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Low angle electron scattering in thermal resistivity

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Abstract. Electrical (ρ) and thermal (W) resistivity measurements in very dilute In alloys between 1 K and 4 K show a temperature dependence expressed by $\rho = \rho_0 + \beta T^5$ and $WT = (WT)_0 + AT^3 + BT^5$. The BT^5 term in the thermal resistivity is due to a contribution from a 'diffusion-like process' of electrons over the Fermi surface, as is the one that describes the temperature dependence of the electrical resistivity. This claim is supported by the experimental fact that the Lorenz number defined as $L = \beta/B$ coincides with the value given by $L = \rho_0/(WT)_0$ which in turn agrees with the free electron value within 15%.

1. Introduction

In a previous paper (Bressan *et al* 1975) we have shown that the only expression that fits the electrical resistivity data for In is of the form

$$\rho = \rho_0 + \beta T^5. \tag{1}$$

The temperature dependence expressed by (1) is usually associated with a low angle electron–phonon scattering. Such 'normal' processes are responsible for a diffusion-like movement of the electrons over the Fermi surface ('horizontal' processes). It is generally accepted that such normal processes contribute a T^2 term to the thermal resistivity. We have measured the thermal resistivity of the same two samples used before (Bressan *et al* 1975) to check the validity of these assumptions. The electrical and thermal measurements were done over the same range of temperatures.

Indium becomes superconducting below 3·4 K. In order to obtain the thermal resistivity at low temperatures we measured the longitudinal thermal magnetoresistance and extrapolated to $H = 0$. A careful and detailed study was required to determine a universal thermal magnetoresistance behaviour that allows for a correct extrapolation.

2. Experimental details

We have measured the electrical and thermal resistivities of two high purity polycrystalline samples of In between 1·4 K and 4·2 K with resistivity ratios of 73 000 and 57 000. A more detailed experimental description of the electrical measurements and

of the doping method used to modify the resistivity ratio has been given elsewhere (Bressan *et al* 1970).

The experimental set up was designed in such a way that the electrical and thermal measurements were performed in the course of the same experiment. The voltage and the thermal drop were measured at the same points 7 cm apart from each other. The samples had square cross sections of 0·063 cm^2. The thermal resistivity was determined by applying a DC heat input at one end of the sample and measuring the temperature drop with two carbon thermometers. The other end of the sample was attached to a ^{4}He bath. The temperature drop, ΔT, was always of the order of 5% of the average sample temperature. The precision obtained was of the order of 2% per experimental point.

A superconducting coil supplied a longitudinal magnetic field up to 15 kOe. The carbon resistor thermometers were calibrated against the ^{4}He vapour pressure. Magnetoresistance effects on the thermometers were corrected when needed.

3. Experimental results and extrapolation procedure

The experimental thermal resistivity data for both samples, measured between T_c and 4·2 K at zero magnetic field, were used to fit a temperature dependence of the form

$$WT = (WT)_0 + AT^3. \tag{2}$$

Using this expression and the corresponding one for the electrical resistivity of the same samples (equation (1)) (Bressan *et al* 1975) we define the zero temperature Lorenz number as the ratio $L = \rho_0/(WT)_0$. The values so obtained for samples B and C are $14{\cdot}4 \times 10^{-8}$ and $-8{\cdot}42 \times 10^{-8}$ $W\Omega K^{-2}$ respectively.

Since the electron–impurity scattering is the dominant one for the electrical and thermal processes at low enough temperature, the ratio $\rho_0/(WT)_0$ should coincide with the free electron Lorenz number (Ziman 1960).

The value of ρ_0 has been well established (Bressan *et al* 1975). Consequently we conclude that expression (2) does not describe properly the temperature dependence of the thermal resistivity in the low temperature limit. To establish the correct temperature dependence we extended the thermal resistivity measurements below T_c.

The main difficulty in obtaining reliable electrical and thermal resistivity data for pure superconducting materials in the normal state is that the measurements have to be done in the presence of magnetic fields higher than the critical field. Strong magnetoresistance effects are present and a correct extrapolation to $H = 0$ is not trivial.

It has been shown previously (Bressan *et al* 1970, 1975) that the use of an incorrect extrapolation procedure for the electrical resistivity gives misleading results.

It was further shown that Kohler's rule, as generalized by Jones and Sondheimer (1964) can be used to describe the magnetoresistance in terms of a universal function F which is independent of purity and temperature. Kohler's modified rule states that

$$\frac{\rho(H,T) - \rho(0,T)}{g\rho(0,T)} = F\left(\frac{H}{g\rho(0,T)}\right) \tag{3}$$

where ρ (H,T) is the resistivity at field H and temperature T and g is a function of temperature to be determined. The function F can be used for extrapolation.

Expression (3) can be extended in a natural way to the thermal resistivity

$$\frac{W(H, T)T - W(0, T)T}{hW(0, T)T} = G\left(\frac{H}{hW(0, T)T}\right) \tag{4}$$

where $W(H,T)$ is the thermal resistance at field H and temperature T and h is another arbitrary function.

When g and h are temperature and purity independent, expressions (3) and (4) coincide with the traditional Kohler's rule.

The procedure for extrapolating to zero magnetic field is accomplished in two steps:

(i) The universality of the function G is established over a range of temperatures greater than T_c to avoid the superconducting state at low fields. $W(H, T)$ was measured as a function of field at seven different, constant temperatures between T_c and 4·2 K. For each temperature $W(H,T)$ was determined at ten different values of H. We plotted

$$\frac{W(H, T) - W(0, T)}{W(0, T)} \text{ against } \frac{H}{W(0, T)T} \tag{5}$$

on log–log paper for each temperature and found that it was possible to superimpose all the resulting curves, within experimental error, by displacing them along a 45° axis. This establishes the universality of the function G since a 45° displacement corresponds merely to a change in the value of h. Choosing an arbitrary temperature T as a reference, the shift that is necessary to superimpose the other curves determines the value of h for each temperature, within an irrelevant constant factor. Once h is known for each temperature a single universal curve represented by expression (4) is plotted on log–log paper.

(ii) Below T_c, neither $W(0, T)$ nor h is known. To determine these values we measured $W(H, T)$ against H at fields larger than the critical field. Measurements of $W(H, T)$ at constant T were taken at 21 different temperatures between 1·4 and 3·4 K. Now, for each set of data, a particular value of $W(0, T)$ was chosen so that when the data were plotted as in (5), the resulting curve had precisely the universal shape, and could be superimposed on the universal curve by a 45° displacement. This method allows for a determination of $W(0, T)$ with an accuracy of 2%.

The universal function obtained using all $W(H,T)$ values for all T and H is shown in figure 1. Figure 2 is a plot of h against T obtained as explained before.

It is important to remark that the universal function shown in figure 1 contains the experimental data not only at different temperatures and fields but from the two samples with different impurity content. Moreover, on a log–log graph, the thermal universal function G (equation (4)) matches exactly the electrical universal function F of expression (3).

4. Discussion

Figure 3 shows a plot of $W(0, T)$ against T^3. Deviations from the expected T^3 dependence are evident. There is no straightforward explanation for these deviations since

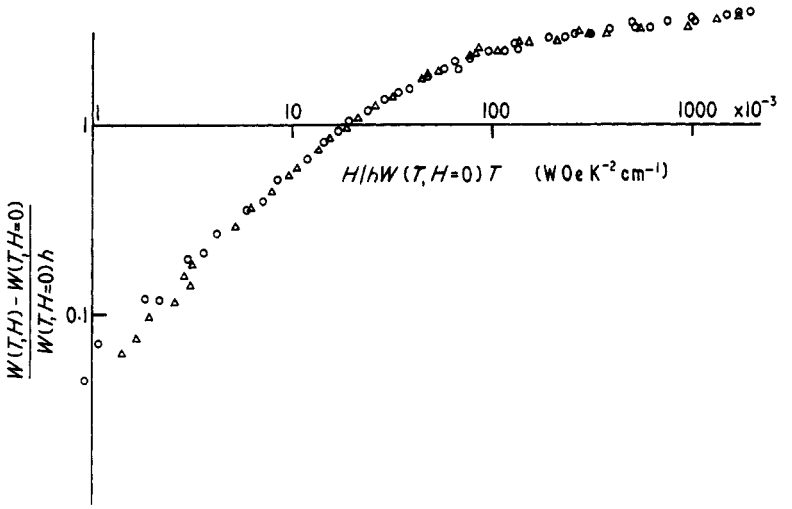

Figure 1. Universal magnetic behaviour of the thermal resistivity. The curve is a result of measurements of magnetoresistance from 10 Oe to 10 kOe at 26 different temperatures. For greater clarity only 20% of the experimental data are actually plotted. Δ, sample B; O, sample C (see table 1).

the 'vertical' processes (thermal case) should be much less sensitive than the diffusion movement (electric case) to the distortion of a real Fermi surface.

A possible interpretation could be that a 'diffusion' process similar to the one mentioned for the electrical case also contributes to the thermal resistance (Klemens 1956). If this is the case the thermal resistivity data should be fitted to an expression of the form

$$WT = (WT)_0 + AT^3 + BT^5. \tag{6}$$

As can be seen in figure 3, a good fit is obtained throughout the temperature range.

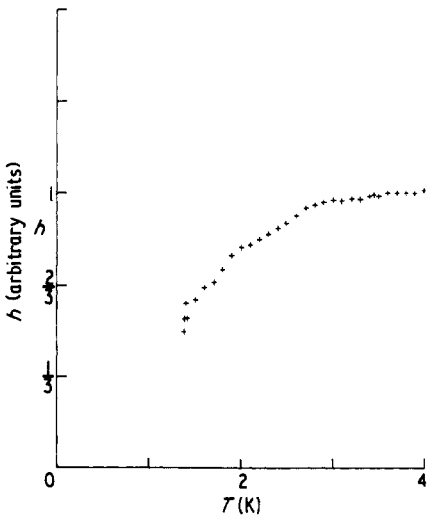

Figure 2. Temperature dependence of the function h, for sample B, obtained as explained in the text. A similar function was obtained for sample C. Kohler's rule should be obeyed in the region h = constant.

Table 1.

Sample	Resistivity ratio $\rho(273\ K)/\rho(0\ K)$	Electrical resistivity ρ_0 $(10^{-11}\ \Omega\,cm)$	β $(10^{-11}\ \Omega cm\,K^{-5})$	Thermal Resistivity $(WT)_0$ $(10^{-3}\ cm\ K^2\ W^{-1})$	A $(10^{-3}\ cm\,K^{-1}\,W^{-1})$	B $(10^{-3}\ cm\,K^{-3}\,W^{-1})$	Lorentz number $\rho_0/(WT)_0$ $(10^{-8}\ W\Omega K^{-2})$	β/B $(10^{-8}\ W\,\Omega\,K^{-2})$
B	73 000	12·45	0·027	4·56	0·62	0·010	2·7	2·8
C	57 000	15·91	0·030	6·76	0·69	0·013	2·4	2·3

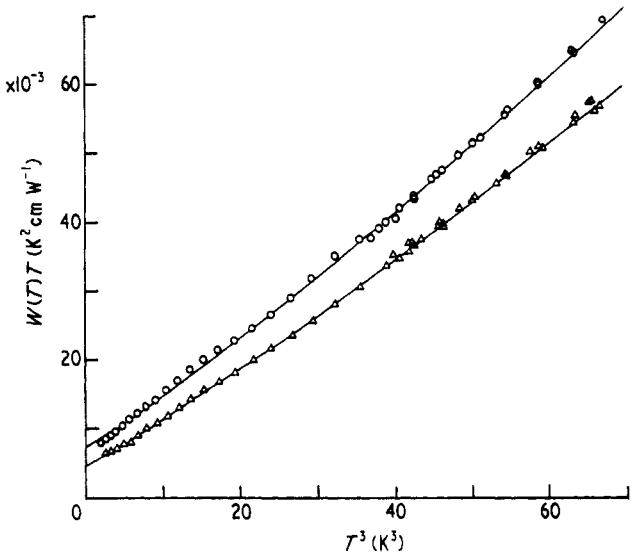

Figure 3. Temperature dependence of the thermal resistivity. The solid line corresponds to $WT = (WT)_0 + AT^3 + BT^5$. The coefficients are given in the table. Δ, sample B; ○, sample C.

If expression (6) represents correctly the temperature dependence of the thermal resistivity, $(WT)_0$ should give the contribution from impurity scattering. The values obtained for the fitting coefficients $(WT)_0$, A and B for both samples are shown in table 1. In the table can also be found the corresponding values of ρ_0 and β of equation (1) and the values for the Lorenz number at $T = 0$. From the fact that the Lorenz number coincides within 15% with the free electron value, we conclude that expression (6) reflects the correct low temperature behaviour. It is striking that the same value is obtained for the ratio of the T^5 dependent terms of the electrical and thermal resistivities. This gives strong support to our earlier suggestion that both terms are due to the same thermalization process. However theoretical calculations of thermal resistivity taking electron–phonon scattering into account (Wilson 1937, Ehrlich 1973) have led to an expression of the same form as (6) but with a negative coefficient B. Using the standard expression for the thermal resistivity, we have

$$B = \frac{4\rho_\theta}{L_0\theta^5}\left(J_5(\theta/T) - \frac{1}{2\pi^2}J_7(\theta/T)\right) \tag{7}$$

where we have used Ziman's notation (Ziman 1960). At low temperatures the J_7 term is dominant. Within the same approximation used to obtain (7), the standard formula for the coefficient β of equation (1) is given by

$$\beta = 4\rho_\theta J_5(\theta/T). \tag{8}$$

In several metals the experimental results give a coefficient β many times greater than those obtained from equation (8). Klemens (1956) has suggested that the coefficient β should increase for metals whose Fermi surface touches the zone boundary.

He reasons that as a consequence of this contact the distance that an electron has to travel to reach equilibrium by 'horizontal' many-step diffusion is effectively shortened. Quite recently Ehrlich and Schriempf (1974) have measured the thermal resistivity of Ag and found deviations from a simple T^3 dependence. They suggested that the term with J_5 in expression (7) could be enhanced by the same factor that multiplies the β coefficient of the electrical resistivity, without substantial modification of the term that contains J_7. However their numerical values obtained using expressions of the form (1) and (6) do not support their assumption. They suggested that this is due to the different electron distribution in momentum space occurring when either the vertical or the diffusion-like process is dominant, in analogy to the electrical case where the elastic and diffusion processes are competing (Ehrlich 1970). In our In samples, at temperatures above 2·0 K, the 'vertical' thermalization process is dominant. However we have proved that the coefficients β and B of expressions (1) and (6) are increased by the same factor, in agreement with the suggestion by Ehrlich and Schriempf (1974).

From our point of view there are two main reasons which explain the different behaviour of these two metals. When real Fermi surfaces are taken into account, variations of the J_5 and J_7 terms in equation (7) should be expected, since both terms originate in the same electron–phonon scattering process. For our samples, β has been increased (following Ehrlich and Schriempf 1974, equations (1′) and (2′)) by a factor of 56 with respect to the value given by equation (8), where $\theta = 109$ K has been used.

Indeed, considering the experimental error, the negative term in equation (7) would need to be multiplied at least by a factor $\pm$ 5 to be detected experimentally. In silver the multiplicative factor for β is around six, so Ag is more sensitive to changes in the J_7 term.

An even more essential difference between these two metals is that the Matthiessen rule deviations (MRD) for In do not alter the temperature dependence of the electrical resistivity. It is reasonable that, independently of what the dominant thermalization process is, the diffusion-like contribution to the thermal resistivity should follow a T^5 dependence. In silver the MRD does change the temperature dependence of the resistivities. As a consequence, the third term of equation (6) could not possibly follow a T^5 dependence.

Our discussion does not cover all possible effects that could contribute to the deviations from a T^2 dependence of the thermal resistivity at low temperatures. However Umklapp scattering processes and possible variation of θ with temperature should not be important. Umklapp contributions may be avoided if the measurements are done at sufficiently low temperature (Lawrence and Wilkins 1972). Our experimental data show that the range of temperature we used was low enough since no deviation was found from a T^5 dependence of the electrical resistivity. These experimental data also show that any contribution from a non-constant behaviour of the Debye temperature can be ruled out, since the electrical resistivity temperature dependence is much more sensitive to this effect than the thermal.

It is interesting to remark that even for a very small change of impurity concentration (0·23 ppm) there appears a marked variation of the coefficients of the T^3 and T^5 terms (see table 1). The MRD for the thermal resistivity of these dilute alloys has a behaviour quite similar to the one found for the electrical resistivity (Bressan *et al* 1975). The origin of this type of deviation is not well explained by present theories.

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