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NO 1	AÑO 1971

02.71.08

LETTERE AL NUOVO CIMENTO

VOL. 2, N. 16

16 Ottobre 1971

The Born Term Subtraction. A Test by Inverse Dispersion Relations.

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(ricevuto il 10 Settembre 1971)

In a recent paper ⁽¹⁾ it has been suggested that *Regge expansions are to be written not for the complete amplitudes, but for the amplitudes stripped of all Born terms in the s and u channels, which have to be added separately.* This proposal arose from a phenomenological analysis of πN finite-energy sum rules (FESR) and continuous-moment sum rules (CMSR) ⁽²⁾. Arguments in its favour, besides the phenomenological ones, can be provided by an analogy with potential scattering ⁽³⁾. This idea was also successfully applied to the analysis of forward Compton-scattering sum rules ⁽⁴⁾. The results were interpreted as evidence for a Born-poles-fixed-Regge-poles duality by CHAYDA ⁽⁵⁾, who extensively discussed the relevance of this approach for all kind of reactions ^(*).

In order to achieve a consistent procedure in the determination of high-energy parameters it is of obvious interest to check the proposal of ref. ⁽¹⁾. Of course, the main difficulty is that it is not easy to decide about the presence of a delta-function at a finite energy by looking at the asymptotic behaviour of the scattering amplitude.

In this note we show that indeed there is a sum rule which provides a nice test of the proposal, which is essentially independent of the assumptions about the Born terms

⁽¹⁾ E. FERRARI and G. VIOLINI: *Nuovo Cimento*, **69 A**, 375 (1970).

⁽²⁾ See, for example, ref. ⁽⁴⁾ to ⁽⁷⁾ or ref. ⁽¹⁾.

⁽³⁾ W. J. ABBE, P. KAUS, P. NATH and Y. N. SRIVASTAVA: *Phys. Rev.*, **140**, B 1595 (1965); **141**, 1513 (1966); **154**, 1515 (1967).

⁽⁴⁾ C. DOMÍNGUEZ, C. FERRO FONTÁN and R. SUAYA: *Phys. Lett.*, **31 B**, 365 (1970).

⁽⁵⁾ L. K. CHAYDA: *On dynamic origin of fixed Regge poles in weak, electromagnetic and strong interactions*, University of Alberta, preprint (1971).

^(*) For example, as pointed out in ref. ⁽⁵⁾, we recall that in particular this philosophy would affect those determinations of the N^3YK coupling constants ^(*) which used FESR or equivalent superconvergent sum rules.

^(*) N. M. QUEEN, M. RESTIGNOLI and G. VIOLINI: *Forts. Phys.*, **17**, 467 (1969).

below the elastic threshold. We find that such sum rule provides a strong support in favour of the Born-term subtraction.

A few years ago, starting from the «inverse» dispersion relation of GILBERT⁽⁷⁾, OLSSON wrote for the crossing-odd forward πN^{\prime} scattering amplitude the superconvergent sum rule^(*) (8)

$$(1) \quad -\frac{M}{\mu} f^2 + \frac{M}{2\pi} \int_{\mu}^{\bar{v}} \frac{d\nu}{k} \operatorname{Re} F^{(-)}(\nu) = \frac{M}{(2\pi\hbar)^2} \beta \left(\frac{\bar{v}}{v_0}\right)^{\alpha} \frac{1}{\alpha} \operatorname{tg} \left(\frac{\pi}{2} \alpha\right).$$

Actually, the derivation of the Olsson sum rule involves an approximation, so that our expression for the r.h.s. of eq. (1) is an underestimate of it.

It is important to remark that this sum rule is almost independent of the Born term. Indeed, this is present in the l.h.s. of eq. (1) as $-(M/\mu)f^2$ and its magnitude is substantial compared to the l.h.s. of that equation (0.73 ± 0.15 , according to the estimate of ref. (8) for $\bar{v} = 5$ GeV), but one has to remember that a Born-term contribution is also present in the integral over the real part of the scattering amplitude.

By writing (9)

$$(2) \quad \operatorname{Re} F^{(-)}(\nu) = \frac{4\nu f^2}{\nu^2 - (\mu^2/2M)^2} + \frac{\nu}{2\pi^2} P \int_{\mu}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2} k'(\sigma_- - \sigma_+),$$

this contribution can be easily evaluated and cancels the $-(M/\mu)f^2$ term up to the order μ/\bar{v} .

One could, following the Olsson procedure, try to determine the high-energy parameters by solving the system of equations (2) and (5) of ref. (8), after the Born-term subtraction; however, in this way possible systematic errors in the sum rules could force the parameters to values of scarce significance. Moreover, the first FESR depends very much on the value of the total πN^{\prime} cross-section measured in the few GeV region⁽¹⁰⁾ for which we prefer to use the corrected values of a recent compilation⁽¹¹⁾. Finally, it is also possible to exploit the information contained in the higher-moment sum rules. Thus we compare the l.h.s. of eq. (1), as calculated by OLSSON, with the r.h.s. values which are obtained by inserting into it the ρ -pole parameters estimated by CMSR and FESR under different assumptions for the Born-term contribution⁽¹⁾.

The results are shown in Table I.

Though the agreement is not impressive, nevertheless we find that the parameters obtained assuming the Born-term subtraction definitely satisfy the sum rule much better than those obtained under the opposite assumption.

We notice that this feature remains valid also if an estimate of possible systematic errors on β is included⁽¹⁾. It is clear that a more accurate evaluation of the l.h.s. of

(7) W. GILBERT: *Phys. Rev.*, **108**, 1078 (1957).

(*) We use the normalization $\operatorname{Im} F^{(-)} = (k/4\pi)[\sigma(\pi^-p) - \sigma(\pi^+p)]$ and define the Regge function by $\operatorname{Im} F_{\text{R}}^{(-)} = (\beta/2\pi)(\nu/v_0)^{\alpha}$ with $v_0 = 1$ GeV; β units are mb GeV, M and μ are the nucleon and pion mass, respectively; $f^2 = 0.081$ and $\hbar^2 = 0.389$ mb (GeV)²; k and ν are the pion momentum and energy in the laboratory system.

(8) M. G. OLSSON: *Phys. Rev. Lett.*, **19**, 550 (1967).

(9) J. HAMILTON and W. S. WOOLCOCK: *Rev. Mod. Phys.*, **35**, 737 (1963).

(10) A. CITRON, W. GALBRAITH, T. F. KYCIA, B. A. LEONTIC, R. H. PHILLIPS, A. ROUSSET and R. M. SHARP: *Phys. Rev.*, **144**, 1101 (1966).

(11) G. GIACOMELLI, P. PINI and S. STAGNI: CERN report HERA 69-1 (1969).

TABLE I. — *The r.h.s. of eq. (1) is given for a set of Regge parameters obtained with the type of fit shown and taking into account or not, as indicated, the Born-term contribution.*

β_p	α_p	Type of fit	Born term	r.h.s. eq. (1)	l.h.s. eq. (1) (*)
2.138	0.712	CMSR	yes	1.19	0.73 ± 0.15
2.538	0.605	CMSR	no	0.95	
2.257	0.675	FESR	yes	1.08	
2.556	0.600	FESR	no	0.94	
2.570	0.597	CMSR and σ	no	0.94	

the sum rule would be very useful. In principle this could be done by using a phase-shift analysis; however, this would imply a lowering of the cut-off in the integral. Thus the Born-term cancellation would be no more complete and the sum rule would lose its nice feature of being essentially independent of the strength of the nucleon pole.

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We wish to thank Dr. C. FERRO FONTÁN for a useful discussion. One of us (G.V.) thanks the Consejo Nacional de Investigaciones Científicas y Técnicas and the Fondazione Angelo Della Riccia for financial support during a visit to Argentina and acknowledges the kind hospitality of Centro Atómico Bariloche and Fundación Bariloche.

(*) We notice that the μ/\bar{v} no-cancellation effect for this value of \bar{v} is of the order of the 1% of the sum rule.

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16 Ottobre 1971

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Serie 2, Vol. 2, pag. 826-828