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## THERMAL STRESSES IN RECTANGULAR PLATES: VARIATIONAL AND FINITE ELEMENT SOLUTIONS

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This paper deals with the development of an approximate method for the analysis of thermal stresses in rectangular plates (plane stress problem) and an evaluation of the relative accuracy of the finite element method. The stress function is expanded in terms of polynomial coordinate functions which identically satisfy the boundary conditions, and a variational approach is used to determine the expansion coefficients. The results are in good agreement with a finite element approach.

### 1. Introduction

As stated by Parkus: 'Although interest in the field of thermoelasticity dates as far back as 1837, when Duhamel published his famous *Memoire Sur les Phénomènes Thermomécaniques*, it was during the last two decades that active and systematic research has been conducted' [1].

Parkus estimated in 1964 that: 'approximately 850 papers and a number of books' had appeared during that period. They have probably doubled today.

Nuclear, space and ocean technological requirements have demanded rapid advances in the field of thermoelasticity and have originated new chapters: thermoplasticity, thermoviscoelasticity, etc.

On the other hand, the development of the digital computer has been the cause during the past twenty years of increased progress in the solution of complex problems where non-linear and/or time dependent characteristics, ablation situations, etc. can now be taken into account.

The powerful finite element method permits almost any problem of modern applied mechanics to be presented in a unified mathematical form suitable for solution on a digital computer. Most investigations of the relative accuracy of the finite element method deal with comparisons between several alternative formulations of the method, or between exact analytical results and finite element values obtained for rather simple stress problems [2]. For the most part, completely independent comparisons for thermoelastic problems have not been made.

The goal of this paper is twofold:

- (a) to develop a simple yet accurate analytical approach for the solution of two-dimensional thermoelastic stress distributions in rectangular plates, and
- (b) to assess the accuracy of the finite element algorithm in the case of two-dimensional thermoelastic problems.

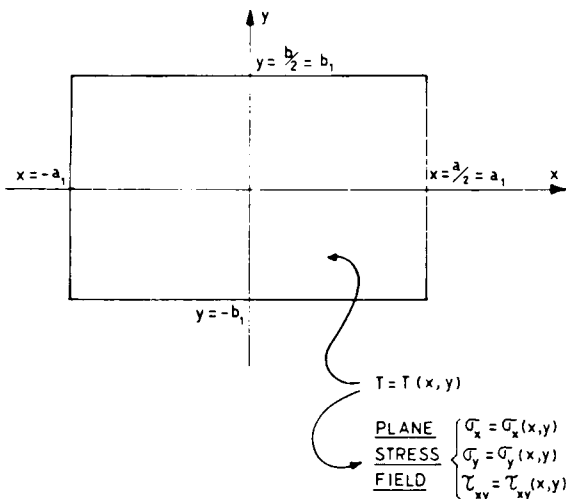


Fig. 1. Mechanical system under study.

Phase (a) consists of expanding Airy’s stress function in a summation of polynomial coordinate functions which identically satisfy the boundary conditions. The coefficients of the expansion are evaluated by means of Galerkin’s minimization procedure. The method is first applied to a problem solved by Przemieniecki [3], and good agreement is shown to exist. Three terms are used in the present investigation while the results obtained in ref. [3] have been calculated using eight ‘vibrating beam functions’.

The finite element approach makes use of a computer code \* which employs constant strain triangular elements. Good agreement is attained between finite element results and the analytical predictions.

2. Statement of the problem in its approximate solution: a polynomial variational approach

The plane, thermal stress problem is governed by the partial differential equation [4]:

$$\nabla^4 F(x, y) + \alpha E \nabla^2 T(x, y) = 0, \tag{1}$$

where  $T(x, y)$  is the two-dimensional steady state temperature distribution and  $F(x, y)$  is the Airy stress function such that

$$\sigma_x = \partial^2 F / \partial y^2; \quad \sigma_y = \partial^2 F / \partial x^2; \quad \tau_{xy} = -\partial^2 F / \partial y \partial x. \tag{2}$$

Assume now an approximate solution of the form

$$F(x, y) \simeq F_a(x, y) = \sum_{n,m} A_{nm} F_{nm}(x, y), \tag{3}$$

where

$$F_{nm}(x, y) = X_n(x) Y_m(y). \tag{4}$$

If each coordinate function  $F_{nm}$  satisfies identically the traction-free boundary conditions, one can easily show

\* Developed at Centro Atómico Bariloche.

that (see fig. 1)

$$F_{nm}(\pm \frac{1}{2}a, y) = F_{nm}(x, \pm \frac{1}{2}b) = 0, \tag{5a}$$

$$-\frac{\partial}{\partial x} F_{nm}(\pm \frac{1}{2}a, y) = \frac{\partial}{\partial y} F_{nm}(x, \pm \frac{1}{2}b) = 0. \tag{5b}$$

Following refs. [5–6] one takes now the polynomial expansion:

$$F_a(x, y) = \sum_{n=0}^N \sum_{m=0}^M A_{nm} (x^2 - a_1^2)^2 (y^2 - b_1^2)^2 x^n y^m, \tag{6}$$

where each polynomial coordinate system satisfies eqs. (5).

Substituting (6) into (1) one obtains an error or residual function  $\epsilon(x, y)$  which, in general, does not vanish since (6) is not an exact solution of the differential system:

$$\begin{aligned} \epsilon(x, y) = & \sum_n \sum_m A_{nm} \{ [24 + 96n + 72n(n-1) + 16n(n-1)(n-2) + n(n-1)(n-2)(n-3)] x^n \\ & - 2n(n-1)[12 + 8(n-2) + (n-2)(n-3)] a_1^2 x^{n-2} + n(n-1)(n-2)(n-3) a_1^4 x^{n-4} \} \\ & \times (y^2 - b_1^2)^2 y^m + 2 \sum_n \sum_m A_{nm} [4(3x^2 - a_1^2) x^n + 8(x^3 - a_1^2 x) n x^{n-1} + (x^2 - a_1^2)^2 n(n-1) x^{n-2}] \\ & \times [4(3y^2 - b_1^2) y^m + 8(y^3 - b_1^2 y) m y^{m-1} + (y^2 - b_1^2)^2 m(m-1) y^{m-2}] + \sum_n \sum_m A_{nm} \\ & \times \{ [24 + 96m + 72m(m-1) + 16m(m-1)(m-2) + m(m-1)(m-2)(m-3)] y^m - 2m(m-1) \\ & \times [12 + 8(m-2) + (m-2)(m-3)] b_1^2 y^{m-2} + m(m-1)(m-2)(m-3) b_1^4 y^{m-4} \} (x^2 - a_1^2)^2 x^n + \alpha E \nabla^2 T. \end{aligned} \tag{7}$$

One requires now that  $\epsilon(x, y)$  be orthogonal with respect to each coordinate function over the domain under consideration. This condition generates a linear system of equations in the  $A_{nm}$ 's. Solving it one has the approximate stress function  $F_a(x, y)$ .

Consider for instance the temperature field [3]

$$T(x, y) = 4T_c(b_1^2 - y^2)/b^2, \tag{8a}$$

where  $b_1 = \frac{1}{2}b$ . One immediately obtains

$$\nabla^2 T = -8T_c/b^2. \tag{8b}$$

Taking the first three even terms of the functional relation (6):

$$F_a(x, y) = A_{00}(x^2 - a_1^2)^2 (y^2 - b_1^2)^2 + A_{20}(x^2 - a_1^2)^2 (y^2 - b_1^2)^2 x^2 + A_{02}(x^2 - a_1^2)^2 (y^2 - b_1^2)^2 y^2, \tag{9}$$

and substituting (8b) and (9) into (1), one obtains, once use is made of Galerkin's orthogonality conditions:

$$(0.1625 + 0.0929\lambda^2 + 0.1625\lambda^4) b^4 A_{00} + (0.0037 + 0.0058\lambda^4) b^6 A_{02}$$

$$+ (0.0058\lambda^2 + 0.0037\lambda^6) b^6 A_{20} = \frac{\alpha E T_c}{b^2} 1.1378,$$

$$\begin{aligned}
& (0.0232 + 0.0148\lambda^4) b^4 A_{00} + (0.0005 + 0.0005\lambda^4) b^6 A_{02} + (0.0174\lambda^2 + 0.0021\lambda^4 + 0.0009\lambda^6) b^6 A_{20} \\
& = \frac{\alpha E T_c}{b^2} 0.1625, \\
& (0.0148 + 0.0232\lambda^4) b^4 A_{00} + (0.0009 + 0.0021\lambda^2 + 0.0174\lambda^4) b^6 A_{02} \\
& + (0.0005\lambda^2 + 0.0005\lambda^6) b^6 A_{20} = \frac{\alpha E T_c}{b^2} 0.1625, \tag{10}
\end{aligned}$$

where  $\lambda = a/b$ .

Solving the linear system (10) and expressing the stress components in dimensionless form one has:

(a) for  $\lambda = 2$

$$\begin{aligned}
\frac{\sigma_x}{\alpha E T_c} &= (12\eta^2 - 1)[5.1088(\xi^2 - \frac{1}{4})^2 + 32.0237(\xi^3 - \frac{1}{4}\xi)^2] + 0.1471(240\eta^4 - 48\eta^2 + 1)(\xi^2 - \frac{1}{4})^2, \\
\frac{\sigma_y}{\alpha E T_c} &= (12\xi^2 - 1)[1.2772(\eta^2 - \frac{1}{4})^2 + 0.2941(\eta^3 - \frac{1}{4}\eta)^2] + 1.0007(240\xi^4 - 48\xi^2 + 1)(\eta^2 - \frac{1}{4})^2, \tag{11} \\
\frac{\tau_{xy}}{\alpha E T_c} &= -64(\xi^3 - \frac{1}{4}\xi)(\eta^3 - \frac{1}{4}\eta)[0.6386 + 0.5004(12\xi^2 - 1) + 0.0184(12\eta^2 - 1)];
\end{aligned}$$

(b) for  $\lambda = 1$

$$\begin{aligned}
\frac{\sigma_x}{\alpha E T_c} &= (12\eta^2 - 1)[2.5860(\xi^2 - \frac{1}{4})^2 + 2.9985(\xi^3 - \frac{1}{4}\xi)^2] + 0.3748(240\eta^4 - 48\eta^2 + 1)(\xi^2 - \frac{1}{4})^2, \\
\frac{\sigma_y}{\alpha E T_c} &= (12\xi^2 - 1)[2.5860(\eta^2 - \frac{1}{4})^2 + 2.9985(\eta^3 - \frac{1}{4}\eta)^2] + 0.3748(240\xi^4 - 48\xi^2 + 1)(\eta^2 - \frac{1}{4})^2, \tag{12} \\
\frac{\tau_{xy}}{\alpha E T_c} &= -8(\xi^3 - \frac{1}{4}\xi)(\eta^3 - \frac{1}{4}\eta)[5.1720 + 2.9985(3\eta^2 - \frac{1}{4}) + 2.9985(3\xi^2 - \frac{1}{4})];
\end{aligned}$$

where  $x = a\xi$  and  $y = b\eta$ .

Consider now a slightly more complicated temperature field:

$$T(x, y) = \frac{16T_c}{a^2 b^2} (a_1^2 - x^2)(b_1^2 - y^2), \tag{13a}$$

where  $a_1 = \frac{1}{2}a$  and  $b_1 = \frac{1}{2}b$ .

Proceeding as previously indicated one determines:

$$\nabla^2 T = \frac{32T_c}{a^2 b^2} (a_1^2 + b_1^2 - x^2 - y^2), \tag{13b}$$

and comparing (8b) and (13b) one concludes that the only change consists in the evaluation of the right-hand side of eqs. (10), which now are

$$\begin{aligned}
& \frac{\alpha E T_c}{\lambda^2 b^2} (\lambda^2 + 1) 0.9752, \\
& \frac{\alpha E T_c}{\lambda^2 b^2} [(\lambda^2 + 1) 0.1625 - \lambda^2 0.0542 - 0.0232],
\end{aligned}$$

$$\frac{\alpha E T_c}{\lambda^2 b^2} [(\lambda^2 + 1) 0.1625 - \lambda^2 0.0232 - 0.0542] .$$

After some straightforward manipulations one can show that the components of the stress tensor are now given by

$$\frac{\sigma_x}{\alpha E T_c} = (12\eta^2 - 1)[5.7862(\xi^2 - \frac{1}{4})^2 + 19.7882(\xi^3 - \frac{1}{4}\xi)^2] + 0.1081(240\eta^4 - 48\eta^2 + 1)(\xi^2 - \frac{1}{4})^2 ,$$

$$\frac{\sigma_y}{\alpha E T_c} = (12\xi^2 - 1)[1.4466(\eta^2 - \frac{1}{4})^2 + 0.2163(\eta^3 - \frac{1}{4}\eta)^2] + 0.6184(240\xi^4 - 48\xi^2 + 1)(\eta^2 - \frac{1}{4})^2 ,$$

$$\frac{\tau_{xy}}{\alpha E T_c} = -64(\xi^3 - \frac{1}{4}\xi)(\eta^3 - \frac{1}{4}\eta)[0.7233 + 0.3092(12\xi^2 - 1) + 0.0135(12\eta^2 - 1)] ,$$

for  $\lambda = 2$ .

### 3. Numerical results

Tables 1–6 depict comparisons of numerical values of the stress parameters  $\sigma_x/\alpha E T_c$ ;  $\sigma_y/\alpha E T_c$  and  $\tau_{xy}/\alpha E T_c$  for  $\lambda = 2$  and 1 in the case where the thermal field is defined by eq. (8a). The values obtained from ref. [3] have been extracted from the curves. In general, the agreement with Przemieniecki's results is quite good.

On the other hand, the finite elements values agree reasonably well with the results obtained following the polynomial-variational approach.

Table 1  
Values of  $\sigma_x/\alpha E T_c$  for  $T = 4T_c(b_1^2 - y^2)/b^2$ ;  $\lambda = 2$ .

Present Approach (Polynomial-Galerkin Solution)

$\eta \backslash \xi$	0	0.125	0.25	0.375	0.5
0	-0.310	-0.300	-0.245	-0.113	0
0.125	-0.257	-0.248	-0.202	-0.093	0
0.25	-0.090	-0.086	-0.068	-0.031	0
0.375	0.210	0.204	0.167	0.077	0
0.5	0.675	0.649	0.521	0.237	0

Finite Elements Solution

$\eta \backslash \xi$	0.014	0.125	0.25	0.375	0.486
0.019	-0.317	-0.298	-0.231	-0.106	-0.0173
0.125	-0.250	-0.246	-0.197	-0.0989	0.0028
0.25	-0.063	-0.0521	-0.0727	-0.0443	0.0081
0.375	0.251	0.207	0.160	0.0736	0.0148
0.481	0.608	0.570	0.477	0.261	0.0538

Reference /3/

$\eta \backslash \xi$	0	0.125	0.25	0.375	0.5
0	-0.28	-0.39	-0.25	-0.10	0
0.125	-0.26				0
0.25	-0.10				0
0.375	0.21				0
0.5	0.64	0.62	0.50	0.23	0

Table 2  
Values of  $\sigma_y/\alpha E T_c$  for  $T = 4T_c(b_1^2 - y^2)/b^2$ ;  $\lambda = 2$ .

Present Approach (Polynomial-Galerkin Solution)

$\eta \backslash \xi$	0	0.125	0.25	0.375	0.5
0	-0.017	-0.046	-0.096	-0.008	0.410
0.125	-0.015	-0.040	-0.076	-0.007	0.361
0.25	-0.010	-0.026	-0.049	-0.004	0.232
0.375	-0.004	-0.009	-0.017	-0.001	0.079
0.5	0	0	0	0	0

Finite Elements Solution

$\eta \backslash \xi$	0.014	0.125	0.25	0.375	0.486
0.019	-0.0296	-0.0480	-0.0725	-0.0033	0.434
0.125	-0.0237	-0.0394	-0.0614	-0.0160	0.334
0.25	-0.0130	-0.0248	-0.0398	-0.0124	0.221
0.375	-0.0046	-0.0085	-0.0138	-0.0044	0.0697
0.481	0.0072	0.0120	0.0103	0.0075	-0.0011

Reference /3/

$\eta \backslash \xi$	0	0.125	0.25	0.375	0.5
0	-0.02	-0.04	-0.06	0.00	0.41
0.125	-0.01				0.36
0.25					0.24
0.375					0.08
0.5	0	0	0	0	0

\* Number of elements used: 384; number of nodes: 221.

Table 3. Values of  $\tau_{xy}/\alpha ET_c$  for  $T = 4T_c(b_1^2 - y^2)/b^2$ ;  $\lambda = 2$ .

Present Approach (Polynomial-Galerkin Solution)

$\eta \backslash \xi$	0	0.125	0.25	0.3125	0.375	0.4375	0.5
0	0	0	0	0	0	0	0
0.125	0	-0.012	-0.044	-0.063	-0.074	-0.061	0
0.25	0	-0.020	-0.072	-0.103	-0.122	-0.098	0
0.3125	0	-0.021	-0.074	-0.106	-0.122	-0.098	0
0.375	0	-0.019	-0.065	-0.092	-0.107	-0.087	0
0.4375	0	-0.012	-0.041	-0.058	-0.068	-0.055	0
0.5	0	0	0	0	0	0	0

Finite Elements Solution

$\eta \backslash \xi$	0.014	0.125	0.25	0.3125	0.375	0.4375	0.486
0.019	-0.0007	-0.0029	-0.0085	-0.0123	-0.0159	-0.0146	-0.0116
0.125	-0.0032	-0.0175	-0.0449	-0.0601	-0.0688	-0.0566	-0.0321
0.25	-0.0038	-0.0263	-0.0693	-0.0945	-0.111	-0.0968	-0.0536
0.3125	-0.0030	-0.0252	-0.0683	-0.0941	-0.113	-0.101	-0.0585
0.375	-0.0008	-0.0205	-0.0578	-0.0817	-0.0995	-0.0902	-0.0572
0.4375	-0.0042	-0.0116	-0.0343	-0.0499	-0.0624	-0.0554	-0.0378
0.481	0.0108	-0.0045	-0.0119	-0.0169	-0.0201	-0.0133	0.0044

Table 4. Values of  $\sigma_x/\alpha ET_c$  for  $T = 4T_c(b_1^2 - y^2)/b^2$ ;  $\lambda = 1$ .

Present Approach (Polynomial-Galerkin Solution)

$\eta \backslash \xi$	0	0.125	0.25	0.375	0.5
0	-0.138	-0.124	-0.065	0.088	0.417
0.125	-0.124	-0.111	-0.058	0.079	0.372
0.25	-0.084	-0.075	-0.038	0.054	0.248
0.375	-0.031	-0.029	-0.014	0.020	0.090
0.5	0	0	0	0	0

Finite Elements Solution

$\eta \backslash \xi$	0.014	0.125	0.25	0.375	0.486
0.019	-0.138	-0.119	-0.0536	0.107	0.398
0.125	-0.119	-0.109	-0.0558	0.0762	0.335
0.25	-0.0739	-0.0728	-0.0390	0.0487	0.231
0.375	-0.0224	-0.0277	-0.0153	0.0195	0.0797
0.481	0.0139	0.0084	0.00755	0.0078	-0.0016

Table 5. Values of  $\sigma_y/\alpha ET_c$  for  $T = 4T_c(b_1^2 - y^2)/b^2$ ;  $\lambda = 1$ .

Present Approach (Polynomial-Galerkin Solution)

$\eta \backslash \xi$	0	0.125	0.25	0.375	0.50
0	-0.138	-0.124	-0.084	-0.031	0
0.125	-0.124	-0.111	-0.075	-0.028	0
0.25	-0.065	-0.058	-0.038	-0.014	0
0.375	0.088	0.079	0.054	0.020	0
0.50	0.417	0.372	0.248	0.090	0

Finite Elements Solution

$\eta \backslash \xi$	0.014	0.125	0.25	0.375	0.486
0.019	-0.142	-0.125	-0.0821	-0.0315	-0.0130
0.125	-0.127	-0.113	-0.0771	-0.0308	0.0056
0.25	-0.0592	-0.0564	-0.0413	-0.0188	0.0087
0.375	0.107	0.0827	0.0542	0.0190	0.0119
0.481	0.361	0.320	0.225	0.0960	0.0292

Table 6. Values of  $\tau_{xy}/\alpha ET_c$  for  $T = 4T_c(b_1^2 - y^2)/b^2$ ;  $\lambda = 1$ .

Present Approach (Polynomial-Galerkin Solution)

$\eta \backslash \xi$	0	0.125	0.25	0.375	0.5
0	0	0	0	0	0
0.125	0	-0.027	-0.048	-0.048	0
0.25	0	-0.048	-0.084	-0.085	0
0.375	0	-0.048	-0.085	-0.083	0
0.5	0	0	0	0	0

Finite Elements Solution

$\eta \backslash \xi$	0.014	0.125	0.25	0.375	0.486
0.019	-0.0007	-0.0065	-0.0115	-0.0129	-0.0174
0.125	-0.0027	-0.0296	-0.0495	-0.0463	-0.0190
0.25	-0.0045	-0.0486	-0.0833	-0.0824	-0.0283
0.375	-0.0039	-0.0448	-0.0793	-0.0816	-0.0300
0.481	0.0104	-0.0098	-0.0189	-0.0173	0.0042

Table 7. Values of  $\sigma_x/\alpha ET_c$  for  $T = 16T_c(a_1^2 - x^2)(b_1^2 - y^2)/a^2b^2$ ;  $\lambda = 2$ .

Present Approach (Polynomial-Galerkin Solution)

$\eta \backslash \xi$	0	0.125	0.25	0.375	0.5
0	-0.355	-0.329	-0.243	-0.101	0
0.125	-0.292	-0.270	-0.199	-0.083	0
0.25	-0.098	-0.090	-0.066	-0.027	0
0.375	0.242	0.224	0.166	0.069	0
0.5	0.750	0.693	0.509	0.210	0

Finite Elements Solution

$\eta \backslash \xi$	0.014	0.125	0.25	0.375	0.5
0.019	-0.356	-0.323	-0.231	-0.0927	0.0058
0.125	-0.281	-0.268	-0.196	-0.0857	0.0041
0.25	-0.0710	-0.0883	-0.0672	-0.0299	0.0047
0.375	0.283	0.224	0.162	0.0699	0.0061
0.481	0.69	0.621	0.464	0.201	0.0039

Table 8. Values of  $\sigma_y/\alpha ET_c$  for  $T = 16T_c(a_1^2 - x^2)(b_1^2 - y^2)/a^2b^2$ ;  $\lambda = 2$ .

Present Approach (Polynomial-Galerkin Solution)

$\eta \backslash \xi$	0	0.125	0.25	0.375	0.5
0	-0.052	-0.062	-0.064	0.023	0.335
0.125	-0.046	-0.054	-0.056	0.020	0.295
0.25	-0.030	-0.035	-0.036	0.013	0.190
0.375	-0.010	-0.012	-0.012	0.005	0.064
0.5	0	0	0	0	0

Finite Elements Solution

$\eta \backslash \xi$	0.014	0.125	0.25	0.375	0.486
0.019	-0.0570	-0.0679	-0.0636	0.0231	0.322
0.125	-0.0486	-0.0558	-0.0507	0.0181	0.270
0.25	-0.0296	-0.0361	-0.0331	0.0119	0.175
0.375	-0.0113	-0.0127	-0.0113	0.0050	0.0597
0.481	0.0041	0.0121	0.0086	0.0039	-0.0004

Table 9.

Values of  $\tau_{xy}/\alpha ET_c$  for  $T = 16T_c(a_1^2 - x^2)(b_1^2 - y^2)/a^2b^2$ ;  $\lambda = 2$ .

Present Approach (Polynomial-Galerkin Solution)

$\epsilon$	0	0.125	0.25	0.375	0.5
0	0	0	0	0	0
0.125	0	-0.025	-0.056	-0.071	0
0.25	0	-0.041	-0.090	-0.115	0
0.375	0	-0.037	-0.081	-0.102	0
0.5	0	0	0	0	0

Finite Elements Solution

$\epsilon$	0.014	0.125	0.25	0.375	0.5
0.019	-0.0009	-0.0035	-0.0076	-0.0105	0.0039
0.125	-0.0055	-0.0277	-0.0556	-0.0676	-0.0184
0.25	-0.0075	-0.0437	-0.0890	-0.109	-0.0311
0.375	-0.0044	-0.0374	-0.0782	-0.0992	-0.0259
0.481	0.0062	-0.0111	-0.0202	-0.0249	0.0031

Tables 7–9 contain comparisons of thermal stress parameters for the case of  $T(x, y) = (16T_c/a^2b^2)(a_1^2 - x^2)(b_1^2 - y^2)$ . Again the agreement can be considered as quite satisfactory. One notices a small ‘boundary stress residual’ in the case of the finite elements results in such a manner that the traction-free condition is not identically satisfied.

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### References

- [1] H. Parkus, in: High Temperature Structure and Materials (Proc. 3rd Symp. on Naval Structural Mechanics), A.M. Freudenthal et al., eds. (Pergamon Press, 1964).
- [2] P.A.A. Laura, J. Reyes and R. Rossi, *Strain*, Apr. (1974) 58.
- [3] J.S. Przemieniecki, *Aeronaut. Quart.*, Feb. (1959) 65.
- [4] B.A. Boley and J.H. Weiner, *Theory of Thermal Stresses* (Wiley, New York, 1960).
- [5] P.A.A. Laura and B.F. Saffel, *J. Acoust. Soc. Amer.* 41 (4) (1967) 836.
- [6] P.A.A. Laura and R. Duran, *J. Sound Vib.* 42 (1) (1975) 129.

