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Semi-local Duality and FESR Problems for Veneziano Models.

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It is well known that dual models with zero-width resonances show Regge behaviour when the asymptotic variables are taken out of a cone around the real axis. Obviously, the asymptotic behaviour on the real axis is meaningless, because of the presence of poles above any physical energy. A way for giving a meaning to this asymptotic behaviour is to take some average over resonances; one may expect that this average is related to the Regge-pole expansion of the amplitude.

A possibility of performing an average is through the global duality idea of finite-energy sum rules (FESR) for the imaginary part of the amplitude. It turned out, however, that in the 4-point case (1) a discrepancy appears between the integral over the resonances and the one over the Regge-pole expansion. This discrepancy appears at the level of the second and lower lying Regge daughters.

In the present note we study the semi-local duality properties as analysed by DE TAR *et al.* (2), including daughters contributions, of Veneziano amplitudes applied to total and one-particles inclusive cross-sections. As a consequence we may clarify the origin of the above-mentioned discrepancies in the FESR.

In order to test the semi-local duality, we perform the average of the imaginary part of the amplitude over one resonance, and compare its asymptotic behaviour with the Regge-pole expansion evaluated at the resonance position.

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(1) A. DELLA SELVA and L. MASPERI: *Progr. Theor. Phys.*, **46**, 190 (1971).
(2) D. GORDON and G. VENEZIANO: *Phys. Rev. D*, **3**, 2116 (1971); M. A. VIRASORO: *Phys. Rev. D*, **3**, 2834 (1971); C. E. DE TAR, K. KANG, C.-I TAN and J. H. WEIS: *Phys. Rev. D*, **4**, 425 (1971); R. J. BIEBL, D. BEBEL and D. EBERT: Berlin Zeuthen preprint No. PHE 71-9.

Beginning with the 4-point amplitude, we define

$$(1)\qquad A_4(N,t)=\int\limits_{N-\frac12}^{N+\frac12}\mathrm{d}\alpha_s\,\mathrm{Im}\,B_4(s,t)=\pi g^2\,\frac{\Gamma(N+\alpha_t+1)}{\Gamma(N+1)\,\Gamma(\alpha_t+1)}\,,$$

where, for all channels, $\alpha_x=\alpha_0+x$, and g is the overall coupling constant. For large N we get

$$(2)\qquad A_4(N,t)\to\pi g^2N^{\alpha_t}\left[\frac{1}{\Gamma(\alpha_t+1)}+\frac{\alpha_t+1}{2\Gamma(\alpha_t)}\frac{1}{N}+\frac{3\alpha_t^2+5\alpha_t+2}{24\Gamma(\alpha_t-1)}\frac{1}{N^2}+\dots\right].$$

We compare this result with the Regge-pole expansion which arises from taking the amplitude for $\alpha_s\to-\infty$ and subsequently continuing it analytically to $\alpha_s\to+\infty$, *i.e.*

$$(3)\qquad \mathrm{Im}\,g^2\,\frac{\Gamma(-\alpha_s)\,\Gamma(-\alpha_t)}{\Gamma(-\alpha_s-\alpha_t)}\to g^2\,\Gamma(-\alpha_t)[\mathrm{Im}\,(-\alpha_s)^{\alpha_t}+\dots]\,.$$

It turns out that the coefficients of expansion (3), once the above prescription is followed, are equal to those of (2) for any power of $\alpha_s=N$. We remark that we are not comparing the average of a resonance amplitude with the average of the Regge amplitude but with its value at the resonance position.

It is now clear why discrepancies in the FESR appear. In fact, if one subtracts two FESR for the imaginary part, with cut-offs in $\alpha_s=N+\frac12$ and $\alpha_s=N-\frac12$, one obtains for the parent Regge-pole contribution

$$(4)\qquad \int\limits_{N-\frac12}^{N+\frac12}\mathrm{d}\alpha_s\,\frac{\pi g^2}{\Gamma(\alpha_t+1)}\,\alpha_s^{\alpha_t}=g^2\pi N^{\alpha_t}\left[\frac{1}{\Gamma(\alpha_t+1)}+\frac{N^{-2}}{24\Gamma(\alpha_t-1)}+O(N^{-4})\right].$$

The second term of r.h.s. of eq. (4) corresponds to a discrepancy at the level of the second daughter, when compared with the N^{α_t} term of eq. (2). The inclusion of the second daughter in the Regge side of the FESR gives a leading contribution equal to the N^{α_t-2} term of eq. (2), and therefore cannot cancel the above discrepancy. Similarly, the inclusion of the first daughter gives rise to a discrepancy of the order $O(N^{\alpha_t-3})$, which cannot be cancelled by taking into account further terms.

Turning now our attention to single-particle inclusive processes, we must consider the dual 6-point tree graph as the simplest contribution (2). We generalize here the above treatment in order to investigate the fulfilment of the sum rules of ref. (3). In this case one considers the triple Regge region for the b-fragmentation of the inclusive reaction $a+b\to c+X$. The sum rule implies a comparison between the integrals over the missing mass of the amplitude described by Fig. 1*a*) and *b*).

We consider first the tree graph with the ordering $a\bar{c}b\bar{b}c\bar{a}$; this is the simplest because it does not contain resonances in the ab-channel. The other permutations will

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[3] A. I. SANDA: NAL-THY-25 (1971) (to be published in *Phys. Rev.*); M. B. EINHORN, J. ELLIS and J. FINKELSTEIN: SLAC-PUB-1006 (1972); *Phys. Rev. D*, 5, 2063 (1972); S. D. ELLIS and A. I. SANDA: NAL-THY-49 (1972); J. KWIECINSKI: Cracow Report 752/PH (1971); P. OLESEN: CERN preprint TH-1376 (1971).

be discussed later. The averaged discontinuity is defined as

$$(5)\qquad A^{(a\bar{c}b)}(s, N, t) \doteq \int\limits_{N-\frac{1}{2}}^{N+\frac{1}{2}} d\alpha_{M^2} \frac{\mathrm{disc}_{M^2} B_6^{(a\bar{c}b)}(s, M^2, t)}{2i} =$$

$$= \frac{\pi g^4}{\Gamma(\alpha_0+1)\Gamma(2\alpha_0)} \frac{\Gamma^2(-\alpha_t)}{\Gamma^2(-\alpha_t-\alpha_0)} \sum_{jkl=0}^{N} \frac{\Gamma(l+2\alpha_0)}{\Gamma(l+1)} \frac{\Gamma(k-\alpha_t-\alpha_0)}{\Gamma(k+1)} \cdot$$

$$\cdot \frac{\Gamma(j-\alpha_t-\alpha_0)}{\Gamma(j+1)} \frac{\Gamma(-\alpha_u+l+k)}{\Gamma(-\alpha_u-\alpha_t+l+k)} \frac{\Gamma(-\alpha_u+l+j)}{\Gamma(-\alpha_u-\alpha_t+l+j)} \frac{\Gamma(N-l-k-j+\alpha_0+1)}{\Gamma(N-l-k-j+1)},$$

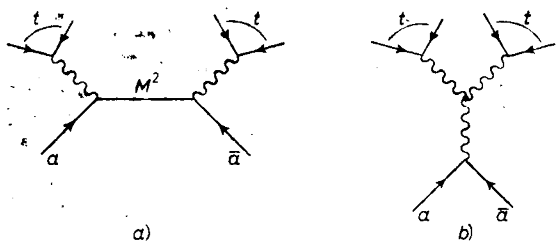

Fig. 1. – Triple Regge region: *a*) resonance amplitude, *b*) reggeized amplitude.

where $s=(p_a+p_b)^2$, $u=(p_a-p_c)^2$. For $s \gg M^2$ the contributions of parent and daughter Regge poles in the t-channels come from the Stirling expansion of the Γ-functions containing α_u. The remaining sum can be evaluated from (4)

$$(6)\qquad \sum_{n=-\infty}^{\infty} \frac{\Gamma(x+n)\Gamma(y+n)}{\Gamma(w+n)\Gamma(z+n)} = \frac{\pi^2}{\sin\pi x \sin\pi y} \frac{\Gamma(w+z-x-y-1)}{\Gamma(w-x)\Gamma(z-x)\Gamma(w-y)\Gamma(z-y)}.$$

As an example, we report the contribution up to first daughter in one of the t-channels (Fig. 1*a*)):

$$(7)\qquad A^{(a\bar{c}b)}(s, N, t) = \pi g^4 \frac{\Gamma^2(-\alpha_t)}{\Gamma(-2\alpha_t+\alpha_0+1)} s^{2\alpha_t} \frac{\Gamma(N-2\alpha_t+\alpha_0+1)}{\Gamma(N+1)} \cdot$$

$$\cdot \left\{1 + \frac{N}{s} \frac{2\alpha_t(\alpha_0-\alpha_t)}{-2\alpha_t+\alpha_0+1} + O\left(\frac{N^2}{s^2}\right)\right\}.$$

where for the triple Regge region $N \to \infty$.

To analyse the Regge asymptotic behaviour we generalize the treatment given by DE TAR *et al.* (2) to include also daughter contributions. Considering a parallel with the above-mentioned work we emphasize two relevant differences. One is to express the variable of the M^2 channel as $y = \exp[z/\alpha_{M^2}]$, its power expansion giving rise to successive Regge daughters in the $a\bar{a}$ channel. The second difference consists in taking the binomial expansion of terms of the type $[1-(\alpha_{M^2}/\alpha_u]y_1]^{\alpha_t+\alpha_0}$ to obtain the Regge daughters in the t-channels. The final result is that this Regge expansion evaluated at $\alpha_{M^2} = N$ ($N \to \infty$) coincides term by term with eq. (7).

(4) *Bateman Manuscript Project, Higher Transcendental Functions*, Vol. 1 (New York, 1954).

In the spirit of ref. ([8]) one is tempted to write for the dual amplitude the sum rule

$$\int_0^N \mathrm{d}\alpha_{M^2} \frac{\mathrm{disc}_{M^2} B_6^{(a\bar{c}b)}(s, M^2, t)}{2i} \approx \sum_{n=0} \beta_n(t) \left(\frac{s}{N}\right)^{2\alpha_t} \frac{N^{\alpha_0 - n + 1}}{\alpha_0 - 2\alpha_t + 1 - n}, \tag{8}$$

where for simplicity only parent Regge poles in t-channels are kept. However, as in the 4-point case, the semi-local duality found above necessarily implies a discrepancy between both sides of eq. (8), at the level $n \geqslant 2$ daughters for the $a\bar{a}$ channel.

Finally, we make some comments about the other permutations of the B_6 tree graph which one would expect to give simply the signature factors of the t-channel Regge poles. Considering the order $ab\bar{c}cb\bar{a}$, one has the additional problem of averaging the resonances in the s-channel to compare with Regge asymptotics. One possibility is to average independently over ab resonances and $\bar{a}\bar{b}$ resonances; the result for the triple Regge region is then the same as for the $a\bar{c}b\bar{b}c\bar{a}$ ordering times the expected phase factors corresponding to t-channel Reggeons. However, another choice seems correct for narrow-width resonance, namely to consider that the product of the resonances in channels ab and $\bar{a}\bar{b}$ gives

$$1/(s - m_R^2 + i m_R \Gamma_R) \times 1/(s - m_R^2 - i m_R \Gamma_R) \approx \pi/(m_R \Gamma_R) \times \delta(s - m_R^2)$$

and then to average over a small range of s ([5]). This implies to neglect the interference between resonances of different mass, which is reasonable only if the widths are small even at high energies. The second choice turns out to be asymptotically not satisfactory because, e.g., for the triple Regge region one obtains

$$\bar{A}^{(ab\bar{c})}(R, N, t) = g^4 \frac{\sin^2 \pi\alpha_t}{m_R \Gamma_R} \frac{\Gamma^2(-\alpha_t)}{\Gamma(-2\alpha_t + \alpha_0 + 1)} R^{2\alpha_t} \frac{\Gamma(N - 2\alpha_t + \alpha_0 + 1)}{\Gamma(N + 1)} \cdot \left\{1 - \frac{N}{R} \frac{2\alpha_t(\alpha_0 - \alpha_t)}{-2\alpha_t + \alpha_0 + 1} + O\left(\frac{N^2}{R^2}\right)\right\}, \tag{9}$$

where

$$\bar{A}^{(ab\bar{c})}(R, N, t) = \int_{R-\frac{1}{2}}^{R+\frac{1}{2}} \mathrm{d}\alpha_s A^{(ab\bar{c})}(s, N, t) .$$

Unless $m_R \Gamma_R$ is independent of R, eq. (9) violates the scaling behaviour and in any case it does not coincide with the corresponding Regge asymptotics. Moreover, studying in this way the behaviour of the $ab\bar{c}cb\bar{a}$ graph in the pionization region one obtains at most a power damping in t, instead of the expected exponential one. These difficulties might be solved if the widths increase with energy, so that the interferences among different resonances become important. In this way one would pass, for increasing energies, from one method of averaging to the other, re-establishing the proper high-energy behaviour and the validity of the semi-local duality.

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([8]) F. ARBAB, J. G. GALLARDO, L. MASPERI and C. REBBI: *Lett. Nuovo Cimento*, **2**, 1245 (1971); J. C. GALLARDO, L. MASPERI and C. REBBI: Trieste Internal Report IC/71/151 (1971).