Low frequency drift dissipative modes in field reversed configurations

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ABSTRACT

Low frequency drift dissipative modes are studied in the parameter regime relevant to field reversed configurations. It is shown that previous theories (Phys. Fluids **30**, 878(1987)) are not correct in that the effect of collisions was not properly included. Analytic and numerical studies with the correct dispersion relation have yielded no unstable modes.

Field Reversed Configurations (FRCs) are elongated compact toroids with a high average beta and ideally only poloidal magnetic field . They have been studied for many years, both experimentally and theoretically, and their properties and different formation methods are summarized elsewhere[1-7].
It is well known that the equilibrium phase of FRCs is characterized by an anomalous energy and flux transport[4,5] that is believed to be the consequence of some active microinstability. FRC transport models based on the lower hybrid drift instability (LHDI) have not been completely successful in reproducing the observed scaling and an attempt to measure LHD induced turbulence in the TRX-2 device yielded negative results[8]. This indicates that the LHDI is not the cause of the anomalous transport and other instabilities must be considered. Low frequency drift dissipative[9] and temperature gradient driven[10] instabilities have recently been proposed as responsibles for the anomalous transport.
In this letter we show that the dispersion relation used in Ref. 9 to study low frequency drift dissipative instabilities is not correct in that the effect of collisions has not been properly included. Using the same assumptions and approximations as in Ref. 9 we derived a new dispersion relation that correctly includes the effect of collisions. This dispersion relation was solved analytically and numerically and the Nyquist technique was used to search for unstable modes. The analytic solution (obtained using the same approximations as in Ref. 9) corresponds to a stable mode with ω_r of the order of the ion diamagnetic drift frequency. The numerical studies consisted on finding the roots of the exact dispersion relation (eq. (8)) and using the Nyquist method with an integration path defined as a semicircle of radius Ω_i and $\mathrm{Im}(\omega) < 0$ (note we used $\exp(i\omega t)$ so that $\gamma < 0$ corresponds to instability). Both methods were applied over a wide range of values of mode number (both components) and equilibrium parameters (density gradient scale lengh, magnetic field, temperature, etc) and no unstable modes were found. These results seem to indicate that within the framework of the basic approximations used in Ref. 9 (uniform magnetic field, local theory, etc) there are no unstable dissipative modes and therefore transport theories based on this model[11] should be revised.
The dispersion relation is derived in slab geometry, considering low frequency ($|\omega| << |\Omega_i|$) electromagnetic modes with $\mathbf{E} = E_y\hat{y}+E_z\hat{z}$ and using the local approximation. The equilibrium magnetic field is assumed to be in the $\hat{z}$ direction and the mode to propagate in the yz plane ($\mathbf{k}=k\hat{y}+K\hat{z}$). The space and time dependence of the perturbed quantities is given by:

$$\exp\{i[ky+Kz+\omega t]\} \qquad (1)$$

where the positive sign in the temporal dependence has been used to agree with Ref. 9. In addition, the density is assumed to vary in the $\hat{x}$ direction, the temperature is considered uniform and the drift due to the magnetic field gradient is neglected in the calculation of the unperturbed particle trajectories. The last approximation is not well justified in a high beta plasma where the scale lengths associated with the density and magnetic field gradients are of the same order and it is used to obtain analytic results and to compare with Ref. 9. Finally, a very simple Krook model is used to introduce the effect of collisions. Using the following equilibrium distribution functions for electrons and ions

$$f_{0_i} = f_{M_i}\left[1+\mathcal{E}_m\left(x+\frac{v_y}{\Omega_i}\right)\right] \qquad f_{0_e} = f_{M_e}\left[1+\mathcal{E}_m\left(x-\frac{v_y}{\Omega_e}\right)\right] \qquad (2)$$

where

$$f_{M_\alpha} = \frac{n_0}{(\pi v_{T_\alpha}^2)^{3/2}}\, e^{-v^2/v_{T_\alpha}^2}\,, \quad \mathcal{E}_m = \frac{n_0'}{n_0}\,, \quad v_{T_\alpha}^2 = \frac{2T_\alpha}{m_\alpha}$$

the solution of the linearized Boltzman equation can be written as[12]

$$f_1(\underline{v}) = -\frac{q}{m}\int_{-\infty}^{0} d\tau \left\{\left[\underline{E}_1 + \frac{\underline{v}'\times\underline{B}_1}{c}\right]\cdot\nabla_{\underline{v}'} f_0 + \frac{n_1}{n_0}\nu f_M\right\} \times$$
$$\times \exp\left\{i\left[k(y'-y) + K(z'-z) + (\omega - i\nu)\tau\right]\right\} \qquad (3)$$

where the time integral is along the unperturbed particle trajectories ν is the collision frequency and we have omitted the species index. Using Maxwell's equations to write the magnetic field in terms of the electric field and considering low frequency perturbations we get:

$$f_1(\underline{v}) = \frac{2q f_0}{m v_T^2}\left\{\frac{E_y}{ik} + \left[E_z v_z\left(1+\frac{\omega^*}{\omega}\right) - \frac{E_y}{k}\left\{\omega - i\nu + K v_z + \omega^*\left(1+\frac{K v_z}{\omega^*}\right)\right\} + \right.\right.$$
$$\left.\left. + \frac{m v_T^2 \nu n_1}{2q n_0}\frac{f_M}{f_0}\right]\sum_\ell J_\ell J_0 \frac{e^{-i\ell\pi/2} e^{i\ell\alpha}}{i(\omega - i\nu + K v_z)}\right\} \qquad (4)$$

where $J_l(kv_\perp/\Omega)$ is a Bessel function and $\omega^* = k\varepsilon_n v_T^2/2\Omega$
The perturbed density and $\hat{z}$ component of the current can be easily calculated now. Introducing,

$$\Gamma_0\left(\frac{k^2a^2}{2}\right) = I_0\left(\frac{k^2a^2}{2}\right)\exp\left\{-\frac{k^2a^2}{2}\right\}$$

where I_0 is the modified Bessel function and a the Larmor radius and

$$W\left(\frac{\omega - i\nu}{|K|\, v_T}\right) = \frac{-1}{\sqrt{\pi}\, v_T}\int_{-\infty}^{\infty} dv\, \frac{Kv\, e^{-v^2/v_T^2}}{\omega - i\nu + Kv}$$

we get

$$n_1 = n_1^K + \frac{2q\, n_0\, \omega^* \nu\, \Gamma_0\, W}{v_T^2\, m\, \omega\left[\omega + i\nu\left(\Gamma_0\,(W+1) - 1\right)\right]}\left(\frac{E_z}{K} - \frac{E_y}{k}\right) \tag{5}$$

$$J_{1z} = J_{1z}^K - \sum_\alpha \frac{\omega_{p\alpha}^2}{2\pi v_{T\alpha}^2}\,\frac{\omega_\alpha^*}{\omega}\,\nu_\alpha\,\Gamma_{0\alpha}\,\frac{W_\alpha}{K}\left(\frac{E_z}{K} - \frac{E_y}{k}\right)\frac{\left[\omega - i\nu_\alpha(1-\Gamma_{0\alpha})\right]}{\omega + i\nu_\alpha\left[\Gamma_{0\alpha}(1+W_\alpha) - 1\right]}$$

where

$$n_1^K = \frac{2q\, n_0\, i}{m\, v_T^2\left[1 + \frac{i\nu\,\Gamma_0(W+1)}{(\omega - i\nu)}\right]}\left\{\frac{E_z\Gamma_0}{K}\left(1 + \frac{\omega^*}{\omega - i\nu}\right)W + \frac{E_y}{k}\left[-1 + \Gamma_0\left(1 + \frac{\omega^*}{\omega - i\nu}\right)\right]\right\} \tag{6}$$

$$J_{1z}^K = \sum_\alpha \frac{\Gamma_0\, i\nu_\alpha\, q_\alpha\, n_{1\alpha}^K\, W_\alpha}{K} - \sum_\alpha \frac{2q_\alpha^2\, n_{0\alpha}\, i\,\Gamma_{0\alpha}}{m_\alpha v_{T\alpha}^2 K^2} W_\alpha\left(\omega - i\nu_\alpha + \omega_\alpha^*\right)E_z$$

are the perturbed density and current given in Ref. 9. It is clear that our results agree with those of Ref. 9 only when

$$\frac{E_z}{K} = \frac{E_y}{k} \tag{7}$$

Since this is satisfied only for electrostatic modes we conclude that the dispersion relation used in Ref. 9 is not correct to study drift dissipative electromagnetic modes. The problem arises because the recipe used in Ref. 9 to calculate the collisional density and current starting from the collisionless results is not correct in the electromagnetic case. The prescription of replacing ω by ω-iν everywhere fails because the use of Maxwell's equation ($\nabla\times E = -c^{-1}\partial B/\partial t$) to eliminate the magnetic field in terms of the electric field introduces an ω (not ω-iν) even if collisions are present. Using the correct form for the perturbed density and current we obtain the following dispersion relation:

$$\left\{-k^2+\sum_\alpha \frac{2\omega_{p\alpha}^2}{V_{T\alpha}^2 A_\alpha}\left[\Gamma_0(\omega-i\nu_\alpha+\omega_\alpha^*)-\omega+i\nu_\alpha+\frac{i\nu_\alpha\omega_\alpha^*\Gamma_0 W_\alpha}{\omega}\right]\right\}\times$$

$$\times\left\{\frac{k^2K^2c^2}{\omega^2}+\sum_\alpha \frac{2\omega_{p\alpha}^2\Gamma_0 W_\alpha}{V_{T\alpha}^2\omega A_\alpha}\left[\omega+\omega_\alpha^*-i\nu_\alpha\left(1+\frac{\omega_\alpha^*}{\omega}\right)\right]\left[i\nu_\alpha(1-\Gamma_0)-\omega\right]\right\}-$$

$$-\left\{-K^2+\sum_\alpha \frac{2\omega_{p\alpha}^2}{V_{T\alpha}^2 A_\alpha} W_\alpha\Gamma_0\left[\omega+\omega_\alpha^*-i\nu_\alpha\left(1+\frac{\omega_\alpha^*}{\omega}\right)\right]\right\}\times \tag{8}$$

$$\times\left\{-\frac{k^2K^2c^2}{\omega^2}+\sum_\alpha \frac{2\omega_{p\alpha}^2\Gamma_0 W_\alpha}{V_{T\alpha}^2\omega A_\alpha} i\nu_\alpha(1-\Gamma_0)\left[\omega+\omega_\alpha^*-i\nu_\alpha\left(1+\frac{\omega_\alpha^*}{\omega}\right)\right]\right\}=0$$

where

$$A_\alpha = \omega - i\nu_\alpha + i\nu_\alpha\Gamma_0(W+1)$$

Assuming $ka_i \ll 1$ and recognizing that $\nu_e \gg \nu_i$ the dispersion relation can be reduced to

$$\left[1-\frac{\omega(\omega+\omega_i^*)}{K^2V_A^2}\right]\sum_\alpha\left(\frac{T_i}{T_\alpha}\right)\frac{(\omega-i\nu_\alpha+\omega_\alpha^*)}{\omega+i\nu_\alpha W_\alpha}\Gamma_{o\alpha}W_\alpha =$$

$$= k^2 a_i^2\frac{(\omega+\omega_i^*)}{\omega}+\left[1-\frac{\omega(\omega+\omega_i^*)}{K^2V_A^2}\right]\frac{i\,T_e W_e \nu_e \omega_e^*}{T_i\,\omega(\omega+i\nu_e W_e)}+$$

$$+\frac{k^2a_i^2}{K^2c^2}\left[2+\left(\frac{c}{V_A}\right)^2\frac{(\omega+\omega_i^*)}{\omega}\right]\left[W_i\Gamma_{oi}(\omega+\omega_i^*)\nu_i+\right. \qquad (9)$$

$$\left.+\;W_e(\omega+\omega_e^*)\nu_e\frac{m_e}{m_i}\left(\frac{\omega-i\nu_e}{\omega+i\nu_e W_e}\right)\right]$$

where

$$V_A^2=\frac{B^2}{4\pi n m_i}\quad (\text{Alfvèn velocity})$$

Considering $|K v_{T_i}|\ll\omega$, $|\omega-i\nu|\ll|K v_{T_e}|$, $\omega\simeq\omega_i^*$ and $\omega^2\gg K^2V_A^2$ we finally get:

$$\omega_r=-\omega_i^*-\frac{K^2V_A^2}{\omega_i^* D}\left(1+\frac{T_e}{T_i}\right)\left\{1+\frac{T_e}{T_i}\left[1-\left(\frac{k a_i K V_A}{\omega_i^*}\right)^2\right]\right\}$$

$$\gamma=\sqrt{\pi}\left(1+\frac{T_e}{T_i}\right)\frac{T_e}{T_i}\;\frac{K^4V_A^4k^2a_i^2}{\omega_i^{*2}|K|v_{T_e}\,D\left[1-\frac{\sqrt{\pi}\,\nu_e}{|K|v_{T_e}}\right]} \qquad (10)$$

where

$$D=\left\{1+\frac{T_e}{T_i}\left[1-\left(\frac{k a_i K V_A}{\omega_i^*}\right)^2\right]\right\}^2+\left\{\frac{\sqrt{\pi}\,T_e k^2a_i^2K^2V_A^2}{|K|v_{T_e}\omega_i^{*2}T_i\left(1-\sqrt{\pi}\nu_e/|K|v_{T_e}\right)}\right\}^2$$

Since γ is always positive we conclude that the modes are stable. Summarizing, we have studied low frequency drift dissipative instabilities in the parameter regime corresponding to FRCs using a dispersion relation that correctly introduces the effect of collisions. Both the analytic calculation and the numerical studies indicate that there are no unstable low frequency drift dissipative modes. This result disagrees with previous theories and the difference is due to the incorrect treatment of collisional effects used in Ref. 9.

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