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GENERAL ASYMMETRIC ROTOR LEVEL STRUCTURE OF SOME EVEN NUCLEI

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Abstract: The quasi-adiabatic general asymmetric rotor model for nuclei was applied to the level structures of ^{56}Fe, ^{96}Mo, ^{106}Pd, ^{110}Cd, ^{132}Xe, ^{148,152}Sm, ^{154,156}Gd, ^{182}W, ^{186,188,190}Os and ^{192}Pt. The agreement of predicted levels with experimental data is satisfactory for nucleides at least 7 % apart from magic numbers. The electric quadrupole transition probabilities are predicted within a factor 2(1.5) in 72 % (53 %) of the cases.

1. Introduction

The application of the general asymmetric rotor model to even nuclei was extensively made by Mallmann [1]). In the present paper, in view of new experimental data, we revise those cases which did not follow the general trend in his systematics (^{152}Sm, ^{154,156}Gd, ^{182}W and ^{186,190}Os).

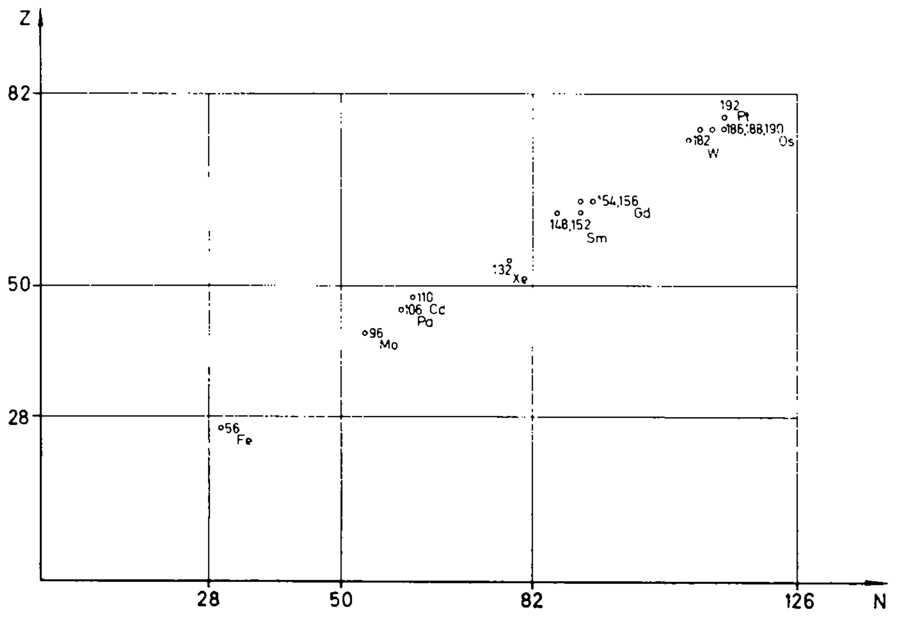

Fig. 1. Relative position of the considered nuclei with respect to magic numbers.

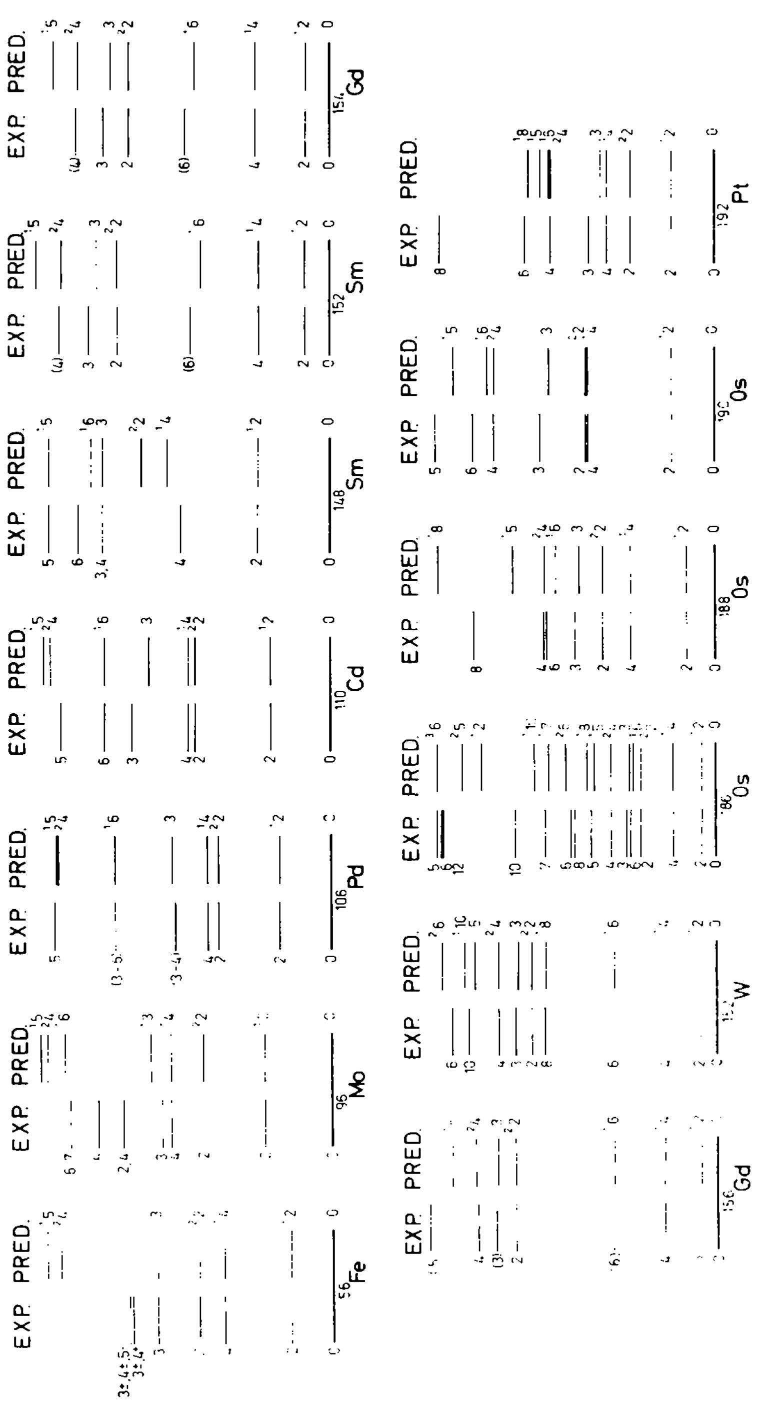

Fig. 2. Levels predicted with the general asymmetric rotor theory (arbitrary scale). For each nucleus only the corresponding experimental levels are indicated.

We also study other nuclei near magic numbers in order to establish the region of applicability of the model (fig. 1). We limit ourselves to cases with $|Z-Z_m| \leqq 0.08\ Z_m$ or $|N-N_m| \leqq 0.08\ N_m$, where Z_m and N_m are magic numbers of protons and neutrons.

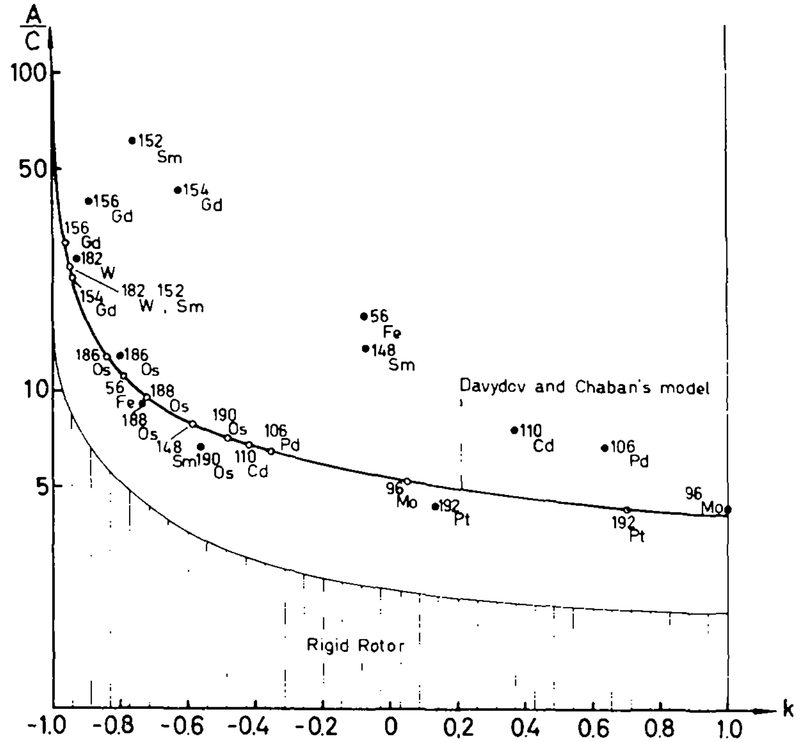

Fig. 3. Plot of the values of k and A/C used to compute the energy levels. The values for the Davydov-Chaban model and the allowed values for rigid rotors are indicated.

2. Notation

Within the framework of the model, the energy ε_b of the n-level with angular momentum J is given in units of hC by

$$\varepsilon_b(k, {}^nJ, A/C) = \varepsilon_0 - b\varepsilon_0^2, \tag{2.1}$$

where

$$\varepsilon_0(k, {}^nJ, A/C) = J(J+1)\{\beta(k, {}^nJ) + A/C[1-\beta(k, {}^nJ)]\} \tag{2.2}$$

corresponds to the energy of the level in absence of the rotation-vibration interaction simulated by the second term in eq. (2.1) and

$$k = (2B-A-C)/(A-C) \tag{2.3}$$

is an asymmetry parameter. The rotational constants A, B and C depend on the moments of inertia I_A, I_B and I_C, which we order according to

$$A = \hbar/4\pi I_A \geqq B = \hbar/4\pi I_B \geqq C = \hbar/4\pi I_C. \tag{2.4}$$

The functions $\beta(k, {}^nJ)$ are tabulated in ref. [2]).

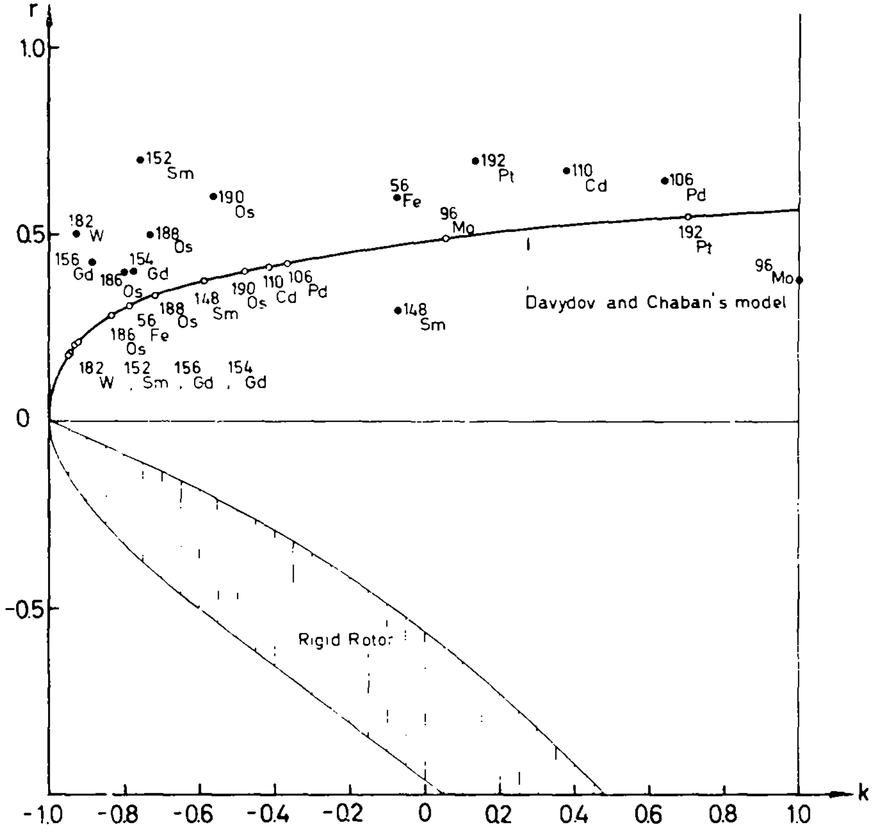

Fig. 4. Plot of the values of k and r which predict the (E2) gamma-ray transition probability ratios (table 2) of the nuclei indicated on the figure. The values for the Davydov-Chaban model and the allowed values for rigid rotors are indicated.

Another parameter measuring the asymmetry is defined

$$r = \sqrt{2}k_2/k_0, \tag{2.5}$$

where k_0 and k_2 are the quadrupole moments. The values of r can be obtained from the ratio between the intensities of two γ-transitions depopulating the same level. Theoretical estimates of the reduced transition probabilities as a function of the parameters k and r have been published in ref. [3]).

TABLE 1
Experimental and predicted energy levels

Nucleus	n_J	E (keV) exp.	E (keV) pred.	Parameters k	A/C	$\hbar C$(keV)	10^4 b	Percentual distance to the nearer Z and N magic numbers	
^{56}Fe	$^{1}2$	846.82	846.82	−0.078	17.05	34.152	4.359	−7.1	7.1
	$^{1}4$	2085.38	2085.38						
	$^{2}2$	2660	2660.0						
	$^{1}3$	3446.14	3446.6	Ref.:	[1]) Nuclear Data Sheets				
	$^{2}4$		5429.0		[2]) H. Pettersson, O. Bergman and C. Bergman, Ark. Fys. **29** (1965) 423				
	$^{1}5$		5694.0						
^{96}Mo	$^{1}2$	779	779	1.00	4.26	65.679	41.60	−16	8.0
	$^{2}2$	1499	1499						
	$^{1}4$	1871	1871						
	$^{1}3$	1979	2103	Ref.:	[1]) S. Monaro, Nuovo Cimento **39** (1965) 442				
	$^{2}4$	2438.5 [a])	3314.9						
	$^{1}5$	2730 [a])	3403						
	$^{1}6$	3040 [a])	3007						
^{106}Pd	$^{1}2$	511.77	511.77	0.638	6.59	32.853	17.47	−8.0	20.0
	$^{2}2$	1127.8	1127.8						
	$^{1}4$	1228.9	1228.9						
	$^{1}3$	1557.4 [a])	1572.1	Ref.:	[1]) E. Y. de Aisenberg and J. F. Suárez, Nuclear Physics **83** (1966) 289				
	$^{1}6$	2076.4 [a])	2076.8						
	$^{2}4$		2699.8						
	$^{1}5$	2756.4	2728.0						
^{110}Cd	$^{1}2$	657.61	657.61	0.374	7.50	41.976	27.51	−4.0	24.0
	$^{2}2$	1475.65	1475.65						
	$^{1}4$	1542.08	1542.08						
	$^{1}3$	2162.24	1983.9	Ref.:	[1]) W. B. Newbolt and J. H. Hamilton, Nuclear Physics **53** (1964) 353				
	$^{1}6$	2479.50	2480.17						
	$^{2}4$		3071.5						
	$^{1}5$	2925.92	3135.6						
^{148}Sm	$^{1}2$	551	551	−0.073	13.59	28.148	32.75	24.0	4.9
	$^{1}4$	1181	1245						
	$^{2}2$		1435						
	$^{1}3$	1735 [a])	1735	Ref.:	C. V. K. Baba, G. T. Ewan and J. F. Suárez, Nuclear Physics **43** (1963) 285				
	$^{1}6$	1908	1821						
	$^{1}5$	2147	2147						
^{152}Sm	$^{1}2$	121.8	121.8	−0.760	61.26	4.5401	2.388	24.0	9.8
	$^{1}4$	366.5	366.5						
	$^{1}6$	712 [a])	666.4						
	$^{2}2$	1087	1087.0	Ref.:	[1]) Nuclear Data Sheets				
	$^{1}3$	1235	1193.9		[2]) Y. Yoshizawa *et al.*, Nuclear Physics **73** (1965) 273				
	$^{2}4$	1385 [a])	1373.6						
	$^{1}5$		1505.0						
^{154}Gd	$^{1}2$	123.1	123.1	−0.767	42.68	6.1853	5.292	−22.0	9.8
	$^{1}4$	371.2	371.2						
	$^{1}6$	718 [a])	677.3						

TABLE 1
(continued)

Nucleus	n_J	E(keV) exp.	E(keV) pred.	Parameters k	A/C	hC(keV)	10^4 b	Percentual distance to the nearer Z and N magic numbers	
	$^{2}2$	997.3	997.3	Ref.:	[1]) Nuclear Data Sheets				
	$^{1}3$	1128.5	1097.0		[2]) Y. Yoshizawa *et al.*, Nuclear Physics				
	$^{2}4$	1265.3 [a])	1257.4		**73** (1965) 273				
	$^{1}5$		1380.0						
^{156}Gd	$^{1}2$	89.0	89.0	−0.890	39.35	7.2833	1.204	−22.0	12.2
	$^{1}4$	288.2	288.2						
	$^{1}6$	584.5 [a])	578.9						
	$^{2}2$	1154	1154.0	Ref.:	[1]) Nuclear Data Sheets				
	$^{1}3$	1249 [a])	1239.8		[2]) Y. Yoshizawa *et al.*, Nuclear Physics				
	$^{2}4$	1357	1362.7		**73** (1965) 273				
	$^{1}5$	1624 [a])	1494.5						
^{182}W	$^{1}2$	100.1	100.1	−0.93	25.98	11.658	1.770	−9.8	−14.3
	$^{1}4$	329.35	329.35						
	$^{1}6$	680.4	677.8						
	$^{1}8$	1138 ± 10	1131.3	Ref.:	[1]) B. Harmatz, T. H. Handley and J. W.				
	$^{2}2$	1221.8	1221.8		Mihelich, Phys. Rev. **123** (1961) 1758				
	$^{1}3$	1331.6	1318.1		[2]) N. L. Lark and H. Morinaga, Nuclear				
	$^{2}4$	1443.3	1450.1		Physics **63** (1965) 466				
	$^{1}5$		1605.2						
	$^{1}10$	1646 ± 20	1675.0						
	$^{2}6$	1757.5	1823.7						
^{186}Os	$^{1}2$	137.2	137.2	−0.802	12.96	14.635	8.827	−7.3	−12.7
	$^{1}4$	434.0	434.0						
	$^{2}2$	767.5	767.5						
	$^{1}6$	868.8	845.2	Ref.:	[1]) B. Harmatz and T. H. Handley,				
	$^{1}3$	910.7	891.2		Nuclear Physics **56** (1964) 1				
	$^{2}4$	1070.5	1070.2		[2]) N. L. Lark and H. Morinaga, Nuclear				
	$^{1}5$	1275.7	1248.6		Physics **63** (1965) 466				
	$^{1}8$	1453.3	1329.0						
	$^{2}6$	1491.3	1540.0						
	$^{1}7$	1751.7	1720.0						
	$^{1}10$	2059 ± 15	1857						
	$^{1}12$	2804 ± 20	2405						
	$^{3}6$	2822.6	2846.2						
	$^{2}5$	2860.0	2682.3						
^{188}Os	$^{1}2$	155.0	155.0	−0.729	9.05	17.197	16.68	−7.3	−11.1
	$^{1}4$	477.9	477.9						
	$^{2}2$	633.0	633.0						
	$^{1}3$	789.9	767.3	Ref.:	[1]) B. Harmatz and T. H. Handley, Nuclear				
	$^{1}6$	948 ± 7	901.2		Physics **56** (1964) 1				
	$^{2}4$	965.4	963.0		[2]) N. L. Lark and H. Morinaga,				
	$^{1}5$		1141.5		Nuclear Physics **63** (1965) 466				
	$^{1}8$	1588 ± 15	1368.7						

TABLE 1
(continued)

Nucleus	n_J	E(keV) exp.	E(keV) pred.	k	Parameters A/C	Parameters $\hbar C$(keV)	Parameters 10^4 b	Percentual distance to the nearer Z and N magic numbers	
^{190}Os	$^{1}2$	186.7	186.7	−0.564	6.60	20.270	26.16	−7.3	−9.5
	$^{1}4$	547.9	547.9						
	$^{2}2$	557.9	557.9						
	$^{1}3$	755.9	714.7	Ref.:	[1]) B. Harmatz and T. H. Handley,				
	$^{2}4$	955.2	953.9		Nuclear Physics **56** (1964) 1				
	$^{1}6$	1050.4	983.5						
	$^{1}5$	1203.5	1128.4						
^{192}Pt	$^{1}2$	316.5	316.5	0.133	4.31	31.759	57.99	−4.9	−9.5
	$^{2}2$	612.4	612.4						
	$^{1}4$	784.5	784.5						
	$^{1}3$	920.9	842.4	Ref.:	[1]) B. Nyman *et al.*, Ark. Fys. **30** (1965) 479				
	$^{2}4$	1201.0	1202.5		[2]) N. L. Lark and H. Morinaga,				
	$^{1}5$		1283.0		Nuclear Physics **63** (1965) 466				
	$^{1}6$	1388±10	1210.6						
	$^{1}8$	2063±20	1363.2						

[a]) Angular momentum assignment uncertain.

The proximity of nuclei to magic numbers and the parameters used in the calculations are indicated.

Note: Using the procedure described in section 3, it was not possible to find a set of parameters to fit the experimental levels of ^{132}Xe which is 8.0 % and −4.9 % distant from the nearer magic numbers (Ref. J. H. Hamilton *et al.*, Nuclear Physics **72** (1965) 625); however, assuming the 2395.6 keV level to be the $^{2}4$ level as suggested by the application of Davydov and Chaban's model, the parameters $k = 1$ $A/C = 4.98$ and $b = 53.33 \times 10^{-4}$ which exactly fit at the $^{1}2$, $^{2}2$ and $^{2}4$ levels are obtained and the $^{1}4$, $^{1}3$ and $^{1}6$ levels result reasonably predicted.

3. Results

Using the experimental energies of the $^{1}2$, $^{1}4$, $^{2}2$ and $^{1}6$ or $^{2}4$ levels the parameter k was obtained; then, the values of A/C and b which exactly fit the $^{1}2$, $^{2}2$ and $^{1}4$ levels were determined (except in the case of ^{148}Sm for which the experimental $^{2}2$ level is unknown). In this way, for all the considered cases, the constant b of formula (2.1) is positive, removing the "anomalous" cases of ref. [1]). We interpreted that the second term of (2.1) must be a correction term and this is not always the case for high angular momenta levels; thus, if one obtains the set of parameters which give the best fit to all known rotational levels including high J (ref. [1])), some negative b-values could appear in disagreement with the systematics. This is not the case if we fix the parameters using levels with $J \leqq 6$, as we have done here.

Table 1 and fig. 2 show the predicted energy levels compared with the corresponding experimental ones, the parameters used in the calculations and the percentual nearness of the nuclei to the magic numbers.

TABLE

Comparison of experimental E2 gamma-ray transition

Nucleus	r	$\frac{I_\gamma(2212)}{I_\gamma(2210)}$	$\frac{I_\gamma(2214)}{I_\gamma(2210)}$	$\frac{I_\gamma(1422)}{I_\gamma(1412)}$	$\frac{I_\gamma(1322)}{I_\gamma(1312)}$	$\frac{I_\gamma(1314)}{I_\gamma(1312)}$
^{56}Fe	0.600	1.1 ± 0.8				$2.3\cdot10^{-1}$ b)
		1.7	$3.8\cdot10^{-4}$		$2.6\cdot10^{-2}$	$2.2\cdot10^{-1}$
^{96}Mo	0.375	2.3 ± 0.4		$(6.9\pm1.2)\,10^{-2}$	$\leqq(1.9\pm0.3)\,10^{-1}$ c)	
		1.3		0	$3.7\cdot10^{-1}$	$1.2\cdot10^{-4}$
^{106}Pd	0.650	2.5 ± 0.8			$(6.1\pm0.9)\,10^{-1}$ b)	$(3.2\pm1.8)\,10^{-2}$ b)
		3.6		$1.7\cdot10^{-7}$	$6.0\cdot10^{-1}$	$8.9\cdot10^{-2}$
^{110}Cd	0.675	1.5 ± 0.5			$(7.0\pm1.2)\,10^{-1}$ b)	$<5\ \ 10^{-1}$ b)
		1.4		$2.4\cdot10^{-8}$	$3.8\cdot10^{-1}$	$1.3\cdot10^{-1}$
^{148}Sm	0.295					5.6
					$6.8\cdot10^{-1}$	5.6
^{152}Sm	0.700	1.3				$2.3\cdot10^{-1}$
		1.3	$2.9\cdot10^{-2}$		$1.1\cdot10^{-4}$	$2.8\cdot10^{-1}$
^{154}Gd	0.400	1.3 ± 0.3	$<3\cdot10^{-2}$			
		1.6	$4.0\cdot10^{-2}$		$3.3\cdot10^{-4}$	$3.9\cdot10^{-1}$
^{156}Gd	0.425	1.3	$1.0\cdot10^{-1}$			$\leqq1.4\cdot10^{-1}$ a)
		1.3	$4.0\cdot10^{-2}$		$2.3\cdot10^{-5}$	$3.0\cdot10^{-1}$
^{182}W	0.500	1.0				$2.1\cdot10^{-1}$
		1.1	$2.6\cdot10^{-2}$		$1.8\cdot10^{-5}$	$2.1\cdot10^{-1}$
^{186}Os	0.400	$9.8\cdot10^{-1}$	$1.7\cdot10^{-2}$		$4.8\cdot10^{-3}$	
		1.0	$5.2\cdot10^{-3}$		$1.8\cdot10^{-3}$	$1.2\cdot10^{-1}$
^{188}Os	0.500	$7.5\cdot10^{-1}$			$7.0\cdot10^{-3}$	$5.9\cdot10^{-2}$
		$7.1\cdot10^{-1}$	$3.2\cdot10^{-4}$		$5.1\cdot10^{-3}$	$4.2\cdot10^{-2}$
^{190}Os	0.600	$5.6\cdot10^{-1}$			$3.4\cdot10^{-2}$	$\approx2.1\cdot10^{-2}$
		$4.8\cdot10^{-1}$	$8.4\cdot10^{-10}$		$2.3\cdot10^{-2}$	$1.2\cdot10^{-2}$
^{192}Pt	0.700	4.6		$2.5\cdot10^{-2}$	$3.2\cdot10^{-1}$	$1.3\cdot10^{-1}$
		$3.5\cdot10^{-1}$		$1.4\cdot10^{-4}$	$3.4\cdot10^{-1}$	$3.4\cdot10^{-3}$

Experimental values for the nuclei indicated in column one are given in the first row within each nucleus. The of k given in table 1. Blocks containing only theoretical information imply that only the transition quoted in the

[a]) M1+E2.
[b]) Assuming both transitions to be pure E2.
[c]) Assuming the (1322) γ-transition to be pure E2.
[d]) Only the transition quoted in the numerator is experimentally known.

2
probability ratios with theoretical predictions

$\frac{I_\gamma(2422)}{I_\gamma(2412)}$	$\frac{I_\gamma(2413)}{I_\gamma(2412)}$	$\frac{I_\gamma(2414)}{I_\gamma(2412)}$	$\frac{I_\gamma(2416)}{I_\gamma(2412)}$	$\frac{I_\gamma(1513)}{I_\gamma(1514)}$	$\frac{I_\gamma(1524)}{I_\gamma(1514)}$	$\frac{I_\gamma(1516)}{I_\gamma(1513)}$	$\frac{I_\gamma(1516)}{I_\gamma(1514)}$
$\frac{I_\gamma(2422)}{I_\gamma(2413)} =$ $(5.1 \pm 0.9)\ 10^{-2}$ b) $5.8 \cdot 10^{2}$							
						$(1.9 \pm 1.2)\ 10^{-1}$ b)	
				$3.2 \cdot 10$ d)		$4.5 \cdot 10^{-2}$	
				$> 1.2 \pm 0.1$ a)			
				2.4		$5.4 \cdot 10^{-2}$	
				$\geqq 2.2$ a)			
				$4.5 \cdot 10^{-1}$			$1.8 \cdot 10^{-2}$
		$2.1 \cdot 10^{-1}$					
$3.9 \cdot 10^{-3}$	$2.3 \cdot 10^{-4}$	3.0	$7.0 \cdot 10^{-2}$				
$3.7 \cdot 10^{-2}$	$2.5 \cdot 10^{-3}$	$1.0 \cdot 10$	$2.3 \cdot 10^{-1}$				
							$\approx 5 \cdot 10^{-1}$ b)
		2.8 d)		$8.1 \cdot 10\ {}^{3}$	$1.6 \cdot 10^{-3}$	$4.1 \cdot 10$ d)	$4.3 \cdot 10^{-1}$
$8.2 \cdot 10^{-1}$		1.9					
$6.8 \cdot 10^{-4}$	$4.8 \cdot 10^{-5}$	1.8	$4.1 \cdot 10^{-2}$				
$1.4 \cdot 10^{-1}$	$< 1.7 \cdot 10^{-2}$ a)	1.3		$1.6 \cdot 10^{-1}$		$3.8 \cdot 10^{-1}$	$6.1 \cdot 10^{-1}$
$1.1 \cdot 10^{-1}$	$9.2 \cdot 10^{-3}$	3.0	$2.4 \cdot 10^{-3}$	$1.4 \cdot 10^{-1}$	$8.2 \cdot 10^{-3}$	$7.3 \cdot 10^{-1}$	$1.0 \cdot 10^{-1}$
$5.1 \cdot 10^{-1}$		1.2					
$2.8 \cdot 10^{-1}$	$2.2 \cdot 10^{-2}$	2.0	$< 1.2 \cdot 10^{-7}$				
1.8	$3.8 \cdot 10^{-2}$	1.6		1.2			
1.8	$9.0 \cdot 10\ {}^{2}$	2.8		1.0	$6.2 \cdot 10^{-2}$	$3.8 \cdot 10\ {}^{3}$	$4.1 \cdot 10^{-3}$
$1.2 \cdot 10$	2.5	3.5					
$4.0 \cdot 10$	$4.7 \cdot 10^{-1}$	6.5					

second row gives the theoretical values calculated with the values of r given in the second column and the values denominator is experimentally known.

The parameters r given in table 2 were obtained by making a best fit to the experimental quadrupole intensity ratios depopulating the rotational levels giving a reasonable weight to the experimental values according to their precision. The transitions labelled as E2 by the authors have been taken as pure quadrupole. When mixing of M1 is reported and information on the amount available, the M1 intensity has been deduced from the total γ-intensity; otherwise it is indicated in the table.

The values obtained for A/C and r are plotted against k in figs. 3 and 4, respectively, together with the predictions of the rigid rotor and the Davydov-Chaban [4]) model. It is apparent from these figures that both asymmetry parameters k and r, estimated with Mallmann's model differ from those calculated with the Davydov-Chaban model.

The values of r drawn in fig. 4 allow the evaluation of the E2 transition probability ratios within a factor 2 (1.5) in more than 72 % (53 %) of the considered cases. These figures should be compared with 49 % (36 %) obtained with the Davydov-Chaban model. It must be remembered that in both cases, the small [4]) contribution to the transition probabilities due to the rotation-vibration coupling has been neglected.

4. Conclusions

From the results given in table 1 it can be stated that the model applies successfully for even nuclei at least 7 % apart from closed shells, in what energy predictions concern.

In spite of the fact that the points in fig. 4 are affected by considerable errors due to uncertainties on the experimental transition intensities, from the total information shown in figs. 3 and 4 it can be concluded that actual even nuclei behave neither as as rigid rotors nor as liquid droplets plus beta vibrations or anything in between.

It is a pleasure to thank Dr. C. A. Mallmann and Dr. D. R. Bès for many fruitful suggestions.

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