

Excited States in Even-Even Nuclei with $40 \leq A \leq 154$ and $180 \leq A \leq 226$

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In recent years different authors have made systematic studies of the first excited states of even-even nuclei and found levels with special characteristics which enable their understanding by means of different models. Among these levels are the following:

(a) Sequence of levels with even parity and spins 0, 2, 4, 6 . . . due to collective rotations of the nucleus.¹⁻⁶ This type of levels is characteristic of strongly deformed nuclei. They have been found for $154 \leq A \leq 180$ (Ref. 1), $A \geq 226$ (Ref. 2) and $19 \leq A \leq 25$ (Refs. 3, 4).

(b) Sequence of levels with even parity and spins 0, 2, 0, 2, 4, . . . whose energy relations are of nearly vibrational type.⁶⁻¹⁵ They have been found while analysing the levels in the regions of the periodic table $40 \leq A \leq 154$ and $180 \leq A \leq 226$.

(c) Levels with character, energy and energy relations characteristic of nuclei with neutron and/or proton numbers which close a shell.^{6, 12, 15-17}

(d) Levels with odd parity whose energy is on the line $E = 67A^{-3/2}$ Mev, which are very similar to the ground states of odd-odd nuclei.^{16, 18, 19}

(e) Levels of odd parity, spin 1 and energy smaller than that of group (d) due to a collective excitation related to a pear-form deformation.^{6, 17, 20}

The present paper is concerned with the levels of even-even nuclei with $40 \leq A \leq 154$ and $180 \leq A \leq 226$. This means level groups (b), (c), (d) and (e), and levels of nuclei in the transition regions between nuclei with level groups (a) and (b).

The experimental data reviewed are taken from the following publications:

(a) J. M. Hollander, I. Perlman and G. T. Seaborg, *Revs. Mod. Physics*, 25, 469 (1953), for all the papers published before the end of 1952.

(b) Nuclear Science Abstracts New Nuclear Data, Vol. 7, 24 B; Nuclear Science Abstracts New Nuclear Data, Vol. 8, 24 B for the information during the years 1953 and 1954.

(c) Nuclear Data Cards. National Research Council U.S.A. of 1955, 1956 and 1957 for the information of these years.

(d) For $40 \leq A \leq 92$, we have also used the Nuclear Level Schemes TID-5300 (June 1955), and from the current literature for the more recent publications. The data may be considered as complete up to the end of March 1958.

In the following we review first the results obtained by other authors, and then analyse the results obtained in this compilation of data. The order followed is:

1. Ground state.
2. Character and energy of the first excited state.
3. Excited states with nearly harmonic pattern.
4. Excited states of nuclei in regions between nearly harmonic-pattern and rotational-pattern regions.
5. Excited states of nuclei with neutron and/or proton numbers which close a shell.
6. Excited states of odd parity.

1. GROUND STATE

Stahelin *et al.*²¹ and Scharff-Goldhaber²² state that the ground state of even-even nuclei has presumably always the character 0+. This observation is based on a small number of nuclei whose spins have been measured²³ and on many experimental facts which indicate that this assumption is correct.²²

We have looked for the measured ground state spins, and found that from the 236 nuclei whose ground state is known, there are 55 with measured spin; 38 of them have spin 0 and 17, ~ 0 . In the following we designate the ground state as $10+$.

2. CHARACTER AND ENERGY OF THE FIRST EXCITED STATE

Character

Previous publications state that with a few exceptions this level has character 2+ (Refs. 12, 21, 22). The exceptions (Ca^{40} , Zr^{90} , Ge^{72} , Pb^{208}) and their explanation have been given by Scharff-Goldhaber.¹²

We have found that out of the 130 nuclei of which one knows the first excited state, in 31 it is $2+$, in 68 it is $2+$ and in 4 cases (those mentioned by Scharff-Goldhaber) it is different from $2+$. In the other 27 cases we have supposed that the first excited state has character 2+ and we indicate it as $(2+)$. $\underline{2+}$ means that the spin of the ground state and first excited state has been measured. $2+$ means that the spin of the

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first excited state has been measured as $2+$ if the assumption is made that the ground state has character $0+$.

In Ge^{70} and Zr^{90} the second level is $2+$. In Ca^{40} the third level may be $2+$ (Ref. 24) and in Pb^{208} the character of the levels known is $3-$, $5-$, $4-$ and $5-$. In the following we call the first excited $2+$ state $12+$.

Energy of the $12+$ State $E(12+ \rightarrow 10+)$

It has been known for a long time that the surface formed by the energies $E(12+ \rightarrow 10+)(N, Z)$ of the $12+$ state has relative maxima for values of N or Z which close a shell and maxima for nuclei with N and Z values which close a shell.^{21, 22} The numbers which close shells are 20, 28, 38, 50, 82 and 126 for protons and for neutrons. The number 38 was pointed out by Stahelin and Preiswerk²¹ and all the others by these authors and by Scharff-Goldhaber.²² Recently the surface has been represented by K. Alder *et al.*⁶ Other previous results are:

(a) The energy for nuclei with N and/or Z values which close a shell oscillates between 1.1 and 6 Mev (Refs. 11, 12).

(b) $E(12+ \rightarrow 10+)$ varies less when one adds two protons to a nucleus than when one adds two neutrons.²²

(c) There are no levels with E smaller than 0.3 Mev (Ref. 11).

(d) For $22 < N < 90$, the product $E(12+ \rightarrow 10+) \times A$ is approximately equal to 60 Mev if one excludes the E values which correspond to nuclei with N and/or Z values which close a shell.^{21, 11} The same product is approximately equal to 75 Mev for $114 < N < 134$ (Refs. 11, 21).

As we have to represent a surface and we have at our disposal only two dimensions, we plot on the N, Z plane the lines of equal energy of the $E(12+ \rightarrow 10+)$ surface (Chart I†). This type of representation enables us to see clearly the simultaneous influence of protons and neutrons on $E(12+ \rightarrow 10+)$. A somewhat different representation has been given by K. Alder *et al.*⁶

To obtain points of the isoergic lines, we represented the curves $E(12+ \rightarrow 10+)(Z, N = \text{const.})$ and $E(12+ \rightarrow 10+)(Z = \text{const.}, N)$ and from them we obtained the points Z, N which have the same energy. We have plotted isoergic lines for the following values of the energy expressed in Mev: 0.050, 0.060, 0.070, 0.080, 0.090, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.350, 0.400, 0.450, 0.500, 0.550, 0.600, 0.650, 0.700, 0.750, 0.800, 0.850, 0.900, 1.000, 1.100, 1.200, 1.300, 1.400, 1.500, 1.600, 1.700, 1.800, 1.900 and 2.000. The lines corresponding to 0.400, 0.800, 1.200, 1.600 and 2.000 are indicated with fuller lines so as to visualize them better.

The energy difference between lines is of the order of 5 times the error of the energy determination of an intense gamma ray with a scintillation spectrometer. The "curves" $E(12+ \rightarrow 10+)(Z, N = \text{const.})$ and

$E(12+ \rightarrow 10+)(Z = \text{const.}, N)$ are formed by segments of straight lines because linear interpolation has been used.

Besides the isoergic lines, the following data have been represented:

I. For even-even nuclei:

(a) The energy of the first $2+$ excited state at the crossing point of the two lines, which indicates the neutron and proton number of the nucleus considered.

(b) At the upper left part of the energy the character of the level.

(c) At the upper right part of the energy the ratio of the experimental reduced E2 transition probability $B(E2; 10+ \rightarrow 12+)$ to the single particle estimate for the same quantity $B(E2; 10+ \rightarrow 12+)_{\text{s.p.}}$. From now on we call this ratio F . To calculate the last quantity we have used the following expression

$$B(E2; 10+ \rightarrow 12+)_{\text{s.p.}} = \frac{45}{100} R_0^4 S,$$

where $R_0 = 1.20 \times A^{1/3} \times 10^{-13}$ cm and S is the statistical factor which we take equal to unity. The last assumption is true within 20% regardless of the particle configuration if we assume two particles responsible for the transition. $B(E2; 10+ \rightarrow 12+)_{\text{s.p.}}$ is then expressed in units of $e^2 \times 10^{48}$ cm⁴. The value of $B(E2; 10+ \rightarrow 12+)_{\text{s.p.}}$ in units of $e^2 \times 10^{-51}$ cm⁴ has been plotted at the end of the lower line indicating the A value.

(d) At the lower left side the character of the ground state. A full triangle \blacktriangle when the character is $0+$. A full triangle with a question mark $\blacktriangle?$ when the character is approximately $0+$ and no sign in case the $0+$ is assigned by systematics.

(e) At the lower right side the half-life of the nucleus and/or the abundance in case it is stable. Both data are given only with two significant figures.

(f) The disintegration form is indicated by arrows.

II. For odd-odd nuclei:

(a) The half-life of the nucleus or the abundance in case it is stable. Two half-lives are given only if the isomer gives information on the neighbouring even-even nuclei because it decays directly to them.

(b) The disintegration form is indicated by arrows.

III. (a) At both ends of each $A = \text{constant}$ line we have indicated the A value.

(b) The numbers for which shells are filled are indicated with fuller broken lines. Only the numbers which are suggested by this systematics are indicated.

(c) We indicate the order proposed by M. G. Mayer and J. H. Jensen in which levels are filled for pairs of particles.

IV. The data of regions different from $40 \leq A \leq 154$ and $180 \leq A \leq 226$ contained in Chart I are described by W. Scheuer and E. Aisenberg in a systematic study of even-even nuclei in rotational regions. Observing Chart I, one sees clearly the form of the surface $E(12+ \rightarrow 10+)(Z, N)$ in the regions

† For Chart I, see insert.

where there are enough data. Specially, we can see the absolute and relative maxima at points where double closed shell nuclei are, and on lines of numbers which close shells, respectively.

The numbers which close shells suggested by this systematic are:

For Neutrons:

- 20 Although one sees the influence of the number 20 we do not consider it here because the information for $A < 40$ is lacking.
- 28 We can see clearly the increase in energy when we approach the number 28, which closes a shell. It would be interesting to know the first excited states of Ca^{46} , Fe^{52} and Ni^{56} to complete the level curves in this region.
- 36? There is an increase in energy on the line
- 38? $Z = 32$ when one comes from N values larger than 38. On $Z = 30$, energy increases slightly when one goes from $N = 34$ to $N = 38$. On $Z = 28$ there is an increase from $N = 34$ to $N = 36$, the value for $N = 38$ is unknown. With this information, and taking into account the absolute value of the energies, we might say that there seems to be a closure of a shell for $N = 36$ and/or $N = 38$. It would be very interesting to determine the first excited states of Ni^{66} , Zn^{70} , Zn^{72} , Ge^{66} , Ge^{68} , Se^{70} and Se^{72} to understand this problem.
- 50 The increase of the energy may be seen on the lines $Z = 38$ and $Z = 40$. There are missing data on the first excited states of Se^{84} , Kr^{86} , Kr^{88} , Zr^{88} , Mo^{90} , Mo^{92} , Ru^{94} and Ru^{96} which would complete the information on the closure of this shell.
- 56? There is an abnormality in the isoergic lines for
- 58? these neutron numbers. The energy sequence on $Z = 42$ seems to indicate that it is at $N = 56$. The effect is very weak. It would be interesting to know the first excited states of Zr^{96} , Mo^{102} , Ru^{96} , Pd^{98} and Pd^{100} .
- 82 There is a strong influence of the closure of a shell at $N = 82$. It may be observed on $Z = 54$, 56, 58 and 60. To complete the information it would be nice to know the first excited state of Te^{132} , Te^{134} , Xe^{138} , Ba^{140} , Nd^{140} , Sm^{144} and Sm^{146} .
- 126 The evidence for the closure of a shell at this neutron number is clear on the line $Z = 84$. For $Z = 82$ the energy decreases from $N = 118$ to $N = 124$ (Ref. 25). The exception is Pb^{208} at $N = 126$ of which one does not know the first 2+ state, which certainly has an energy larger than 2.62 Mev. On $Z = 80$ the energy remains nearly constant from $N = 116$ to $N = 124$, the value for $N = 126$ is unknown. It would be interesting to determine the first excited state of Hg^{206} , Rn^{210} and Rn^{212} .

For Protons:

- 20 Although one sees the influence of the number 20

we do not consider it here because the information for $A < 40$ is missing.

- 28 The closure of a shell for this proton number is seen clearly for $N = 30, 32, 34$ and 36. It would be interesting to know the first excited states of Ni^{56} , Ni^{66} , Zn^{60} , Zn^{62} and Fe^{60} .
- 40 The influence of this number is weak and may be seen for $N = 50, 52$ and 54. To complete the information on the closure of this shell it would be nice to know the first excited states of Sr^{84} , Sr^{92} , Sr^{94} , Zr^{86} , Zr^{88} , Zr^{96} , Mo^{90} and Mo^{92} .
- 50 The closure of this shell is seen clearly for $N = 62, 64, 66, 68, 70, 72$ and 74. It would be interesting to know the first excited states for Cd^{106} , Sn^{108} , Sn^{110} , Sn^{126} , Sn^{128} , Sn^{130} , Sn^{132} , Te^{116} , Te^{118} , Te^{132} and Te^{134} .
- 82 The increase of the energy surface due to the closure of this shell is seen clearly for $N = 118, 120, 122$ and 124. For $N = 126$ one does not know the first 2+ state in Pb^{208} but as it is certainly higher than 2.62 Mev, we may say that there is a strong increase in energy from $Z = 84$ to $Z = 82$. The determination of the first excited states of Hg^{194} , Hg^{206} , Pb^{196} , Pb^{198} , Pb^{212} , Pb^{214} , Po^{198} , Po^{200} , Po^{202} , Po^{204} and Po^{206} would be very interesting.

Knowing the numbers which close shells, we may now say that nuclei with these numbers have $0.92 \leq E(2+ \rightarrow 10+) \leq 3.90$ Mev for $40 \leq A \leq 154$, and $0.78 \leq E(2+ \rightarrow 10+) \leq 1.2$ Mev for $180 \leq A \leq 226$, if we exclude Pb^{208} for which we do not know the first 2+ state.

Apart from these shell closures, there are some irregularities which are worth mentioning:

(a) The energy of the first excited state of Se^{78} is such that it produces an irregularity in the energy surface. The value seems to be experimentally confirmed by various means.

(b) The first excited state of Ba^{136} produces an irregularity in the energy surface. The systematics suggest a value of 0.830 Mev instead of 1.041 Mev as it was published.²⁶ An experimental check of this value is desirable.

(c) There is a small irregularity in the energy surface at $N = 84, 86, 88$ and $Z = 60, 62$ for which we have no explanation.

(d) There is an irregularity in the energy surface around Hg^{200} . The energy of this state has been checked by various methods.

(e) The energy of the first excited state of Pb^{210} seems to be more likely 0.783 Mev than 2.36 Mev. This agrees with the theoretical arguments given by Th. Mayer Kuckuk.²⁷

The influence of pairs of protons and pairs of neutrons²² on $E(2+ \rightarrow 10+)$ has been reinvestigated using the two-dimensional representation. To compare this influence we have evaluated the $\Delta E(2+ \rightarrow 10+)$ produced when one varies $N(Z)$ in $\Delta N = \pm 2$ ($\Delta Z = \pm 2$) starting from a number which closes a shell and on a line $Z = \text{const.}$ ($N = \text{const.}$), to which

Table 1

Number of nucleons which close a shell	ΔN	Z	ΔE (Mev)	ΔZ	N	ΔE (Mev)
28	26-28	22	0.60	26-28	30	0.60
					32	0.52
28	28-30	24	0.61	28-30	34	0.20
		26	0.57		36	0.30
50	48-50	38	0.77	48-50	66	0.72
50	50-52	40	0.82	50-52	70	0.62
82	80-82	54	0.54	80-82	122	0.46
		58	0.97		124	0.37
82	82-84	60	0.87	82-84	—	—

the isoergic lines are approximately perpendicular. The values obtained are given in Table 1.

Comparing the influence of pairs of protons and pairs of neutrons for the same number which closes a shell we see that there is a somewhat larger influence of neutrons. We do not think that this is due to a specific property of neutrons but to the fact that the number which closes a shell for protons is related to a larger A than the same number for neutrons and also to the fact that the increase in $E(^{12}+ \rightarrow ^{10}+)$ depends not only on $N(Z)$, but also on the $Z(N)$ values which may or may not be near a closed shell or subshell.

3. EXCITED STATES WITH NEARLY HARMONIC PATTERN

We shall give here (a) a short review of the theoretical predictions for this kind of levels; (b) the reduced E2 transition probability for the $^{10}+ \rightarrow ^{12}+$ transition; (c) the energy and energy relations of the higher excited states; (d) the transitions from the $^{14}+$, $^{20}+$ and $^{22}+$ levels to the lower levels; and (e) the transitions from the $^{30}+$, $^{32}+$, $^{13}+$, $^{24}+$ and $^{16}+$ levels to the lower-lying levels.

Theoretical Predictions

The original analysis of nearly harmonic-pattern excited states in even-even nuclei was given by Scharff-Goldhaber and Weneser⁸ using the Bohr-Mottelson model in the region of weak to moderate coupling. Later on Willets and Jean¹⁰ made an analysis applying the strong-coupling Bohr-Mottelson approximation introducing a γ -unstable potential. Recently Raz¹⁵ examined the results of adding a weak or intermediate surface interaction to the typical two-particle interactions. A review of the theoretical work done with weak coupling is contained in the review article by Alder *et al.*⁶ We shall now give the theoretical prediction of these three different approaches.

*Bohr-Mottelson model in the region of weak to moderate coupling.*⁸—In the limit of zero coupling the lowest excited states are expected to be due to quadrupole vibrations of the core. The excitation quanta are called phonons and have total angular momentum 2 and even parity.

The levels are at $\hbar\omega$, $2\hbar\omega$, $3\hbar\omega$, etc., above the ground state. $\hbar\omega$ is given by

$$\hbar\omega = \hbar(C_2/B_2)^{\frac{1}{2}},$$

where C_2 represents an effective surface tension and B_2 the mass transport associated with the vibration. As the level $2\hbar\omega$ is a degenerate triplet of characters $^{20}+$, $^{22}+$, $^{14}+$ and $3\hbar\omega$ is a quintuplet of characters $^{30}+$, $^{32}+$, $^{13}+$, $^{24}+$, $^{16}+$, M1 radiation is forbidden since current in the nucleus is directly proportional to the velocity of the fluid. For the transitions from the second to the first excited state, we obtain pure E2 radiation and the $^{22}+ \rightarrow ^{10}+$ E2 transition is very weak because of the favored nature of the one-phonon transitions ($^{22}+ \rightarrow ^{12}+$). The collective nature of the excitation gives rise to fast E2 transitions. The F value is given by

$$F = \left[\frac{9}{16\pi^2} Z^2 \left(\frac{\hbar^2/2B_2}{\hbar\omega} \right) R^4 \right] / \left(\frac{R^4}{25} \right).$$

For a nucleus with $Z \sim 40$ and $\hbar\omega \sim 0.75$ Mev, F is ~ 25 . The ratio $B(E2; ^{22}+ \rightarrow ^{12}+)/B(E2; ^{12}+ \rightarrow ^{10}+)$ is two.

In the weak to moderate coupling region the results on transition probabilities are the same with the exception that a weak M1 component is allowed. The degeneracy of the second excited triplet disappears, giving a sequence of levels in the following order: $^{12}+$, $^{22}+$, $^{20}+$, although consideration of anharmonic terms could change the sequence. The ratios of the energies of the $^{14}+$, $^{22}+$, $^{20}+$ states to the energy of the $^{12}+$ state increase with increasing coupling strength and take a value of ~ 2.3 for moderate coupling.

*Bohr-Mottelson model in the region of strong coupling with unstable potential.*¹⁰—In this approach the usual phonon spectrum is obtained when

$$V(\beta) = C/2\beta^2 = C/2 \sum_{\mu=2}^2 |\alpha_{\mu}|^2,$$

with $\hbar\omega = \hbar(C/B)^{\frac{1}{2}}$, where B is the mass parameter.

(a) Energy potential $V(\beta)$ (γ -unstable) gives the following rules for the transitions:

The $^{22}+ \rightarrow ^{10}+$ transition and M1 transitions are forbidden. The $^{10}+ \rightarrow ^{12}+$ transition probability is enhanced and the expression for $B(E2; ^{10}+ \rightarrow ^{12}+)$ is identical with the expression for rotational states. The ratio $B(E2; ^{22}+ \rightarrow ^{12}+)/B(E2; ^{12}+ \rightarrow ^{10}+)$ is two.

(b) Any deviation from gamma instability results in the failure of the rule that the $^{22}+ \rightarrow ^{10}+$ transition is forbidden.

(c) For γ -unstable potentials the position of the $^{20}+$, $^{22}+$, $^{14}+$ levels of the triplet is influenced by the form of $V(\beta)$. If one assumes for $V(\beta)$ a displaced harmonic oscillator $V(\beta) = \frac{1}{2}C(\beta - \beta_0)^2$, one obtains that the $^{22}+$, $^{14}+$ levels form a doublet whose energy varies from 2 to 2.5 times that of the $^{12}+$ state when one passes from small to large equilibrium deforma-

tions. The $^{20+}$ level is near to the $^{22+}$, $^{14+}$ doublet for small deformations and lies considerably higher for larger deformations.

(d) Small deviations from γ instability remove the degeneracy of the $^{22+}$ and $^{14+}$ states depressing the energy of the $^{14+}$ state with respect to that of the $^{12+}$ state.

*Combination of interparticle and collective interactions.*¹⁵—In this approach Raz examines the result of adding a variable surface interaction (weak to intermediate) to a two-particle interaction which may also be varied within the typical two-particle interactions. The strength parameter for the particle interaction is called D and the form of the two-body interaction is:

$$H_{12} = 3Dh\omega[3 - \sigma_1 \cdot \sigma_2] \exp(-r^2/r_0^2),$$

with $r_0 \sim 2.7 \times 10^{-13}$ cm.

The deformation parameter x as defined by Bohr and Mottelson²⁸ is used for the surface interaction. Their results are:

(a) For $x > 0.25$ and all values of D , the $^{22+}$ state and the $^{14+}$ state are at about twice the energy of the first excited state and the $^{20+}$ state is at a higher energy.

(b) For $D \leq 0.4$, the energy of the $^{12+}$ state increases with x and the second excited state has spin $4+$ for all values of x .

(c) For $D > 0.4$, the energy of the $^{12+}$ state decreases as x increases, and when $D = 1.0$ the energy of the $^{22+}$ state is lower than that of the $^{14+}$ level for $x < 0.7$.

(d) For $x > 1.0$, the spectrum becomes almost independent of D .

(e) The transition probability $B(E2; ^{12+} \rightarrow ^{10+})$ is a rapidly increasing function of x even for small x and does not depend sensitively on D .

The nuclei in which we are going to look for levels with nearly harmonic pattern are those for which (α) $Z > 19$, $N > 19$ and $N < 89$. We exclude nuclei which have a closed shell of neutrons (20, 28, 36?, 38?, 50, 82) and/or of protons (20, 28, 40, 50); (β) $Z > 76$, $N > 116$ and $Z < 88$, $N < 126$ and we exclude nuclei which have a closed shell of neutrons (126) and/or of protons (82).

In the following we call the nuclei of the first group α -nuclei and those of the second group β -nuclei. The reason for this division and the limits of the regions will be given later.

Reduced E2 Transition Probability of the $^{10+} \rightarrow ^{12+}$ Transition

It is well known that for α - and β -nuclei the $B(E2; ^{10+} \rightarrow ^{12+})$ value is one order of magnitude larger than the single-particle estimate for that quantity.^{6, 11, 12, 15, 25} The previous results are:

(a) F is approximately 25 for $22 < N < 90$ (Ref. 11) and approximately 30 for $114 \leq N \leq 134$

(Ref. 11). It varies between 10 and 80 for $66 \leq A \leq 150$ (Ref. 12). A representation of F/Z^2 as a function of $E(^{12+} \rightarrow ^{10+})$ for different Z values has been given by Raz.¹⁵ He observed a correlation of F/Z^2 with energy for practically all the Z values which is in agreement with his theoretical predictions.

(b) There is a qualitative correlation between the quadrupole moment of nuclei and their $B(E2; ^{10+} \rightarrow ^{12+})$ values.¹¹

We have represented the F values for α -, β - and closed-shell nuclei as a function of $E(^{12+} \rightarrow ^{10+})$ in Fig. 1. It is interesting to note that the majority of the points for α -nuclei and closed-shell nuclei of the α -region fall on the curve

$$F = 11 + 320 \exp\{-5.1 E(^{12+} \rightarrow ^{10+})\},$$

which is represented by a full line in the figure; E is expressed in Mev. This is true if one takes into account that the $B(E2; ^{10+} \rightarrow ^{12+})$ values are determined with $\pm 25\%$ of error in this region (Coulomb excitation). The two broken-line curves in Fig. 1 represent the F curve with plus and minus 25% .

The closed-shell nuclei for which in the α -region $E(^{12+} \rightarrow ^{10+}) \gtrsim 0.92$ Mev follow the single-particle expression for $B(E2; ^{10+} \rightarrow ^{12+})$, although its value is 11 times larger. These nuclei will be analysed later.

The points corresponding to the different Xe and Kr isotopes have a question mark because their absolute $B(E2; ^{10+} \rightarrow ^{12+})$ values were not determined experimentally; only the relative values were determined. In the case of Xe (Ref. 29) the absolute $B(E2; ^{10+} \rightarrow ^{12+})$ values were obtained using for Xe^{32} the absolute $B(E2; ^{10+} \rightarrow ^{12+})$ value of Te^{126} which has the same energy $E(^{12+} \rightarrow ^{10+})$ and is very near it in the periodic table. For the Kr isotopes Pieper *et al.*³⁰ used the $B(E2; ^{10+} \rightarrow ^{12+})$ value of Se^{82} for Kr^{84} . We have used the value of Mo^{96} for Kr^{82} because Se^{82} is one of the nuclei for which $B(E2; ^{10+} \rightarrow ^{12+})$ falls out of the general trend.

Po^{212} and Po^{214} also have question marks because their $B(E2; ^{10+} \rightarrow ^{12+})$ values have been calculated using the α - γ branching of the $^{12+}$ level and the α -decay theory. This means that they are not measured directly.

From the fifty-seven F values of nuclei in the α -region there are 45 which fall on the curve within their errors. The other 12 values pertain to the nuclei Se^{78} , $\text{Kr}^{78?}$, $\text{Kr}^{80?}$, Mo^{94} and Mo^{100} , which seem to be too high; Ti^{48} , Se^{82} , Ba^{130} , Nd^{146} , Nd^{148} , Sm^{148} and Sm^{150} , which seem to be too low. It would be interesting to remeasure these values to confirm their $B(E2; ^{10+} \rightarrow ^{12+})$ values.

The values for nuclei of the β -region have been indicated with a circle and it is interesting to note that all ten do not fall on the curve for α -nuclei. Because of the small number of values it is difficult to say if they follow some other law; they are always smaller than those of the α -region.

Knowing that $B(E2; ^{10+} \rightarrow ^{12+})$ is a function of $E(^{12+} \rightarrow ^{10+})$ for α -nuclei, it is unnecessary to plot it as a function of N and Z because the trends are going

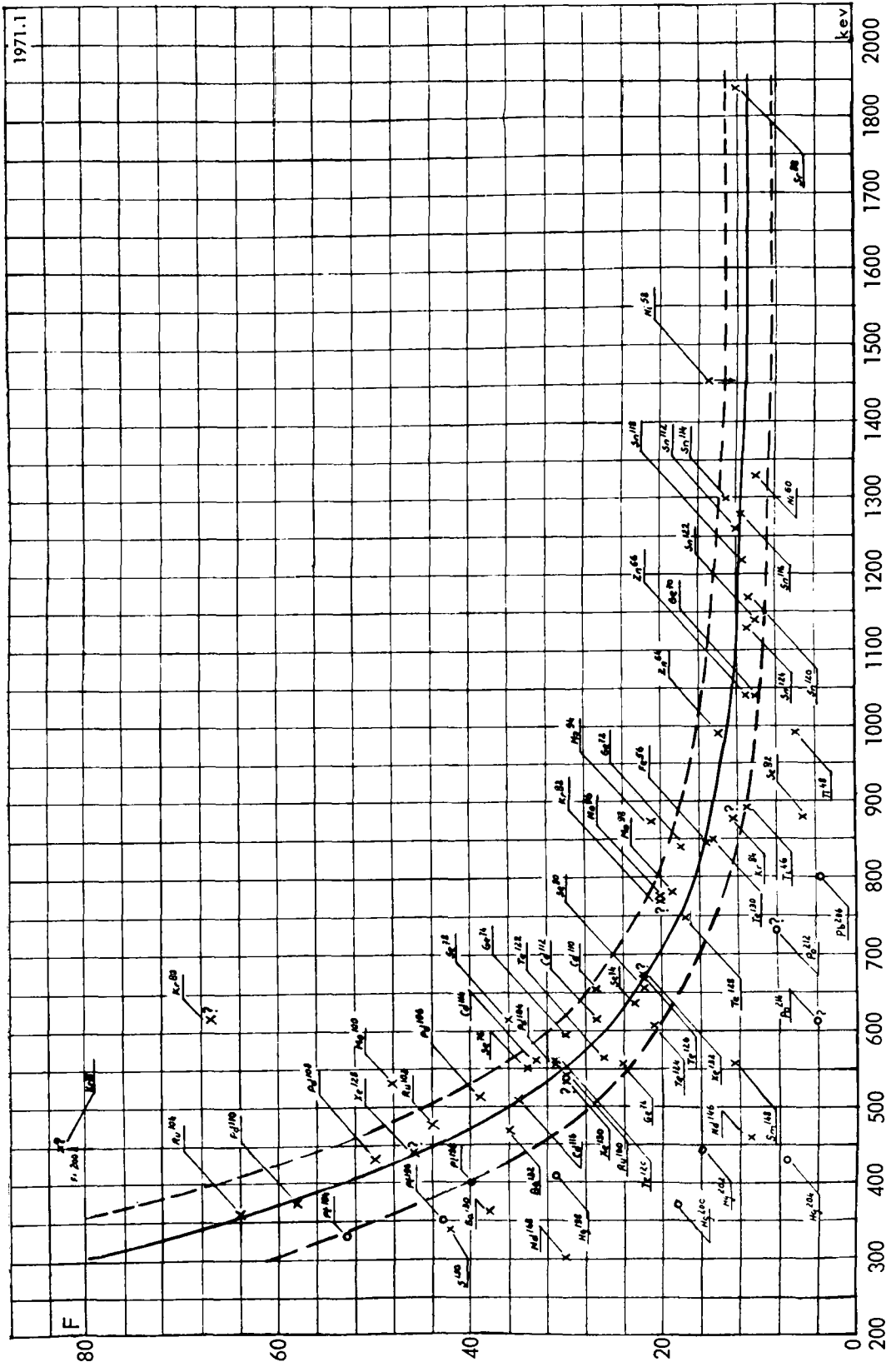


Figure 1. F values for α , β and closed-shell nuclei as a function of the energy $E(12+ \rightarrow 10+)$

to be given by those of $E(10+ \rightarrow 12+)$ which have already been studied.

For β -nuclei²⁵ we see in Chart I and Figs. 2 and 3 that the influence of numbers which close a shell ($N = 126$ and $Z = 82$) on F is stronger than on the energy, specially for $N = 126$. Knowing the energy and $B(E2; 10+ \rightarrow 12+)$ values, we may calculate the parameters C_2 and B_2 .

As we have seen, both are related to the shape oscillations of a spherical nucleus.⁶ C_2 is expressed in Mev and represents an effective surface tension. B_2 represents the mass transport associated with the vibration and is compared with the same parameter (B_2^*) for the surface oscillation of an irrotational and incompressible liquid drop. C_2 and B_2/B_2^* have been tabulated by Alder *et al.*⁶ and represented as a function of N by G. M. Temmer *et al.*³¹ Values for C_2 much lower than those estimated from the liquid drop model are found away from numbers which close a shell and values in excess near closed shells.³¹

We are not giving the dependence of C_2 and B_2/B_2^* as a function of N , Z or the energy $E(12+ \rightarrow 10+)$ because their trends are contained in the trends of $E(12+ \rightarrow 10+)$ and F , already discussed.

Energy and Energy Relations of the Higher Excited States

In nearly harmonic-pattern nuclei we should find at approximately twice the energy of the $12+$ state a

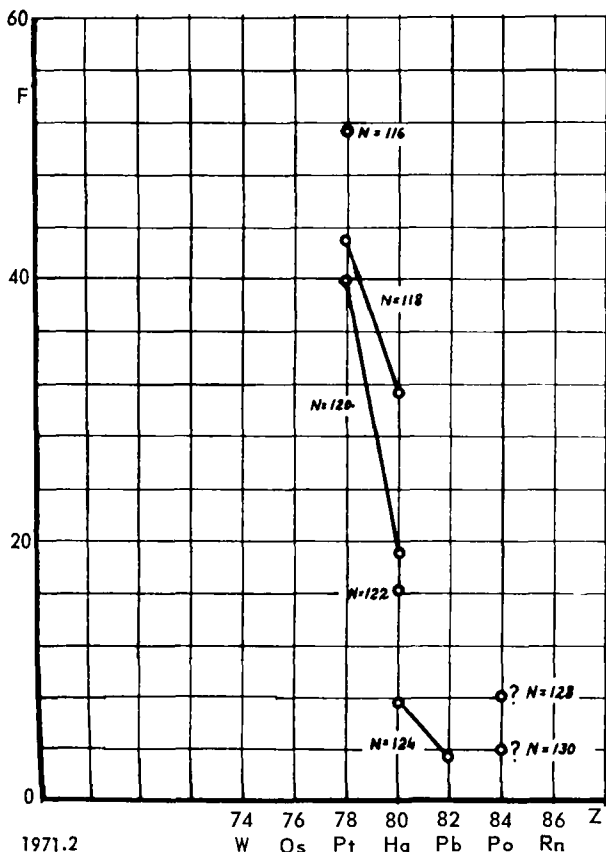


Figure 2. Values of the parameter F as a function of the proton number near a closed shell ($N = 126$, $Z = 82$)

triplet of levels with characters $0+$, $2+$, $4+$ and at approximately three times that energy, levels with characters 0 , 2 , 3 , 4 and $6+$. We look first into the results obtained on the second excited triplet and afterwards on the third excited quintuplet. For the second excited triplet $20+$, $22+$, and $14+$ levels, previous authors arrived at the following conclusions:

(a) The character of the second excited state is predominantly $2+$ and $4+$. Occasionally it is $0+$ and $3+$ (Refs. 8, 9, 11).

(b) The relation between the energy of the second excited state and that of the first excited state fluctuates around the value 2.2 and takes values which vary between 2 and 2.5 (Refs. 6-8, 10-12, 15).

(c) The energy $E(12+ \rightarrow 10+)$ in this region is not smaller than 330 keV and reaches values up to 2 MeV. The ratio of the energies of the second to the first excited state decreases as $E(12+ \rightarrow 10+)$ increases.^{8, 10, 12}

(d) For nuclei with $30 \leq N \leq 40$, E_2/E_1 drops below two (Ref. 13). For nuclei with $22 \leq N \leq 30$ the spin of the second excited state is always $4+$, when measured.¹⁴ For $32 \leq N \leq 50$ the second excited state is always $2+$ when measured, omitting the low-lying $0+$ states in Ge^{70} and Ge^{72} (Ref. 14).

We have looked for all the levels of even parity which have energy $E \leq 3 \times E(12+ \rightarrow 10+)$ and represented them in Chart II \ddagger as a function of N and Z . At the

\ddagger See second insert.

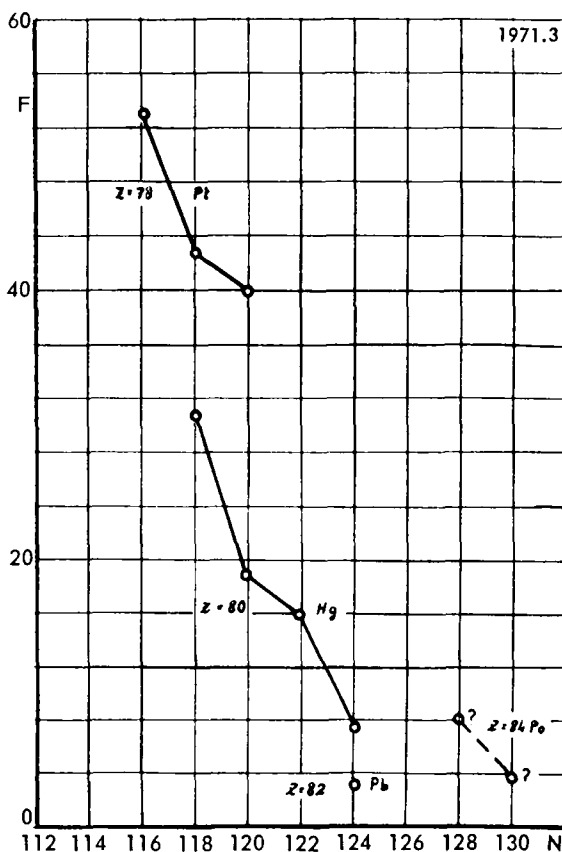


Figure 3. Values of the parameter F as a function of the nuclear number near a closed shell ($N = 126$, $Z = 82$)

Table 2

Nucleus	$E^{(2+ \rightarrow 0+)}$ (MeV)	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$
$^{32}\text{Ge}^{72}$	0.840	0.82	0+	1.74	(2+)	2.07	(4+)	2.46	(2+)	2.84	(2+)
$^{36}\text{Kr}^{82}$	1.777	1.90	(2+)	2.36	(4+)	2.68	3+
$^{36}\text{Kr}^{84}$	0.879	2.16	(2+)	2.47	(?)	2.69	(?)
$^{42}\text{Mo}^{96}$	0.778	2.08	(4+)	2.37	(?)	2.52	(?)
$^{48}\text{Cd}^{110}$	0.656	2.16	(?)	2.25	2+	2.35	(?)
$^{48}\text{Cd}^{114}$	0.559	2.17	2+	2.31	4+	2.35	0+	2.46	2+
$^{78}\text{Pt}^{192}$	0.316	1.93	2+	2.48	4+	2.90	3±
$^{84}\text{Po}^{212}$	0.729	1.83	(?)	2.21	(?)	2.47	(?)
$^{84}\text{Po}^{214}$	0.609	1.35?	(?)	2.11?	(?)	2.27	(2+)	2.32	0+	2.54	(?)
		2.74	(?)	2.85	(?)	2.89	(?)

upper left of the energy of the first excited state the energy ratios of the energy of the known 2+ states to the energy of the $^{12+}$ state are given; at the upper right those for the 4+ levels, at the lower left those for the 0+ levels and at the lower right those for levels with different characters than 0+, 2+ or 4+ and those for levels with unknown character. A ratio without a parenthesis indicates that the spin of the levels involved is measured. We consider the ground state as if it would be always measured and 0+. A ratio with a parenthesis means that at least one of the spins is not measured but inferred. A question mark indicates that the character of the level is unknown and when it appears beside the 2+ levels it means that the stop and crossover gamma rays have been measured. Two question marks indicate that the level position is doubtful.

Observing Chart II we can realize that the values are larger than two, with very few exceptions: Ge^{72} , Kr^{82} , Mo^{94} , Ba^{134} , Ba^{136} , Pt^{192} , 194, 196, Po^{212} , 214.

There are nuclei which have three or more levels at approximately twice the energy of the first excited state. They are given in Table 2, together with the

Table 3

Nucleus	$E^{(2+ \rightarrow 0+)}$ (MeV)	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$	Observations
$^{32}\text{Ge}^{72}$	0.840	1.74	(2+)	2.07	(4+)	First level is 0+
$^{36}\text{Kr}^{92}$	0.7769	1.90	(2+)	2.36	(4+)	—
$^{48}\text{Cd}^{114}$	0.559	2.17	2+	2.31	4+	—
$^{56}\text{Ba}^{134}$	0.605	1.93	2+	2.33	4+	—
$^{78}\text{Pt}^{192}$	0.316	1.93	2+	2.48	4+	—
$^{80}\text{Hg}^{198}$	0.412	2.43	(4+)	2.63	2+	—
		or				
		2.56				

Table 4

Nucleus	$E^{(2+ \rightarrow 0+)}$ (MeV)	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$
$^{44}\text{Ru}^{100}$	0.542	2.10	(0+)	2.50	2+
$^{46}\text{Pd}^{106}$	0.513	2.19	2+	2.22	0+
$^{46}\text{Pd}^{108}$	0.430	2.20	2+	2.40	(0+)
$^{48}\text{Cd}^{114}$	0.559	2.17	2+	2.35	0+
$^{84}\text{Po}^{214}$	0.609	2.27	(2+)	2.32	0+

energy ratios and the energy of the $^{12+}$ state. We have excluded $^{56}\text{Ba}^{136}$ because the energy of the first excited state and the ratios disagree with the systematics. Ge^{72} is also a special case because it has a 0+ first excited state. A possible explanation for this has been given by Scharff-Goldhaber.¹²

Kr^{82} has two levels ((2+) and (4+)) which could be members of the vibrational triplet, the third level 3+ could be a member of the group of levels which should be at three times the energy of the first excited state.³² We find a similar situation in Pt^{192} . In Po^{212} the ratio is smaller than 2 and so also in Po^{214} , although in this case the level is uncertain. In Po^{214} there are 6 well-known levels with energy ratios between 2 and 3. Of the remaining nuclei, Kr^{84} , Mo^{96} , Cd^{110} and Cd^{114} , only in the last one is the character of the levels well known; three of them (0+, 2+, 4+) could form the vibrational triplet. We shall see later that the first three levels are the ones which form the vibrational triplet. These results show that there is some evidence for the existence of the second excited triplet.

Table 5

Nucleus	$E^{(2+ \rightarrow 0+)}$ (MeV)	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$S\pi$
$^{48}\text{Cd}^{114}$	0.559	2.31	0+	2.35	4+

Table 6

Nucleus	$E^{(2+ \rightarrow 0+)}$ (MeV)	$\frac{E}{E^{(2+ \rightarrow 0+)}}$	$\frac{E}{E^{(4+ \rightarrow 0+)}}$
$^{22}\text{Ti}^{46}$	0.892	...	2.25
$^{22}\text{Ti}^{48}$	0.996	...	2.32
$^{24}\text{Cr}^{54}$	0.835	...	(2.22)
$^{26}\text{Fe}^{56}$	0.845	...	2.47
$^{32}\text{Ge}^{72}$	0.840	...	(2.07)
$^{36}\text{Kr}^{82}$	0.777	...	(2.36)
$^{36}\text{Kr}^{84}$	0.879	...	(2.69)
$^{42}\text{Mo}^{96}$	0.778	...	(2.08)
$^{44}\text{Ru}^{98}$	0.655	...	(2.14)
$^{48}\text{Cd}^{114}$	0.559	...	2.31
$^{54}\text{Xe}^{130}$	0.535	...	2.24
$^{54}\text{Xe}^{132}$	0.670	...	(2.17)
$^{54}\text{Xe}^{134}$	0.860	...	(2.28)
$^{56}\text{Ba}^{134}$	0.605	...	2.33
$^{58}\text{Ce}^{142}$	0.630	...	(2.38)
$^{60}\text{Nd}^{146}$	0.460	...	(2.63)
$^{78}\text{Pt}^{192}$	0.316	...	(2.48)
$^{80}\text{Hg}^{200}$	0.368	...	2.58

Table 7

Nucleus	$E(12+ \rightarrow 10+)$ (MeV)	$\frac{E(12+ \rightarrow 10+)}{E(12+ \rightarrow 10+)}$
$^{26}\text{Fe}^{58}$	0.802	2.04
$^{32}\text{Ge}^{74}$	0.596	(2.07)
$^{34}\text{Se}^{76}$	0.550	2.18
$^{36}\text{Kr}^{82}$	0.777	(1.90)
$^{36}\text{Kr}^{84}$	0.879	(2.16)
$^{44}\text{Ru}^{100}$	0.542	2.50
$^{44}\text{Ru}^{102}$	0.475	2.31
$^{44}\text{Ru}^{104}$	0.358	2.50
$^{46}\text{Pd}^{104}$	0.555	(2.39)
$^{46}\text{Pd}^{106}$	0.513	2.19
$^{46}\text{Pd}^{108}$	0.430	2.20
$^{46}\text{Pd}^{110}$	0.370	2.19
$^{48}\text{Cd}^{110}$	0.656	2.25
$^{48}\text{Cd}^{112}$	0.615	2.10
$^{48}\text{Cd}^{114}$	0.559	2.17
$^{48}\text{Cd}^{116}$	0.510	2.38
$^{52}\text{Te}^{122}$	0.564	2.23
$^{52}\text{Te}^{126}$	0.670	(2.09)
$^{54}\text{Xe}^{126}$	0.386	(2.23)
$^{54}\text{Xe}^{128}$	0.440	(2.23)
$^{56}\text{Ba}^{134}$	0.605	1.93
$^{78}\text{Pt}^{192}$	0.316	1.93
$^{78}\text{Pt}^{194}$	0.329	1.89
$^{78}\text{Pt}^{196}$	0.354	1.94
$^{80}\text{Hg}^{198}$	0.412	2.63
$^{84}\text{Po}^{214}$	0.609	(2.27)

Table 8

Nucleus	$E(12+ \rightarrow 10+)$ (MeV)	$\frac{E(10+ \rightarrow 10+)}{E(12+ \rightarrow 10+)}$
$^{44}\text{Ru}^{100}$	0.542	(2.10)
$^{46}\text{Pd}^{106}$	0.513	2.22
$^{46}\text{Pd}^{108}$	0.430	(2.40)
$^{48}\text{Cd}^{114}$	0.559	2.35
$^{84}\text{Po}^{214}$	0.609	2.32

nuclei which have a sequence of levels $10+$, $12+$ and $20+$ are given in Table 8 and the values of the ratio given in the last column are plotted as a function of $E(12+ \rightarrow 10+)$ in Fig. 6.

Figures 4, 5 and 6 show that there is no correlation of the ratios with $E(12+ \rightarrow 10+)$. The known doublets suggest that generally the sequence of levels is $22+$, $20+$, $14+$; this is supported by the mean values of the ratios for sequences $10+$, $12+$, $22+$; $10+$, $12+$, $20+$ and $10+$, $12+$, $14+$ which are respectively 2.18, 2.26 and 2.30. This result is against the predictions of the Bohr-Mottelson model in the regions of strong coupling and of weak to moderate coupling. The position of the $22+$, $14+$ levels agrees with Raz's predictions for $D = 1.0$ and $x < 0.7$, but does not seem to be compatible with the results on the position of the $20+$ level.

Third excited quintuplet.— $30+$, $32+$, $13+$, $24+$, $16+$ levels. We have looked for nuclei in which there is some evidence for the existence of levels of known or inferred character which have positive parity and approximately three times the energy of the first excited $2+$ state. In Table 9 we give these data together with the information for the two phonon excitations in the same nuclei.

It is interesting to note that only the $(5+)$ level of Fe^{56} has a character which does not agree with the character of the possible three phonon excitations. This is an evidence for the presence of the three phonon excited states in nuclei in these regions.

Transitions from the $14+$, $20+$ and $22+$ Levels to the Low-Lying Levels

(1) $14+$ state.—The $14+$ level decays via E2 gamma rays to the first excited $2+$ state. The E4 crossover

Let us now look for the known $12+$, $14+$ doublets. They are given in Table 3. With the exception of Hg^{198} , in all of them the $2+$ level is lower than the $4+$ level by an amount which varies from 10% to 20% . We have excluded Ge^{72} .

The known $20+$, $22+$ doublets are given in Table 4. With the exception of Ru^{100} all the $2+$ levels are lower than the $0+$ levels and within 10% of their energy. The $20+$, $14+$ doublet is known only in Cd^{114} (see Table 5).

Nuclei which have a sequence of levels $10+$, $12+$, $14+$ are given in Table 6. The values of the ratio in the last column have been plotted as a function of $E(12+ \rightarrow 10+)$ in Fig. 4. The nuclei which have a sequence of levels $10+$, $12+$ and $22+$ are given in Table 7. The values of the ratio in the last column are plotted in Fig. 5, as a function of $E(12+ \rightarrow 10+)$. The

Table 9

Nucleus	$E(10+ \rightarrow 12+)$ (keV)	2+	0+	4+	0+	2+	3+	4+	6+
$^{22}\text{Ti}^{48}$	990	—	—	2.32	—	—	—	3.21	3.35
$^{26}\text{Fe}^{56}$	845	—	—	2.47	—	3.15	—	—	3.75(5+) 3.51(?)
$^{36}\text{Kr}^{82}$	777	(1.90)	—	(2.36)	—	—	2.68	—	—
$^{42}\text{Mo}^{96}$	780	—	—	(2.08)	—	—	—	—	(3.10) (3.57) ^{5, 6+} 2.37(?) 2.51(?)
$^{46}\text{Pd}^{106}$	513	2.20	2.21	—	—	3.00	—	—	—
$^{48}\text{Cd}^{114}$	559	2.17	2.35	2.31	—	—	(3.33)3, 4+	(3.33)3, 4+	—
		2.46	—	—	—	—	—	—	—
$^{52}\text{Te}^{124}$	603	2.18	—	—	—	(3.25)2, 4+	—	(3.25)2, 4+	—
$^{54}\text{Xe}^{130}$	535	—	—	(2.24)	—	—	—	—	(3.62)
$^{56}\text{Ba}^{134}$	604	1.93	—	2.33	—	—	—	3.26	—
$^{78}\text{Pt}^{192}$	316	1.93	—	2.48	—	—	2.90(±)	(3.8)	—

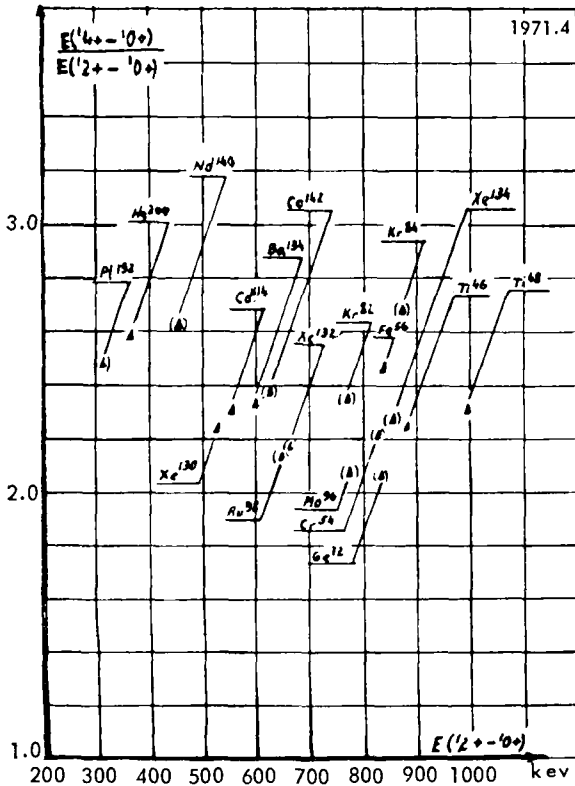


Figure 4. Values of the ratio $E(4+ - 10+)/E(2+ - 10+)$, as a function of $E(2+ - 10+)$

transition is forbidden from the vibrational point of view and is a factor of approximately 10^9 less probable, assuming single-particle matrix elements. The E4 transition has never been measured. In Ti^{46} a γ transition of $1.67 < E_\gamma < 2.33$ Mev and intensity $1.2 \times 10^{-5}\%$ of that of the stop-over transition of a $4+$ level lying at 2.010 Mev energy to a $2+$ level at 0.892 Mev, has been measured.³³ This gamma ray could be the crossover of a very weakly fed $2+$ level in the decay of Sc^{46} ; both calculated probabilities are of the same order of magnitude.

(2) $20+$ state.—This state decays mainly via an E2 gamma ray to the first excited $2+$ state. It may also decay through the emission of an E0 conversion electron to the ground state. The latter transition has been found in Pd^{106} , Cd^{114} and Po^{214} . In Table 10 we give the energy of this state, the position of the level, the ratio of its energy to that of the first excited state, the nuclear "strength parameter" ρ for the E0 conversion electron transition³⁴ and in the last column observations on the assumptions or results of measurements made to calculate or determine ρ . For the sake of completeness we also give data on other $0+ \rightarrow 0+$ transitions which have been measured in these regions of the periodic table and the known $0+$ levels for which there are no measurements available. The ρ values are followed by a question mark when they have been calculated using some theoretical assumption.

An analysis of the ρ values obtained and their significance has been given by M. Deutsch.³⁶

(3) 2^2+ state.—This state obeys certain rules for

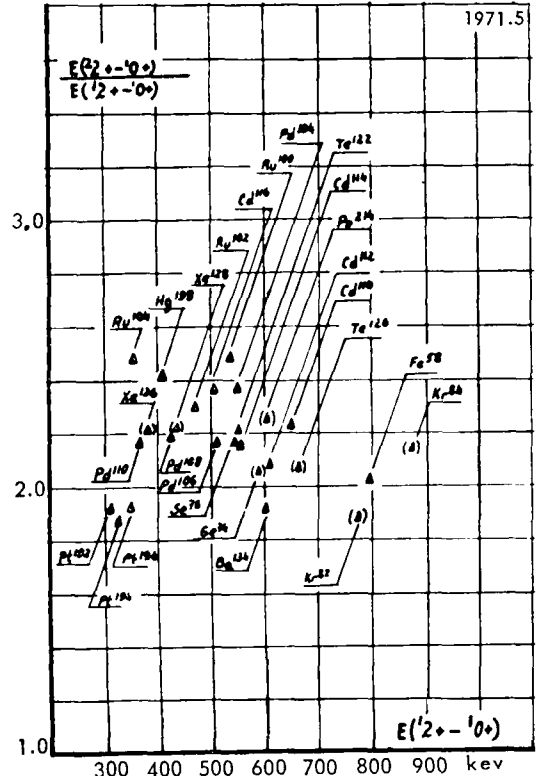


Figure 5. Values of the ratio $E(2+ - 10+)/E(2+ - 10+)$, as a function of $E(2+ - 10+)$

the transition probabilities and for the multipole orders of the transitions which were first pointed out by Kraushaar, Goldhaber⁴⁰ and Sheffen.⁴¹ Reviews of these properties^{6, 11, 12, 42} give the following results:

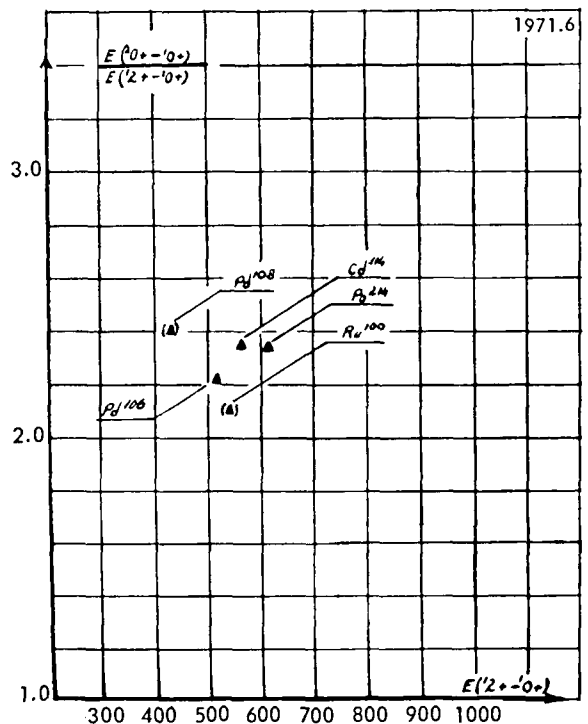


Figure 6. Values of the ratio $E(20+ - 10+)/E(2+ - 10+)$ as a function of $E(2+ - 10+)$

Table 10

Nucleus	$E(^{20+} \rightarrow ^{10+})$ (Mev)	Position	$\frac{E(^{20+} \rightarrow ^{10+})}{E(^{12+} \rightarrow ^{10+})}$	$\rho(E0; ^{20+} \rightarrow ^{10+})$	Observations
$^{20}\text{Ca}^{40}$	3.35	1st level	—	—	—
$^{32}\text{Ge}^{70}$	1.215	2nd level	1.17	0.09	(Ref. 35) $(I_{\pi}/I_{\alpha})_{\text{exp.}} \leq 0.02$; Theor. = 0.01.
$^{32}\text{Ge}^{72}$	0.690	1st level	—	0.11	(Refs. 36, 30) $(I_{\pi}/I_{\alpha})_{\text{theor.}} = 0$
$^{40}\text{Zr}^{90}$	1.750	1st level	—	0.06	(Refs. 35, 36) $(I_{\pi}/I_{\alpha})_{\text{exp.}} = 0.2$
$^{44}\text{Ru}^{100}$	1.140	2nd level	2.10	—	Spin not certain; c.e. not measured
$^{46}\text{Pd}^{106}$	1.137	3rd level	2.22	$\lesssim 0.14?$	(Ref. 5) Based on assumptions on rates of competing gamma-ray transitions
$^{46}\text{Pd}^{108}$	1.043	3rd level	2.40	—	(Refs. 37, 27) Spin not certain; c.e. not measured
$^{48}\text{Cd}^{114}$	1.308	4th level	2.35	$\sim 0, 6?$	(Ref. 34) $K/L = 9$; theory $K/M = \infty$. (Ref. 38) Based on assumptions on rates of competing gamma-ray transitions
$^{84}\text{Po}^{214}$	1.416	6th level	2.32	0.045?	(Refs. 39, 36) Based on $(I_{\alpha}/I_{\alpha})_{\text{exp.}}$ and α -theory

(a) The gamma transition to the $^{12+}$ state has a ratio of M1 to E2 of ~ 0.1 (Refs. 11, 12) which is approximately 10^3 times smaller than that expected by single-particle estimates (Refs. 12, 41) and 17 times smaller than expected from empirical transition probabilities.¹¹

(b) The ratio of intensities of the transition to the ground state to the E2 part of the transition to the $^{12+}$ state is frequently about 0.1 instead of 25 as expected for single-particle transitions.¹² The ratio of the transition matrix elements varies from $\frac{1}{3}$ to $\frac{1}{25}$, averaging about $\frac{1}{7}$, instead of being unity.¹¹

(c) In Cd^{114} and Pt^{194} one knows the ratios of $B(E2; ^{22+} \rightarrow ^{12+})$ to $B(E2; ^{12+} \rightarrow ^{10+})$ and they are 1.4 and 0.66 respectively.¹²

Church and Weneser³⁴ have pointed out that the $^{22+} \rightarrow ^{12+}$ transitions are very interesting because they may proceed via nuclear E0 conversion electron emission. Lindqvist and Marklund⁴³ have given a review of the experimental information about these E0 transitions.

We give the results available on $^{10+}$, $^{12+}$, $^{22+}$ level sequences in Table 11.

The first three columns give the energies in kev of the $^{12+} \rightarrow ^{10+}$, $^{22+} \rightarrow ^{10+}$ and $^{22+} \rightarrow ^{12+}$ transitions, respectively. The fourth column gives the ratio of $E(^{22+} \rightarrow ^{10+})$ to $E(^{12+} \rightarrow ^{10+})$; this ratio has no parenthesis when all the spins involved are measured and a parenthesis when one of the spins at least is not measured but inferred. Column five gives the experimental reduced E2 transition probability over the single-particle estimate, from the $0+$ ground state to the $^{12+}$ state. The numbers with a question mark are those for which only a relative value of their transition probabilities has been measured. Column six gives the experimental reduced E2 transition probability from the $0+$ ground state to the n th $2+$ state over the single-particle estimate for the same quantity. The 0.72 value which is given in parentheses is the mean

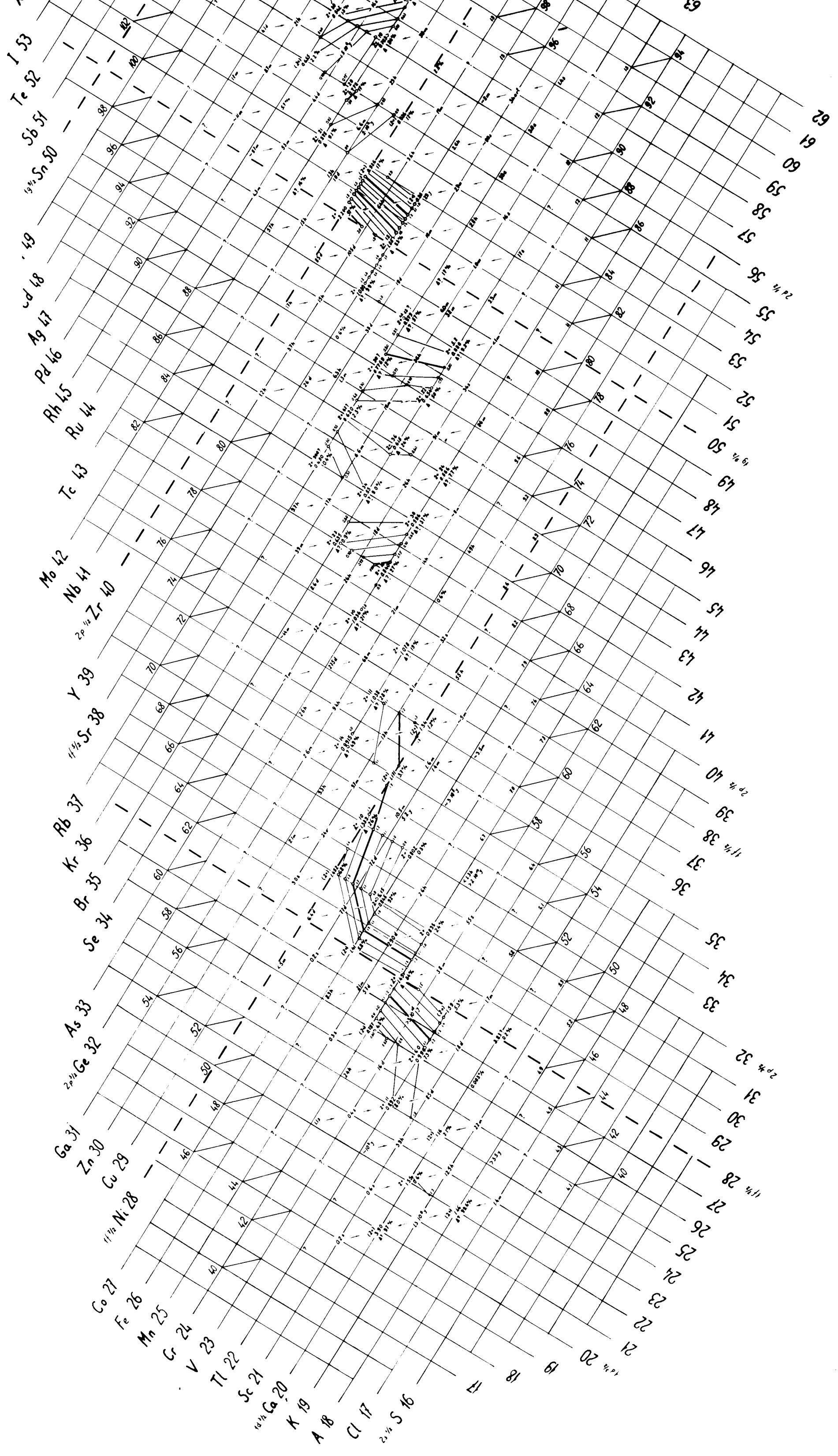
of the measured values for all the α nuclei. Column seven gives the ratio of the reduced E2 transition probabilities of the transitions $^{22+} \rightarrow ^{12+}$ and $^{10+} \rightarrow ^{22+}$ times $(1 + \delta^2)$. δ^2 is the ratio of M1 to E2 radiation intensity. This product is given by

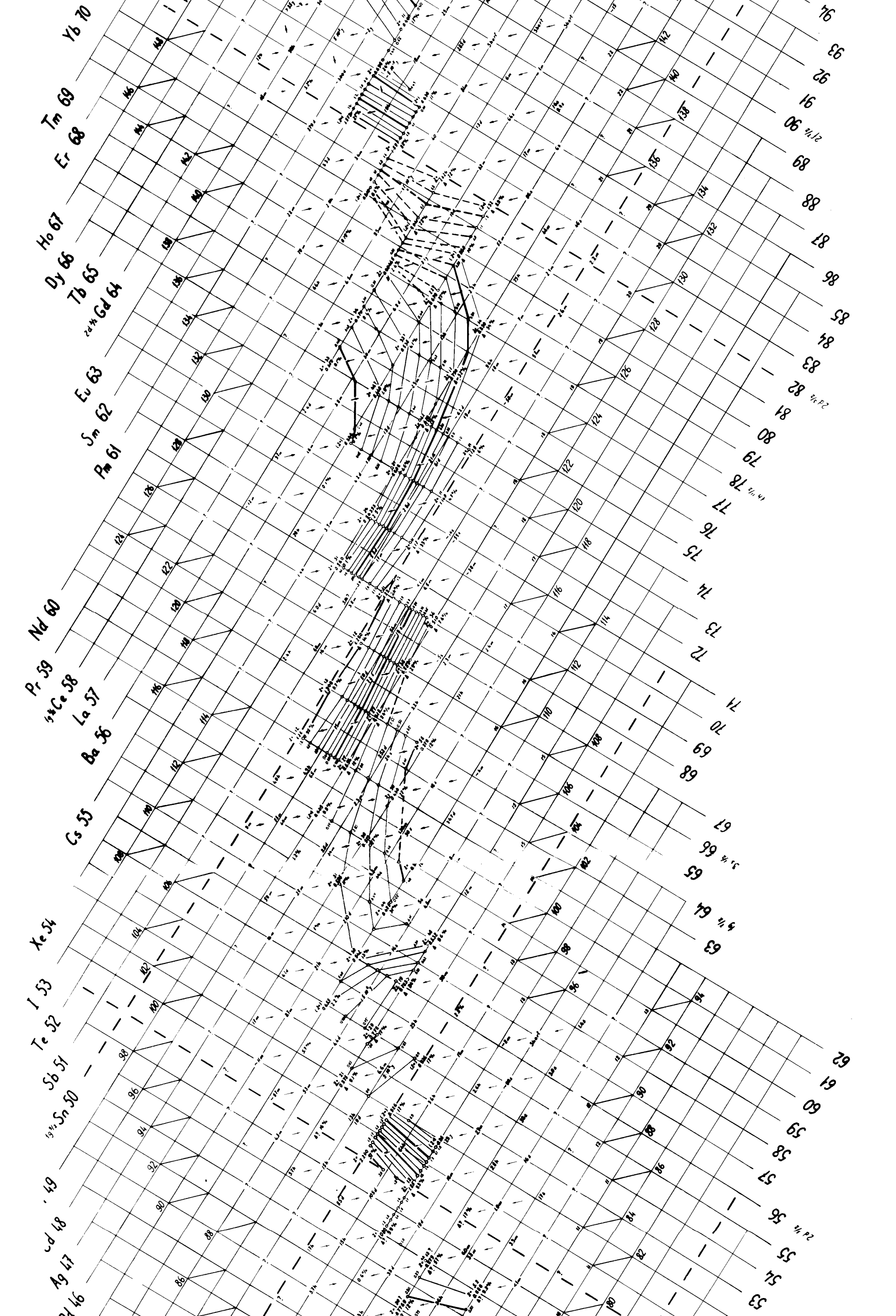
$$\frac{I(M1 + E2; ^{22+} \rightarrow ^{12+})}{I(E2; ^{22+} \rightarrow ^{10+})} \left[\frac{E(^{22+} \rightarrow ^{10+})}{E(^{22+} \rightarrow ^{12+})} \right]^5,$$

where I indicates intensity of gamma radiation. Its value is written down only when δ^2 is not measured. In column eight we give the δ^2 values and in column nine δ^2 times $A^3 E^2 / 4 \times 10^5$ (E in Mev), which should be unity if the M1 admixture to the E2 radiation would follow the single-particle estimates. Column ten gives the ratio between the reduced E2 transition probabilities of the $^{22+} \rightarrow ^{12+}$ and $^{10+} \rightarrow ^{22+}$ γ transitions. Column eleven gives the ratio between the reduced E2; $^{22+} \rightarrow ^{12+}$ transition probability to the single-particle estimate. The parentheses indicate that we have used $(1 + \delta^2) \times B(E2; ^{22+} \rightarrow ^{12+}) / B(E2; ^{10+} \rightarrow ^{22+})$ instead of $B(E2; ^{22+} \rightarrow ^{12+}) / B(E2; ^{10+} \rightarrow ^{22+})$; the question mark indicates that we have used (0.72) for $B(E2; ^{10+} \rightarrow ^{22+}) / B(E2)_{\text{s.p.}}$ which is the mean value for all the α -nuclei. The same nomenclature has been used for the ratio between the reduced E2; $^{22+} \rightarrow ^{12+}$ and $^{10+} \rightarrow ^{12+}$ transition probabilities which is given in column twelve. Column thirteen is the ratio of the reduced M1; $^{22+} \rightarrow ^{12+}$ transition probability to the single-particle estimate. The expression for it is

$$\frac{\delta^2 A^3 E^2}{4 \times 10^{15}} \times \frac{B(E2; ^{22+} \rightarrow ^{12+})}{B(E2; ^{10+} \rightarrow ^{22+})}$$

and we use the same nomenclature as in column eleven. In column fourteen we give the K conversion coefficient for the $^{22+} \rightarrow ^{12+}$ transition and in column fifteen the references to the publications on measurements of conversion electron ($^{22+} \rightarrow ^{12+}$) gamma ($^{12+} \rightarrow ^{10+}$) angular correlations. The latter

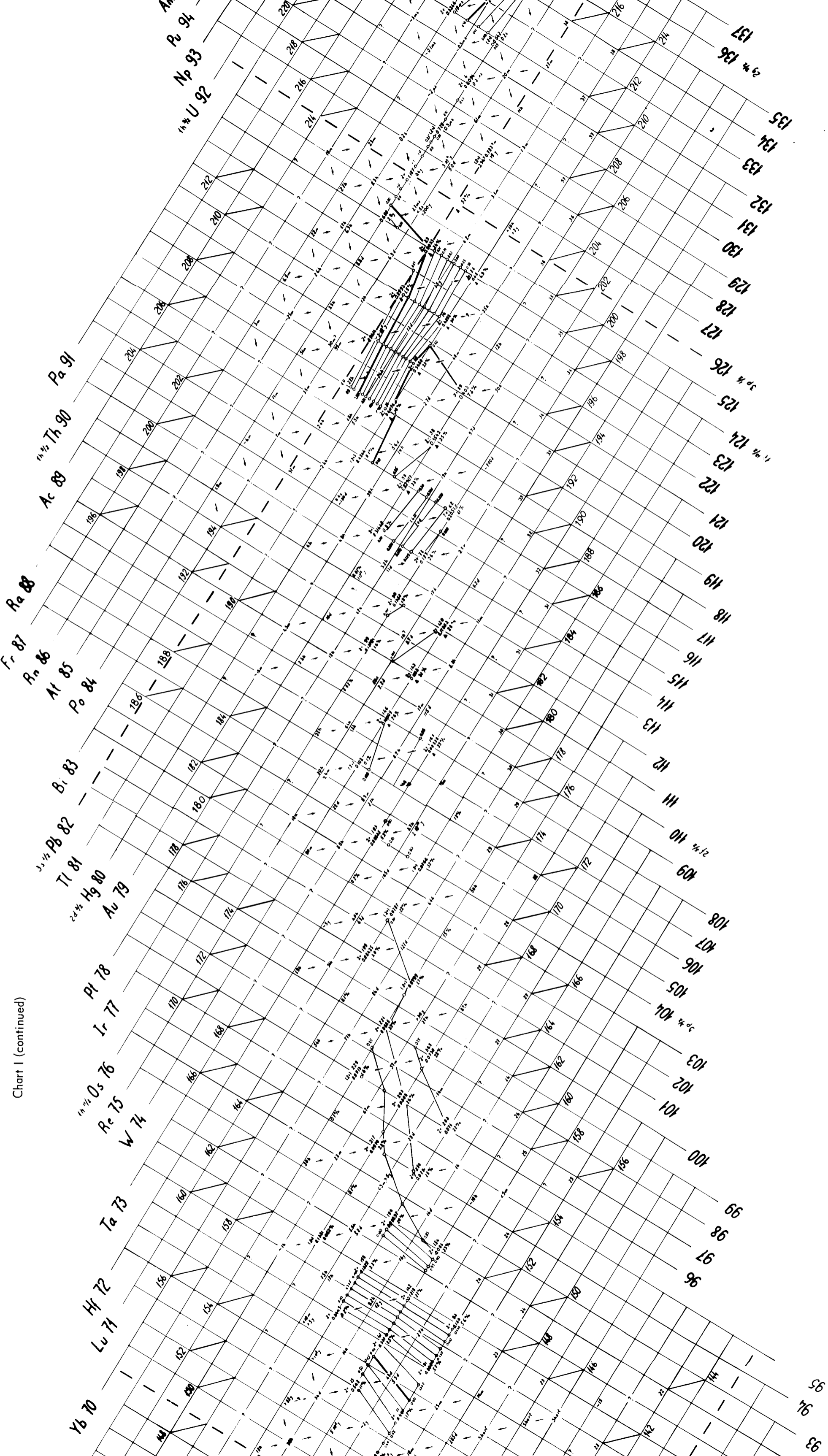




Yb 70
Tm 69
Er 68
Ho 67
Dy 66
Tb 65
Gd 64
Eu 63
Sm 62
Pm 61
Nd 60
Pr 59
Ce 58
La 57
Ba 56
Cs 55
Xe 54
I 53
Te 52
Sb 51
Sn 50
Pd 48
Ag 47
Au 46

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Chart I (continued)



Ac 88

1/2 Th 90

Pa 91

1/2 U 92

Np 93

Pu 94

Am 95

Cm 96

Bk 97

Cf 98

F 99

Fm 100

Mv 101

102

103

104

121

122

123

124

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128

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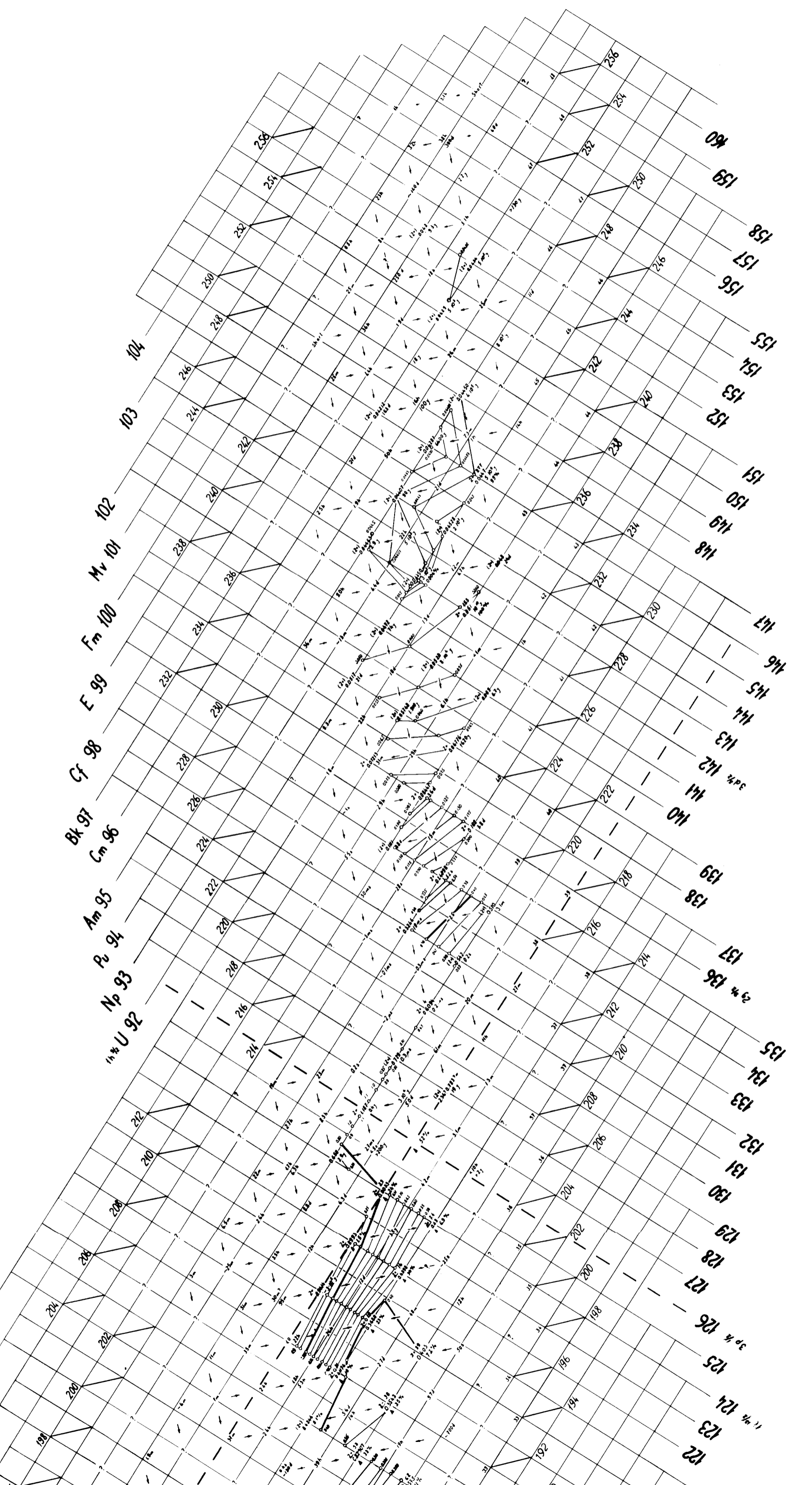
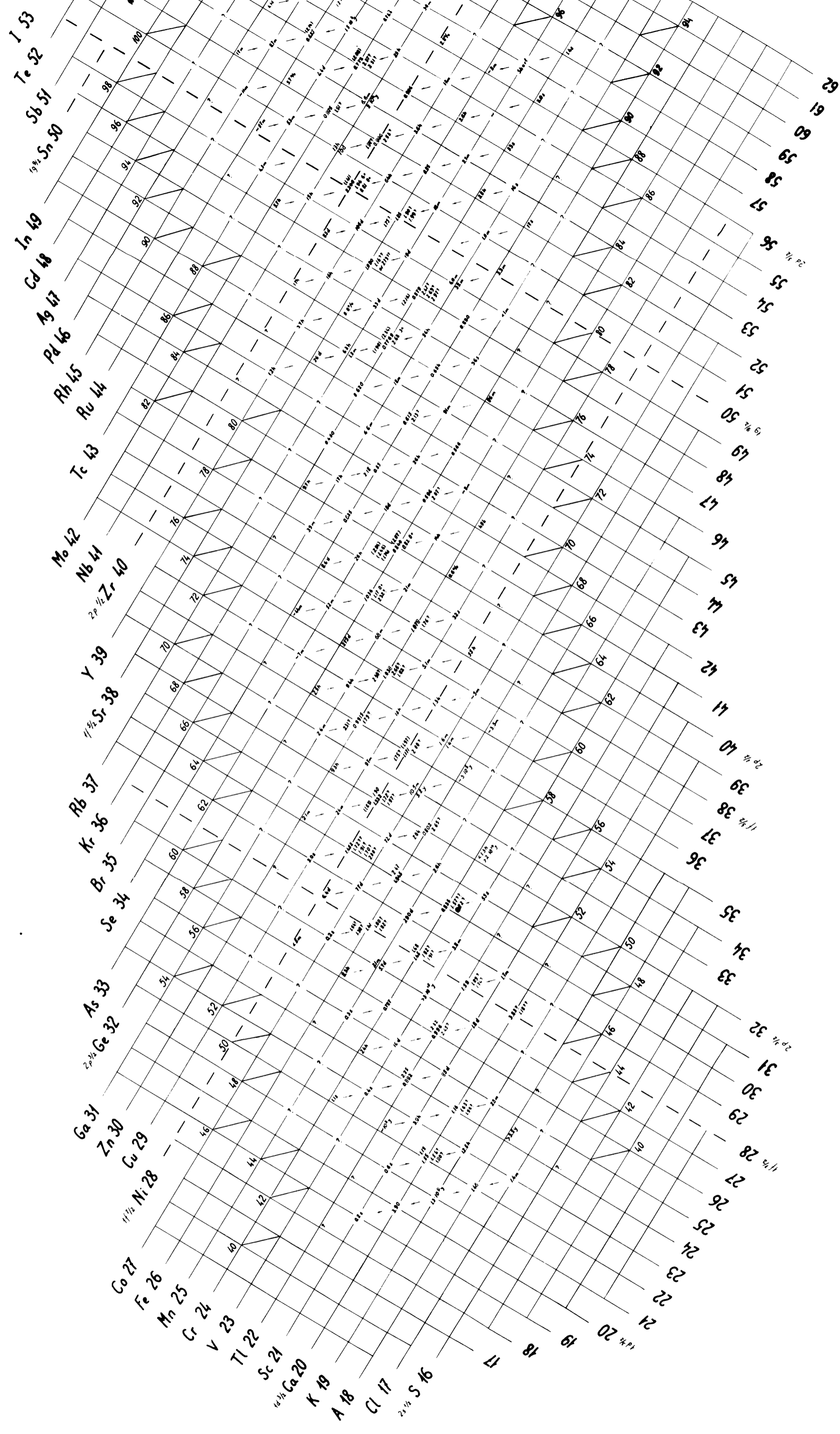
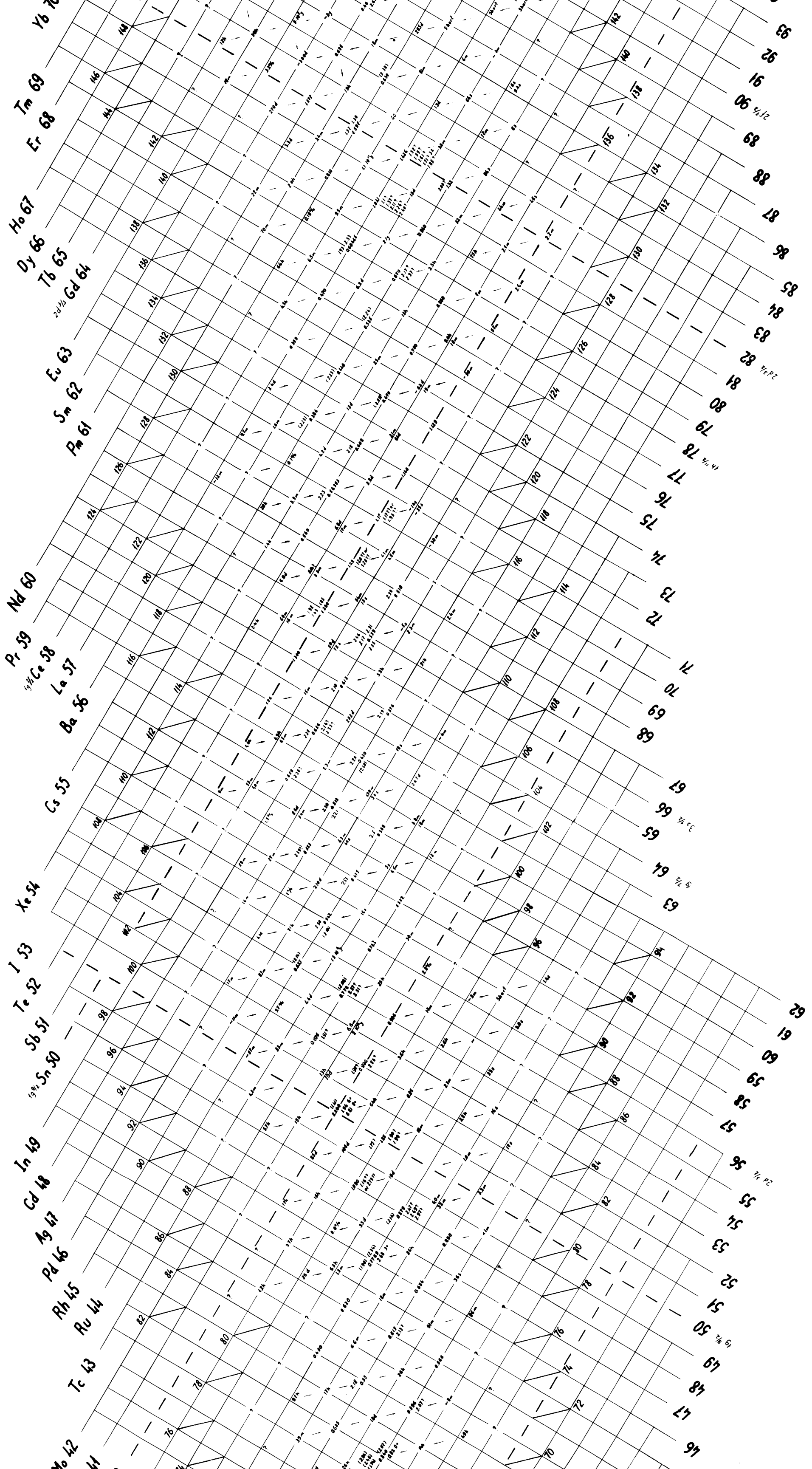


Chart II





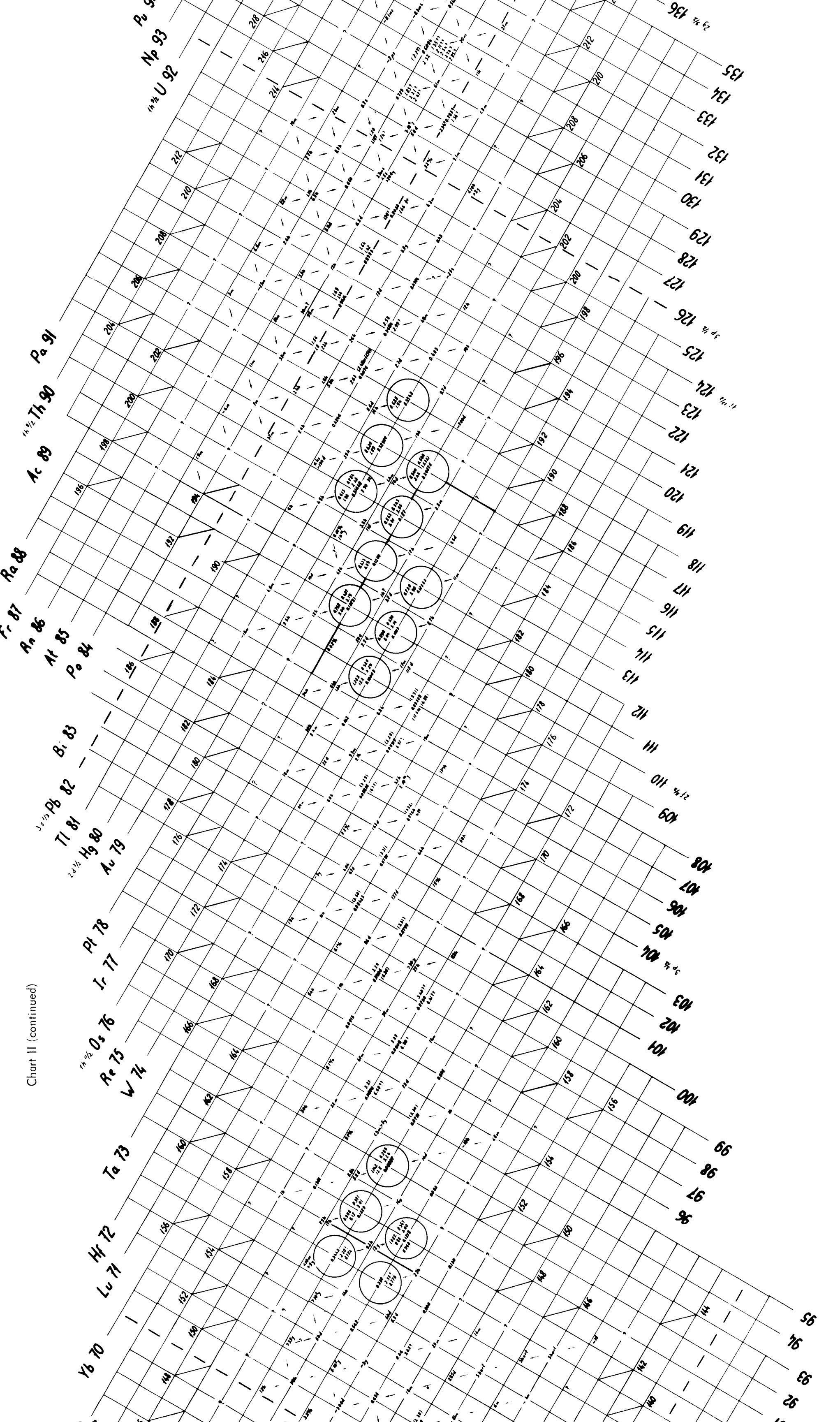
Mo 62
Pm 61
Sm 62
Eu 63
Gd 64
Tb 65
Dy 66
Ho 67
Er 68
Tm 69
Yb 70

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Yt 54
Cs 55
Ba 56
La 57
Ce 58
Pr 59
Nd 60

Tc 43
Ru 44
Rh 45
Pd 46
Ag 47
Cd 48
In 49
Sn 50
Sb 51
Te 52
I 53
Xe 54

Chart II (continued)



Ac 89

Th 90

Pa 91

1/2 U 92

Np 93

Pa 94

Am 95

Cm 96

Bk 97

Cf 98

E 99

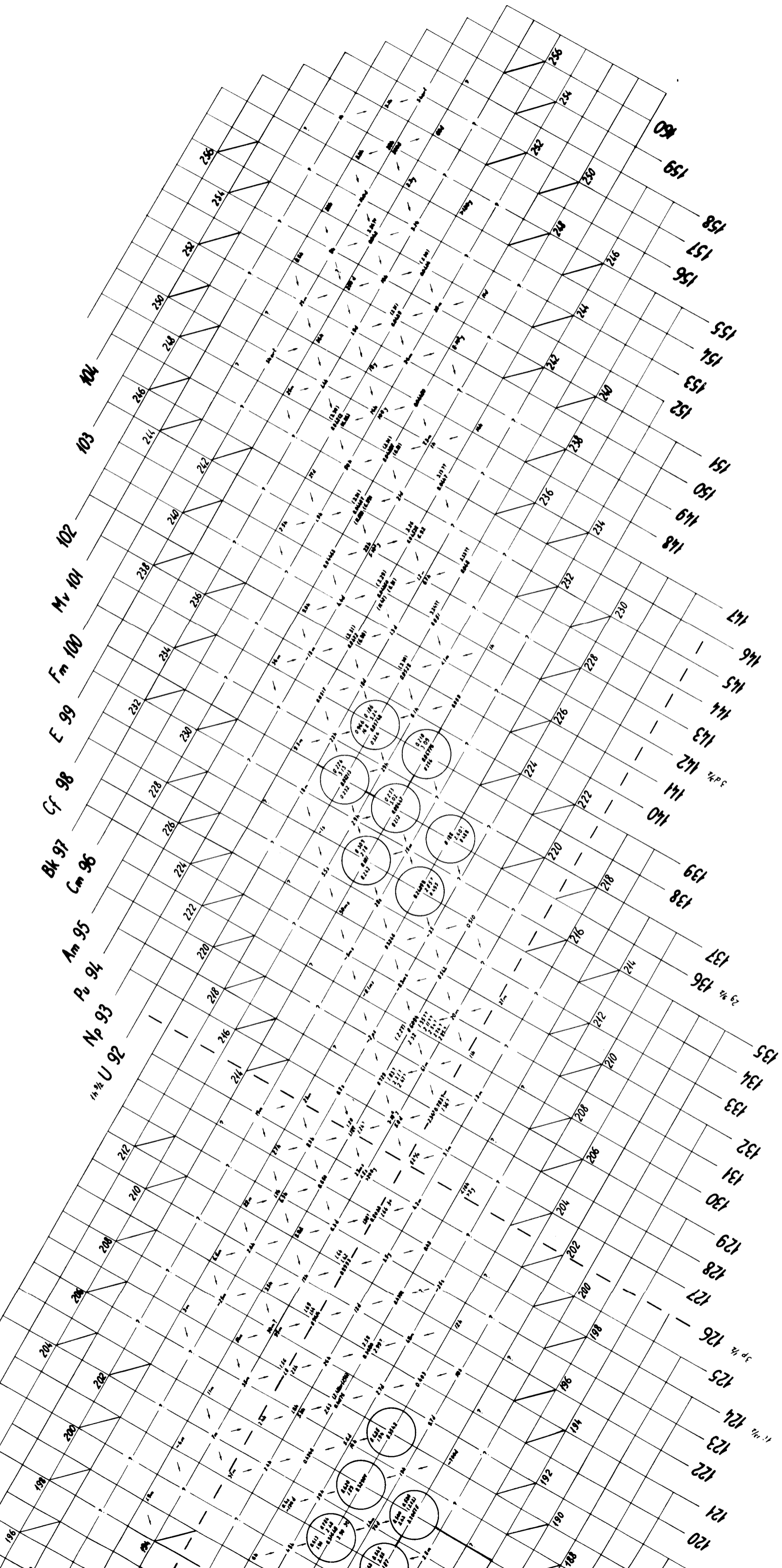
Fm 100

Mv 101

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Table 13. Energy and Character of Some Levels of Closed-Shell Nuclei

Nucleus	E (Mev)	$S\pi$	E (Mev)	$S\pi$	E (Mev)	$S\pi$	E (Mev)	$S\pi$
$^{20}\text{Ca}^{40}$	3.35	0+	3.73	3-	3.90	(2+)	—	—
$^{20}\text{Ca}^{42}$	1.53	2+	1.84	4+	—	—	—	—
$^{24}\text{Cr}^{52}$	1.45	2+	2.43	4+	—	—	—	—
$^{26}\text{Fe}^{54}$	1.41	(2+)	1.95	(2+)?	2.17	(2+)?	—	—
$^{28}\text{Ni}^{60}$	1.33	2+	1.48	(2+)	—	—	—	—
$^{28}\text{Ni}^{62}$	1.17	(2+)	2.05	(2+)?	2.30	(4+)	—	—
$^{32}\text{Ge}^{70}$	1.04	2+	1.21	0+	—	—	—	—
$^{38}\text{Sr}^{88}$	1.85	2+	2.76	3-	3.24	(2+)?	—	—
$^{40}\text{Zr}^{90}$	1.77	0+	2.20	2+	2.33	5-	3.10	(4+)
$^{40}\text{Zr}^{92}$	0.93	(2+)	1.83	(2+)?	—	—	—	—
$^{50}\text{Sn}^{116}$	1.28	2+	2.09	2+	2.36	4+	2.50	2+
$^{50}\text{Sn}^{120}$	1.17	2+	1.37	(4+)	—	—	—	—
			or 2.21					
$^{54}\text{Xe}^{136}$	1.32	(2+)	2.64	(2+)?	—	—	—	—
$^{58}\text{Ce}^{140}$	1.60	2+	2.09	4+	2.42	3-	2.52	2+
$^{82}\text{Pb}^{200}$	~ 1.00	2+	~ 1.56	4+	—	—	—	—
$^{82}\text{Pb}^{202}$	0.96	2+	1.38	4+	1.62	4+	—	—
$^{82}\text{Pb}^{204}$	0.90	2+	1.27	4+	1.56	4+	—	—
$^{82}\text{Pb}^{206}$	0.80	2+	1.34	3+	1.44	(2+)?	—	—
$^{82}\text{Pb}^{208}$	2.61	3-	3.20	5-	3.48	4-	3.71	5-
$^{84}\text{Po}^{210}$	1.19	2+	1.43	4+	1.48	(4+)	—	—

In Ba^{134} the $24+ \rightarrow 14+$ and $24+ \rightarrow 22+$ transitions have larger intensity than the $24+ \rightarrow 12+$ transition as predicted for vibrational levels. In Ti^{48} (Ref. 52) only the $24+ \rightarrow 12+$ transition is known, which is almost pure E2. The fact that the energy relation to the $12+$ level is 3.21 has made Van Noijen *et al.*⁵² suggest that it is a rotational level.

In the same nucleus there is a well-known $16+$ state which decays to the $14+$ level and could be a level of vibrational type. The case of Ti^{48} is worth further investigation.

There is no information about $30+$ levels.

Excited states of nuclei in regions between nearly harmonic-pattern and rotational-pattern regions.—Many authors have studied the transition from nearly harmonic-pattern regions to rotational-pattern regions: Mottelson and Nilsson⁵³ and Scharff-Goldhaber and Weneser⁸ have studied the transition from the α -region to the rotational region $156 \leq A \leq 178$. Scharff-Goldhaber⁹ has studied the transition from the β region to the rotational region $A \geq 228$ and that from the β region to the rotational region $156 \leq A \leq 178$ (Ref. 12). Sheline¹⁷ has recently studied the way in which the levels of spherical nuclei go over into the levels of nonspherical nuclei in all the previously mentioned transition regions. Alder *et al.*⁶ gave an expression for the energy of the $12+$ state for which the transition should take place.

The results of all these authors are summarized below:

(a) The $12+$ vibrational level goes over into the $2+$ first excited state of rotational type.¹⁷

(b) The $14+$ vibrational level goes over into the $4+$ second excited state of rotational type.¹⁷

(c) The $20+$ and $22+$ vibrational levels probably go over into the beta $0+$ and gamma $2+$ vibrational band heads.¹⁷ As we know, the beta band has $K = 0$

and spins $0+, 2+, 4+$ and the γ band has $K = 2$ and spins $2+, 3+, 4+$.

(d) There is a trough in the energy of the quadrupole vibrations above the ground state and in the energy of the octupole vibrations.¹⁷ This is evidence for a "soft" nucleus in the transition regions. As we know, the octupole vibrations have $K = 0$ and spins $1-, 3-$ and $5-$.

(e) Transitions take place for the neutron numbers 88–89 (Refs. 11, 12, 17); 134–136 (Ref. 17); 114–116 (Ref. 17) and proton numbers 86–88 (Refs. 12, 17). The energy of the $12+$ state for which the transition takes place is given by⁶

$$E(12+ \rightarrow 10+) \approx 13 \times \hbar^2 / I_{\text{rig.}}$$

where $I_{\text{rig.}} = \frac{2}{5} AMR_0^2(1 + 0.31\beta + \dots)$,

with $R_0 = 1.20A^{\frac{1}{3}} \times 10^{-13}$ cm,

and $\beta = 0.6v$ where $v = 1.8 \times A^{\frac{1}{3}}$.

We have represented in Chart II, within a circle centered at the coordinates of the nucleus considered, the following data for the nuclei whose energy levels contribute to the knowledge of the transition region:

(a) At the upper left part of the energy of the $12+$ state, the ratio of the energy of the $12+$ state and the energy of the $22+$ level.

(b) At the upper right part of the energy of the $12+$ state, the ratio of the energy of the $12+$ state and the energy of the $14+$ level.

(c) At the lower left part of the energy of the $12+$ state, the energy of the $1-$ level (octupole vibration level).

(d) At the lower right part the energy ratio and energy of levels whose character is not known with certainty.

The parentheses and question marks have the same meaning as in the rest of Chart II.

In the Os region we can see that the minimum energy for the 2^2+ level is obtained for $Z = 76$ ($82-6$) and $N = 116$ ($126-10$). There is no information about $1-$ states. The 1^4+ states go over with continuity. The energy ratios for the different levels also go over with continuity.

In the Ra region we see that the minimum energy for the $1-$ state is obtained for $Z = 88$ ($82+6$) and $N = 136$ ($126+10$). There is not enough information about 2^2+ states to draw any conclusion. The 1^4+ levels vary in a continuous manner and so do their energy ratios.

In the Gd region the transition is due to neutrons alone and the most "soft" nuclei seem to be those with $N = 89$ because at $N = 88$ the energy ratios are 2.2 and 2.3 and at $N = 90$ they are 3.0. There is not enough information about the 2^2+ and $1-$ levels to obtain the minimum energy point.

The foregoing results show that the trough in the vibrational energy is obtained for a nucleon number which is approximately 8% in excess (or defect) of the last shell which has been filled (or which is being filled). It is observed if both nucleon numbers are $\sim 8\%$ apart from a shell closure or if one is $\sim 8\%$ apart and the other one is apart more than 8%.

This conclusion is supported by the results of Racavy⁴ in his analysis of rotations in light nuclei. If one represents his parameter ϵ , which is proportional to the deformation, as a function of N and Z , one may see that his statement that rotations take place for $\epsilon \leq 0.35$ is equivalent to saying that transition takes place at approximately 8% away from the nucleon numbers 8 and 14, which close a shell. There, one does not see vibrational pattern nuclei because between rotational nuclei and closed shell nuclei there are no even-even nuclei. This situation changes for N and Z larger than 20.

There are two other regions in the periodic table where one is very close to a transition region; these are around Ba^{128} and Ru^{106} . In Ba^{128} some evidence for large deformations has been found.⁵⁴

The lines of numbers for which the minimum vibration energy is obtained are drawn fuller in Chart II, and nuclei within but not on these lines are those in which we have looked for levels of nearly harmonic pattern.

Excited states of nuclei with neutron and/or proton numbers which close a shell.—As we have seen before, the nuclei of this group which are in the α -region of the periodic table have $E(1^2+ \rightarrow 1^0+) \gtrsim 0.92$ Mev and follow the single-particle expression for $B(E2; 1^0+ \rightarrow 1^2+)$ although its value is 11 times larger. Those of the β -region have $E(1^2+ \rightarrow 1^0+) \geq 0.78$ Mev and have $B(E2; 1^0+ \rightarrow 1^2+)$ values smaller than those of the α -region.

Another characteristic which distinguishes these nuclei from the nearly harmonic-pattern ones and which also contributes towards clarifying them in a separate group is that the ratio of the energy of the second excited state to that of the 1^2+ state is smaller than 1.8. This statement contains the observation

Table 14^a

Nucleus	$E(1^2+ \rightarrow 1^0+)$ (kev)	$E(1^2+ \rightarrow 1^0+)$ (kev)	$E(1^2+ \rightarrow 1^2+)$ (kev)	$\frac{E(1^2+ \rightarrow 1^0+)}{E(1^2+ \rightarrow 1^2+)}$	$\frac{E(1^2+ \rightarrow 1^0+)}{B(E2)_{\alpha,p}}$	$\frac{E(1^2+ \rightarrow 1^2+)}{B(E2)_{\alpha,p}}$	$\frac{B(E2; 1^2+ \rightarrow 1^2+)}{B(E2; 1^0+ \rightarrow 1^2+)}$	δ^2	$\frac{\delta^2 A^{1/3} E^2}{4 \times 10^3}$	$\frac{B(E2; 1^2+ \rightarrow 1^2+)}{B(E2; 1^0+ \rightarrow 1^2+)}$
²⁶ Fe ⁵⁴	1410	1950	570	(1.38)	—	670	—	—	—	—
		2170	770	(1.54)	—	100	—	—	—	—
²⁸ Ni ⁶⁰	1332	2160	850	(1.63)	—	285	—	—	—	—
²⁸ Ni ⁶²	1171	2047	880	(1.75)	—	γ 2047 not certain	—	—	—	—
³⁰ Zn ⁶⁶	1038	2400	1370	(2.30)	11	> 25	—	—	—	—
³⁸ Sr ⁸⁸	1850	3240	1390	(1.75)	—	300	—	—	—	—
⁴⁰ Zr ⁹²	926	1830	900	(1.97)	—	18	—	—	—	—
⁵⁰ Sn ¹¹⁶	1280	2090	800	1.43	12	—	—	0.11	1×10^{-4}	500
		2760	1480	2.16	—	—	No γ 's 2760 and 1480	99-1000	$2 \times 10^{-1-2}$	0.3-0.03
⁵⁸ Ce ¹⁴⁰	1597	2515	920	1.57	—	—	No γ -ray intensities measured	—	—	—
⁵⁴ Xe ¹³⁶	1320	2640	1320	(2.00)	—	—	No γ -ray intensities measured	—	—	—
⁸² Pb ²⁰⁸	803	1440	660	(1.80)	3.9	—	No γ -ray intensities measured	—	—	—

^a As there are no values to be entered, the columns equivalent to the last six of Table 11 have been suppressed.

Table 15

Nucleus	E (Mev)	$S\pi$	Nucleus	E (Mev)	$S\pi$
$^{18}\text{A}^{40}$	4.21	(3-)	$^{66}\text{Dy}^{160}$	1.26	2-
$^{20}\text{Ca}^{40}$	3.73	3-	$^{68}\text{Er}^{166}$	1.69	(? -)
$^{32}\text{Ge}^{72}$	2.91	(1-)	$^{72}\text{Hf}^{180}$	1.14	(9-)
$^{36}\text{Kr}^{82}$	2.65	4-	$^{74}\text{W}^{182}$	1.25	(1-)
$^{36}\text{Kr}^{84}$	3.35	(? -)	$^{76}\text{Os}^{190}$	1.70	(10-)
$^{38}\text{Sr}^{88}$	2.76	3-	$^{82}\text{Pb}^{202}$	2.04	5-
$^{40}\text{Zr}^{90}$	2.33	5-	$^{82}\text{Pb}^{204}$	2.19	9-
$^{40}\text{Zr}^{92}$	2.35	(? -)	$^{82}\text{Pb}^{206}$	2.20	7-
$^{44}\text{Ru}^{100}$	2.10	(? -)	$^{82}\text{Pb}^{208}$	2.61	3-
$^{46}\text{Pd}^{106}$	2.80	(? -)	$^{84}\text{Po}^{212}$	2.20	(? -)
$^{52}\text{Te}^{124}$	2.30	3-	$^{88}\text{Ra}^{222}$	0.24	1-
$^{54}\text{Xe}^{130}$	2.34	(5-)	$^{88}\text{Ra}^{224}$	0.21	1-
$^{58}\text{Ce}^{140}$	2.42	3-	$^{88}\text{Ra}^{226}$	0.26	1-
$^{60}\text{Nd}^{144}$	2.18	1-	$^{90}\text{Th}^{226}$	0.23	1-
$^{60}\text{Nd}^{146}$	1.96	(? -)	$^{90}\text{Th}^{228}$	0.33	(1-)
$^{62}\text{Sm}^{152}$	0.96	1-	$^{92}\text{U}^{232}$	0.52	(1-)
$^{64}\text{Gd}^{152}$	1.12	3-	$^{92}\text{U}^{236}$	0.21	(1-)
$^{64}\text{Gd}^{156}$	1.50	(2, 3-)	$^{94}\text{Pu}^{238}$	0.60	(1-)

made by Scharff-Goldhaber¹² that for nuclei which have $E(12+ \rightarrow 0+) > 1.2$ Mev the ratio is smaller than two. This may be verified in Chart II where we have represented the ratios for $0+$, $2+$ and $4+$ and other positive parity levels in the same way as for α - and β -nuclei. Only levels with $E \leq 2 \times E(12+ \rightarrow 0+)$ are written down. The exception to this rule is Xe^{136} for which it seems to be 2.00, although because of the great number of non-assigned gamma rays in the decay⁵⁵ of I^{130} , it is possible that there are levels in between this and Zr^{92} for which it is 1.97.

In Table 13 we give the energy and character of the successive levels of closed shell nuclei for which one knows these data.

Of the 17 nuclei which have first-excited $2+$ states, 8 have second-excited $4+$ states, 6 have second-excited $2+$ states and the other three have second-excited states with characters $0+$, $3-$ and $3+$. The three nuclei which have first-excited state with character different from $2+$ are double-closed-shell nuclei. The shell model predictions⁵⁶ for the character and position of the levels are:

(a) Same parity as that of the ground state for levels which arise from the same configuration. For two nucleons in a shell the spin should be even and take values from $J = 0$ to $J = 2j - 1$. Their energy should increase with increasing J . For more than two nucleons in a shell the predicted spins are 0, 2, 3, 4, etc., with preponderance of spins 2 and 4.

(b) For levels which arise from different configurations, this is the only possibility to obtain excited states for double-closed-shell nuclei.

One sees that the experimental facts seem to agree with these very general predictions.

In Table 14 we give the same data which were given for the $2+$ state in α -nuclei and for $2+$ excited states in closed-shell nuclei. The ratio of M1 to E2 radiations is of the order of unity for Ce^{140} . The $(1+\delta^2) \times B(\text{E}2; 22+ \rightarrow 12+)/B(\text{E}2; 10+ \rightarrow 22+)$ values are of the same order as those for α - and β -nuclei.

Excited states of odd parity.—Morinaga¹⁸ studied these states and arrived at the conclusion that they have odd angular momentum, predominantly 3, and that they appear not too far from the line $E = 67 A^{-\frac{2}{3}}$ Mev (a semi-empirical formula which is used to describe the separation of two mass parabolas in even A nuclei). There are two kinds of exceptions: Pb isotopes and low-lying $1-$ states which are considered to be of collective type.

We have plotted in Fig. 7 the energy of the first odd parity state as a function of A . They are indicated with a triangle for α - and β -nuclei and with a circle for nuclei in the rotational region. A parenthesis indicates that the spin is not certain and a question mark indicates that it is unknown. The spin is given with the nucleus considered. The curve represents $E = 67 A^{-\frac{2}{3}}$ Mev. With the data now available we conclude that for α - and β -nuclei there is no definite trend of the energy as a function of A . It assumes values between 2 and 3 Mev.

For nuclei in the rotational regions the energies are smaller than 1.7 Mev and particularly small for the $1-$ states which are supposed to be due to an octupole vibration of the nucleus.²⁰ One arrives at similar conclusions if one plots the energy as a function of N and Z .

In Table 15 we give the energy and character of the levels represented in Fig. 7.

There are 6 nuclei with first excited states of odd parity with spin $3-$; 6 with $1-$; 2 with $5-$ and 1 each of $2-$, $4-$, $7-$ and $9-$. The other nuclei have uncertain or unknown spins.

CONCLUSIONS

According to the analysis of some of the properties of the low-lying even-parity excited states which we have made, the nuclei with A values considered may be classified in three groups, (1) transition-pattern nuclei, (2) nearly harmonic-pattern nuclei, and (3) closed-shell nuclei. The following general comments may be made:

1. The transition-pattern nuclei are those which are in the region between nearly harmonic- and rotational-pattern regions. The region starts with the nuclei for which the vibrational energies (quadrupole and octupole) are minimum and ends where the nuclei have excited states of rotational origin. The vibrational energy surface has relative minima when one of the nucleon numbers (N or Z) is approximately 8% in excess or in deficit of a closed shell number and the other nucleon number is larger than 8% in excess or in deficit of the number which closes a shell. At the point where both numbers are 8% away from a closed shell, the energy has an absolute minimum.

2. The nearly harmonic-pattern nuclei are found in the regions which are between nuclei which have a nucleon number (proton or neutron or both) which closes a shell, closed-shell nuclei, and transition-pattern nuclei. For $40 \leq A \leq 154$, the nuclei form a coherent group and their properties agree fairly well with the theoretical predictions for "vibrational" levels. This agreement is worse for $180 \leq A \leq 226$.

Low-lying odd-parity states seem to have the same behaviour in both groups.

3. Closed-shell nuclei also form a coherent group which have characteristic properties for their low-lying even-parity excited states. Low-lying odd-parity states seem to have the same behaviour as in nearly harmonic-pattern regions.

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