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LETTER TO THE EDITOR

Separable interactions and excited states in open-shell nuclei

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Abstract. The relevant matrix elements of the Hamiltonian for a RPA description of collective states in open-shell nuclei are determined. For separable interactions it is found necessary to include the particle–particle and particle–hole interactions simultaneously. The energy-weighted sum rule for the electromagnetic operator (with angular momentum I) is greatly reduced by the use of the pairing interaction with the same angular momentum.

Separable interactions have been used widely (see Bohr and Mottelson 1975) in the description of nuclear collective and single-particle excitations. The introduction of the pairing interaction (Bohr *et al* 1958, Belyaev 1959) and the quadrupole–quadrupole (Elliott 1958) one made it possible to understand qualitative features of the low-energy spectra of medium and heavy nuclei (Kumar and Baranger 1968).

The excitations of double-closed-shell nuclei have been analysed extensively using particle–hole and/or particle–particle collective degrees of freedom (Bohr and Mottelson 1975). Both types of excitation are usually described microscopically using the RPA or TDA solutions of the nuclear Hamiltonian

$$\begin{aligned} H &= H_{\mathrm{sp}} + H_{\mathrm{tb}} \\ &= \sum_i \varepsilon_i C_i^+ C_i + \sum_{ijkl,J} V^J_{ij,kl}\{[C_i^+ C_j^+]^J[\bar{C}_k \bar{C}_l]^J\}^0_0 \\ &= \sum_i \varepsilon_i C_i^+ C_i + \sum_{ijkl,\lambda} Q^\lambda_{ij,kl}\{[C_i^+ \bar{C}_k]^\lambda[C_j^+ \bar{C}_l]^\lambda\}^0_0 \end{aligned} \qquad (1)$$

where C^+_{im} creates a particle in the state (i, m_i) while $\bar{C}_{nm_n}$ transforms under rotations as (n, m_n).

When performing a RPA description of the collective excitations with angular momentum I (neglecting exchange terms), only some of the matrix elements V^J or Q^λ are relevant. In particular, for particle–hole excitations only the $Q^\lambda_{ik,jl}$ with the $\lambda = I$ are needed while for particle–particle excitations the necessary information is contained in $V^J_{ij,kl}$ with $J = I$.

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The two-body Hamiltonian may be simulated by a separable pairing interaction

$$H_{\text{p-p}} = -4\pi \sum_{JM} G_J P^+_{JM} P_{JM}(2J+1)$$
$$P^+_{JM} = \sum_{j_1 j_2} \frac{\langle J_1 \| Y_J \| J_2 \rangle}{(2J+1)^{1/2}} [C^+_{j_1} C^+_{j_2}]^J_M \qquad (2a)$$

where all the pairing strengths G_J (Broglia *et al* 1974) (obtained, for example, by fitting the experimental energies in ^{208}Pb) can be considered as equal.

It is also possible to simulate the Hamiltonian by a separable particle–hole interaction, i.e.

$$H_{\text{p-h}} = -4\pi \sum_{\lambda\mu} \chi_\lambda Q^*_{\lambda\mu} Q_{\lambda\mu}$$
$$Q_{\lambda\mu} = -\sum_{j_1 j_2} \frac{\langle J_1 \| r^\lambda Y_\lambda \| J_2 \rangle}{(2\lambda+1)^{1/2}} [C^+_{j_1} \bar{C}_{j_2}]^\lambda_\mu . \qquad (2b)$$

The Hamiltonians (2*a*) and (2*b*) are simple versions of the nuclear Hamiltonian and they are helpful in describing bosonic excitations in systems where the concept of particle and hole is clear, as in the case of nuclei near a double closed shell.

The aim of this Letter is to study which are the relevant terms of the two-body Hamiltonian for an RPA description of the bosonic excitations in open-shell nuclei. We consider only separable interactions, like the ones defined in equations (2), and a system where only one type of particle is active, as in the Sn isotopes.

The first step in the description of these spherical superconductive nuclei consists of a Bogoliubov–Valatin transformation to quasiparticles:

$$\beta^+_{jm} = U_j C^+_{jm} - V_j \bar{C}_{jm} . \qquad (3)$$

In the RPA, the microscopic structure of the bosonic excitations (Γ^+_{nJM}) is given in terms of pairs of quasiparticles as

$$\Gamma^+_{nJM} = \sum_{j_1 j_2} \lambda^n_{j_1 j_2} [\beta^+_{j_1} \beta^+_{j_2}]^J_M - \mu^n_{j_1 j_2} [\bar{\beta}_{j_1} \bar{\beta}_{j_2}]^J_M . \qquad (4)$$

The amplitudes λ and μ as well as the boson excitation energy are determined by solving, in a linear approximation, the equations

$$[\Gamma_{nIM}, \Gamma^+_{n'I'M'}] = \delta_{nn'} \delta_{II'} \delta_{MM'} \qquad (5a)$$

$$[H, \Gamma^+_{nIM}] = \omega_{nI} \Gamma^+_{nIM} . \qquad (5b)$$

In order to impose (5*b*) it is necessary to write the Hamiltonian in terms of quasiparticles. Because of the linear approximation the only terms in the Hamiltonian that are relevant when performing the RPA must have one of these structures:

$$\{[\beta^+\beta^+]^I [\beta^+\beta^+]^I\}^0_0 \qquad (6a)$$

$$\{[\beta^+\beta^+]^I [\bar{\beta}\bar{\beta}]^I\}^0_0 \qquad (6b)$$

$$\{[\bar{\beta}\bar{\beta}]^I [\bar{\beta}\bar{\beta}]^I\}^0_0 . \qquad (6c)$$

For example, we can study the contributions of a separable Hamiltonian to terms of the form (6*a*) for a given I. If we consider the particle–hole version, the only term contributing is the one with $\lambda = I$, i.e. terms like

$$\frac{\chi_I \pi}{(2I+1)} \sum_{\mu j_1 j_2 j_3 j_4} \langle j_1 \| r^I Y_I \| j_2 \rangle \langle j_3 \| r^I Y_I \| j_4 \rangle (U_{j_1} V_{j_2} + U_{j_2} V_{j_1}) \times (U_{j_3} V_{j_4} + U_{j_4} V_{j_3})(-1)^{I-\mu} [\beta^+_{j_1} \beta^+_{j_2}]^I_\mu [\bar{\beta}_{j_3} \bar{\beta}_{j_4}]^I_{-\mu}. \tag{7a}$$

The particle–particle Hamiltonian can be obtained from the (p–h) one through a recoupling of the operators†. This gives the contribution to the RPA of all the λ terms equivalent to the $J = I$ term of the (particle–particle) Hamiltonian (2*a*). Therefore, terms of the type (6a) such as

$$4\pi G_I \sum_{\mu j_1 j_2 j_3 j_4} \langle j_1 \| Y_I \| j_2 \rangle \langle j_3 \| Y_I \| j_4 \rangle U_{j_1} U_{j_2} V_{j_3} V_{j_4} (-1)^{I-\mu} [\beta^+_{j_1} \beta^+_{j_2}]^I_\mu [\beta^+_{j_3} \beta^+_{j_4}]^I_{-\mu} \tag{7b}$$

will arise.

One must therefore consider for the RPA a Hamiltonian of the form

$$H = \sum_i E_i \beta_i^+ \beta_i - 4\pi\chi_I \sum_\mu Q^*_{I\mu} Q_{I\mu} - 4\pi G_I (2I+1) \sum_\mu P^+_{I\mu} P_{I\mu}. \tag{8}$$

The inclusion of the two interactions introduces a small double counting. If one considers, for example, a simple shell with degeneracy Ω, the contribution of the particle–hole Hamiltonian with $\lambda = I$ to the particle–particle one with $J = I$ is of order $1/\Omega$, which is of the order of the Racah coefficient involved. The sum of all the recouplings corresponding to all λ's is, instead, of order one and, as the RPA must be exact to this order, it must therefore include both types of interaction.

Solving equations (5) for this Hamiltonian (Bes *et al* 1966), the RPA energies are determined by the condition

$$\Delta(\omega) = \begin{vmatrix} 1-x & z & -u \\ z & 1-y & v \\ -u & v & 1-t \end{vmatrix} = 0 \tag{9}$$

where

$$x = -2\pi G_I \sum_{1,2} \langle 1 \| Y_I \| 2 \rangle^2 \left(\frac{U_1^2 U_2^2}{\omega - E_1 - E_2} - \frac{V_1^2 V_2^2}{\omega + E_1 + E_2} \right)$$

$$y = -2\pi G_I \sum_{1,2} \langle 1 \| Y_I \| 2 \rangle^2 \left(\frac{V_1^2 V_2^2}{\omega - E_1 - E_2} - \frac{U_1^2 U_2^2}{\omega + E_1 + E_2} \right)$$

$$z = -2\pi G_I \sum_{1,2} \langle 1 \| Y_I \| 2 \rangle^2 U_1 U_2 V_1 V_2 \frac{2(E_1 + E_2)}{\omega^2 - (E_1 + E_2)^2}$$

$$u = -\left(\frac{\pi G_I \chi_I}{(2I+1)} \right)^{1/2} \sum_{1,2} \langle 1 \| Y_I \| 2 \rangle \langle 1 \| r^I Y_I \| 2 \rangle (U_1 V_2 + U_2 V_1) \left(\frac{U_1 U_2}{\omega - E_1 - E_2} + \frac{V_1 V_2}{\omega + E_1 + E_2} \right)$$

$$v = -\left(\frac{\pi G_I \chi_I}{2I+1} \right)^{1/2} \sum_{1,2} \langle 1 \| Y_I \| 2 \rangle \langle 1 \| r^I Y_I \| 2 \rangle (U_1 U_2 + U_2 V_1) \left(\frac{V_1 V_2}{\omega - E_1 - E_2} + \frac{U_1 U_2}{\omega + E_1 + E_2} \right)$$

$$t = \frac{\chi_I}{2(2I+1)} \sum_{1,2} \langle 1 \| r^I Y_I \| 2 \rangle^2 (U_1 V_2 + U_2 V_1)^2 \frac{2(E_1 + E_2)}{\omega^2 - (E_1 + E_2)^2}.$$

† It must be noted that even if (2*a*) and (2*b*) are not related by a recoupling (as they are the separable interactions that reproduce the low-energy data), they can be considered as the particle–particle or particle–hole versions of the nuclear Hamiltonian.

Table 1. Energies, the square of the matrix elements of Q_2 (in fm^4), P_2^+ and P_2 and the summation of the square of the backward amplitudes for the five lowest 2^+ states obtained in ^{114}Sn are displayed. In the last line the EWSR for the three operators are given. The results are given without ($G_2=0$) and with ($G_2=19/A$ MeV) the quadrupole pairing interaction.

	$G_2=0$					$G_2=19/A$ MeV				
n	ω_n	$\vert\langle n\Vert Q_2\Vert 0\rangle\vert^2$	$\vert\langle n\Vert P_2^+\Vert 0\rangle\vert^2$	$\vert\langle n\Vert P_2\Vert 0\rangle\vert^2$	$\sum_{12}(\mu_{12}^n)^2$	ω_n	$\vert\langle n\Vert Q_2\Vert 0\rangle\vert^2$	$\vert\langle n\Vert P_2^+\Vert 0\rangle\vert^2$	$\vert\langle n\Vert P_2\Vert 0\rangle\vert^2$	$\sum_{12}(\mu_{12}^n)^2$
1	1.30	4500	0.10	0.047	0.23	1.30	2500	0.22	0.14	0.03
2	2.87	72	0.013	0.0001	0.002	2.84	41.2	0.032	0.002	0.0006
3	3.21	9.80	0.015	0.003	0.0002	3.13	3.0	0.0003	0.003	0.001
4	3.25	3.90	0.010	0.012	0.0001	3.23	0.61	0.0003	0.005	0.0002
5	3.33	4.71	0.003	0.0001	0.0001	3.33	0.80	0.007	0.003	0.00002
$\sum_n \omega_n\vert\langle n\Vert\hat{O}\Vert 0\rangle\vert^2$		8700	5.4	3.0			4300	5.3	3.2	

Table 2. The same as table 1 for ^{120}Sn.

	$G_2=0$					$G_2=19/A$ MeV				
n	ω_n	$\lvert\langle n\Vert Q_2\Vert 0\rangle\rvert^2$	$\lvert\langle n\Vert P_2^+\Vert 0\rangle\rvert^2$	$\lvert\langle n\Vert P_2\Vert 0\rangle\rvert^2$	$\sum_{12}(\mu_{12}^n)^2$	ω_n	$\lvert\langle n\Vert Q_2\Vert 0\rangle\rvert^2$	$\lvert\langle n\Vert P_2^+\Vert 0\rangle\rvert^2$	$\lvert\langle n\Vert P_2\Vert 0\rangle\rvert^2$	$\sum_{12}(\mu_{12}^n)^2$
1	1.17	4600	0.047	0.069	0.22	1.17	2800	0.12	0.16	0.044
2	2.69	26	0.0020	0.0029	0.0007	2.68	11	0.0024	0.0079	0.00004
3	2.81	32	0.0013	0.0086	0.0008	2.79	25	0.0028	0.024	0.0004
4	3.10	72	0.3×10^{-7}	0.021	0.016	3.05	27	0.6×10^{-4}	0.044	0.0002
5	3.65	61	0.75×10^{-3}	0.054	0.011	3.54	10	0.0020	0.083	0.0006
$\sum_n \omega_n \lvert\langle n\Vert \hat{O}\Vert 0\rangle\rvert^2$		9500	3.6	4.4			5200	3.9	4.2	

The amplitudes that give the boson structure can be obtained from (5), and using these amplitudes it is possible to evaluate the matrix elements of $Q_{I\mu}$, $P^+_{I\mu}$ and $P_{I\mu}$ which are related to the electromagnetic transitions, (t, p) and (p, t) reactions respectively.

To estimate the influence of the pairing terms (2*a*) on the description of the excited states, the 2^+ states of ^{114}Sn and ^{120}Sn were evaluated with and without the pairing interaction (2*a*). The single-particle energies of Uhrer and Sorensen (1966) were used and G_0 was chosen equal to $19/A$ ($=G_2$ when (2*a*) was included). In both cases χ_2 was chosen to reproduce the experimental energy of the first 2^+ state.

Bes *et al* (1966) found that the inclusion of the $I=2$ pairing interaction diminished the backward amplitudes and therefore we also evaluate the $\Sigma_{1,2}\,(\mu^n_{12})^2$ which are related to the ground-state correlations.

Tables 1 and 2 show the frequencies, the squares of the reduced matrix elements for Q_2, P^+_2 and P_2 and $\Sigma_{1,2}\,(\mu^n_{12})^2$ for the five lowest states in ^{114}Sn and ^{120}Sn as well as the energy-weighted sum rule for Q_2, P^+_2 and P_2.

There are two main features introduced by taking terms of type (2*a*) into account. One is a rather large increase in the two-particle transfer spectroscopic amplitudes and the decrease of the matrix element associated with the electromagnetic transition to the first excited state. The other feature is the drastic decrease of the energy-weighted sum rule for the operator Q. These two features may be considered as due to the almost complete disappearance of the backward amplitudes and therefore the ground-state correlations.

The conclusions of this work are that the relevant matrix elements of the Hamiltonian for a RPA calculation in open-shell nuclei of excitations of angular momentum I are $V^I_{ij,kl}$ and $Q^I_{ik,jl}$, and that both sets of matrix elements must be considered simultaneously. The results obtained show that the consideration of the particle–particle channel has a strong influence on the matrix elements of the specific particle–hole operator and its energy-weighted sum rule.

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