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Transport Calculations of Neutron Wave Experiments in Subcritical Assemblies

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A recent neutron wave experiment in a thermal-neutron multiplying assembly with and without control rods has been analyzed numerically in terms of transport theory. The code TASK was used for this purpose. The present study dealing with a highly ^{235}U-enriched, compact, neutron-multiplicative uranium system indicates that the dispersion law of the assembly is very sensitive to transport effects and to the estimation of the leakage of the fast neutron population. The present calculations of neutron wave propagation in multiplicative systems show that this technique can be used as a highly sophisticated experiment for integral checks of neutron cross-section sets.

INTRODUCTION

A recent work[1] on neutron wave propagation in a compact light-water-moderated subcritical assembly, fueled with 90%-^{235}U-enriched uranium contained in materials testing reactor-type fuel elements, showed large discrepancies between the experimental results and a four-group diffusion calculation.

The purpose of the present investigation is twofold: One desires first to reinterpret the experimental results in terms of transport theory calculations to evaluate the importance of transport effects in light-water compact systems. Second, and as an aftermath of the calculation, one can investigate the neutron wave propagation technique as an integral experiment to check neutron cross-section sets and the calculation of the effect of control rods on the neutron flux in the assembly. A detailed account of the experimental technique, as well as the description of the subcritical system, is given in Ref. 1.

DESCRIPTION OF THE CALCULATIONAL SCHEME

The transport code TASK (Refs. 2 and 3) was utilized in the present calculations. Because in its present form TASK performs only one-dimensional calculations, it was necessary to estimate the transverse neutron leakage to properly mock up the actual assembly (transverse dimensions 30.97×32.71 cm^2). The main steps in the calculational procedure were the generation of group cross sections, calculation of the transverse leakage, and the calculation of the dispersion law of the neutron waves with and without control rods in the assembly sketched in Fig. 1. Table I gives the pertinent composition of the system. The positions and dimensions of the two cadmium control rods used in Ref. 1 were according to the nomenclature of Fig. 1: $c_i = 0$; $d_i = \pm 0.60$ cm; $h = 4$ cm; and $e = 0.8$ mm. The control rods extend in the z direction throughout the multiplicative region (region III of Fig. 1).

Generation of Group Cross Sections

The group cross-section set utilized as input for the TASK calculation was a five-group set

[1]F. C. DIFILIPPO, *Ann. Nucl. Energy*, **2**, 483 (1975).

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[2]A. R. BUHL et al., "A User's Manual for Task. A Generalized One-Dimensional Transport and Diffusion Kinetic Code," ORNL-TM-3811, Oak Ridge National Laboratory (1972).

[3]H. L. DODDS, J. C. ROBINSON, and A. R. BUHL, *Nucl. Sci. Eng.*, **47**, 262 (1972).

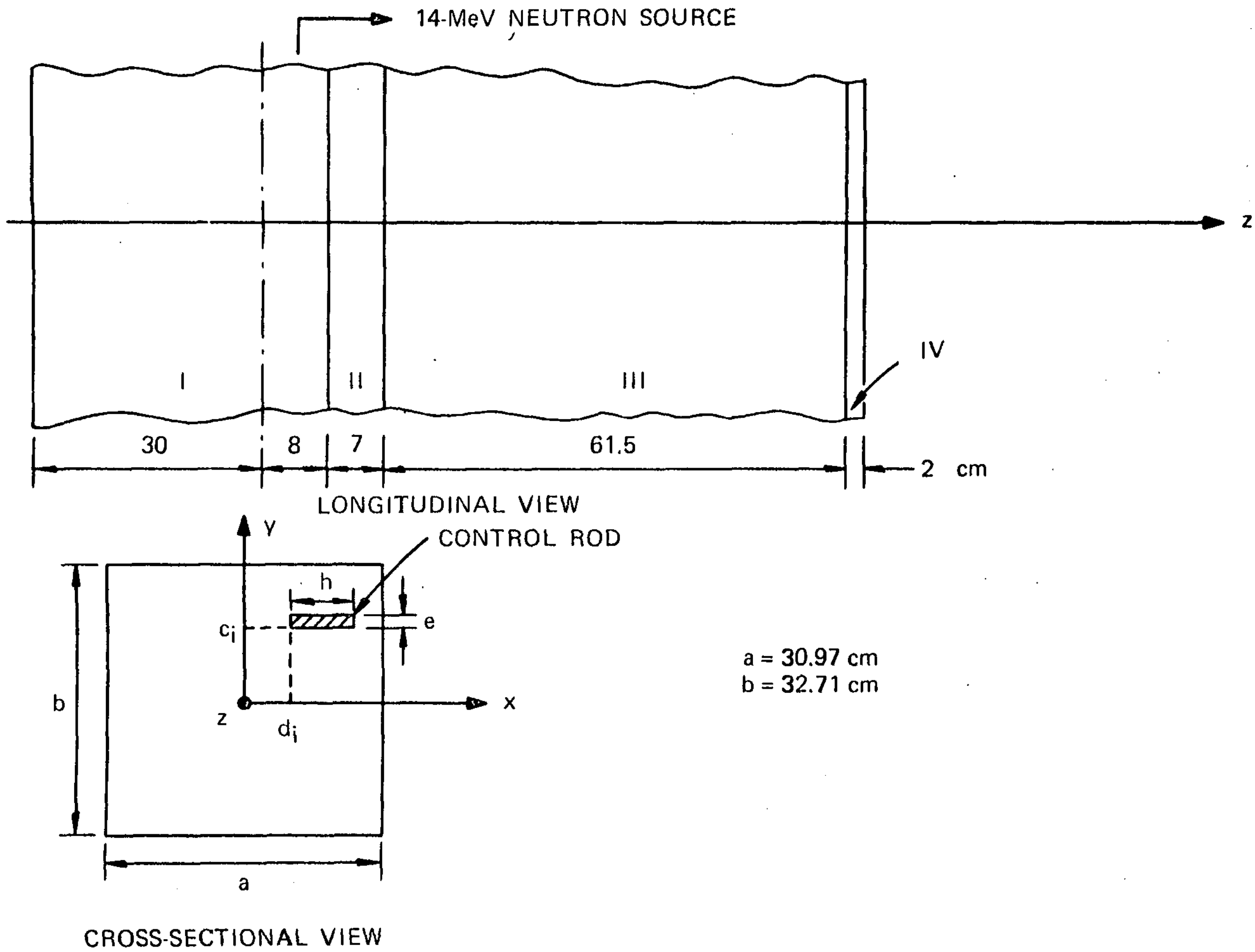

Fig. 1. Schematic description of the calculated subcritical system. The regions are: I, water; II and IV, water and structural aluminum; and III, multiplicative zone.

obtained[4] for each region by collapsing a 123-group set[5] for hydrogen, aluminum, ^{235}U, and ^{238}U. The ^{16}O cross sections were obtained from the ENDF/B-IV evaluation. The flux-weighting factors were obtained via a P_3, S_8 transport calculation with the XSDRNPM program[6] for a source of 14-MeV neutrons. Table II shows the group structure utilized in the present calculation. In this calculation, the geometry and compositions specified in Fig. 1 and Table I, respectively, were used.

Calculation of the Transverse Leakage

The transverse neutron leakage for the one-dimensional TASK calculation was introduced by an additional term to the absorption cross section, $\Sigma_\perp$, defined as

$$\Sigma_\perp = \Sigma_x + \Sigma_y \ , \qquad (1)$$

where Σ_x is an effective absorption cross section that accounts for the leakage in the x direction, given (in a one-dimensional calculation) by

$$\Sigma_x = \int_\mu f(x = a/2,\mu)\mu d\mu \bigg/ \int_0^a \phi^2 dx \ , \qquad (2)$$

with a similar expression for Σ_y. In Eq. (2), $f(x,\mu)$ is the angular flux per unit μ, μ is the cosine of the angle formed by the neutron direction and the positive x axis, ϕ is the scalar flux, and $a/2$ is the half-thickness in the x direction. The vector and scaler fluxes were also computed with the code TASK in the P_3, S_8 approximation. Table III compares the values obtained for $\Sigma_\perp$ with the absorption and total macroscopic cross sections for each energy group in region III of the assembly. These results show that for highly compact systems such as the one studied here, the transverse effects are by no means negligible. Nevertheless, these effects were approximated as a spatially independent correction to the cross

[4] W. E. FORD III, Personal Communication (Sep. 1975).

[5] C. C. WEBSTER and N. M. GREENE, "506 Nuclide 123 Group Neutron Cross Section Library," Oak Ridge National Laboratory Internal Correspondence (Jan. 23, 1975).

[6] N. M. GREENE et al., "AMPX: A Modular Code System for Generating Coupled Multigroup Neutron-Gamma Libraries from ENDF/B," ORNL-TM-3706, Oak Ridge National Laboratory (1976).

sections to make possible the use of the one-dimensional code TASK for lack of a better treatment.

Calculation of the Dispersion Law in the Rodless Assembly

The dispersion law, which gives the values of α and ξ, the real and imaginary parts, respectively, of the complex inverse relaxation length,[1] was computed by fitting the amplitude of the $1/v$ reaction rate to an exponential in the z direction of the assembly and its phase shift to a linear function of the position along the same direction. Appropriate end-effect corrections were made in both cases in the manner described on p. 489 of Ref. 1.

The reaction rates along the direction of wave propagation were computed by TASK, in the P_3, S_8 approximation, with a neutron source in group 1, to simulate the external injection of 14-MeV neutron pulses.

The dispersion law was also computed in the diffusion theory approximation on the basis of the same group cross-section set and structure and the inclusion of the effective absorption $\Sigma_{\perp}$ as before. This calculation proceeds in the manner indicated in Ref. 1 except that presently the inclusion of $\Sigma_{\perp}$ in the removal cross-section term makes unnecessary the consideration of a transverse buckling, which is set equal to zero in the present calculation.

TABLE I

Composition of Each Zone

Zone	Nucleus	Concentration (units of 10^{24} cm^{-3})
I	H_2O $\{$H	0.06692
	^{16}O	0.03346
II	H_2O $\{$H	0.05412
	^{16}O	0.02706
	^{27}Al	0.01152
III	H_2O $\{$H	0.03972
	^{16}O	0.01986
	^{27}Al	0.02389
	^{235}U	0.00009833
	^{238}U	0.00001067
IV	H_2O $\{$H	0.03968
	^{16}O	0.01984
	^{27}Al	0.02451

TABLE II

Energy Limits of the Five Energy Groups Used in Task Calculations

Group	E_{max}	E_{min}
1	14.918 MeV	10 MeV
2	10 MeV	1.0026 MeV
3	1.0026 MeV	961.12 eV
4	961.12 eV	0.65 eV
5	0.65 eV	0.005 eV

Calculation of the Dispersion Law for the Subcritical Assembly with Control Rods

In this instance, the calculation of the dispersion law proceeded in the same way as in the previous case. The effect of the control rods was mocked up by the introduction of an effective thermal-neutron absorption cross section, Σ'_{ath}, which is given in terms of the thermal-neutron absorption cross section, Σ_{ath}, for the case with no control rods, by the expression

$$\Sigma'_{ath} = \Sigma_{ath} + 2\kappa C_{00} \ , \tag{3}$$

where κ is a coefficient defined by Maynard[1,7] and C_{00} is a geometric factor calculated in our previous work.[1]

[7] C. W. MAYNARD, "Blackness Theory for Slabs," *Naval Reactor Physics Handbook*, Vol. I, p. 409, U.S. Atomic Energy Commission (1964).

TABLE III

Comparison of $\Sigma_{\perp}$ with Σ_a and Σ_t for Zone III

Group	Σ_a (cm^{-1})	Σ_t (cm^{-1})	$\Sigma_{\perp}$ (cm^{-1})	$\Sigma_{\perp}/(\Sigma_a + \Sigma_{\perp})$ (%)	$\Sigma_{\perp}/(\Sigma_t + \Sigma_{\perp})$ (%)
1	0.009257	0.10335	0.023662	71.8	18.6
2	0.001070	0.20631	0.018932	94.7	8.41
3	0.000384	0.64236	0.013272	97.2	2.02
4	0.006454	0.93131	0.011679	64.4	1.24
5	0.061913	1.8062	0.004480	6.75	0.247

RESULTS

Subcritical Assembly Without Control Rods

First, we discuss the results concerning the assembly without control rods. Figure 2 shows the experimental and calculated results for the phase shift as a function of distance from the source for two frequencies, 200 and 600 Hz, respectively. The slopes of the curves correspond to the values of ξ, the imaginary part of the inverse complex relaxation length. In general, there is good agreement between the shapes of the theoretical and experimental values. Figure 3 shows the experimental values of the real and imaginary parts of the inverse complex relaxation length, α and ξ, respectively, as a function of frequency. Also shown in the figure are the corresponding theoretical values calculated both by transport theory and the diffusion approximation. As expected in such a compact system as

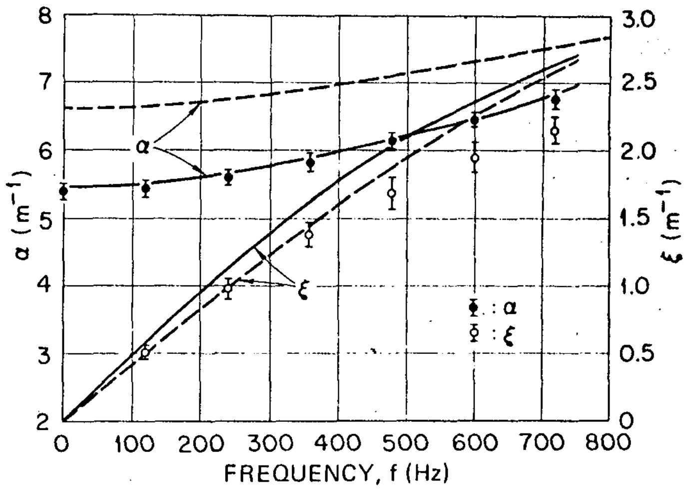

Fig. 3. System without control rods, calculated and experimental dispersion law. ΦΦ = α and ξ experimental values; —— = transport calculation; and ---- = diffusion theory calculation.

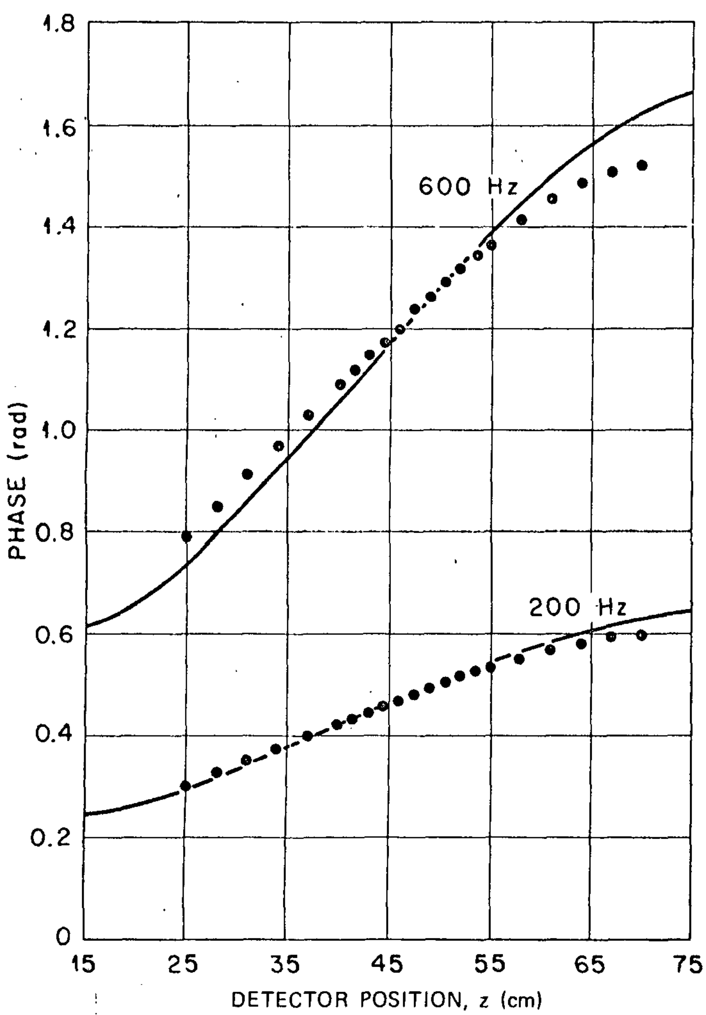

Fig. 2. System without control rods. Phase as a function of detector position; z, measured from the source plane; —— = TASK calculation; and ● = experimental values. The calculated values are normalized to the experimental one at z = 52 cm.

the present one, the transport calculation affords a much improved description of the dispersion law than the one provided by diffusion theory in the sense that the complex number $p = \alpha(\omega) + j\xi(\omega)$ agrees better with the experiment. Especially remarkable is the difference in the predictions of the shape of the frequency dependence for the function $\xi(\omega)$ at high frequencies. In this region, diffusion theory predicts an almost linear behavior, while the transport calculation tends to reproduce the curvature exhibited by the experimental results.

In general, the agreement of the transport calculation, as far as it concerns the values of α, is within experimental error, while the error in the prediction of ξ is ~14%. This behavior does not necessarily imply that the calculation of ξ is not done correctly. The integral parameters α and ξ are very sensitive to both cross-section values and spectral effects. For instance, a change of the values of ν and Σ_f by -1% coupled to an increase of Σ_a by 1% increases α by 6% and decreases ξ by 9%. After these hypothetical changes, the agreement between calculation and experiment would be opposite to the one found previously.

The above example indicates the usefulness of the neutron wave propagation technique for integral experiment. Not only does it provide two parameters for each frequency, but the sensitivity of the two measured parameters α and ξ to cross-section changes is of opposite sign. This fact decreases the possibility of fortuitous cancellations in theoretical calculations.

Figures 4 and 5 show the sensitivity of α and ξ to systematic changes in cross sections for the present system. The calculations were done in

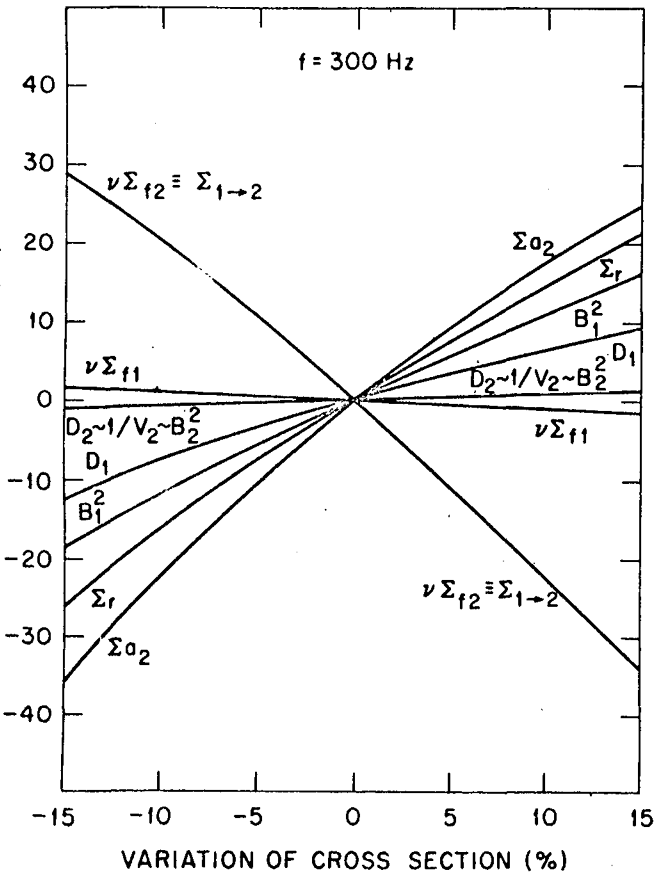

Fig. 4. Sensitivity of α to cross-section changes in a two-group diffusion approximation. The figure gives the percent variation of α for a given percent variation of cross sections. Subscripts 1 and 2 indicate the fast- and thermal-neutron groups, respectively; Σ_r is the removal cross section in group 1; standard nomenclature describes the other cross sections.

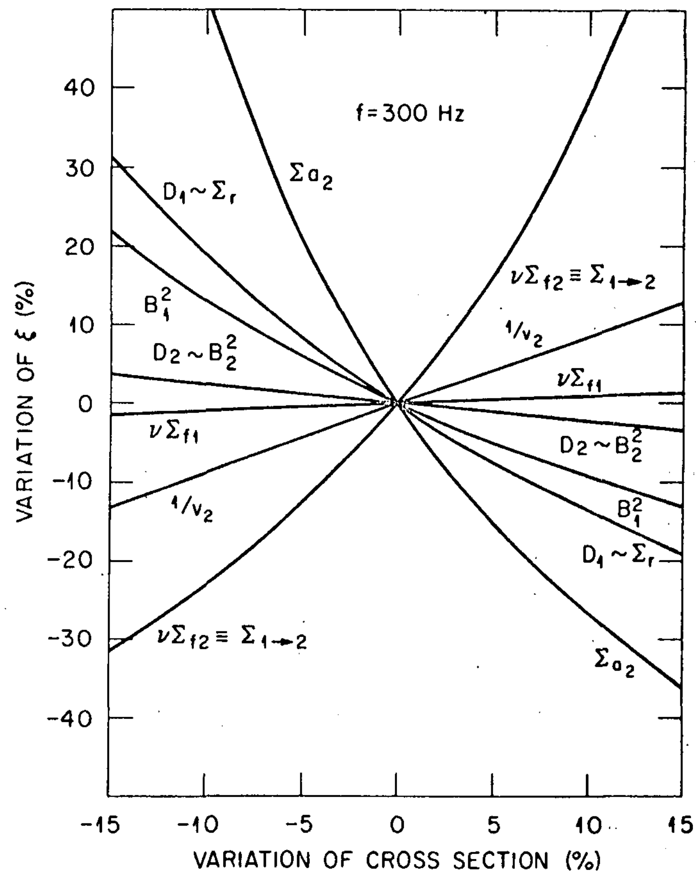

Fig. 5. Sensitivity of ξ to cross-section changes in a two-group diffusion approximation. The figure gives the percent variation of ξ for a given percent variation of cross sections. Subscripts 1 and 2 indicate the fast- and thermal-neutron groups, respectively; Σ_r is the removal cross section in group 1; standard nomenclature describes the other cross sections.

the two-group diffusion approximation. According to these calculations, the sensitivities of parameters α and ξ have opposite signs. Although it is believed that the same situation will be encountered in multigroup transport calculations, this has not yet been rigorously proven.

Subcritical Assembly with Control Rods

In this instance, the parameter κ was utilized as a free parameter in the calculation of the dispersion law until agreement with experimental dispersion law was achieved within the 14% accuracy obtained in the analysis of the rodless case. Convergence within this accuracy was reached for $\kappa = 0.135$.

The results of these calculations are shown in Fig. 6. To check the reliability of the value for the κ parameter obtained in the present calculation, the value of κ versus energy was computed for the control rods used in this experiment (0.8-mm-thick cadmium rods). The results of this calculation, shown in Fig. 7, demonstrate that the value of κ used to reproduce the dispersion law is compatible with considerable self-shielding of the thermal flux in the edge of the rod. If we consider that the rod is completely black for neutrons of energy <0.3 eV ($\kappa = 0.5$), this result is reasonable.

A further confirmation of the κ value is afforded by comparison with actual measurements of the calculated shape of the amplitude of the neutron flux in the transverse direction to the control rods. The calculation utilizes an expression previously derived in Ref. 1. The results in Fig. 8 show a remarkable consistency between the experimental and calculated shapes.

The frequency behavior of the neutron wave across the control rod was computed by the same expression, except that in this case the experimental values of the dispersion law were introduced in the calculation. The results of this procedure are shown in Figs. 9 and 10. There is a good agreement for the frequency dependence of the amplitudes. The phase is well reproduced in shape but not in absolute magnitude. Figure 11

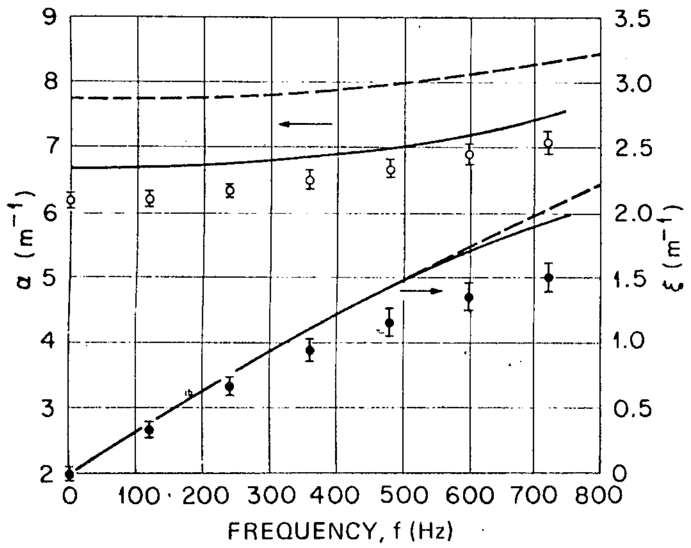

Fig. 6. System with control rods, calculated and experimental dispersion law. ΦΦ = α and ξ experimental values; —— = transport calculation; and - - - - = diffusion theory calculation.

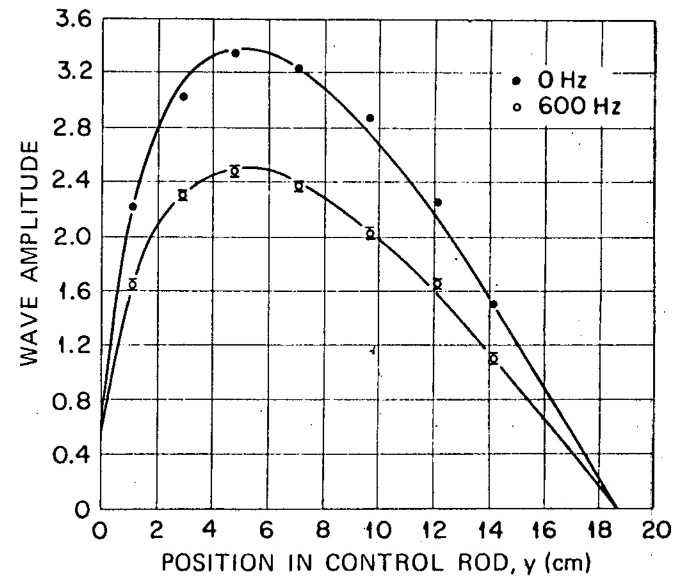

Fig. 8. Amplitude of the wave across the control rods; •Φ = experimental values; ——, = calculated values with $\kappa = 0.135$.

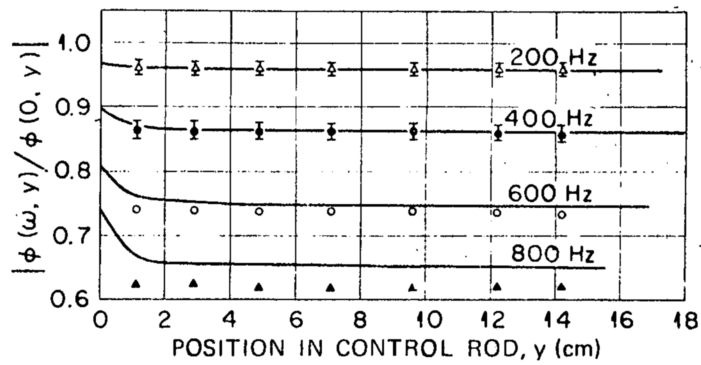

Fig. 9. Frequency behavior of the amplitude of the wave across the control rods; experimental (▲∘ΦΦ) and calculated (——) values. The calculated values include the experimental dispersion law and $\kappa = 0.135$.

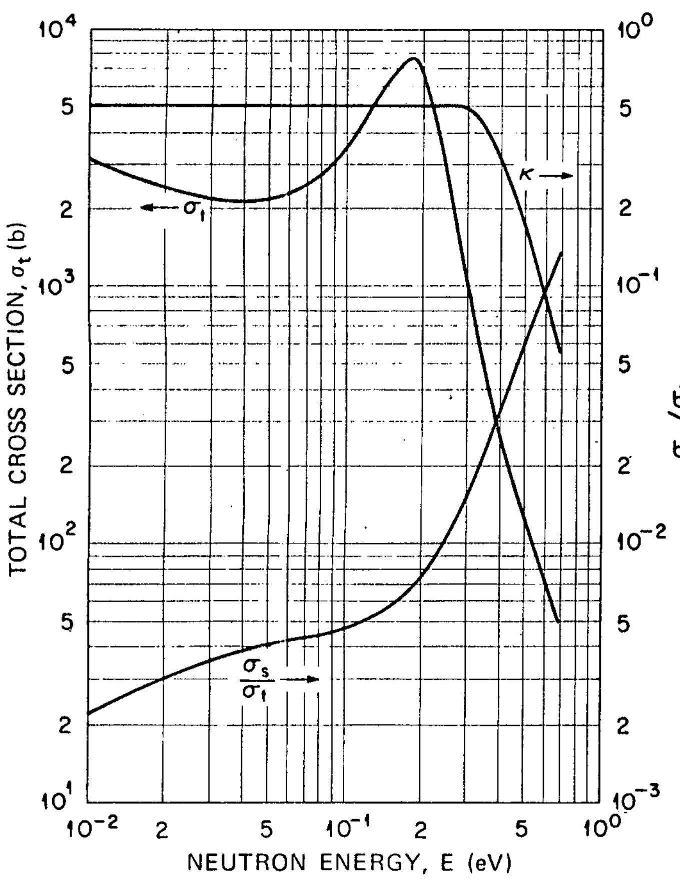

Fig. 7. Values of σ_t and σ_s/σ_t for natural cadmium and the Maynard coefficient (κ) of the control rods used in the analyzed experiment.

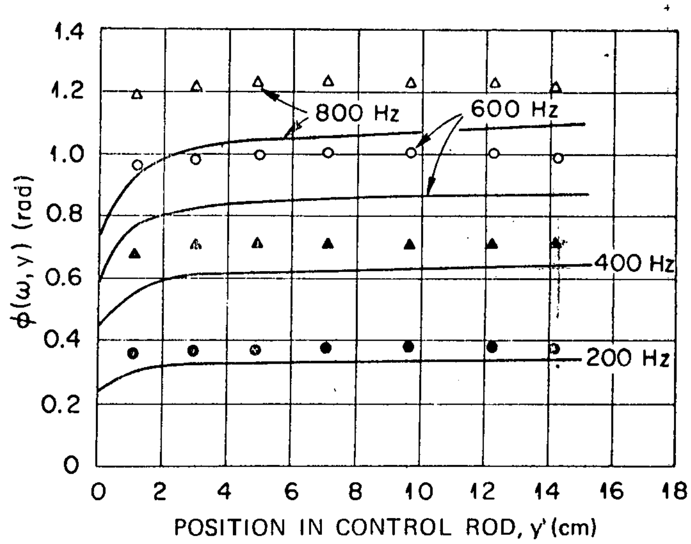

Fig. 10. Frequency behavior of the phase of the wave across the control rods; experimental (•▲∘Δ) and calculated (——) values. The calculated values include the experimental dispersion law and $\kappa = 0.135$.

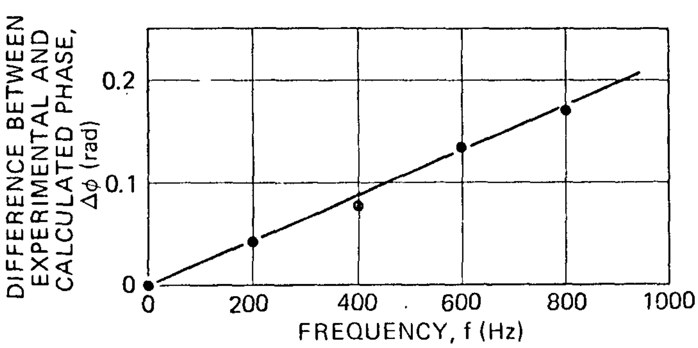

Fig. 11. Difference, as a function of frequency, of the experimental and calculated values of the phases shown in Fig. 10. The continuous line is the phase shift for a delay in the neutron pulse of 34.8 μs.

shows a plot of the difference between the magnitude of the computed phase and the experimental results as a function of the frequency. The observed deviations are compatible with a 34.8-μs delay in the trigger of the neutron pulse; this value is consistent with the delay of the neutron source utilized in the experimental work.[8]

CONCLUSIONS

The present results have shown the usefulness of the code TASK for the calculation of neutron wave experiments provided that careful attention is given to a good estimation of transverse leakage effects. The present transport calculation greatly improved the theoretical description of the experimental results, which had been given previously[1] on the basis of diffusion theory. Due to the high transverse leakage, the present treatment is still affected by the complications involved in the treatment of a three-dimensional system by a one-dimensional transport calculation.

Especially interesting is the fact that a good representation of the amplitude and phase of the neutron flux transverse to the control rods could be implemented by the use of the dispersion law and the theory given in Ref. 1. The present work confirms the consistency of the 123-group cross-section set. The sensitivity of the neutron wave experiment to transport effects, spectral changes, and cross-section sets indicates that this technique affords a new type of integral experiment that has the inherent advantage over the usual static integral methods of yielding both amplitude and phase information.

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[8]C. W. ELENGA and O. REIFENSCHWEILER, "The Generation of Neutron Pulses and Modulated Neutron Fluxes with Sealed-Off Neutron Tubes," *Proc. Symp. Pulsed Neutron Research*, Karlsruhe, 1965, Vol. I, p. 609, International Atomic Energy Agency, Vienna (1965).