

## Spatially Dependent Resonance Neutron Spectra in a Slab of Depleted Uranium

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*Received February 12, 1968*

Neutron spectra have been measured by the time-of-flight method from 1 eV to 2 keV at various positions across depleted uranium and borated polyethylene in slab geometry with an energy resolution of 0.9% at 6.68 eV. The observed behavior around the 6.68-eV resonance in  $^{238}\text{U}$  has been compared with the predictions of the transport theory code 1DF in  $S_4$  and  $S_6$  approximations using different quadratures and spatial representations. The space-averaged flux has been calculated with the slowing down code GAROL. The 1DF calculations agree reasonably well with experiment and the GAROL values.

### I. INTRODUCTION

In this paper, we describe position-dependent spectral measurements in the resolved resonance energy range across a thick slab of depleted uranium placed between two slabs of boron-loaded polyethylene. These measurements were undertaken to demonstrate that techniques developed at Gulf General Atomic<sup>1,2</sup> and elsewhere<sup>3-5</sup> for measuring thermal-neutron spectra in heterogeneous systems could be refined for use in media

having strong resonances and large flux anisotropies. The results of the measurements were of interest because there were very little differential spectral data that could be used to confirm the adequacy of the calculational methods commonly used for reactor design in the resolved resonance energy region. It was of particular interest<sup>6</sup> to compare the experimental data to the predictions of the discrete ordinates transport theory code<sup>7</sup> 1DF—and to intercompare that code's predictions to those of the resonance absorption code,<sup>8</sup> GAROL. We wished to check certain simplifying assumptions of the GAROL code: 1) that the inelastic scattering by the absorber medium can be neglected and 2) that the collision escape probabilities for each region in the cell can be computed on the basis of a flat source distribution.

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<sup>1</sup>J. M. NEILL and J. C. YOUNG, "Spatially Dependent Thermal Neutron Spectra in Slab and Rod Geometries," GA-7508, General Atomic (1966).

<sup>2</sup>J. C. YOUNG and D. HUFFMAN, "Experimental and Theoretical Neutron Spectra," GA-5319, General Atomic (Updated May 1966).

<sup>3</sup>"Neutron Spectra," EACRP-L-62 and EACRP-L-62, Supp. 1, European-American Committee on Reactor Physics (1966).

<sup>4</sup>R. H. JONES, "A Compilation of Experimental and Theoretical Neutron Spectra," AERE-R4776, United Kingdom Atomic Energy Authority (1964).

<sup>5</sup>E. JOHANSSON, "Some Studies of Thermal and Epithermal Neutron Spectra in Heterogeneous Systems," Paper SM-96/43, *Proc. Symp. Neutron Thermalization and Reactor Spectra, Ann Arbor, Michigan, July 1967*. To be published by the International Atomic Energy Agency.

<sup>6</sup>E. HELLSTRAND, "Measurements in Resonance Integrals," in *Reactor Physics in the Resonance and Thermal Regions*, Vol. II, MIT Press, Cambridge, Mass. (1966).

<sup>7</sup>Private Communication from B. ROOS to R. DAHLBERG (December 1966); see also K. D. LATHROP, "DTF-IV Code, a FORTRAN IV Program for Solving the Multigroup Transport Equation with Anisotropic Scattering," LA-3373, Los Alamos Scientific Laboratory (1965).

<sup>8</sup>C. A. STEVENS and C. V. SMITH, "GAROL, A Computer Program for Evaluating Resonance Absorption Including Resonance Overlap," GA-6637, General Atomic (1965).

The latter would be tested with the use of a thick (3.14-cm) absorber slab of depleted uranium. It was also deemed worthwhile to explore the circumstances under which an infinite medium code such as GAROL could be applied to the solution of the space-integrated, energy-dependent neutron spectra in a finite geometry such as the one employed here.

The experiment has also motivated studies of various quadrature sets in 1DF. This was of special interest here because the highly anisotropic fluxes observed in this experiment are not source induced, but arise as a consequence of the different properties of the adjacent media. In addition, the analysis of the experimental data has led to studies of the sensitivity of the calculated fluxes to the spatial and energy representation in 1DF and to the cross-section averaging procedures. Finally, the strong flux gradients in the system were expected to amplify any perturbations in the spectra caused by the presence of the reentrant hole from which the flux was extracted. There has long been a dearth of reliable information on this perturbation, and this knowledge would support the heterogeneous spectral measurements presently being made<sup>9</sup> at thermal energies.

## II. EXPERIMENT

The time-integrated spectral measurements reported here were performed by standard time-of-flight means<sup>1,2,10</sup> utilizing the Gulf General Atomic Electron Linear Accelerator to provide a pulsed source of neutrons. The experimental configuration, illustrated in Fig. 1, consisted of a 30.5- × 30.5- × 3.14-cm-thick slab of depleted uranium (0.23% <sup>235</sup>U) that was placed between two slabs of polyethylene each 30.5- × 30.5- × 2.49-cm thick. A slab geometry was chosen to simplify the analysis of the problem. The polyethylene was loaded with boron to 5.3 wt%. Uranium-238 was chosen as the resonance absorber in this work because of its ready availability, convenient resonance width, and level spacing, and because its cross sections were well known. The moderator (borated polyethylene) was chosen to combine a small slowing down time and high absorption to ensure a short mean emission time of the neutrons from the system and, hence, adequate energy resolution for these studies. In addition, the borated polyethylene made the leakage small relative to the absorption. This configuration did not represent a cell in a semi-infinite array, but

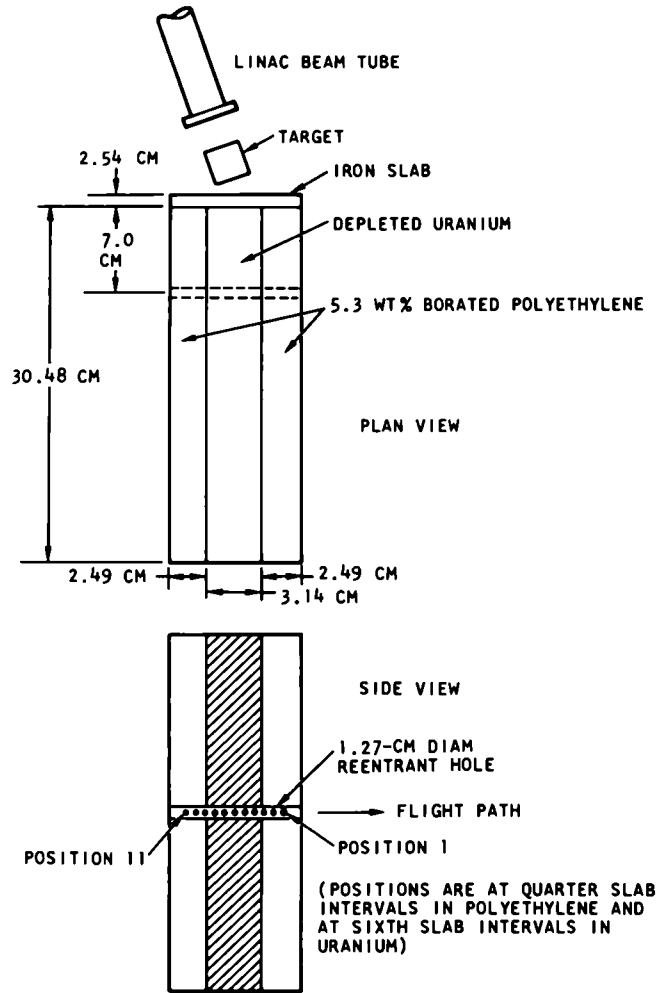


Fig. 1. Geometry for resonance region spectral studies in depleted uranium.

there were insufficient uranium slabs available to approximate that array.

The pulsed photoneutron source was produced by impinging electrons from the LINAC onto a water-cooled fansteel (tungsten alloy) target. These neutrons were spread spatially by a 1.27-cm-thick slab of iron before exciting the assembly. The fast neutrons were slowed down in the system and extracted from a 1.27-cm-diam reentrant hole located at 7.0 cm from the edge. A precollimator diameter of 1.05 cm was found to provide satisfactory counting rates. (A 0.95-cm-diam reentrant hole in conjunction with a 0.76-cm-diam precollimator was used initially and found to give unacceptably low count rates.) The position of the reentrant hole was chosen to be as close as possible to the source, to increase the observed flux intensities, but far enough so that the higher flux harmonics would have small magnitudes. The adequacy of the compromise of

<sup>9</sup>G. D. TRIMBLE et al., "Lattice Physics Studies, Quarterly Progress Report to March 31, 1967," GA-7934, General Atomic (1967).

<sup>10</sup>J. C. YOUNG et al., *Nucl. Sci. Eng.*, **28**, 259 (1967).

these two conditions is reflected in Fig. 2 by comparison at the measurement position of the axial flux plots using dysprosium-aluminum and cadmium-covered manganese and indium foils. The slopes of the curves are not greatly dissimilar at the measurement position of 7.0 cm, indicating that the flux harmonics are small in the energy response range of these foils. Flux plots with cadmium-covered indium foils were measured in the other two directions to permit the local buckling to be determined. In addition, the flux plot in the measurement direction, shown in Fig. 3, provided a spatial distribution for the slowing source in subsequent calculations.

The reentrant hole was, in fact, a through hole into which plugs of the appropriate material and thickness were inserted to obtain the desired measurement positions. The neutrons were extracted from this hole and timed over an evacuated 16-m flight path to their detection by a bank of 32 BF<sub>3</sub> counters. The data were recorded on a TMC-1024 channel analyzer using channel widths of 1  $\mu$ sec. A background was measured by putting a <sup>10</sup>B plug over the precollimator. The data were reduced to neutron spectra by means

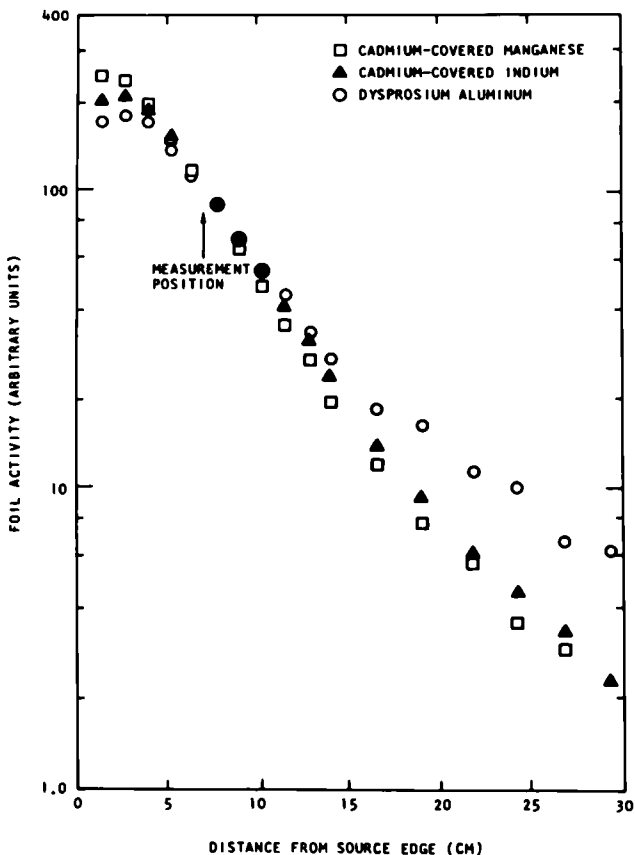


Fig. 2. Foil activations along the source-assembly axis.

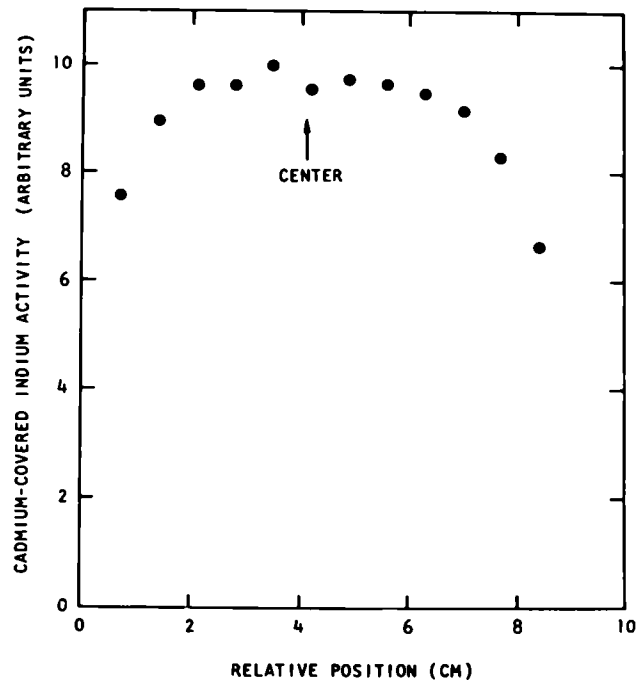


Fig. 3. Cadmium-covered indium foil activations in the direction of the spectrum measurements.

of the code HECTO,<sup>11</sup> which corrects for the mean emission time, subtracts background, corrects for count rate losses, and incorporates the energy dependence of the detector sensitivity and the flight path transmission.

The spectral data were reduced incorporating the measured time-dependent background and a small constant background based on the residual counts at the bottom of the flux dip due to the 6.68-eV resonance in <sup>238</sup>U. It was of interest to identify the sources of this residual background and the following mechanisms were postulated:

1) Direct slowing down contribution to this energy from inelastic scattering in <sup>238</sup>U. This postulate was rejected since the computed energy transfer matrix indicated no such contribution.

2) Streaming effect in the reentrant hole. This postulate was examined by performing a subsidiary experiment in which the neutron spectrum at the center of the uranium slab was measured by time-of-flight with and without a depleted uranium sleeve (0.127-cm wall) placed inside the reentrant hole. This sleeve covered the polyethylene and prevented resonance energy neutrons from streaming into the uranium and scattering back down the flight path. No significant spectral differences were observed.

<sup>11</sup>H. M. ANTUNEZ et al., "HECTO Code," General Atomic Private Data (December 1966).

3) Delayed neutrons from fission in  $^{235}\text{U}$  and  $^{238}\text{U}$ . A dieaway experiment was performed to long times after the neutron burst, which permitted measurement of the ratio of prompt to delayed neutrons. This ratio was much too small to provide a significant background at the 6.68-eV flux dip. The asymptotic dieaway of the system was found to be  $10.1 \mu\text{sec}$ , which is larger than the infinite medium dieaway of the borated polyethylene. In view of the very low multiplication of the system, the long dieaway must have been due to the long lifetime of thermal neutrons in the depleted uranium slab.

4) Pulsing overlap neutrons. These neutrons had been removed by a  $\text{B}_4\text{C}$  filter in the flight path.

5) Resolution broadening effects. The resolving time was estimated to be  $\sim 2 \mu\text{sec}$ , derived from  $1\text{-}\mu\text{sec}$  resolution time for the  $\text{BF}_3$  bank,  $1 \mu\text{sec}$  from the analyzer channel width and  $0.3 \mu\text{sec}$  from the standard deviation of the mean emission time. The latter was calculated with the methods given by Beckurts and Wirtz.<sup>12</sup> The FWHM of the flux dip at the 6.68-eV resonance is  $\sim 20\%$  in energy, and this corresponds to a time spread of  $45 \mu\text{sec}$ . This value is so much larger than the resolving time ( $2 \mu\text{sec}$ ) of the measurement system that resolution broadening can be dismissed as a significant contributor to the residual counts at the resonance flux dip.

6) Alignment errors. Considerable care was taken in aligning the reentrant hole to the pre-collimator and flight-path axis. Had there been much misalignment, the observed neutron spectra with and without the uranium sleeve would have displayed noticeable differences.

7) Room return neutrons. These neutrons would give rise to a signal with an apparent energy shift and, moreover, would not be observed during the background run. The effects of these room return neutrons had been observed previously<sup>13</sup> in  $\sigma_T$  measurements near the Bragg cutoff of various crystalline materials.

In view of the negligible shielding around the small experimental assembly, it is reasonable to ascribe the small residual background to room return neutrons. The value of the constant residual background subtracted was adjusted at each position so that the measured and 1DF calculated neutron spectra were in agreement at the 6.68-eV

resonance flux dip. Fortunately, the agreement mentioned in Sec. III between the GAROL computed flux and the 1DF space-averaged values gave some confidence in the magnitude of the computed flux dip. In addition, the value of the residual background was found consistently to be proportional to the monitor counts.

Experimental results for all measurement positions are compared with the calculations in Sec. III. However, we show here, in Fig. 4, the time-of-flight neutron spectrum at the center of the depleted uranium slab throughout the whole energy range of the measurements. The points shown correspond to the  $1\text{-}\mu\text{sec}$  time channels of the multichannel analyzer, except that they have been grouped below  $135 \text{ eV}$  to obtain an energy resolution of  $\sim 2\%$ . The actual energy resolution, based on the  $2\text{-}\mu\text{sec}$  resolution time discussed previously is proportional to  $E^{-1/2}$  and amounts to  $15\%$  at  $2 \text{ keV}$  and  $0.9\%$  at  $6.68 \text{ eV}$ . The sensitivity of the near-zero flux at  $6.68 \text{ eV}$  to the value of the small constant background that is subtracted explains the very large statistical uncertainties observed at that energy.

The measured spectra show a significant departure from a  $1/E$  behavior between resonances because of the high leakage and strong absorption in the boron-loaded polyethylene. The flux is also strongly perturbed by the  $^{238}\text{U}$  resonances below  $100 \text{ eV}$ . The flux depression due to the  $8.8\text{-eV}$  resonance in  $^{235}\text{U}$  can also be observed despite the low concentration ( $0.23 \text{ wt}\%$ ) of this isotope. (However, this depression is not displayed clearly in Fig. 4.)

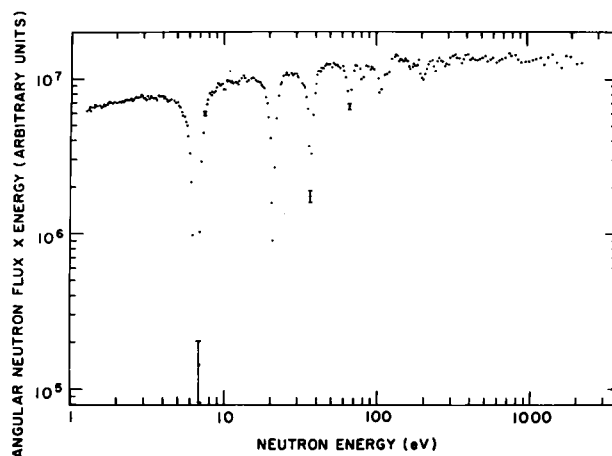


Fig. 4. Time-of-flight neutron spectrum at position 6 (the center of the uranium slab).

### III. CALCULATION

Limitations of computer memory precluded consideration of more than one dimension in cal-

<sup>12</sup>K. H. BECKHURTS and K. WIRTZ, *Neutron Physics*, Sec. 9.1.1, Springer-Verlag, New York (1964).

<sup>13</sup>J. R. BEYSTER et al., "Integral Neutron Thermalization, Annual Summary Report, October 1, 1965 through November 30, 1966," GA-7480, General Atomic (1966).

culating the space-dependent neutron spectrum in our system. These computations were therefore performed with 1DF, an  $S_n$  transport theory code. The planning of the transport calculations was based on a thorough study of the physics of the problem with the aid of preliminary experimental results. This led us to use a linearly anisotropic ( $P_1$ ) scattering approximation, an isotropic volume-distributed neutron source, and a compromise of the largest order of  $S_n$  and the best space and energy resolution possible. This compromise was achieved by first limiting the energy range of the calculation to a band around the strongest  $^{238}\text{U}$  resonance, namely, that at 6.68 eV. Symmetry about the uranium midplane was assumed. It was also considered desirable to restrict the spacing  $\Delta x$  according to the relationship below

$$\Delta x < 3\lambda_{\min}/n,$$

where  $n$  is the order of quadrature and  $\lambda_{\min}$  is the minimum total mean-free-path. The resulting compromise permitted ten energy groups (down-scattering only), whose boundaries are given in Table I. This also allowed 608 space intervals (588 in the uranium and 20 in the polyethylene), in an  $S_6$  calculation for a total of 64 400 words in memory. The space intervals were constant in the uranium but were variable in the polyethylene, decreasing close to the interface to allow for the increasing resonance flux gradient.

The code 1DF has the limitation that a "first interval vacuum, last reflective," boundary condition cannot be used. It was therefore desirable to use a quadrature that included the direction cosines,  $\mu = +1, -1$ , in order to calculate in the half-system geometry all of the measured angular spectra. The Lobatto quadrature<sup>14</sup> was selected

TABLE I  
Broad Energy Group Boundaries  
for the 1DF Calculations

Group Number	Energy Interval (eV)
1	11.0 - 9.0
2	9.0 - 7.6
3	7.6 - 7.2
4	7.2 - 6.99
5	6.99 - 6.76
6	6.76 - 6.6
7	6.6 - 6.1
8	6.1 - 5.8
9	5.8 - 4.5
10	4.5 - 3.3

<sup>14</sup>H. H. MICHELS, "Abscissas and Weight Coefficients for Lobatto Quadrature," *Math. Computation*, **17**, 263 (1963).

because it meets this requirement. For the sake of checking it against frequently used quadratures, comparative 1DF runs were made with the set of direction cosines and corresponding weights<sup>15</sup> built into 1DF and with the Gauss-Legendre ( $P_n$ ) set,<sup>16</sup> which takes as direction cosines the zeros of the Legendre polynomials. Two other comparative 1DF runs were performed using  $S_4$  rather than  $S_6$ ; one of them with only 352 space intervals (336 in uranium and 16 in polyethylene), obtained from a less stringent largest interval condition:

$$\Delta x < 4\lambda_{\min}/n.$$

These 1DF runs were made with the same broad group cross sections and with the same energy dependence for neutron source. The parameters, which varied from one to another, and the largest total differences observed in the scalar and angular neutron fluxes calculated in the five runs are summarized in Table II. These maximum differences are not large and the typical differences have much smaller values. The point at which the calculated flux anisotropy was found to be largest is in the uranium, at 0.0053 cm from the interface and in energy group 6. Figure 5 displays the values of the angular flux as a function of the direction cosine at that energy and radius for the different runs having an interval mesh of 608 points. No interpolation curve has been drawn on

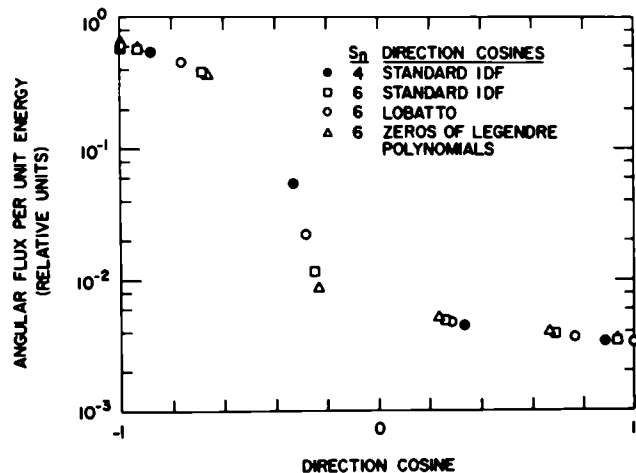


Fig. 5. The effect of different quadratures on the angular flux at the position and energy of maximum anisotropy.

<sup>15</sup>B. G. CARLSON and C. E. LEE, "Mechanical Quadrature of the Transport Equation," LAMS-2573, Los Alamos Scientific Laboratory (1961).

<sup>16</sup>NATIONAL BUREAU OF STANDARDS, *Handbook of Mathematical Functions*, Applied Mathematics Series, No. 55, Table 25.4, U.S. Government Printing Office, Washington, D.C. (1964).

TABLE II

Largest Total Differences Observed in the Neutron Fluxes Calculated with the Code 1DF Around the 6.68-eV Uranium Resonance for Different Spatial Meshes,  $S_n$  Approximations and Quadratures

Run	Mesh	$S_n$	Quadrature	Scalar Flux		Angular Flux, $\mu = -1$		
				Uranium (%)	Polyethylene (%)	Group 5 (%)	Group 6 (%)	Group 7 (%)
4	352	4	1DF	10	2	<1	20	<1
5	608	4	1DF					
6	608	6	1DF	10	1.5	<1	4	<1
7	608	6	Lobatto					
8	608	6	$P_n$	2.5	<1	<1	2	<1

the plot, since it is clear that the points lie approximately on a smooth curve. Therefore, an increase of the quadrature order to  $S_8$  would not have been justified in view of the degree of agreement already achieved. The errors resulting from having used a quadrature of  $<S_6$  are already comparable to, or smaller than, those from other uncertainties in the problem, such as energy resolution and knowledge of the source and cross sections.

The cross sections used for  $^{238}\text{U}$  were Doppler-broadened values obtained with the code<sup>17</sup> FASDOP using the parameters of Joanou and Stevens.<sup>18</sup> The  $^{235}\text{U}$  cross sections were from Joanou and Drake.<sup>19</sup> The broad group averaging process was performed for the uranium slab with the two-region slowing down code GAROL. This procedure did not provide the slowing down kernel, and so this was separately computed in a  $P_0$  approximation and averaged with the GAROL spectrum. An approximate  $P_1$  slowing down kernel for the uranium was obtained by multiplying the corresponding  $P_0$  term by a constant  $\bar{\mu}$ , the average cosine of the scattering angle.

In the borated polyethylene region there are no rapidly changing cross sections in our range of interest. As a consequence, satisfactory broad group averages could be obtained in a single step with the infinite medium slowing down code GAR.<sup>20</sup>

A correction to the total and absorption cross sections was made to account for transverse

leakage, since 1DF is a one-dimensional code. The correction  $D(E)B^2$  was based on the epicadmium indium activation measurement and was found to be negative. This correction was found to be unimportant since it made a difference of at most 2% in the 1DF calculated flux, even though  $D(E)B^2/\Sigma_a$  was  $>0.06$  in the uranium for the broad energy groups 1 and 10. This justified the omission of transverse leakage considerations in the GAROL calculations, which will be discussed later.

Epicadmium indium activation measurements provided a spatial flux distribution that was used to calculate the volume distributed slowing down neutron source in both regions by means of the code DSZ.<sup>21</sup> This code also gave a  $\phi(x,E)$  estimate as a byproduct, which allowed convergence of the 1DF runs to an overall precision of  $10^{-4}$  after only 3 outer iterations involving 91 inner (energy group) iterations. Improvement of this flux estimate by replacing it with an actual 1DF output reduced the number of inner iterations to 65. The 1DF runs were made with satisfactory results using a value of  $10^{-4}$  for the overall convergence precision. No oscillations or any other kind of qualitatively unexpected results were observed even for the 352-interval mesh.

The calculated fluxes were found to be sensitive to the weighting function used to average the cross sections in the broad groups only in the resonance wings. About 6% change in the width of the resonance angular flux dip was observed in a 1DF run for which the group cross sections had been averaged with a preliminary measured spectrum. The depth of the angular flux dip changed from  $4.0 \times 10^{-6}$  to  $5.6 \times 10^{-6}$ .

Figure 6 shows the spatial variation of the scalar flux calculated with the 1DF code. The

<sup>17</sup>C. A. STEVENS, "FASDOP, A FORTRAN IV Computer Program for Computing Cross Sections From Resonance Parameters," GAMD-6562, General Atomic (1965).

<sup>18</sup>G. D. JOANOU and C. A. STEVENS, "Neutron Cross Sections for  $^{238}\text{U}$ ," CR-54290, General Atomic (1965).

<sup>19</sup>G. D. JOANOU and M. K. DRAKE, "Neutron Cross Sections for  $^{235}\text{U}$ ," CR-54263, General Atomic (1964).

<sup>20</sup>C. A. STEVENS and J. R. ARCHIBALD, "GAR, A Computer Program for Evaluating Leakage Dependent Resonance Absorption," GA-6952, General Atomic (1966).

<sup>21</sup>J. M. NEILL, "DSZ and PREDSZ, FORTRAN IV Codes for Computing Slowing Down Sources," GA-8556, Gulf General Atomic (1968).

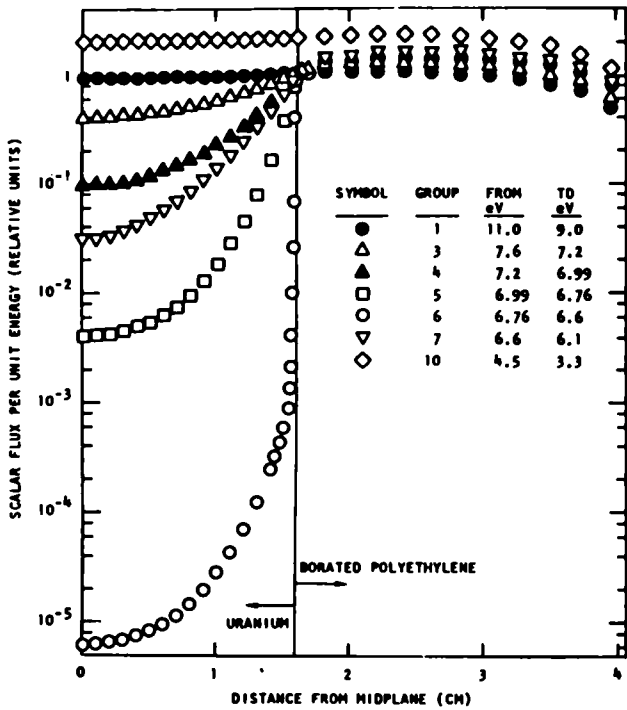


Fig. 6. 1DF calculations of the spatially dependent scalar neutron flux across the uranium and borated polyethylene slabs.

small difference in the shape of the flux for groups 1 and 10 justifies the use of the episcadmium indium activation data to generate the spatial distribution of the neutron source for 1DF. The spatial dependence shown in Fig. 6 did not change appreciably for different 1DF runs in which cross sections based on different averaging fluxes had been used. The most striking feature of this figure is perhaps the drastic change (factor of 600) in the scalar flux in the first 0.03 cm of the uranium at the resonance energy, broad group 6.

The angular fluxes calculated by 1DF using GAROL cross sections are compared to the measured spectra in Figs. 7 and 8. Monitor normalization has been retained at each measurement position and the single normalization between theory and all the experimental data is obtained from the fit in the highest energy group at position 3 only. The experimental values have been averaged over the broad energy group intervals for a more accurate comparison. The standard deviation of the measured fluxes is smaller than the radius of the dot except at the points where the error bars are drawn. The agreement between experiment and theory is shown to be reasonably good considering the broad group structure of the

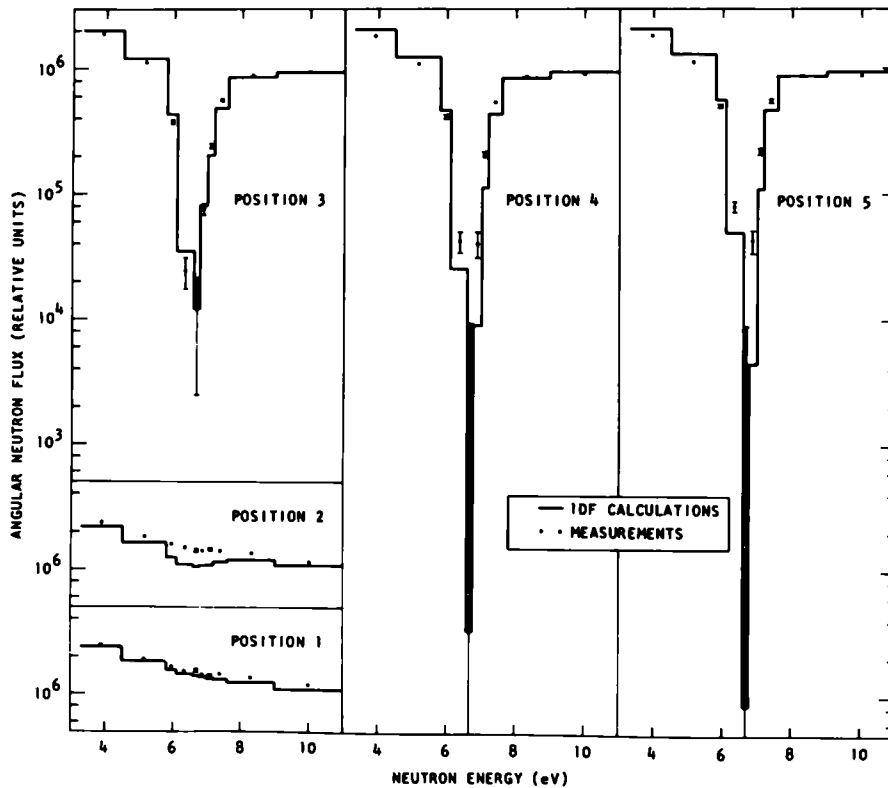


Fig. 7. Comparison of calculated and measured neutron fluxes at positions 1 through 5.

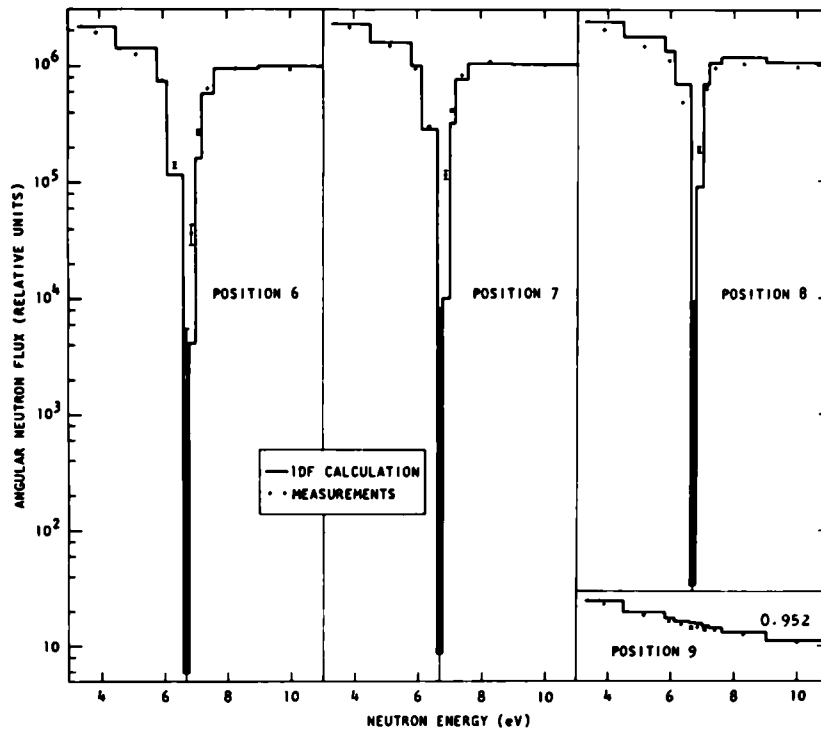


Fig. 8. Comparison of calculated and measured neutron fluxes at positions 6 through 9.

calculation. The figures display, in both theory and measurement, the systematic broadening of the resonance flux dip as the position moves further from the polyethylene source. There are, however, some significant discrepancies in energy groups 4 and 5, experiment being generally higher. The calculated fluxes below the resonance are consistently high by some 3%, which is large compared to the resonance absorption probability.

The space dependence of the resonance energy angular flux (group 6,  $\mu = +1$ ) is shown in Fig. 9 and can be compared with the spatial behavior of groups 4, 5, and 7, which are on the resonance wings. Group 6 shows little of the spatial asymmetry away from the interface displayed by the other energy groups. This is because neutrons that downscatter in the uranium predominate in group 6. Nevertheless, at the uranium-polyethylene interface, the resonance flux anisotropy is extreme as shown in Figs. 7 and 8 by the difference between the flux at positions 3 and 9.

There is a different spatial behavior of the flux in energy groups 4 and 5 (upper resonance wing) from that in group 7 (lower resonance wing), although the average cross sections for these wings are roughly similar. This has been interpreted as a consequence of only a few neutrons being downscattered in the uranium past the resonance into group 7 while groups 4 and 5 have a much greater downscattering source. Thus, the

angular flux behavior in energy group 7 is dominated by neutrons leaking in from the polyethylene and the calculated flux therefore shows a strong exponential behavior over a large portion of the uranium slab. This calculated spatial distribution has an exponent that corresponds exactly to the average total cross section for energy group 7. The measured fluxes in group 7 show a somewhat different behavior. If interpreted in the same way, the measured exponents would lie between the point total cross sections corresponding to the energy group 7 boundaries. This indicates how the "actual" broad group cross section decreases as the neutron spectrum changes shape deeper into the slab. This effect is characteristic of the sensitivity of the group cross sections in the wings of a strong resonance to the averaging flux spectrum. The observed effect is not shown by the 1DF calculation because the broad group cross sections were taken there as a constant in each region. The difficulty of using constant group cross sections in regions where the spectrum is changing very rapidly has been encountered by others.<sup>22</sup>

The experimental information affords some comment on the reentrant hole perturbation. The effect of the reentrant hole is expected to be most

<sup>22</sup>J. M. RAVETS and L. I. KOPP, *Trans. Am. Nucl. Soc.*, 8, 303 (1965).

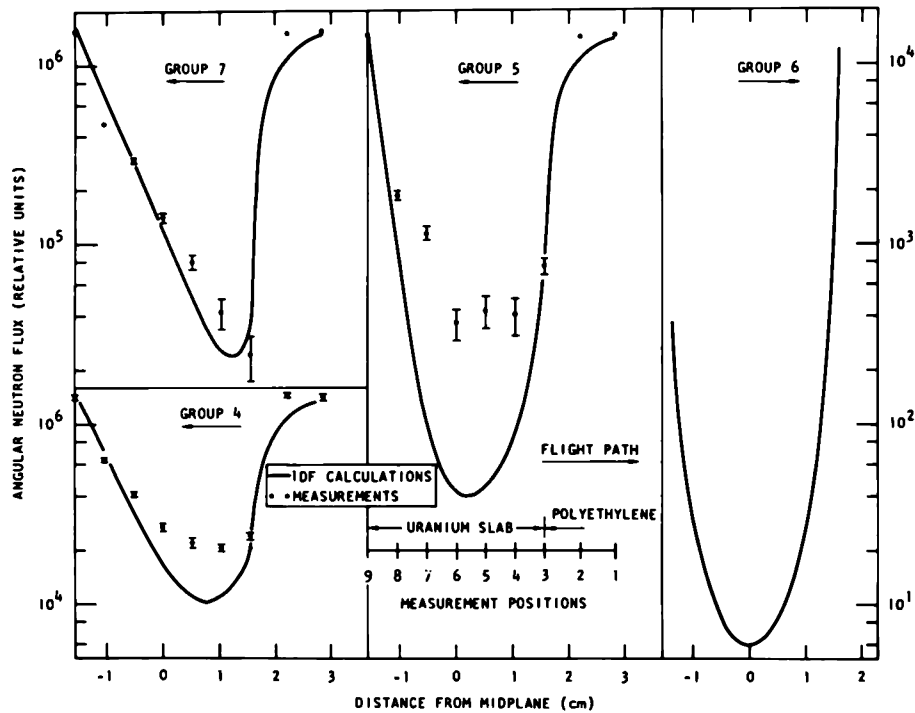


Fig. 9. Comparison of the measured and calculated spatial distribution of the angular fluxes around 6.68 eV.

pronounced at the interface in energy group 7 because there are few downscattered neutrons in this energy group. The removal of the polyethylene in creating the reentrant hole therefore removes some of the neutron source at this point. Since the measured flux at this point and energy is not much lower than the calculated value, the reentrant hole perturbation would appear to be small.

Unfortunately, it was not possible to use the code GAROL to compare to experiment because no position-dependent scalar fluxes were measured which could be spatially integrated for comparison to the predictions of this code. In addition, GAROL can be used only to calculate the volume-averaged scalar flux in each region of an infinite lattice array, while we have merely a single cell. Nevertheless, we wanted to check some of the assumptions in GAROL that make it a very fast code, and we wished to take advantage of its greater energy detail. This was done by comparing the GAROL predictions to 1DF results that in turn were compared to experimental data.

The code GAROL computes for several hundred energies the average flux in two adjacent media by solving the coupled integral equations for the flux in each region. These equations include external and slowing down sources and account for the transfer of neutrons between both media by means of their collision escape probabilities. A

space-independent source is assumed in each region for the calculation of these collision escape probabilities. The code GAROL does not permit external leakage and all these limitations were examined before applying the code to the calculation of our problem. In Fig. 6, the 1DF calculations show that the flux in the borated polyethylene is surprisingly flat, so that the flat source assumption in GAROL is not an unreasonable approximation for polyethylene. The observed epicadmium flux in the depleted uranium is fairly constant so that the flat source approximation is satisfactory in that medium. Consequently, we are concerned only with the effect on the GAROL calculation of the leakage normal to the slabs. (We disregard the small leakage perpendicular to that direction which was discussed earlier.) The leakage in our finite geometry can be crudely approximated as the leakage from a borated polyethylene medium having the same overall dimensions and characterized by a fundamental mode distribution. This gives a buckling  $B^2$  that permits us to compare the leakage  $D(E)B^2$  to the absorption  $\Sigma a(E)$  in the uranium. For energy groups 3 to 8, the ratio of leakage to absorption is  $< 7\%$ . Thus, only outside this energy range would leakage effects be expected to change the values of the GAROL flux computations.

The polyethylene in our system is very strongly poisoned so that the flux in the uranium will be insensitive to the effects of finite external

boundaries. This has been examined by comparing two GAROL calculations; one for an infinite lattice of alternate uranium and polyethylene slabs identical to those in our geometry, and the other for a lattice in which the polyethylene slabs were taken to be 250 cm thick (the width of the uranium slabs still being 3.14 cm). The ratio of the average flux in the uranium for the second case to the first case was found to be different by at most 9.5%, thus indicating an insensitivity of the average flux in the uranium to neutron events beyond 2.5 cm of the borated polyethylene. It has thus been possible to correct the GAROL flux computations and apply them to our finite geometry by assuming that the change per unit lethargy of the flux ratio for the two lattice calculations at the 6.68-eV resonance energy provides a measure of the leakage in the uranium. This approach is based on the fact that at 6.68 eV the vacuum boundary conditions of the actual system are matched by those of the infinite lattice calculation. We have used this measure of the leakage effect to correct the GAROL computations; other details of the correction procedure are given by Beyster et al.<sup>23</sup>

The fluxes computed by GAROL are shown in Fig. 10. The effect of the finite geometry on the spectrum is seen to be small. The GAROL calculated average scalar flux in the uranium shows the presence of the <sup>235</sup>U resonances that were observed in the measured data but could not be calculated by 1DF due to the limitations of the broad energy group structure used.

In Fig. 11, we compare the GAROL and 1DF fluxes as normalized in energy group 1 (11.0 to 9.0 eV). For the sake of presenting them in a comparable fashion, the former have been averaged in energy within each 1DF broad group, and the latter have been averaged in each material region. The agreement of GAROL with 1DF, and thus indirectly with experiment, is good, the maximum difference over the entire energy range being only 7%. It is possible that this difference could have been reduced if more energy groups could have been used in the 1DF calculations. The coincidence of the flux dip at the resonance energy is remarkable. It would appear that the average scalar spectrum in the resonance absorber in systems similar to ours can be calculated reasonably well with GAROL and with much more energy detail than with 1DF. In addition, the agreement between 1DF and GAROL when such greatly different methods are used in each code lends confidence to the adequacy of the approximations in each code, those in GAROL being much more severe.

<sup>23</sup>J. R. BEYSTER et al., "Integral Neutron Thermalization, Quarterly Progress Report for the Period Ending June 30, 1967," GA-8085, General Atomic (1967).

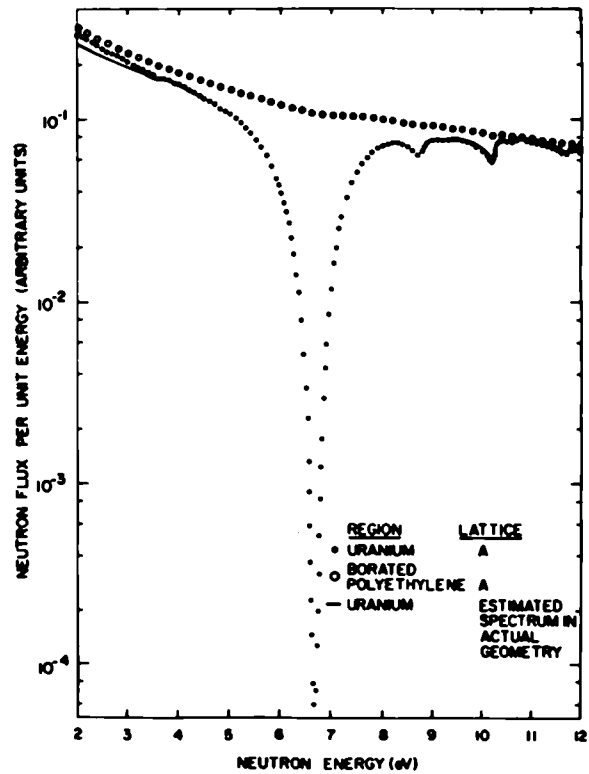


Fig. 10. Region-averaged fluxes calculated with GAROL.

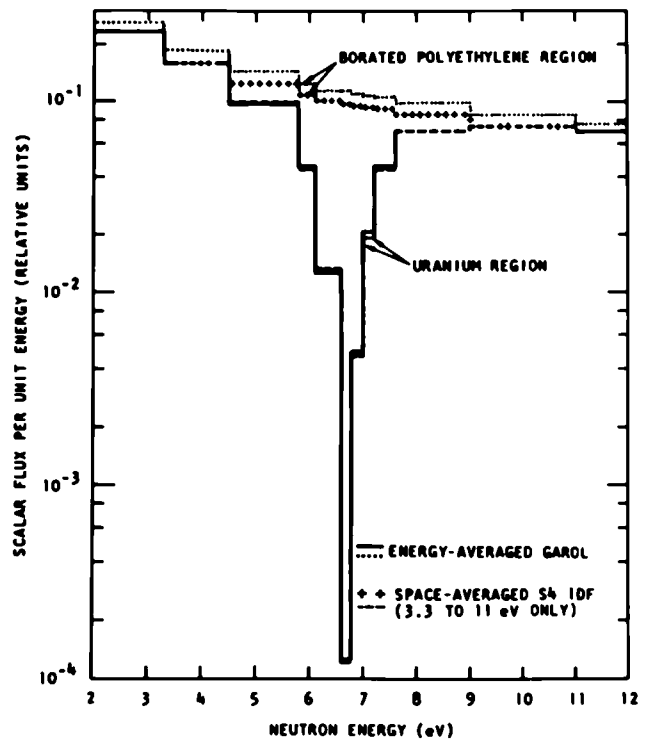


Fig. 11. Comparison of the GAROL and 1DF calculated fluxes in the uranium and borated polyethylene regions.

## IV. CONCLUSIONS

Spatially dependent neutron spectra have been measured in the resolved resonance energy region in a slab system of depleted uranium and borated polyethylene with very good spatial and energy resolution. The measurements have demonstrated that time-of-flight techniques are suitable for making detailed observations of the resonance flux in such heterogeneous systems. Spectral calculations have been performed in the neighborhood of the 6.68-eV resonance in  $^{238}\text{U}$  separately with high spatial and energy resolution. The agreement of measurements with theory and the self-consistency of the calculations lends confidence to the nuclear cross sections used and to the ability of the codes that were used to account for flux variations in strongly anisotropic situations.

The studies have shown that careful analysis of the background is necessary to obtain a reasonable description of the flux at the resonance energies. In addition, little indication of reentrant hole perturbation has been found. The different  $S_n$  transport theory calculations performed with 1DF have shown surprisingly small dependence on the order of quadrature  $n$ . Some sensitivity to the weighting flux used to get the group cross sections in the resonance wings was observed. The spatial representation in 1DF was difficult and limiting. Nevertheless, no spatial or angular oscillations were observed at quite coarse meshes. It was found that the space-averaged fluxes in our finite geometry could be calculated

reasonably well with the infinite lattice slowing down code GAROL. The agreement of GAROL predictions with 1DF results and the agreement of the latter to the experimental data support the adequacy of the approximations in the GAROL code. Specifically, the flat source approximation for computing collision escape probabilities was found to be satisfactory even for a uranium slab as thick as 3.14 cm. Neglect of inelastic scattering was also a good assumption in this code at the 6.68-eV resonance in  $^{238}\text{U}$ .

The experimental methods demonstrated here provide means for evaluating simplified methods for computing resonance absorption in laminated shields of hydrogenous and heavy metal slabs. In addition, the methods described here may be used to check:

- 1) files of resonance cross sections below a few hundred eV
- 2) the approximations used in reactor analysis to describe self-shielding effects
- 3) the energy meshes that are necessary to account for strong resonance absorption in reactor materials.

## ACKNOWLEDGMENTS

The authors are indebted to Drs. C. A. Preskitt and J. R. Beyster for advice and helpful discussions during the course of these studies, and to Mr. G. D. Trimble for his assistance in performing the experiments.

This work was sponsored by the USAEC.