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Phenomenological Failure of Sum Rules for Inclusive Distributions Based on PCAC.

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In the last years a few papers have appeared referring to the application of the PCAC condition ([1]) (or Adler's theorem) to hadronic processes of multiparticle production ([2-5]). Comforted by the good results obtained from the analysis of some low-multiplicity exclusive reactions ([3]), in the present note PCAC is applied to available inclusive distributions; the result is however negative in the sense that the sum rules predicted by PCAC are not phenomenologically satisfied.

We reproduce below the sum rules which can be derived, disregarding interference terms, for the inclusive processes $\text{ap} \to \pi^{\mp} + \text{anything}$

$$\text{(1)} \qquad \lim_{|\boldsymbol{k}|\to 0} \omega_k \frac{\mathrm{d}\sigma(\text{ap}\to\pi^-)}{k^2\,\mathrm{d}k} = \frac{1}{4\pi^2}\frac{g^2}{2m^2}\int \mathrm{d}\boldsymbol{p}\,\frac{|\boldsymbol{p}|^2}{E_\text{p}^2}\left[\frac{\mathrm{d}\sigma(\text{ap}\to\text{n})}{\mathrm{d}\boldsymbol{p}} + \frac{\mathrm{d}\sigma(\text{ap}\to\bar{\text{p}})}{\mathrm{d}\boldsymbol{p}}\right],$$

$$\text{(2)} \qquad \lim_{|\boldsymbol{k}|\to 0} \omega_k \frac{\mathrm{d}\sigma(\text{ap}\to\pi^+)}{k^2\,\mathrm{d}k} = \frac{1}{4\pi^2}\frac{g^2}{2m^2}\left\{\frac{|\boldsymbol{p}_\text{inc}|^2}{E_\text{inc}^2}\sigma_\text{an}^\text{tot} + \int \mathrm{d}\boldsymbol{p}\,\frac{|\boldsymbol{p}|^2}{E_\text{p}^2}\left(\frac{\mathrm{d}\sigma(\text{ap}\to\text{p})}{\mathrm{d}\boldsymbol{p}} + \frac{\mathrm{d}\sigma(\text{ap}\to\bar{\text{n}})}{\mathrm{d}\boldsymbol{p}}\right)\right\},$$

where we have assumed the existence of final nucleons and antinucleons as the only possible baryons into which insertions of axial currents are considered. $g^2/4\pi \approx 15$ is the πN coupling constant and m the nucleon mass. $(E_\text{inc}, \boldsymbol{p}_\text{inc})$ represents the momentum of the initial proton. For the case a = p, an additional factor 2 appears in the first term of the r.h.s. of eq. (2). Some comments are in order:

i) Whereas the l.h.s. of (1) and (2) correspond to the experimental distributions for $|\boldsymbol{k}| \to 0$, $\omega_k \to \mu$ (pion mass), the r.h.s. result from Adler's theorem, *i.e.* for $(k_0, \boldsymbol{k}) \to 0$. Therefore (1) and (2) can be expected to hold only if the continuation from $\omega_k = \mu$ to $\omega_k = 0$ is smooth.

([1]) S. L. ADLER and R. F. DASHEN: *Current Algebras and Applications to Particle Physics* (New York, 1968).
([2]) N. SAKAI and M. YAMADA: *Phys. Lett.*, **37** B, 505 (1971).
([3]) F. ARBAB, J. C. GALLARDO and L. MASPERI: *Lett. Nuovo Cimento*, **2**, 1069 (1971).
([4]) K. FABRICIUS: Bielefeld preprint Bi-72/08 (1972).
([5]) K. FABRICIUS: Bielefeld preprint Bi-72/13 (1972).

ii) In the r.h.s. of (1) and (2) the interference terms corresponding to insertions into two different final (or initial) nucleons or into one initial and one final nucleon are neglected. These terms cannot be easily evaluated, but it is very likely that, since they are not positive definite, their inclusion is not able to restore the systematic strong disagreement which is described below between both sides of eqs. (1) and (2).

iii) As they stand, sum rules (1) and (2) should be valid for soft pions in any reference frame. Following the general practice, the values reproduced below correspond to soft pions in the c.m. frame. It must be noted however that the application of PCAC in laboratory system does not change things too much: the l.h.s. decreases only a small amount because, for large energy, soft pions in the laboratory frame correspond to $x \approx 0.15$ instead of $x \approx 0$; as for the r.h.s., the only relevant change is in eq. (2), where the first term disappears, leaving however unchanged the general conclusions described below.

iv) There could be objections against the possible validity of (1) and (2) on the basis that Adler's theorem seems to suggest the emission of soft pions only from external nucleons. However, it must be stressed that, as shown in ref. [6], the insertion of an axial current into an external line corresponds to the sum of a true nucleon pole plus an internal-emission term which may take over at high energy (see Fig. 1).

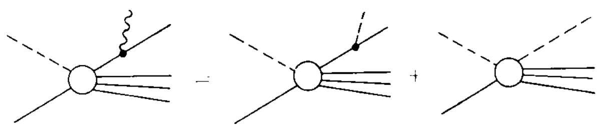

Fig. 1. – Adler's insertion equivalent to pole term with pseudoscalar πN coupling plus internal bremsstrahlung.

v) Another objection can be that, if scaling is valid, for high energy the l.h.s. of (1) should be independent of s, whereas the r.h.s. should increase as $\ln s$. But it must be remembered that interference terms have been neglected and that, moreover, the approach to scaling is slow for the pionization region which corresponds to the l.h.s. Therefore it does not seem possible to test such fine details with sum rules (1) or (2).

Regarding the evaluation of r.h.s., the major problem is the absence of experimental data for the process $ap \to n + \text{anything}$. In Table I this contribution is computed by means of the one-pion exchange (OPE) model of ref. [7] plus factorization (see Fig. 2):

$$\frac{d\sigma}{dt\, dM'^2/s} = \frac{1}{2\pi}\frac{g^2}{4\pi}\frac{(-t)}{(t-\mu^2)^2}\frac{M'^2}{s}\left(\frac{t_0-\mu^2}{t_0-t}\right)^{\alpha+1}\sigma^{tot}_{\pi a}\,. \tag{3}$$

It is remarked that with another mechanism, the novel model of ref. [8], the contribution of $ap \to nX$ turns out to be around 50% of the value reported here, so that it seems this choice is not critical.

It is clear from Table I that the l.h.s. of the PCAC sum rules is systematically larger than the r.h.s. by about an order of magnitude. This result is so define that it seems

(6) E. Schrauner: *Phys. Rev.*, **131**, 1847 (1963).
(7) M. Bishari: *Nucl. Phys.*, **48** B, 325 (1972).
(8) R. Slansky: *A diffraction dissociation model of hadron production*, Yale report (1972).

therefore to be independent of the rather strong assumptions made in the evaluation of the r.h.s. It must be noted moreover that a similar result has been obtained in a recent paper (5) for the reaction $pp \to \pi^0 +$ anything.

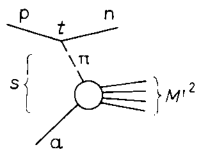

Fig. 2. – One-pion exchange model. According to ref. (7) $t_0 = 0.7\ (\text{GeV})^2$, $\bar{\alpha} = 0.7$.

Table I. – *Numerical evaluation fo sum rules eqs.* (1) *and* (2) *for soft pions in the* c.m. *frame.*

Reaction	p_{lab} (GeV)	l.h.s. (mb/(GeV)^2)	Contributions to r.h.s. (mb/(GeV)^2)	r.h.s. (mb/(GeV)^2)
$pp \to \pi^- X$	28.5	400 (9)	$pp \to nX$ 37.6 (OPE)	37.7
			$pp \to \bar{p}X$ 0.1 (9)	
	1000	900 (9)	$pp \to nX$ 49 (OPE)	56
			$pp \to \bar{p}X$ 7 (9)	
$pp \to \pi^+ X$	28.5	600 (9)	$pn \to X$ 196	267
			$pp \to pX$ 71 (9)	
	1000	800 (9)	$pn \to X$ 232	387
			$pp \to pX$ 148 (11)	
			$pp \to \bar{n}X$ 7 (assuming $pp \to \bar{n} = pp \to \bar{p}$)	
$K^+p \to \pi^- X$	12	230 (10)	$K^+p \to nX$ 9 (OPE)	9
$\pi^- p \to \pi^- X$	16	700 (10)	$\pi^- p \to nX$ 12 (OPE)	12
$\pi^- p \to \pi^+ X$	16	550 (10)	$\pi^- n \to X$ 57	125
			$\pi^- p \to pX$ 68 (estimation with $p \approx E_p$ and $\int (d\sigma/d\boldsymbol{p})\, d\boldsymbol{p} \approx \sigma_{\pi p}$)	

The experimental reason for this discrepancy can be understood by looking at Fig. 3, where typical curves for $(\sqrt{s}/M)(1/k)d\sigma/dM$ are plotted; the extrapolations for $k \to 0$ give the l.h.s. of the sum rules. As a comparison, when an exclusive reaction of low multiplicity is considered, *e.g.* $\pi^- p \to \pi^+(\text{soft})\pi^0 p$, the corresponding graph is almost

(9) M. Jacob: report of *XVI International Conference on High-Energy Physics* (Batavia, Ill., 1972), TH. 1570-CERN, NAL-THY-63 (1972), and references therein.
(10) M. Law, J. Kasman, R. Panvini, W. Sims and T. Ludlam: *A compilation of data on inclusive reactions*, Particle Data Group LBL-80 (1972).
(11) J. C. Sens: report of *IV International Conference on High-Energy Collisions* (Oxford, 1972), CERN preprint (1972), and references therein.

constant for small k. The reason of the comparatively large value of $d\sigma/dM$ for high M in inclusive processes is clearly due to the large-multiplicity final states. These final states contribute considerably to the l.h.s. because they affect particularly the large-M region; their influence on the r.h.s. is much smaller since the main contribution, because of the factor $(p/E_p)^2$, comes from the opposite region of small M.

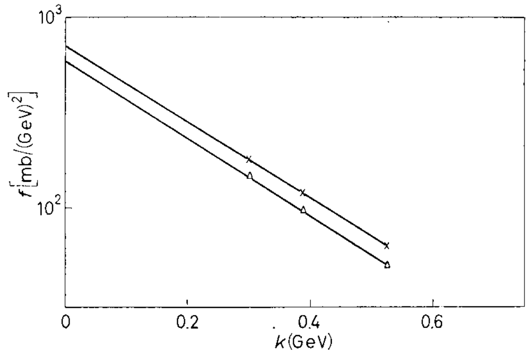

Fig. 3. – $f=(\sqrt{s}/m)(1/k)d\sigma/dM$, where M is missing mass, *vs.* k, momentum of the soft π, at $p_{lab}=16$ GeV. × $\pi^-p\to\pi^-$+anything; △ $\pi^-p\to\pi^+$+anything.

From the point of view of Adler's theorem, the explanation of the discrepancy of the sum rules is that the analytic extension of the l.h.s. from the experimental limit $\omega_k=\mu$ to $\omega_k\to 0$ is not smooth. The possibility of such a problem had been pointed out in ref. [12]. This difficulty can be attributed to the contributions of the resonances from which the soft π is obtained (see also ref. [5]). The effect of these insertions into resonances would vanish for $\mu\to 0$, as considered in the derivation of the sum rules. It is clear that for low-multiplicity exclusive reactions the influence of low-mass resonances is negligible at high energy and, as a matter of fact, it was found in ref. [3] that the PCAC sum rules were well satisfied. On the contrary, in an inclusive process the large-multiplicity final states allow subenergies containing the soft pion to be near the mass of the prominent resonances enhancing their contributions. Therefore, discrepancies in the sum rules should appear for increasing-multiplicity final states and, as a consequence, for the inclusive distributions.

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[12] E. A. PASCHOS: *Phys. Rev. D*, **6**, 379 (1972).

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