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INCOMPATIBILITY OF π N CEx GROSS EXPERIMENTAL FEATURES AND NAIVE VENEZIANO MODELS

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Abstract: This is an expanded version, not intended for publication, of a previous paper (1).

A model for π N Cex is obtained which reproduces the forward and backward peaks and the widths of the most prominent low-lying resonances at the expense of strong cancellations. It is shown that the angular distribution cannot be predicted by this or similar models assigning an imaginary part to s-channel trajectories. Finite energy sum rules for this model are discussed.

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1 - Introduction

The Veneziano[2] model has been applied to πN scattering by several authors. The first intent by Igi[3] succeeded in giving a parametrization of high energy forward and backward peaks but failed in the prediction of low-lying resonance widths. In fact his widths were too small and non existing parity partners were necessary. On the other hand Virasoro [4] without considering high energy peaks, found a qualitative agreement of predicted and observed low energy widths.

The thorough examination of models without satellites by Berger and Fox[5] confirmed the results of Igi with regard to the extrapolation of high energy fits to resonance widths and in particular the failure in the Δ_δ (1236) case.

Agreement between any one of the high energy peaks and low energy data can be achieved within particular models. Thus, giving up the fit of the backward peak, a reasonable agreement between forward peak and low energy resonances may be got as done by Namysłowski and Swiecki[6] and, using a modified Veneziano model, by Chu and Desai[7], though difficulties according to the conditions imposed to the model can arise[8]. Conversely, Amann[9] fitted the backward peak and the widths of resonances on the Δ - trajectory with infinite series of Veneziano blocks, at the expense of allowing small parity partners.

Fenster and Wali[10] increased the number of terms of Igi's model requiring a more detailed fit of low energy widths and forward peak but in so doing a huge backward peak was predicted. Even with a more elaborated model[11], including secondary meson trajectories aiming at reproducing the high energy angular distribution, the backward peak trouble is still present.

Finally, Hendry, Jones and Wyld[12] obtained a fit of the high energy angular distribution in the near to forward direction (again with disagreement in the backward direction) through the introduction of an imaginary part for baryon trajectories and evaluating the Γ-functions explicitely.

The above quoted listing, certainly not exhaustive, shows as a common feature the difficulty of fitting simultaneously the forward and backward peaks and the low energy widths.

Our purpose is to construct the least complicated Veneziano model compatible with the previous requirements. One expects that a model which reproduces the most relevant features of πN CEx scattering has the best chances to give an acceptable description at all angles.

We shall prove in Section 2 that it is possible, including adequate satellites, to obtain a model which reproduces the forward and backward peaks as well as the widths of the most important low lying resonances at the expense of too strong cancellations which will lead to the drawbacks enumerated in Section 3. In Section 4 we shall show that the naive assignement of an imaginary part to s-channel trajectories is unable to predict the angular distribution of πN CEx at high energy unless one admits the violation of other high energy data and the appearance of terms whose meaning is at least unclear. Finally, in Section 5, looking for an averaged interpretation of the model, we shall analyze the finite energy sum rules.

2 - The model

Our hope is to find a minimal Veneziano model for πN CEx which describes the angular distribution in a range of energy, requiring a fit of

a) forward peak ,

b) backward peak ,

c) residues of N_α (939), N_γ (1518), Δ_δ (1236), with zero residues for the parity partners of N_γ and Δ_δ .

As starting model we choose one with blocks beginning at the position of resonances to be fitted, i.e.

$$\alpha_\Delta = 3/2 \ , \ \alpha_N = 1/2 \ , \qquad \alpha_\gamma = 3/2 \quad \text{and} \quad \alpha_\rho = 1$$

(through forward peak) with the further restriction of leading asymptotic behaviour in both trajectories involved. No pole is allowed in $A^{(-)}$ for $\alpha_N = 1/2$. We get, with degenerate N_α and N_γ trajectories,

$$\begin{aligned} A^{(-)} &= a_1\, C^-_{\Delta\rho}\ (3/2,1) + a_2\ C^-_{N\rho}\ (3/2,1) + a_3\ C^-_{N\Delta}\ (3/2,3/2), \\ B^{(-)} &= b_1\ B_{NN}\ (1/2,1/2) + b_2\ C_{NN}\ (3/2,3/2) + b_3\ C_{\Delta\Delta}\ (3/2,3/2) \\ &\quad + b_4\ B^+_{N\rho}\ (1/2,1) + b_5\ C^+_{N\Delta}\ (3/2,3/2). \end{aligned} \tag{1}$$

At this point one realises that every term contributes to at least two of the above mentioned requirements for the model, except C_{NN} (3/2,3/2) which contributes only to the N_γ residue at $\alpha_N = 3/2$ (provided backward behaviour is approximated just by Λ-leading terms) and can therefore be dropped on the ground of avoiding trivial fits.

We can prove that the previous model cannot simultaneously fit the magnitude of Δ (1236) width Γ_{1+} and the high energy backward peak (given by Δ -leading terms). At J = 3/2, the invariant isospin amplitudes behave like (See e.g. Ref. 10)

$$\lim_{s \to s_{J=3/2}} A\,(s,\cos\,\theta_s) = \frac{2\,\pi}{q_J^3\,(s_J - s)} \Big\{ -\Gamma_{2_-} \left[W_-\,(W_J + M) + 3\,W_+\,(W_J - M)\cos\,\theta_s \right] +$$

$$+ \Gamma_{1_+} \left[W_+\,(W_J - M) + 3\,W_-\,(W_J + M)\cos\,\theta_s \right] - \Gamma_{1_-}\,W_+\,(W_J - M) + \Gamma_{0+} W_-(W_J + M) \Big\} \quad ,(2)$$

$$\lim_{s \to s_{J=3/2}} B\,(s,\,\cos\,\theta_s) = \frac{2\,\pi}{q_J^3 (s_J - s)} \Big\{ \Gamma_{2_-}\,(-\,W_- + 3\,W_+\cos\,\theta_s) +$$

$$+ \Gamma_{1+}\,(-\,W_+ + 3\,W_-\cos\,\theta_s) + \Gamma_{1_-}W_+ + \Gamma_{0+}\,W_- \Big\} \,, \qquad (3)$$

where

$$W_\pm = (W_J \pm M)^2 - \mu^2 \quad , \qquad (4)$$

and q_J and W_J are the centre of mass momentum and energy of the corresponding resonance. Setting $\Gamma_{2_-} = 0$, and leaving aside daughters we have for the $L^P = 1^+$, Δ resonance width the following relations:

$$a_1 + a_3 = \frac{\pi}{q_\Delta^5}\, W_-\,(W_\Delta + M)\,\Gamma_\Delta \simeq 88 \quad 1/\mathrm{GeV} \quad , \qquad (5)$$

$$b_3 + b_5 = -\,\frac{\pi}{q_\Delta^5}\, W_-\,\Gamma_\Delta \simeq -\,40 \quad 1/\mathrm{GeV}^2 . \qquad (6)$$

On the other hand from the differential cross-section at u = 0 (near backward direction)

$$\frac{d\,\sigma}{d\,\Omega}\,\mathrm{CEx}\Big|_{u=0} = \frac{1}{32\,\pi^2} \Big\{ \left[1 - \frac{(M^2 - \mu^2)^2}{s^2} \right] \left| A^{(-)} + M\,B^{(-)} \right|^2 + $$

$$+ \frac{(M^2 - \mu^2)^2}{s} \left| B^{(-)} \right|^2 \Big\} \,, \qquad (7)$$

and keeping only Δ-leading terms, we get:

$$a_1^2 + \left[a_3 + M\,(b_3 + b_5) \right]^2 = 128\,\pi\, \frac{s}{s_o}\, \frac{d\sigma_{CEx}}{d\Omega}\Big|_{u=0} \simeq 32 \quad 1/\mathrm{GeV}^2 \, , \qquad (8)$$

where $s_o = 1\ \mathrm{GeV}^2$ is the usual scale factor.

The numbers were got using the trajectories $\alpha\,(x) = \alpha_o + \alpha'\,x$ with

$$\alpha_{o_\rho} = .5;\ \alpha_{o_N} = -.38;\ \alpha_{o_\Delta} = -.02$$

and $\alpha' = 1$, and experimental data (13) for backward peak.

Relations (5), (6) and (8) are clearly inconsistent since from (8),

$$\left[a_3 + M\,(b_3 + b_5) \right]^2$$

can be at most $\sim 32\ \ 1/\mathrm{Gev}^2$ whereas from (5) and (6) we can at best obtain

$$\left[a_3 + M\,(b_3 + b_5) \right]^2 \sim 1800 \quad 1/\mathrm{Gev}^2 \, .$$

This is obviously related to the smallness of the number describing backward peak compared to those related to resonance residues. If we now accept the presence of a parity partner for Δ (1236) it is surprising to find that an extremely small width for it, $\Gamma_{2-} \simeq 1.4$ MeV, makes the above relations consistent. Even if it would be hard to detect such a narrow resonance experimentally, one is induced to leave it out since the meaning of this width is at least questionable for it comes from strong cancellations among large numbers (+). This could certainly explain the success of the model of Ref. 9.

In an attempt to solve this incompatibility between backward peak and Δ width we will therefore look for the minimun number of Veneziano blocks which must be added in order to find a way out.

One could think of enlarging the model allowing terms with leading asymptotic behaviour in only one trajectory. There are no such terms for the $A^{(-)}$ amplitude whereas for the $B^{(-)}$ one has

$$B^{\pm}_{NN}\ (1/2,3/2)\ ;\ B^{\pm}_{N\rho}\ (3/2,1)\ ;\ B^{\pm}_{N\Delta}\ (1/2,3/2);\ B^{\pm}_{\Delta\rho}\ (3/2,1) \quad .$$

It is straightforward to check that none of these contribute either to Δ-leading backward behaviour or to Δ (1236) residue.

We are therefore forced to abandon the restriction of considering only blocks beginning at the position of resonances to be fitted, and introduce satellite terms starting at 5/2 for Δ and 2 or 3 for ρ. This could be justified on the basis of a bit larger energy interval to be fitted (up to approximately 1.6 GeV which is below the $\alpha_N = 5/2$ resonance). Of course we simultaneously impose a vanishing residue of the Δ parent trajectory at 5/2.

If we again restrict ourselves to blocks with leading behaviour in both trajectories the only useful terms turn out to be $D^-_{\Delta\rho}$ (5/2,2) for $A^{(-)}$ and $D_{\Delta\Delta}$ (5/2,5/2) and $D^+_{\Delta\rho}$ (5/2,3) for $B^{(-)}$ with the definition

$$D_{ij}\ (m,n) = \frac{\Gamma(m - \alpha_i)\ \Gamma\ (n - \alpha_j)}{\Gamma(m + n - 2 - \alpha_i - \alpha_j)} .$$

In fact, the other possible contribution to $B^{(-)}$, i.e. $C^+_{\Delta\rho}$ (3/2,2) cannot still solve our trouble, as sketched in the following. Actually if we add it to model Eqn. 1 with a coefficient b^* we get as conditions for determining the parameters

$$(a_1 - M\ b^*)^2 + \left[a_3 + M\ (b_3 + b_5)\right]^2 \simeq 32 \quad 1/\mathrm{GeV}^2, \tag{9}$$

from backward peak and

$$\begin{aligned} b_3 + b_5 - b^* &\simeq -40 \quad 1/\mathrm{GeV}^2 , \\ a_1 + a_3 &\simeq 88 \quad 1/\mathrm{GeV} , \end{aligned} \tag{10}$$

from $\alpha_\Delta = 3/2$ residues, instead of the previous ones (5), (6) and (8), which are still inconsistent.

On the other hand if we accept terms with leading behaviour in only one trajectory, several among them are useless because they contribute neither to backward behaviour nor to Δ (1236) residue. The remaining ones are $C^-_{\Delta\Delta}$ (3/2,5/2) for $A^{(-)}$ and $C^+_{\Delta\Delta}$ (3/2,5/2) for $B^{(-)}$ which contribute both to backward leading behaviour and to Δ (1236) residue but in a trivial way in the sense that their contributions simply add to those of $C^-_{N\Delta}$ (3/2,3/2) and $C^+_{\Delta\Delta}$ (3/2,3/2).

One could even think of a model with any number of terms of the general type:

$$C_{\Delta\rho}^{\pm(n)} = \frac{\Gamma(3/2 - \alpha_\Delta(s))\,\Gamma(n - \alpha_\rho(t))}{\Gamma(n + 1/2 - \alpha_\Delta(s) - \alpha_\rho(t))} \pm \quad s \leftrightarrow u \ ,$$

$$C_{N\Delta}^{\pm(n)} = \frac{\Gamma(n - 1/2 - \alpha_N(s))\,\Gamma(3/2 - \alpha_\Delta(u))}{\Gamma(n + 1 - \alpha_N(s) - \alpha_\Delta(u))} \pm \ s \leftrightarrow u, \tag{11}$$

$$C_{\Delta\Delta}^{\pm(n)} = \frac{\Gamma(3/2 - \alpha_\Delta(s))\,\Gamma(n + 1/2 - \alpha_\Delta(u))}{\Gamma(n + 1 - \alpha_\Delta(s) - \alpha_\Delta(u))} \pm \quad s \leftrightarrow u,$$

for $n \geqslant 1$. However their contributions to backward peak and Δ residue add trivially to those of $C_{\Delta\rho}^{-}$ (3/2,1), $C_{\Delta\rho}^{+}$ (3/2,2), $C_{N\Delta}^{\mp}$ (3/2,3/2), $C_{\Delta\Delta}^{-}$ (3/2,5/2) and $C_{\Delta\Delta}$ (3/2,3/2) which have already been considered.

Therefore, we are led to the model

$$\begin{aligned} A^{(-)} &= a_1\, C_{\Delta\rho}^{-}(3/2,1) + a_2\, C_{N\rho}^{-}(3/2,1) + a_3\, C_{N\Delta}^{-}(3/2,3/2) + \\ &\qquad + a_4\, D_{\Delta\rho}^{-}(5/2,2)\ , \\ B^{(-)} &= b_1\, B_{NN}(1/2,1/2) + b_3\, C_{\Delta\Delta}(3/2,3/2) + b_4\, B_{N\rho}^{+}(1/2,1) \\ &\quad + b_5\, C_{N\Delta}^{+}(3/2,3/2) + b_6\, D_{\Delta\Delta}(5/2,5/2) + b_7\, D_{\Delta\rho}^{+}(5/2,3)\ . \end{aligned} \tag{12}$$

The conditions imposed to fix the 10 parameters are:

Nucleon pole residue of $B^{(-)}$

$$b_1 + b_4 = \alpha' G^2 = 182.5 \ \ 1/\mathrm{GeV}^2\ . \tag{13}$$

Δ (1236) residues of $A^{(-)}$ and $B^{(-)}$

$$a_1 + a_3 = 87.5 \quad 1/\mathrm{GeV}, \tag{14}$$

$$b_3 + b_5 = -40.3 \quad 1/\mathrm{GeV}^2\ . \tag{15}$$

N_γ (1518) residues of $A^{(-)}$ and $B^{(-)}$

$$a_2 - a_3 = \frac{\pi}{q_\gamma^5}\, W_+ (W_\gamma - M)\Gamma_\gamma = 86.8 \ \ 1/\mathrm{GeV}, \tag{16}$$

$$- b_1 + b_4 + b_5 = \frac{\pi}{q_\gamma^5}\, W_+\, \Gamma_\gamma = 200.0 \ \ 1/\mathrm{GeV}^2\ . \tag{17}$$

Vanishing parent pole residue at $\alpha_\Delta = 5/2$, of $A^{(-)}$ and $B^{(-)}$

$$-a_1 + a_3 + a_4 = 0, \tag{18}$$

$$-b_3 - b_5 + b_6 + b_7 = 0 \quad . \tag{19}$$

Forward peak

$$\left[a_1 + a_2 + \frac{1}{2} a_4 - \frac{s_o}{2M} (b_4 + \frac{3}{4} b_7)\right]^2 = \frac{s\, s_o}{M^2} \left.\frac{d\sigma}{dt} \mathrm{CEX}\right|_{t=0} = \tag{20}$$

$$= 13.6 \quad 1/\mathrm{GeV}^2,$$

and backward peak

$$(a_1 + \frac{3}{2} a_4 - \frac{3}{2} Mb_7)^2 + \left[a_3 + M(b_3 + b_5 + \frac{3}{2} b_6)\right]^2 = 32.2 \quad 1/\mathrm{GeV}^2 . \tag{21}$$

We have nine conditions to determine 10 parameters. We exploit the quadratic character of relation (21) to split it into two

$$a_1 + \frac{3}{2} a_4 + \frac{3}{2} M\, b_7 = E_1 \ , \tag{22}$$

$$a_3 + M\, (b_3 + b_5 + \frac{3}{2} b_6) = E_2 \ , \tag{23}$$

with E_1 and E_2 constrained by

$$E_1^2 + E_2^2 = 32.2 \quad 1/\mathrm{GeV}^2 \tag{24}$$

and uncorrelated signs. There is no indetermination coming from relation (20) since the ρ Regge pole residue must be positive from total cross-section high energy data. The resulting coefficients $a_i \left[1/\mathrm{GeV}\right]$ and $b_i \left[1/\mathrm{GeV}^2\right]$ are:

$$a_1 = 25.2 \pm \frac{1}{5} E_1 \mp \frac{1}{5} E_2 \qquad b_1 = -134.1 \mp \frac{E_1}{M}$$

$$a_2 = 149.1 \mp \frac{1}{5} E_1 \pm \frac{1}{5} E_2 \qquad b_3 = 210.4 \pm \frac{E_1}{M}$$

$$a_3 = 62.3 \mp \frac{1}{5} E_1 \pm \frac{1}{5} E_2 \qquad b_4 = 316.6 \pm \frac{1}{2} \frac{E_1}{M}$$

$$a_4 = -37.1 \pm \frac{2}{5} E_1 \mp \frac{2}{5} E_2 \qquad b_5 = -250.7 \mp \frac{E_1}{M} \tag{25}$$

$$b_6 = -17.4 \pm \frac{2}{15} \frac{E_1}{M} \pm \frac{8}{15} \frac{E_2}{M}$$

$$b_7 = -22.9 \mp \frac{2}{15} \frac{E_1}{M} \mp \frac{8}{15} \frac{E_2}{M} \quad .$$

3 - Alternatives and drawbacks of the model.

At this stage one could think of an alternative model which includes Veneziano terms leading in both trajectories and starting at $\alpha_\Delta = 1/2$ instead of introducing terms beginning at 5/2. Of course one must impose a vanishing residue for $\alpha_\Delta = 1/2$ since there is no particle there. There are three such terms to be included in $B^{(-)}$ i.e. $B^{+}_{N\Delta}$ (1/2,1/2), $B^{+}_{\Delta\rho}$ (1/2,1) and $B_{\Delta\Delta}$ (1/2,1/2) and none for $A^{(-)}$. We would therefore have a new model with ten parameters as that of Eqn. 12. The explicit calculation shows an extreme numerical analogy between both models except for the fact that in the last case one cannot have vanishing parent residues of $A^{(-)}$ and $B^{(-)}$ at $\alpha_\Delta = 5/2$. In this way, with the same number of parameters this new model satisfies less physical requirements. Besides this defect, it is less satisfactory than that of Eqn.12 because it includes terms with poles at a point where no physical particle appears(+).

Returning therefore to model Eqn.12, we can however enumerate several fundamental drawbacks. First, with the assumed requirements for the model, there is no way of controlling the widths of daughter trajectory poles. On the other hand, it is difficult to establish with certainty a correspondence between the large number of Veneziano daughters and the few experimental candidates known at present.

Secondly, a defect peculiar to this particular model is the too large residue of the ρ daughter trajectory. In fact for $A'^{(-)}$ it comes out to be three times larger than the ρ residue itself. This is a consequence of the very severe concellations which hold true only for the parent pole.

Finally and again due to cancellations, an even worse situation occurs in backward direction. According to Eqn 7, after the Δ-leading contribution to $A^{(-)} + M\,B^{(-)}$ two competitive contributions come, i.e. that of Δ trajectory to $B^{(-)}$ and that of N trajectory to $A^{(-)} + M\,B^{(-)}$ (besides the interference terms). Compared to the main term the residue of the first correction is about ten times larger whereas that of the second one is about hundred times larger. This means that even at $p_{lab} \simeq 18$ GeV the N trajectory contribution to the amplitude is greater than the Δ-leading contribution by a factor of twenty.

4 - Angular distribution

In order to predict the differential cross-section at all angles one could evaluate the amplitudes (12) at energies half-way between two neighbouring poles. However, since N and Δ resonances alternate it turns out that every selected energy is too close to some pole, giving rise to big errors. Moreover the already mentioned strong cancellations among the asymptotic terms are destroyed at subasymptotic energies giving values for the amplitude completely outside of any control. This difficulty is magnified by the fact that one has to consider the (s, u) terms, neglected in the asymptotic evaluation on the basis of a tacitly assumed imaginary part of trajectories, some of which even increase more rapidly than the leading Regge behaviour, e.g. $D_{\Delta\Delta}$ (5/2,5/2). It is therefore clear that a finite imaginary part is necessary to give a meaning to the model.

One must assign an imaginary part α_I to trajectories above threshold and in so doing the attempt to modify the model in the physical region of e.g. s-channel causes an obvious violation of crossing. One has also to be aware of the influence such an imaginary part has on (s, t) terms. For instance the term $B^{+}_{N\rho}$ (1/2,1) contributing to $B^{(-)}$ can be expanded near the forward direction, where Stirling's formula converges more rapidly, up to the second order as

$$B^{+}_{N\rho}\ (1/2,1) = \frac{\pi}{\Gamma\left[\alpha_\rho\ (t)\right]}\ (2\,M\,\alpha'\,\nu)^{\alpha_\rho(t)\ -\ 1} \Bigg\{ \frac{1 - e^{-i\,\pi\alpha_\rho(t)}}{\sin\pi\ \alpha_\rho(t)}$$

$$+ \frac{1-\alpha_\rho\ (t)}{2\,M\ \alpha'\ \nu}\left[\alpha_{O_N} + \alpha'\ (M^2+\mu^2) + \frac{\alpha_{O_\rho} - 1}{2}\right] \frac{1 + e^{-i\,\pi\,\alpha_\rho(t)}}{\sin\ \pi\,\alpha_\rho\ (t)} \tag{26}$$

$$- 2\,i\,e^{-2\,\pi\ \alpha_I}\ \ e^{2\,\pi\,i\,(\alpha_{O_N} +\ \alpha'\,s)}$$

$$+ i\ \frac{1-\alpha_\rho\ (t)}{2\,M\,\alpha'\ \nu}\ \alpha_I\ \frac{e^{-i\,\pi\,\alpha_\rho\ (t)}}{\sin\ \pi\,\alpha_\rho\ (t)} \Bigg\}\ ,$$

where $\nu = (s-u)/4M$. The first term corresponds to the parent ρ-trajectory and is the one used to fit the asymptotic peak. The second one is the first daughter term which contributes to the dip at $\alpha_\rho\ (t) = 0$ (note that this daughter term has the same killing mechanism as that of the parent trajectory at $\alpha_\rho\ (t) = -1$) but with a too fast decrease with energy.

The experimental behaviour at this dip corresponds in fact to that of the parent trajectory and this is consistent with standard Regge pole fits which use a sense choosing mechanism for the ρ-coupling [14]. The third is the non-Regge part which is usually neglected in the asymptotic expressions of Veneziano terms, on the assumption of an increasing imaginary part for trajectories. It decays exponentially with s and vanishes at $\alpha_\rho = 0$.

The last one is a spurious term which appears because of the introduction of an imaginary part in the baryon trajectories. It does not vanish at the dip position and with the assumption $\alpha_I \sim s$ asymptotically it behaves with energy like the parent Regge term. This spurious term has no signature factor, since the imaginary part was attached only to s-channel trajectories, and in principle cannot be treated on the same footing with a Regge pole. However as its energy behaviour is the same as that of the parent, it has the effect of breaking the Gell-Mann dip mechanism usually attributed to Veneziano model. This feature is common to the spurious part of every term contributing to $B^{(-)}$, whereas those contributing to $A^{(-)}$ are proportional to $\alpha_\rho(t)$ instead of $(1 - \alpha_\rho(t))$ and consequently do vanish at the dip position. The "mechanism" originated in this way (vanishing $A^{(-)}$ and non-vanishing $B^{(-)}$ at the dip) is opposite to the sense-choosing implied by experimental fits.

Though (s, u) terms do not vanish at the dip position, their energy behaviour is exponentially decreasing as soon as one allows an imaginary part on s-channel trajectories. Therefore, they cannot fill in the dip in a finite range of energy.

Returning to the model Eqn. 12, we took $\alpha_I = .1\ s$ which is a reasonable choice to fit the total widths of low energy resonances. The numerical results show that the (s, u) terms, non-Regge and spurious part of (s, t) terms give contributions which may be larger than that of the parent trajectory even at fairly high energy (e.g. $p_{lab} \approx 18$ GeV). This is of course due to the very severe cancellations mentioned above. In this sense the spurious term is the most dangerous one since it does not decay exponentially with energy. In fact at the dip position, the lack of appropriate cancellation due to the absence of $A^{(-)}$ contribution at this point, as already explained, gives rise to a predicted dip value several orders of magnitude larger than the observed one. Also at $t = 0$, since the $B^{(-)}$ contribution of the spurious term reverses the sign with respect to the corresponding contribution of the parent pole, the forward peak is considerably enlarged.

The drawbacks enumerated at the end of the previous section, together with the difficulties coming from the introduction of an imaginary part in s-channel trajectories invalidates completely the predictive power of our model.

This failure is not inherent to our model. One could think that these troubles arise because of the strong cancellations which in turn come mainly from the requirement of fitting the backward peak. In this sense, if one gives up the fit of this peak aiming only at reproducing the angular distribution in the near to forward region one can obtain a model with cancellations in fair agreement with the observed ones. For instance, Hendry et al [(12)] obtained in this spirit, and using the exact Γ-functions, a rather impressive fit of differential cross-section data near the forward region in a fairly wide energy interval. However, their spurious term contribution is still too large. In fact this term by itself is responsible for the filling up of the dip at $\alpha_\rho = 0$ whereas the (s, u) terms are negligible at this point, at variance with the interpretation of the authors of Ref. 12. As a consequence, the amount of spurious term that comes into their model for $t = 0$ is even larger than the parent Regge pole contribution to $A'^{(-)}$ and therefore, since it is purely real at this point (see Eqn. 26), their model would predict a high energy total cross-section $\sigma_{\pi-} - \sigma_{\pi+}$ about 40% of the experimental value.

One may remark that these problems are a common feature of any naive Veneziano model with complex trajectories. Actually, only the spurious terms could explain the filling up of the dip, but this is of course prevented by a proper simultaneous fit to forward peak and asymptotic total cross-section. These troubles do not disappear when unitarizing Veneziano model with Lovelace K-matrix method, as discussed in Ref. 15.

5 - Finite energy sum rules (FESR)

Since the Veneziano formula corresponds to a narrow-width approximation, it is reasonable to think that in order to give a meaning to it one must somehow perform an energy average of the amplitudes, for instance through FESR. In so doing one avoids any assumption about imaginary parts of trajectories apart from the usual $i\varepsilon$ prescription for resonances.

It has been proved [16] that FESR for the imaginary part of $\pi\pi$ Veneziano amplitude hold when including contributions of parent and first daughter trajectories provided the energy cut-off is chosen half-way between two neighbouring resonances.

In the πN model of Eqn. 12 there is no possibility of satisfying FESR up to first daughter by choosing the cut-off in this way since N and Δ trajectories alternate their resonances.

For the zeroth moment sum rule a typical (s, t) terms gives

$$\int_0^{\nu_c} d\nu \ \mathrm{Im}\, C^-_{\Delta\rho} \ (3/2,1) = -\frac{\pi}{2M} \frac{1}{(\alpha_\rho(t)+1)\Gamma(\alpha_\rho(t))} \frac{\Gamma(\alpha_\rho(t)+n+2)}{\Gamma(n+1)} , \quad (27)$$

where $\nu_c = \frac{1}{2M}(n+\tau+\frac{t}{2})$ with $.62 \leq \tau \leq .98$, so that we include n resonances on the N trajectory and n on the Δ one. Taking into account all (s, t) terms contributing to $A'^{(-)}$ the zeroth moment FESR is satisfied for the parent ρ trajectory whereas a discrepancy appears for the first non leading term. In fact while the ρ-daughter residue for $t = 0$ is

$$\beta_1(0) = 28.0 \pm 1.4 \quad 1/\mathrm{GeV} ,$$

from the left hand side of FESR one gets

$$\beta_1^{SR}(0) = 35.0 \pm 1.6 \quad 1/\mathrm{GeV}$$

with the choice $\tau = .8$ which corresponds to a cut-off half-way between a N and a Δ resonance. As remarked above the daughter residue is too large and at $\nu_c \simeq 4$ GeV, where FESR are experimentally well satisfied with ρ-pole alone, its contribution is still twice as large as that of the parent ρ-pole. Consequently, the mismatch between both sides of the sum rule is relevant.

On the other hand a typical (s, u) term gives

$$\int_0^{\nu_c} d\nu \ \mathrm{Im}\ C^-_{N\Delta} \ (3/2,3/2) = \frac{\pi}{2M} \frac{(-)^{n+1}\,\Gamma(\alpha_t+n+1.1)}{\Gamma(\alpha_t+.1)\,\Gamma(n+1)} . \quad (28)$$

Since this contribution to the sum rule vanishes for $\nu_c \dashrightarrow \infty$ and $t < -.6$ we find that $C^-_{N\Delta}$ (3/2,3/2) does not contain right signature fixed pole at -1, as one would expect. A similar result is obtained for all the (s, u) terms contributing to $A'^{(-)}$. However one must remark that the higher the satellite is the lower is the limit of t one has to take to ensure its vanishing contribution to zeroth moment FESR for $A'^{(-)}$ with $\nu_c \dashrightarrow \infty$ (e.g. the contribution of $D_{\Delta\Delta}$ (5/2,5/2) tends to zero for $t < -3.25$). In this sense satellite terms spoil FESR near the forward direction.

6 - Conclusions

We have tried to build up a model which reproduces the gross experimental features of πN charge exchange scattering. The inclusion of the requirement of fitting the backward peak is determinant in the sense that strong cancellations must occur, at least in a model with a reasonable number of Veneziano blocks. In particular in the model proposed here, it is hopeless to try to reproduce the angular distribution (even with an imaginary part for s-channel trajectories) since those strong cancellations give a wild behaviour outside the peaks where they are constrained to hold.

Giving up the fit of the backward peak it is possible to get a model with reasonable cancellations and reproducing the angular distribution near $t = 0$ when an imaginary part α_I is assigned to s-trajectories. However the bulk of this success is due to a spurious term which arises because of the introduction of α_I, has no signature factor and, albeit its Regge energy behaviour, breaks the Gell-Mann ghost killing mechanism peculiar to Veneziano models.

Smoothing out the model with a FESR one does not need to consider imaginary parts for trajectories. However in order to damp the oscillations caused by (s, u) terms one must exclude a forward cone which increases as the number of satellites becomes larger.

We aknowledge many discussions with Prof. D. Wong and Prof. J. Fulco.

FOOTNOTES

(+) Daughters residues do not affect these considerations .

(∓) Nobody thinks of a $\pi\pi$ model with terms starting at zero.

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