C, △.

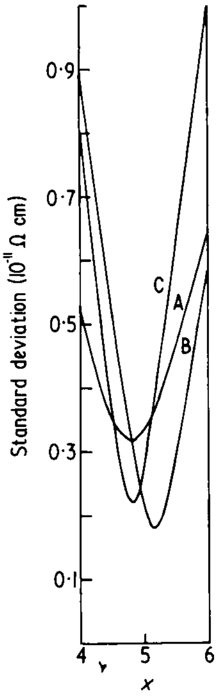

Figure 2. Standard deviation as a function of the exponent x in equation (1).

A temperature dependence of the form

$$\rho = \rho_0 + \beta T^x \tag{1}$$

was proposed. By using the standard least squares method we have found that the minimum standard deviation is within $x = 5{\cdot}0 \pm 0{\cdot}2$. Figure 2 shows a plot of the standard deviation as a function of different exponents for the samples investigated. On the other

Table 2. Coefficients for different linear combinations of integer powers of T we have tried ($\rho = \rho_0 + \Sigma_i a_i T^i$). Only some representative combinations are reported.

Sample	ρ_0	a_2	a_3	a_4	a_5	Standard error
A	8·865				0·0235	0·336
B	12·448				0·0272	0·217
C	15·906				0·0302	0·279
A	7·794	0·1549			0·0220	0·325
B	12·986	−0·1173			0·0287	0·163
C	15·591	0·0886			0·0290	0·246
A	8·102		0·0471		0·0212	0·325
B	12·805		−0·0394		0·0295	0·172
C	15·667		0·0365		0·0280	0·239
A	8·249			0·0213	0·0187	0·325
B	12·710			−0·0188	0·0319	0·180
C	15·702			0·0210	0·0249	0·227
A	7·203	0·3760		−0·0299	0·0266	0·591
B	13·707	−0·4738		0·0662	0·0169	0·330
C	16·384	−0·4070		0·1007	0·0106	0·266

hand if other terms with different integer powers in T are proposed to fit the data we find that the standard deviation is not improved, and what is more important, the coefficient of these terms do not change monotonically with impurity concentration. Moreover for all combinations of different integer powers of T that we have tried, the sign of the coefficients changes at random from sample to sample (see table 2). We conclude that no term in T^2, T^3, T^4 or linear combinations thereof together with the T^5 term should be included to explain the behaviour of the electrical resistivity of In in our range of temperature.

As a further confirmation that our extrapolation method is correct the thermal resistivity of samples B and C was measured. It was possible to find a correspondent Jones and Sondheimer (1969) universal behaviour for the thermal magnetoresistance. Using the same procedure as the one described for the electrical resistivity, the data for the thermal resistivity below the critical temperature were obtained. Using these values to extrapolate to $T = 0$ a Lorenz number for both samples was found that corresponds to the free electron model within 10%. If the Kohler rule is used for thermal and

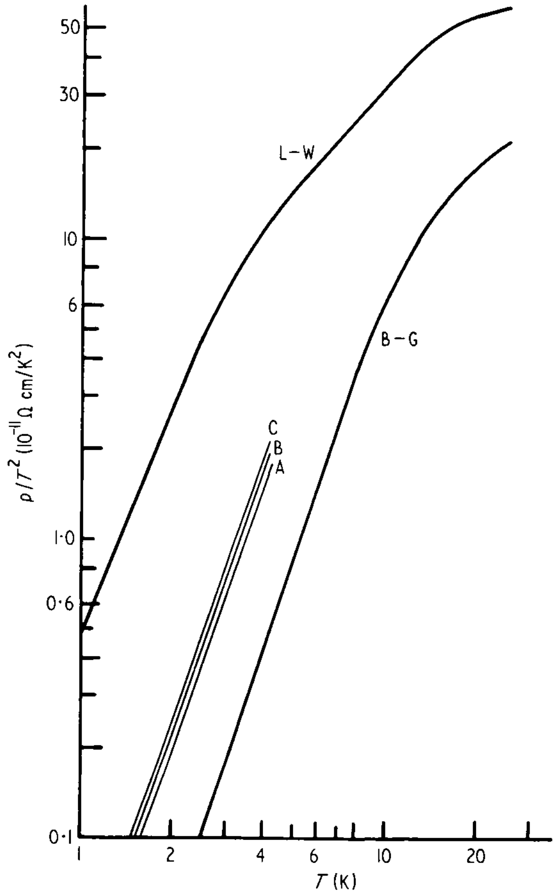

Figure 3. Theoretical results from Lawrence and Wilkins for comparison with our experimental data. The parallel shift indicates deviations from Mathiessen's rule. (B G, Bloch Grüneisen.)

electrical extrapolations to $H = 0$ a Lorenz number $L = 5{\cdot}6 \times 10^{-8}$ $W\Omega K^{-2}$ is obtained (de la Cruz *et al* 1968).

The experimental results for the thermal conductivity and the effect of the impurity concentration on it will be the subject of a forthcoming article.

Figure 1 also shows that although the T^5 dependence is obeyed strong deviations from Matthiessen's rule are evident.

An important parameter that is used by the theorist is the ratio between the residual resistivity and the temperature dependent term as an indication of which is the dominant scattering process. In our experiment the ratio between ρ_0 and βT^5 (equation (1)) is approximately 0·5 at 4 K and 60 at $T = 1{\cdot}5$ K. It is clear from these ratios that indium is a very convenient metal to be used for this investigation.

3. Discussion

From our results we arrive at some general conditions for any theoretical model used to explain the temperature dependence and the Matthiessen rule deviations in indium at low temperatures:

(*a*) the Matthiessen rule deviations should have a T^5 dependence.

(*b*) this dependence should be followed independently of the ratio $\rho_0/\beta T^5$ (60 to 0·5).

The only explicit calculation for indium is due to Lawrence and Wilkins (1972). They predict a T^5 behaviour for the resistivity of polyvalent 'impure' metals at low temperatures. They show that for indium at temperatures below 7 K the main contribution to the resistivity is due to the distortion of the Fermi surface. Figure 3 shows their theoretical results together with the experimental temperature dependent part of the resistivity of our three samples as described by equation (1) with $x = 5$. The experimental points at 4 K are well below the theoretical prediction but there is no reason to expect agreement because the impure limit has not been reached. When the temperature is decreased the impurity scattering becomes dominant and at 1·5 K we can consider that the impure limit is reached for all samples. At this temperature agreement with theory should be expected.

The theory predicts that once the impure limit has been reached, the temperature dependent part of the resistivity should be the same for all samples, regardless of the intrinsic impurity, if a plot such as the one in figure 3 is used. Obviously this will not happen if the DMR follows a T^5 behaviour. We show that there is neither quantitative nor qualitative agreement with the theory in the range of temperature we have studied.

This work is part of a more comprehensive one. Experimental details and data on the electrical and thermal resistivities as a function of field and temperature will be published elsewhere.

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