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PERTURBATION THEORY IN A DEFORMED BASIS

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I. Motivation

The total Hamiltonian H is written in two parts $H = H_0 + x\, H_{res}$ $(0 \leq x \leq 1)$ We consider the case in which the basic set of states (eigenfunctions of H_0) is deformed, i.e., $[H,L] = 0$ and $[H_0,L] \neq 0$, where L can be the angular momentum, the number of particles, etc. In such cases perturbation theory of H_{res} does not converge. This may be seen as follows:

1) Since the inclusion of H_{res} $(x = 1)$ restores the original symmetry of the problem, a perturbative treatment of H_{res} would imply a perturbative transformation from the deformed to the spherical basis. This cannot exist, since the opposite transformation (from the normal to the deformed basis) is not possible to perform perturbatively.

2) In simple systems (particles moving in a degenerate shell and coupled by pairing interaction) we have verified that the radius of convergence of the corresponding perturbative series in x is always smaller than one (and tends to one as the degeneracy of the shell increases).

3) In a deformed basis, there exist zero frequency modes (Goldstone bosons) which couple with the fermion degrees of freedom with a coupling strength that diverges as $w^{-1/2}$.

The origin of these difficulties can be traced back to the abscence of restoring force in the angular direction for a freely rotating system, giving rise to infrarred catastrophes.

II. Procedure

In Ref.[1]) a procedure to overcome these difficulties is derived. It is based on techniques used in field theory for quantification of constrained mechanical systems. For a system of fermion fields b_m^+, b_m, the amplitude for vacuum to vacuum transitions is written

$$Z = \pi_m \int Db_m^+ \, Db_m \, e^{i\int |i \sum_m b_m^+ \dot{b}_m - H| \, dT} \tag{1}$$

Let $R(\theta)$ be a group of transformations that leave H invariant, and $a_m^+ = R(\theta)\, b_m^+$. For simplicity we assume that there is only one generator $L(a_m^+, a_m)$. We obtain after manipulations

$$Z = \pi_m \int Da_m^+ \, Da_m \, DID\theta \, \delta(I - L)\, \delta(\phi - \phi_0)\, \{L,\phi\}\, e^{i\int |i \sum_m a_m^+ \dot{a}_m + I\dot{\theta} - H| \, dt} \tag{2}$$

The kinetic energy terms in the Lagrange indicate that the collective degrees of freedom I, have been incorporated to the original set of fermion degrees of freedom. We choose (a_m^+, a_m) to be the angular variable conjugate to L, so that the Poisson bracket $\{L,\phi\} = 1$. Since the properties of the system cannot depend on the parameter ϕ_0, we multiply Z by $e^{-i\phi_0^2/2\alpha}$ and integrate over ϕ_0. The angular momentum constraint is exponentiated as the limit of a gaussian. Up to a normalization constant, we obtain

$$Z = \lim_{\beta \to 0} \pi_m \; Da_m^+ \, Da_m \, DID\phi \; e^{i\int |i \sum_m a_m^+ \dot{a}_m + I\dot{\phi} - H - (I-L)^2/2\beta - \phi^2/2\alpha| dT} \tag{3}$$

Therefore the original Hamiltonian H has been replaced by

$$H' = \lim_{\beta \to 0} |H + (I-L)^2/2\beta + \phi^2/2\alpha| \tag{4}$$

Since the basis set of states of H' is of the form $e^{iL_0\phi}\, u(a_m^+, a_m)$, the collective operator I in H' can be replaced by its eigenvalue L_0.

In (4), the restoring force in the angular direction is a complicated (many-body) function of the fermion coordinates. Moreover, the term $(L-L_0)^2/2\beta$ represents a two-body interaction which becomes infinite. However, it is possible to treat perturbatively such an interaction if the perturbation is made in terms of a parameter other than the strength of the interaction. The NFT provides for such an expansion.

III. Application

We consider the problem defined by particles moving in a two-dimensional harmonic oscillator potential and coupled by quadrupole forces. It has an exact solution (SU2 algebra) which may be interpreted in terms of two-dimensional rotational bands. For the ground state band, the energies are written

$$H\ (L_0) = \frac{1}{2} \times \sum (\sum + 2) + \frac{1}{2} \times L_0^2 \qquad (5)$$

Here Σ is the maximum angular momentum or the maximum quadrupole moment that can be obtained for a given number of particles in the h.o. shell, and X is the strength of the interaction. The corresponding Nilsson single-particle states are the h.o. cartesian states which do not carry the cylindrical symmetry of the problem. The Nilsson plus RPA contributions reproduce the value (5) but H_{res} yields additional contributions of $0\ (X\Sigma^0)$ which do not appear in (5) and which are removed by the use of H'. An expansion of $\theta^2/2\alpha$ in powers of Σ^{-1} produces is the same order in 2 two, three and four-body terms. The Goldstone boson has now infinite energy, i.e., $w \rightarrow \Sigma\ (2/\beta\alpha)^{1/2}$. It must be included diagramatically according to the NFT rules. Each graph diverges as $\beta \rightarrow 0$ but the sum of all contributions (of a given order in Σ^{-1}) yields the correct results to that order. In this limit, the results are also independent of the parameter α, as they should. This work has been done in collaboration with Drs. V.Alessandrini, G.G.Dussel, B.Machet and R.P.J.Perazzo.

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[1]) V.Alessandrini, D.R.Bes and B.Machet; Nucl.Phys. B142 (1975) 489.