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Systems with four and six particles outside closed-shell cores are described in a basis which uses the vector-coupled states from the two- and four-particle systems. The method is applied to calculate the spectra of ^{204}Pb and ^{202}Pb.

A recurrent theme in nuclear spectroscopy has recently been the inclusion of collective states in describing nuclear spectra [1–7]. The main achievement of models that contain these correlated states in the basis is that with only a few basis states a rather large amount of experimental data is explained. A method of this kind was recently proposed which is rather simple and its application is therefore not much time consuming [7]. In this method the matrix elements of the interaction and those of the overlap among the basis states are very similar in some cases. Thus the actual computation of both matrices is performed at the same time and the wavefunctions are readily obtained making use of the overlap matrix [7].

In this paper we present an extension of the method given in ref. [7] to analyse four- and six-particle systems outside closed-shell cores. The basic idea of the method presented here is to use the two-particle spectrum components as building blocks to describe the four-particle system (the so-called two-phonon states [3,5,6]). Once the four-particle spectrum is obtained, we use this spectrum and that of the two-particle system as building blocks to describe the six-particle system. Actually, other partitions are also possible. For instance, one can solve the six-particle system in terms of the five- and the one-particle spectra. We chose the partition mentioned above because we have in mind to analyze the (p, t) experimental data of ref. [8] for which our choice seems natural.

Introducing the two-particle TDA equation into the four-particle TDA equation one gets [3]

$$(W_\beta - \omega_{\alpha_1} - \omega_{\alpha_2})\langle\beta|(P^\dagger(\alpha_1)P^\dagger(\alpha_2))_\beta|0\rangle = -\sum_{\alpha_3\alpha_4}\sum_{ijkl}(\omega_{\alpha_3} - \epsilon_j - \epsilon_k)A(ijkl, \alpha_1\alpha_2\alpha_3\alpha_4)\langle\beta|(P^\dagger(\alpha_3)P^\dagger(\alpha_4))_\beta|0\rangle, \tag{1}$$

with

$$A(ijkl, \alpha_1\alpha_2\alpha_3\alpha_4) = Y^*(ij;\alpha_1)Y^*(kl;\alpha_2)Y(ik;\alpha_3)Y(jl;\alpha_4)\hat{\alpha}_1\hat{\alpha}_2\hat{\alpha}_3\hat{\alpha}_4\hat{\beta}\begin{Bmatrix} i & j & \alpha_1 \\ k & l & \alpha_2 \\ \alpha_3 & \alpha_4 & \beta \end{Bmatrix}. \tag{2}$$

In these equations small latin letters indicate single-particle states. The indices α_i label two-particle and the index β

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four-particle states as well as the corresponding angular momenta, for instance $\hat{\alpha}_1 = (2\lambda_{\alpha_1} + 1)^{1/2}$. The single (two)-particle energies are indicated by the letter $\epsilon(\omega)$ as usual. W_β is the four-particle energy. The operators $P^\dagger$ create the TDA two-particle states, i.e.

$$P^\dagger(\alpha) = \sum_{i \leqslant j} X(ij;\alpha)(c_i^\dagger c_j^\dagger)_\alpha/(1+\delta_{ij})^{1/2} , \tag{3}$$

where $c^\dagger$ is the single-particle creation operator. Finally, $Y(ij;\alpha) = (1+\delta_{ij})^{1/2} X(ij;\alpha)$.

Eq. (1) is relatively simple (cf. eq. (12) of ref. [5]) but its diagonalization is not very meaningful because although it provides the physical energies (plus a number of spurious energies) the physical eigenvectors cannot be obtained directly from (1) since the basis elements are not orthogonal to each other. In addition, eq. (1) is not hermitean and complex eigenvalues can also be obtained [3]. To overcome these problems we evaluate the overlap matrix of the two-phonon states to get

$$\langle 0|(P^\dagger(\alpha_1)P^\dagger(\alpha_2))_\beta^\dagger (P^\dagger(\alpha_3)P^\dagger(\alpha_4))_\beta|0\rangle = \delta_{\alpha_1\alpha_2}\delta_{\alpha_3\alpha_4} + (-1)^{\alpha_1+\alpha_2-\beta}\delta_{\alpha_1\alpha_4}\delta_{\alpha_2\alpha_3} - \sum_{ijkl} A(ijkl,\alpha_1\alpha_2\alpha_3\alpha_4), \tag{4}$$

which can be evaluated at the same time as (1). This property and the simplicity of eq. (1) allow to solve the four-particle system rather fast.

With the overlap matrix (4) one can make use of any of the available methods to construct a set of orthonormal states. One can then obtain the four-particle wavefunctions as [5,7]

$$|\beta\rangle = P^\dagger(\beta)|0\rangle = \sum_{\alpha_1 \leqslant \alpha_2} X(\alpha_1\alpha_2;\beta)(P^\dagger(\alpha_1)P^\dagger(\alpha_2))_\beta|0\rangle . \tag{5}$$

We use in (5) the same symbols as in (3) and (6) to denote the creation operators P and the wavefunction amplitudes X. However, no confusion can arise since the labels clearly identify the various quantities.

Once the diagonalization of the hermitean matrix obtained through (1) and (4) is performed and the energies W_β and wavefunctions (5) are obtained, one proceeds to calculate the six-particle wavefunctions

$$|\gamma\rangle = P^\dagger(\gamma)|0\rangle = \sum_{\alpha\beta} X(\alpha\beta;\gamma)(P^\dagger(\alpha)P^\dagger(\beta))_\gamma|0\rangle , \tag{6}$$

where the letter γ labels six-particle states. As before, the amplitudes in (6) and the corresponding energies W_γ are provided by the six-particle TDA equation which after some algebra is found to have the form

$$(W_\gamma - W_{\beta_1} - \omega_{\alpha_1})\langle\gamma|(P^\dagger(\alpha_1)P^\dagger(\beta_1))_\gamma|0\rangle = \sum_{\alpha_2\beta_2\alpha_3} (W_{\beta_2} - \omega_{\alpha_1} - \omega_{\alpha_3}) Y^*(\alpha_3\alpha_1;\beta_2)$$

$$\times \langle\beta_1|(P^\dagger(\alpha_3)P^\dagger(\alpha_2))_{\beta_1}|0\rangle \hat{\beta}_1\hat{\beta}_2 \begin{Bmatrix} \alpha_1 & \alpha_3 & \beta_2 \\ \alpha_2 & \gamma & \beta_1 \end{Bmatrix} \langle\gamma|(P^\dagger(\alpha_2)P^\dagger(\beta_2))_\gamma|0\rangle , \tag{7}$$

where the symbols have a similar meaning as in eq. (1) but $Y(\alpha_1\alpha_2;\beta) = (1+\delta_{\alpha_1\alpha_2})X(\alpha_1\alpha_2;\beta)$. The corresponding overlap matrix is given by

$$\begin{aligned}
&\langle 0|(P^\dagger(\alpha_1)P^\dagger(\beta_1))_\gamma^\dagger (P^\dagger(\alpha_2)P^\dagger(\beta_2))_\gamma|0\rangle \\
&= \delta_{\alpha_1\alpha_2}\delta_{\beta_1\beta_2} + \sum_{\alpha_3} \langle\beta_2|(P^\dagger(\alpha_1)P^\dagger(\alpha_3))_{\beta_2}|0\rangle^* \langle\beta_1|(P^\dagger(\alpha_2)P^\dagger(\alpha_3))_{\beta_1}|0\rangle \hat{\beta}_1\hat{\beta}_2 \begin{Bmatrix} \alpha_1 & \alpha_2 & \beta_2 \\ \alpha_2 & \gamma & \beta_1 \end{Bmatrix} \\
&+ \sum_{\alpha_3\alpha_4\alpha_5} \sum_{\beta} Y(\alpha_3\alpha_4;\beta_2)\langle\beta_1|(P^\dagger(\alpha_3)P^\dagger(\alpha_5))_{\beta_1}|0\rangle [\langle\beta|(P^\dagger(\alpha_1)P^\dagger(\alpha_5))_\beta|0\rangle^* \langle\beta|(P^\dagger(\alpha_2)P^\dagger(\alpha_4))_\beta|0\rangle \\
&- \delta_{\alpha_1\alpha_2}\delta_{\alpha_4\alpha_5} - (-1)^{\alpha_1+\alpha_5-\beta}\delta_{\alpha_1\alpha_4}\delta_{\alpha_2\alpha_5}]\hat{\beta}^2\hat{\beta}_1\hat{\beta}_2 \begin{Bmatrix} \alpha_3 & \alpha_4 & \beta_2 \\ \alpha_2 & \gamma & \beta \end{Bmatrix} \begin{Bmatrix} \alpha_3 & \alpha_5 & \beta_1 \\ \alpha_1 & \gamma & \beta \end{Bmatrix} .
\end{aligned} \tag{8}$$

Notice that all the quantities in the matrices (7) and (8) have been calculated when solving the two-particle or four-particle system. It is also worthwhile to point out that the last term in (8) would vanish if the operators $P^{\dagger}(\alpha)$ were real boson creation operators.

An important feature of eq. (7) is that it is even simpler than eq. (1). Thus, in this method, the calculation of the six-particle system seems easier than that of the four-particle system. However, the overlap matrix (8) is more difficult to evaluate than the overlap (2), specially since (2) is evaluated at the same time as (1).

To check our formalism (and our computer code), we applied it to the ^{88}Sr region, where there are only two single-particle states and all possible configurations can easily be included. We thus found complete agreement with the shell-model results of ref. [9]. However, our main purpose is to analyse the ^{204}Pb and ^{202}Pb spectra and (p, t) reactions [8]. A complete calculation in this region would be impossible. There would be, for instance, several hundred 2^+ states in ^{204}Pb and several thousand in ^{202}Pb. Still, we expect that the lowest states in both nuclei are contained in a rather small subspace of the total space spanned by the corresponding correlated basis (which also spans the shell-model space).

We took the two-particle energies and wavefunctions from the calculation by Blomqvist [10] which, including six single-particle levels, fits well most of the experimental data in ^{206}Pb. To calculate ^{204}Pb we took the first 19

Table 1
Experimental [8] and theoretical energies (in MeV) and (p, t) cross sections [relative to the ^{208}Pb(p, t)^{206}Pb(J_1^{π}) cross section]. In (a) only σ_{rel}(theor) > 0.003 are given while in (b) σ_{rel}(theor) > 0.03. The calculated (experimental) ground state energies, relative to the ^{208}Pb core, are 28.883 (28.925) MeV for ^{204}Pb and 43.983 (44.112) MeV for ^{202}Pb. The cross sections were calculated using the code DWUCK [12] with optical parameters as in ref. [8]. In parentheses are doubtful experimental data.

(a) ^{206}Pb(p, t)^{204}Pb(J^{π})						(b) ^{204}Pb(p, t)^{202}Pb(J^{π})					
experiment			theory			experiment			theory		
J^{π}	E	σ_{rel}	J^{π}	E	σ_{rel}	J^{π}	E	σ_{rel}	J^{π}	E	σ_{rel}
0_1^+	0	1.74	0_1^+	0	1.54	0_1^+	0	2.31	0_1^+	0	2.02
(0_2^+)	1.584	(0.10)	0_2^+	1.561	0.004	2_1^+	0.961	0.56	2_1^+	0.884	0.44
0_3^+	1.728	0.17	0_3^+	1.655	0.28	4_1^+	1.383	0.48	4_7^+	1.322	0.65
2_1^+	0.899	0.63	2_1^+	0.893	0.54	6_1^+	2.747	0.55	6_{18}^+	2.512	0.15
2_2^+	1.351	0.015	2_3^+	1.446	0.088	5_1^-	2.040	1.28	5_5^-	2.093	0.64
2_3^+	1.663	0.12	2_5^+	1.739	0.005	–	–	–	5_{12}^-	2.577	0.49
2_4^+	1.958	0.05	2_6^+	1.844	0.089	9_1^-	2.172	0.93	9_2^-	1.975	0.66
2_5^+	2.103	0.015	2_9^+	2.156	0.0074	–	–	–	9_4^-	2.273	0.18
4_1^+	1.274	0.65	4_1^+	1.156	0.68						
4_2^+	1.563	0.22	4_2^+	1.431	0.21						
4_3^+	1.816	0.05	4_3^+	1.781	0.18						
4_4^+	2.897	0.20	4_{12}^+	2.562	0.14						
6_1^+	2.808	0.77	6_2^+	2.557	0.79						
(5_1^-)	2.257	(1)	5_1^-	2.233	1.26						
5_2^-	2.505	0.48	5_2^-	2.432	0.83						
(7_1^-)	2.257	(0.70)	7_1^-	2.127	0.57						
(7_2^-)	2.399	0.11	7_2^-	2.381	0.054						
9_1^-	2.186	0.88	9_1^-	2.148	0.88						

states in ^{206}Pb plus the state 6_1^+ to build the basis used in (5). Of all the possible elements of the basis thus formed we chose for each spin and parity the first 50 states with lowest zeroth order energy [7]. In this basis we solved eqs. (1) and (4) as previously described. We took this rather large basis because we wanted to have a good description of ^{204}Pb to calculate ^{202}Pb.

To describe ^{202}Pb we proceeded in the same fashion. We took the states in ^{206}Pb as mentioned above plus the 18 experimental states of table 1a in ^{204}Pb, but in this case we reduced our basis for each spin and parity to the first 20 elements. In table 1 we present the calculated spectra. In general, the agreement with the experimental data is reasonable for ^{204}Pb. The worst states are the 2_3^+, 4_4^+ and 6_1^+. All of them may feel strongly the effect of the truncation. Yet, other excitations (like particle–hole) may be relevant for some of the ^{204}Pb states [11]. In ^{202}Pb our truncation was so drastic that, in general, only the first state of each spin and parity is rather well explained. Again here the state 6_1^+ is poorly given, but notice that it is the *eighteenth* calculated state which has the largest (and still small as compared with the experimental value) two-particle transfer cross section.

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