

with one-group spherical harmonics theory (5). It is interesting to compare with Conkie's result as our calculations have been done similarly. He also obtains one eigenvalue that is less than but quite close to one. The only difference between our calculations is that we here regard a moderator with a mass greater than one, which makes the kernel more complicated. However, as Conkie points out, it might be that the use of the  $P_1$ -approximation distorts the eigenvalue spectrum. The rapid oscillations in the higher Techebycheff polynomials certainly makes it doubtful if the use of Simpson's rule is the best choice.

The equations (17) have been solved for  $x_0 = 10$  and  $x_0 = 100$  for different values of  $r_0$  between 2.5 and 4 in order to make the difference between the thermal density and the slowing down density as small as possible. For  $x_0 = 10$  it was found that the point of connection should be close to  $r_0 = 2.96$  and for  $x_0 = 100$ ,  $r_0 = 3.50$  (the errors at these points are 0.003 and 0.009 respectively). We obtain at the interface

$$N_0(x_0, r) = r^2 \exp(-r^2) \sum_{k=0}^9 a_k T_k^*(y) \quad (18)$$

and we get for the coefficients  $a_k$ :

$x_0$	$a_0$	$a_1$	$a_2$	$a_3$	$a_4$
10	14.3	9.37	3.56	5.28	3.09
100	312.0	122.0	-30.2	31.6	-6.00
	$a_5$	$a_6$	$a_7$	$a_8$	$a_9$
	3.38	1.99	1.79	0.97	0.37
	13.4	-0.70	6.12	0.18	1.68

For  $x_0 = 100$  the convergence is good at the interface. Inside the slab, for  $x = 0$ , the density is a pure Maxwellian, given by

$$N_0(x = 0, r) = 30600r^2 \exp(-r^2), \quad x_0 = 100. \quad (19)$$

However, it is certainly much more interesting to obtain the neutron distribution for a slab that is only a couple of mean free paths thick. Unfortunately, even for  $x_0 = 10$ , the convergence is not good enough at the interface as is easily seen if one draws the corresponding curve, which shows small oscillations near the maximum. Conkie already remarked that the expansion variable used might not be the optimum choice and even the use of the Techebycheff polynomials is somewhat arbitrary. It would also be convenient if one could use fewer terms in the expansion although that is not a major trouble with the big calculating machines. The problem of finding more suitable functions is now being considered and the hope is to let these be generated by the Boltzman equation. There is another limitation to the method that will distort the distribution, and that is the  $P_1$ -approximation in  $\mu$ . This should be of importance at the interface of thin slabs. In our case of  $x_0 = 10$  the density inside the slab rapidly becomes convergent and it would therefore be worth while to try these calculations in a  $P_3$ -approximation. The computations are not very complicated, and once more suitable expansion functions have been found it should be possible to study more realistic problems, e.g., including absorption.

#### ACKNOWLEDGMENTS

This work started during a one-year visit to the Chalk River Project on an N.R.C. fellowship. It was suggested

to me in a discussion with Drs. B. Davison, W. R. Conkie, and S. A. Kushneriuk, all of whom have given me considerable help and encouragement. I am further much indebted to Drs. L. G. Elliott and T. D. Newton for their kind interest and their efforts to make my visit at Chalk River a pleasant experience. My thanks are also due to the members of the Theoretical Physics Branch for many stimulating discussions and for help in performing the numerical calculations.

#### REFERENCES

1. W. R. CONKIE, Velocity dependent neutron transport theory. *Nuclear Sc. and Eng.* 7, 295<sub>k</sub> (1960).
2. E. GREULING, F. CLARKE, AND G. GOERTZEL, A multi-group approximation to the Boltzmann equation for critical reactors. NDA 10-96, 1953.
3. E. R. COHEN, *Proc. Intern. Conf. Peaceful Uses Atomic Energy, Geneva, 1955*, p. 1.
4. A. CLAESSON, Report RFR-150, 1961, AB Atomenergi, Sweden.
5. See B. DAVISON, "Neutron Transport Theory," p. 124 ff., Oxford University Press, 1958.

ARNE CLAESSON

AB Atomenergi  
Studsvik  
Tystberga, Sweden  
Received March 12, 1962

## A Note on the Measurement of the $U^{238}$ Cadmium Ratio

It is customary to obtain the  $U^{238}$  cadmium ratio (CR) in a lattice by measuring the ratio of the  $U^{238}$  neutron capture induced activities in two uranium foils, irradiated bare and cadmium covered respectively in split fuel rods. The activity of the former is proportional to the sum of the thermal (subcadmium) and the epithermal (epicadmium) capture rates, the activity of the latter being proportional to the epithermal capture rate only.

It is well known that, because of the high  $U^{238}$  neutron resonance capture cross section, the epithermal neutron flux is more strongly depressed in the center of the fuel rods (and in the center of the foils) than is the thermal neutron flux. It follows that (roughly speaking) the thermal captures occur in the whole volume of the foils, whereas the epithermal captures are more concentrated near their outer perimeter. It was remarked some time ago (1) that the different spatial distributions of the capture in the two foils might be a source of error in the measurement of the CR. Self-absorption and geometry effects could cause the response of the counter to radiation produced near the foil periphery to be different from the response to radiation produced near the foil center. Nevertheless, recently published experimental results (2, 3) obtained both by measuring foil activity directly and by homogenizing the foils first seem to agree reasonably well.

We have carried out a technique study with a light water moderated lattice of 1.15 wt. %  $U^{238}$  enriched metal rods of 0.387 in. diameter, at a 3:1  $H_2O/U$  volume ratio. We have

found a noticeable increase of the CR in this case when the activities of the two foils have been homogenized before being counted.

#### Methods

Bare and cadmium covered metal uranium foils were located at the same height in two flux-symmetric fuel rods of a miniature lattice (4), which was irradiated by neutrons from the BNL reactor for 40 min. The "covered" foils were placed between 0.050 in. thick buttons of fuel material, surrounded by a 0.020 in. thick cadmium sleeve and by two 0.028 in. thick cadmium covers.

The following techniques were used to count the activity of the foils.

(a) After the exposure, the foils (0.005 in. thick natural uranium) were dissolved in hot nitric acid, and a solvent extraction process (5, 6) was used to separate the uranium from the fission products. The uranium was converted to 1 to 4 mg/cm<sup>2</sup> thick ammonium diuranate deposits on filter paper, covered with mylar tape to absorb the natural alpha activity. In each pair of samples, the deposits differed by less than 0.5 mg/cm<sup>2</sup>. The 23.5 min half-life  $\beta$ -decay of the U<sup>235</sup> was followed with a proportional gas flow counter. It was experimentally determined that no significant self-absorption effect existed in the range of sample thickness used. The amount of uranium in each sample was determined by colorimetric analysis and by weighing the uranium dioxide obtained after burning the samples.

(b) After the same chemical process as above, samples weighing 20 to 30 mg were prepared. Their activities were measured by following the 23.5 min half-life, 74 keV gamma ray accompanying decay of the U<sup>235</sup>. A scintillation counter and a 256 channel pulse analyzer were used, an integration between 50 and 80 keV being taken. The uranium mass in each sample was found as in (a) above.

(c) After exposure, each foil (0.007 in. thick, 24.3 ppm U<sup>235</sup>) was slowly dissolved in 1 ml of nitric acid at room temperature, in a polyethylene container of 0.690 in. diameter and 0.900 in. height, which was then sealed. The activities were measured by following for 180 hr the 56 hr half-life of the coincidences between the 106 keV Np<sup>239</sup> gamma ray and the 103 keV Pu<sup>239</sup> internal conversion x-ray. Two single channel scintillation spectrometers and a coincidence unit were used, each channel covering a 50 keV energy interval. A Chase multiple coincidence circuit (7) was used, with a resolving time of 0.16  $\mu$ sec. No correction for fission products was made.

(d) After exposure, the foils (0.007 in. thick, 6 ppm U<sup>235</sup>) were counted by following the 56 hr half-life decay of the combination of the 106 keV Np<sup>239</sup> gamma ray and the 103 keV Pu<sup>239</sup> internal conversion x-ray, by means of a single channel scintillation spectrometer. A small 1.5% fission product correction factor was arrived at by simultaneously counting the foil activity with a second single channel analyzer biased at 750 keV, and by counting later, in the same energy ranges, the activity of aluminum catchers exposed in the same way as the uranium foils.

(e) The foils (same characteristics as in (d)) were coincidence counted in the same way as the solution in (c).

In measurements (c) and (e), the measured activities were corrected for background, small differences in sample weight, decay time, and accidental coincidences. Least

TABLE I  
EXPERIMENTAL VALUES OF CR

Activity homogenized		Activity not homogenized	
Method (a)	1.990 $\pm$ 0.02	Method (d)	1.923 $\pm$ 0.02
Method (b)	1.990 $\pm$ 0.02	Method (e)	1.910 $\pm$ 0.02
Method (c)	1.974 $\pm$ 0.02		

squares fits to the final values always yielded the expected half life.

#### Results

The experimental values of the CR are listed in Table I. The errors are standard deviations from the mean of several runs.

With the mass absorption coefficient of uranium for 105 keV gamma rays assumed to be 1 cm<sup>2</sup>/gm (8), a self-absorption factor of 0.856 has been calculated for 0.007 in. thick foils. Approximate calculations show that if the epithermal activity is distributed only on the lateral surface of the foils, the 2 in. diameter crystals used should have a relative difference of 6% in the counting efficiency for the thermal and epithermal activities. The relative difference in the (CR-1), as obtained from the two sets of measurement, is of this expected order of magnitude.

The theoretical value for the CR in the case tested has been calculated (9) to be 2.02.

We wish to thank Drs. H. Kouts and G. Price for their kind suggestions.

#### REFERENCES

1. H. KOUTS, BNL internal memorandum 2077 (1953), unpublished.
2. D. KLEIN, A. Z. KRANZ, G. G. SMITH, W. BAER, AND J. DE JUREN, *Nuclear Sci. and Eng.* **3**, 403 (1958).
3. R. SHER, *Nuclear Sci. and Eng.* **7**, 479 (1960).
4. H. KOUTS AND R. SHER, BNL 486 (1957).
5. J. M. FLETCHER, *Prog. Nuclear Energy, Ser. III*, **1**, 105 (1956).
6. F. R. BRUCE, *Prog. Nuclear Energy, Ser. III*, **1**, 128 (1956).
7. R. L. CHASE, *Rev. Sci. Instr.* **31**, 9, 945 (1960).
8. B. T. PRICE, C. C. HORTON, AND K. T. SPINNEY, "Radiation Shielding," p. 34. Pergamon Press, London, 1957.
9. R. L. HELLENS, BNL private communication.

BIAGIO ARCIPIANI\*  
DELFINA RICABARRA†  
GUILLERMO H. RICABARRA†

Brookhaven National Laboratory  
Upton, New York

Received April 6, 1962

Revised June 18, 1962

\* Fulbright fellow from Italy. Now at Euratom, CCR, Ispra, Italy.

† On leave from the Argentinian AEC, Buenos Aires, Argentina, under the sponsorship of the IAEA.