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Nonvanishing Asymptotic Total π-$\mathcal{N}$ Cross-Sections from Continuous Moment Sum Rules.

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The possibility of an asymptotic vanishing of total cross-sections has recently been suggested ([1,2]). The conventional high-energy Regge-pole fits are not able to support or reject such a proposal ([3]).

Implementing the information deriving from the high-energy total cross-section data with the results of a continuous family of sum rules by means of which information from lower-energy data is extracted, we have succeeded in determining α_p (the $t=0$ intercept of the Pomeranchuk trajectory) to be 1 with an uncertainty of $0.02\div0.03$.

The sum rules we use are essentially a generalization of the conventional finite energy sum rules ([4]) and have been derived independently by OLSSON ([5]) and the authors of this note ([6]) (see also refs. ([7,8])). However, we should stress that although Olsson's and our sum rules are very similar, there are significant differences when total cross-

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section data are used in their evaluation. In Olsson's approach, indeed, to see the maximum in $\varphi(\gamma)$ (see eq. (1) below and ref. ([5,6])(*)) corresponding to the dominant Regge poles requires giving too much weight to the near-to-threshold data. In this way large errors are made to appear just in the region most sensitive to the relevant Regge parameters. In the study of the antisymmetric part of forward π-$\mathcal{N}$ scattering, this fact has prevented the above author from recognizing the existence of a sizable background ([9]) which was instead evident from our sum rules.

We want to apply here the same technique to the symmetric part of the forward π-$\mathcal{N}$ scattering. We are now faced with a difficulty which was absent in the antisymmetric case. In fact, the real part of the amplitude which enters the sum rule has to be evaluated using dispersion relations and the latter need a subtraction in the symmetric case. This would mean putting the precise total cross-section data beside a rather uncertain scattering-length value which would spoil the precision of the sum rule. To avoid this trouble we have reconsidered our approach on the basis of the two following observations.

i) Given the total cross-section half sum $\sigma^{(+)} = \frac{1}{2}(\sigma_- + \sigma_+)$ from threshold to ∞, one may define ([10]) as well as the physical symmetric amplitude, an antisymmetric one, both having as their imaginary parts $q_L\sigma^{(+)}$ for $\nu > 0$. In the Regge model the high-energy parts of these amplitudes are described by essentially the same Regge poles (*i.e.* same trajectory intercepts, same residues) but with opposite signatures. The use of an antisymmetric amplitude has the advantage that it vanishes for $\nu = 0$ and this may help us in eliminating the subtraction constant.

ii) If we define the imaginary part of the amplitude for $\nu > 0$ not as $q_L\sigma^{(+)}$ but as $q_L\sigma^{(+)}/\nu^\delta$ (with δ not too large, see below) the complete amplitude is described in its high-energy part by the same Regge parameters but with the trajectory intercepts shifted down by δ. If some Regge trajectory in this way crosses -1, this fact may no longer be true. In fact, the Regge background contribution $(O(\nu^{-1}))$ changes with δ and in the antisymmetric case, for instance, when a trajectory approaches $\alpha = -1$ it develops a counter contribution to cancel the divergence. All this may easily be understood by looking, for example, at dispersion relations.

These considerations allow us to extract the relevant Regge parameters not from the physical symmetric amplitude, the determination of which implies the knowledge of a subtraction constant, but from the antisymmetric amplitude whose imaginary part for $\nu > 0$ is $q_L\sigma^{(+)}/\nu^\delta$ with some not-too-large δ. It is to this amplitude that we apply exactly the same procedure used in the treatment of the antisymmetric case ([6]). Moreover, there is a new parameter δ on which the sum rule depends and, as we shall see, the use of this parameter is of the utmost importance in our considerations.

One may wonder what are the advantages of extracting Regge parameters in such a complicated way and not directly from experimental-data fits. Some advantages are the following:

a) One can extract information from intermediate-energy data which are not directly exploitable because they are affected by nonnegligible Regge background.

(*) All symbols and notations are defined as in ref. ([6]).

([9]) Its weight at 6 GeV was $\sim 5\%$ in the physical amplitude and $\sim 20\%$ in the curve $\varphi(\gamma)$ for some regions of the moment γ.

([10]) Let us neglect convergence problems for a moment.

b) The function $\varphi(\gamma)$ (see eq. (1)) is more sensitive to the positions of the Regge trajectories than the physical amplitude. This is due to the presence of the factor $1/(\alpha+\gamma-1)$ which enhances a particular Regge contribution for $\gamma - 1 - \alpha$.

c) Again, for the presence of the factor $1/(\alpha+\gamma-1)$ the Regge background contributes in a simpler form to $\varphi(\gamma)$ at a given ν_0 than to the physical amplitude at the same energy. In fact, if we assume that the physical amplitude is given by a few Regge-pole contributions plus an integral in the J-plane along the axis which goes from $(-1-i\infty)$ to $(-1+i\infty)$, the contribution to $\varphi(\gamma)$ of the various pieces of this integral are weighted by $1/(J+\gamma-1)$, which damps down the contributions far from the real axis. This fact allows one to simulate the background contribution to $\varphi(\gamma)$ more simply (*i.e.* with less parameters) than the one to the physical amplitude, and this may be particularly useful when only low-energy data are available and the cut-off ν_0 is therefore rather small.

d) The sum rule results are not critically affected by possible large errors in the data in some restricted region of the energy.

The detailed form of the sum rule used is

$$(1)\qquad \varphi_\delta(\gamma) = -\nu_0^{(1-\gamma)}\int_0^{\nu_0} \mathrm{d}\nu\,\nu^\gamma\,\mathrm{Im}\left[\exp\left[-i\frac{\pi}{2}\gamma\right]\frac{F_\delta(\nu)-F'_\delta(0)\cdot\nu}{\nu^2}\right] + F'_\delta(0)\cdot\nu_0\cdot\frac{\sin(\pi/2)\gamma}{\gamma} =$$

$$= \sum_n \frac{\beta_n}{\cos(\pi/2)(\alpha_n-\delta)}\,\frac{\sin(\pi/2)(\alpha_n-\delta+\gamma-1)}{\alpha_n-\delta+\gamma-1}\,\nu_0^{(\alpha_n-\delta)} + O(\nu_0^{-1})$$

where $O(\nu_0^{-1})$ represents the Regge background contribution, $F_\delta(\nu)$ is defined by [11]

$$(2)\qquad F_\delta(\nu) = \frac{\nu}{\pi}\int_\mu^\infty \mathrm{d}\nu'\,\frac{q'_L}{\nu'^2-(\nu+i\varepsilon)^2}\left(\frac{\sigma_-+\sigma_+}{\nu^\delta}\right),$$

$F'_\delta(0)$ by

$$(3)\qquad F'_\delta(0) = \left(\frac{\partial F_\delta(\nu)}{\partial\nu}\right)_{\nu=0}$$

and the Regge parameters are normalized by

$$\tfrac{1}{2}(\sigma_-+\sigma_+) \simeq \sum_n \beta_n \nu^{\alpha_n-1}$$

(total cross-sections in mb and ν in GeV).

Two values of the cut-off have been considered: $\nu_0 = 3$ and $\nu_0 = 6$ GeV. For the

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([11]) We have neglected the nucleon pole, because it gives (at $(3\div6)$ GeV) a pure background contribution.

evaluation of the sum rule we have used the currently known total cross-section data ([12]), including some recent measurement in the interval $(0.750 \div 2.30)$ GeV. Beyond 20 GeV we have extrapolated the total cross-section to infinity by means of the fit of FOLEY *et al.* ([12c]) in order to compute the real part of the amplitude. However, our results are largely independent of this particular extrapolation.

Two values of δ have been considered ($\delta = 0.3$ and $\delta = 0.6$) and the resulting $\varphi_\delta(\gamma)$ have been best-fitted simultaneously. A best fit of this kind is much more sensitive to the Regge parameter values than a best fit to only one $\varphi_\delta(\gamma)$. In fact, if for $\delta = 0.3$ the ratio of the relative weights in $\varphi_\delta(\gamma)$ of P' and P is $r = W_{P'}/W_P$; for $\delta = 0.6$ this ratio becomes $\sim 1.5r$ and, moreover, its exact value is determined purely by α_P and $\alpha_{P'}$ which are present in the denominators $\cos(\pi/2)(\alpha_n - \delta)$ of eq. (1).

The best fit has been performed in the interval $\gamma = 1 \div 5$. We have neglected the points with γ less than 1 because they depend too strongly on the near-to-threshold total cross-section data affected by non-negligible uncertainties. The weight of these data is particularly enhanced by the denominator ν^δ present in eq. (2). The points with γ greater than ~ 5 give no essentially new information, because for this value of γ the various denominators $1/(\alpha_n + \gamma - 1)$ present in eq. (1) have in practice reached their asymptotic form (supposing that the lowest sizable singularity has $\alpha \sim -1$) and therefore, in varying γ, the relative weight of the various pole contributions does not change.

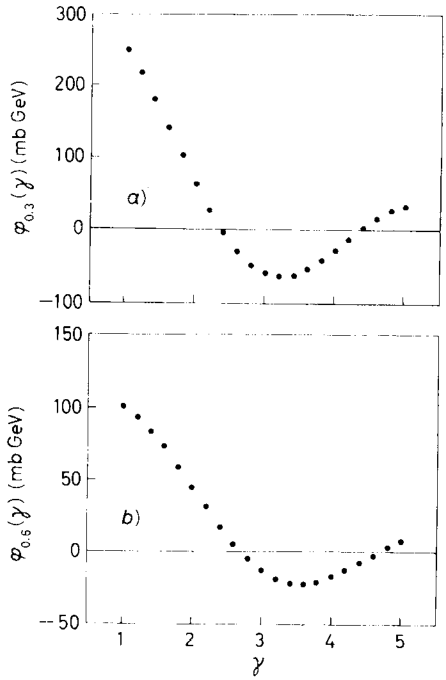

Fig. 1. – Plots of *a*) $\varphi_{0.3}(\gamma)$ and *b*) $\varphi_{0.6}(\gamma)$ for $\nu_0 = 6$ GeV, as an illustration.

We have made a three-pole fit (P, P' and a pole with fixed $\alpha = -1$ simulating the Regge background). The total number of free parameters was six (the two background coefficients at $\delta = 0.3$ and $\delta = 0.6$ were independent). The background contribution was, however, found to be negligible both at 3 and 6 GeV (below 1%).

What essentially differentiates our determination of P, P' parameters from those preceding is that we are able to determine $\alpha_P(0)$, the Pomeranchuk intercept at $t = 0$. This is possible because the intermediate-energy data are brought into the game by use of the sum rule and, furthermore, in the sum rule one may better determine the

([12]) The data with $T_\pi \leqslant 750$ MeV have been extracted from the references quoted in the review by: *a*) M. N. FOCACCI and G. GIACOMELLI: CERN Report 66-18 (April 1966). The data have been selected looking at errors and continuity. In the interval $0.750 \leqslant T_\pi \leqslant 2.30$ GeV we have used data from recent measurements at Brookhaven. (G. GIACOMELLI: private communication to Prof. L. BERTOCCHI.) The data for $2.30 \leqslant T_\pi \leqslant 7$ GeV were taken from *b*) A. CITRON, W. GALBRAITH, T. F. KYCIA, B. A. LEONTIĆ, R. H. PHILLIPS, A. ROUSSET and P. H. SHARP: *Phys. Rev.*, **144**, 1101 (1966). Beyond 7 GeV we used the Brookhaven data *c*) K. J. FOLEY, R. S. JONES, S. J. LINDENBAUM, W. A. LOVE, S. OZAKI, E. D. PLATNER, C. A. QUARLES and E. H. WILLEN: *Phys. Rev. Lett.*, **19**, 330 (1967).

relative weight of P, P' varying with δ the relative weight of low- and high-energy data.

We have at first used the sum rule looking for least-square configurations independently at $\nu_0 = 3$ and $\nu_0 = 6$ GeV. At this stage one does not obtain a unique solution [13]; however, all solutions have an $\alpha_P(0)$ near to 1 (the lowest $\alpha_P(0)$ being 0.97 [14]).

We then compared the solutions at 3 and 6 GeV, eliminating those which were clearly not acceptable for both ν_0's. In this way we were left with a more restricted set of configurations. Among the latter the favoured one both at 3 and 6 GeV was

$$a) \qquad \alpha_P = 1.00\,, \qquad \beta_P = 20.25\,, \qquad \alpha_{P'} = 0.60\,, \qquad \beta_{P'} = 16.25\,.$$

The parameters of all other accepted configurations were contained between those of the following:

$$b) \qquad \alpha_P = 1.015\,, \qquad \beta_P = 23.3\,, \qquad \alpha_{P'} = 0.45\,, \qquad \beta_{P'} = 11.9\,,$$

$$c) \qquad \alpha_P = 0.99\,, \qquad \beta_P = 19.1\,, \qquad \alpha_{P'} = 0.65\,, \qquad \beta_{P'} = 16.25\,.$$

The parameter correlation is strong and it is such that all accepted configurations give very similar predictions for the sum of $\pi^{\pm}$p total cross-sections and the symmetric real part $D^{(+)}$ at high energy. In Fig. 2 and 3 these predictions are compared with the data of FOLEY *et al.* [12c,15] between 8 and 20 GeV. $D^{(+)}$ is well reproduced within the errors ($\sim 15\%$), while the prediction for $\sigma_- + \sigma_+$ is constantly higher by 4% with respect to the Brookhaven data which are quoted with a precision $\sim 0.2\%$.

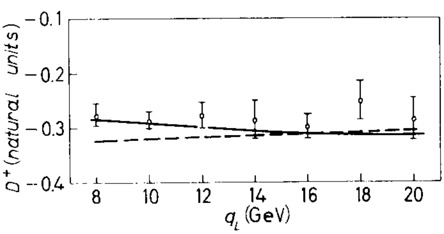

Fig. 2. – Comparison with the Brookhaven data on the real part of the forward c.m.s. amplitude [15]. The continuous line represents the prediction of solution *a*). The dashed line represents the prediction of solution *d*). (See text.)

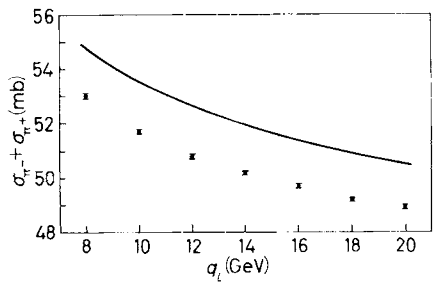

Fig. 3. – Comparison with the Brookhaven data on total πp cross-sections. The continuous line represents the prediction of solution *a*).

One way of looking at this discrepancy is to consider 4% as a reasonable error for our sum rule predictions. However, the fact would then be inexplicable that both for $\nu_0 = 3$ and $\nu_0 = 6$ GeV and for all solutions (also for those successively rejected) one gets this discrepancy. Indeed in the 6 GeV sum rule, data with a considerable weight appear which essentially do not enter into the determination of the 3 GeV sum rule.

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[13] We retained as «solutions» all the configurations for which the minimizing function was not larger than twice the absolute minimum found.

[14] The higher bound on α_P for these solutions was 1.06. We should observe that, because of the range in γ available, the sum rule is better able to reject configurations with $\alpha_P < 1$ than with $\alpha_P > 1$.

[15] K. J. FOLEY, R. S. JONES, S. J. LINDENBAUM, W. A. LOVE, S. OZAKI, E. D. PLATNER, C. A. QUARLES and E. D. WILLEN: *Phys. Rev. Lett.*, 19, 193 (1967).

To obtain an indication of which parameters are mainly responsible for the discrepancy, we remade the fit with the constraint of reproducing within the errors the Brookhaven total cross-sections. The fit to the two $\varphi_\delta(\gamma)$ provided by the various solutions were worse than before but still reasonable. Still, all solutions had α_P very near to 1 (the solution with the lowest α_P had $\alpha_P = 0.98$). Because the minimum was rather narrow in α_P and flatter in the other parameters, small variations of α_P were correlated to appreciable variations of the other parameters within the set of accepted solutions. To have clearer indications about β_P, $\beta_{P'}$ and $\alpha_{P'}$ we therefore performed the analysis anew with α_P fixed at 1, and obtained an essentially unique solution with

$$d) \qquad \beta_P = 21.5 \pm 1 \,, \qquad \beta_{P'} = 16.8 \pm 0.7 \,, \qquad \alpha_{P'} = 0.41 \pm 0.1 \,,$$

where the errors (still correlated) are only indicative and have been determined considering both the variance matrix and the spread of the various parameters in the set of solutions.

Comparing the solution *d*) and *a*), one finds that the constraint of reproducing the Brookhaven data has produced a considerable lowering of $\alpha_{P'}$ leaving the other parameters essentially unchanged. Now this fact is not isolated and may be put in connection with other facts. Indeed, first, it is correlated with the circumstance that the best solutions (without constraint) at 3 GeV had $\alpha_{P'} \simeq 0.75$ and at 6 GeV had $\alpha_{P'} \simeq 0.45$, both kinds of solution having been rejected as unacceptable at both ν_0. Second, it is correlated with the fact that the fit of FOLEY *et al.* ([12c]) to their total cross-section data gives $\alpha_{P'} = 0.31 \pm 0.1$. That is, it seems that $\alpha_{P'}$ shifts to lower and lower values when in its determination the weight of higher-energy data is more and more enhanced.

This fact, if really true, would mean that P' simulates a more complex syxtem of singularities ([16]). In particular, the quoted deviations could be explained by the presence of a third Regge pole P'' with $\alpha_{P''}$ lower than $\alpha_{P'}$ and with $\beta_{P''}$ negative. One could equally well reproduce the system $P' + P''$ by a cut.

We believe that one cannot definitely exclude the possibility that all the above systematics concerning $\alpha_{P'}$ is only an effect of the errors; however, we should observe that the previous $\alpha_{P'}$ determinations using sum rules (*i.e.* low-energy data) give systematically an $\alpha_{P'}$ considerably higher than those using only high-energy fits and this looks like a confirmation of our quantitative considerations. (See Table I in which a representative sample of previous determinations is presented.)

In conclusion we are able to quote the following results:

i) $\alpha_P(0)$ has been found to be 1 with an uncertainty $\sim (0.02 \div 0.03)$. This is in contrast with the recent suggestion ([1,2]) of a dominant pole with α less than 1 ($\alpha \sim 0.93$), which would imply total cross-sections vanishing at very high energy.

ii) There seems to be some evidence that the system $P + P'$ simulates a more complex singularity structure. In terms of Regge poles, the observed deviations could be explained by a third pole P'' with $\alpha_{P''}$ lower than $\alpha_{P'}$ and with residue opposite to that of P'.

([16]) Working in the nonforward direction and using differential cross-section and polarization data, HÖHLER and EISENBEISS ([17]) also found deviations from the $P+P'$ model in π-$\mathcal{N}$ scattering.

([17]) G. HÖHLER and G. EISENBEISS: *Zeits. Phys.*, **202**, 397 (1967).

TABLE I. – *A representative sample of previous determinations of $\pi\mathcal{N}$ P, P' parameters.*

	β_P (mb GeV)	$\alpha_{P'}$	$\beta_{P'}$ (mb GeV)
FOLEY *et al.* ([a])	22.12 ± 0.95	0.31 ± 0.1	18.5 ± 4.25
BARGER *et al.* ([b])	21.2 ± 0.3	0.51 ± 0.03	14.7 ± 0.6
RARITA *et al.* ([c])	20.48	0.57	14.6
RESTIGNOLI *et al.* ([d])	17.48	0.658 ± 0.033	17.85
SCANIO ([e])	16.2	0.69	19.8
MIYAMURA *et al.* ([f])	$19.6^{+1.3}_{-1.8}$	$0.61^{+1.3}_{-1.8}$	$15.4^{+2.3}_{-1.4}$

(*a*) Only the Brookhaven $\pi^{\pm}$p total cross-section data are used ([12c]).

(*b*) From a general fit to all total cross-section and forward charge-exchange data ([18]).

(*c*) From a general fit to πp, pp and $\bar{\text{p}}$p scattering at small momentum transfer ([19]). (Solution 3 of ref. ([19]), determined from Brookhaven data is considered here.)

(*d*) From a high-energy fit with sum rule constraints to $\pi\mathcal{N}$, K$\mathcal{N}$ and $\bar{\text{K}}\mathcal{N}$ total cross-section data ([20]).

(*e*) High-energy fit with sum rule constraint to $\pi\mathcal{N}$ total cross-section data ([21]). Beyond 8 GeV the data of GALBRAITH *et al.* ([22]) are used.

(*f*) Same as in *e*) with the more recent Brookhaven data ([23]).

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([18]) V. BARGER, M. OLSSON and D. REEDER: *Asymptotic projections of scattering theories*, to be published. See also ref. ([9]).

([19]) W. RARITA, R. RIDDEL jr., C. CHIU and R. PHILLIPS: Lawrence Radiation Laboratory, UCRL-17523 (11 April 1967).

([20]) M. RESTIGNOLI, L. SERTORIO and M. TOLLER: *Phys. Rev.*, **150**, 1389 (1966).

([21]) J. J. G. SCANIO: *Phys. Rev.*, **152**, 1337 (1966).

([22]) W. GALBRAITH, E. W. JENKINS, T. F. KYCIA, B. A. LEONTIĆ, R. H. PHILLIPS and A. L. READ: *Phys. Rev.*, **138**, B 913 (1965).

([23]) O. MIYAMURA and F. TAKAGI: Tohoku University, preprint 22 (1967).

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