FLUXOID QUANTUM NUMBER AT H_{c3}

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For a cylinder in an axial magnetic field the largest field at which superconductivity nucleates is the same as the surface nucleation field of the semi-infinite half-space when the radius R of the cylinder is much larger than the temperature-dependent coherence length ξ. From the solution of the multiply connected surface sheath one obtains the number of fluxoid quanta which are locked-in at the nucleation field as a function of R/ξ.

Using energy arguments it was shown [1] for a cylinder in an axial magnetic field that at H_{c3} [2,3] the number of fluxoids which are locked-in at the nucleation field ($R \gg \xi$) is

$$n \approx (1.7/2)(R/\xi)^2 - R/\xi \,. \tag{1}$$

We show here, using one of the Ginzburg-Landau (GL) [4] differential equations in cylindrical [3,5] and cartesian [2,6] coordinates, that eq. (1) follows directly by comparing this equation in these coordinate systems in the limit $R/\xi \gg 1$.

Assuming a second-order phase transition at the nucleation field and a single-valued order parameter $\Psi = F(\rho)\mathrm{e}^{-in\theta}$, where n is an integer, one of the linearized GL equations in cylindrical coordinates (ρ, θ, z) is

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,\frac{\mathrm{d}F}{\mathrm{d}r}\right) = \chi^2\left[Q^2(r) - 1\right]F(r)\,. \tag{2}$$

We used the following definitions: the superfluid velocity $Q_\theta \equiv Q = \xi(2eA/c\hbar + \nabla\Phi)_\theta$, where $A_\theta = H_0\rho/2$, H_0 the applied magnetic field; $\Phi = -n\theta$, the phase of the order parameter; $r = \rho/R$; $\chi = R/\xi$; $\mu^2 = \sqrt{2}\kappa H_c/H_0$, where H_c is the temperature-dependent thermodynamic critical field and κ is the GL κ-value. Therefore

$$Q(r) = \frac{\chi}{2\sqrt{2}\kappa}\left[\frac{H_0}{H_c}r - \frac{2\sqrt{2}\kappa}{\chi^2}\,\frac{n}{r}\right]. \tag{3}$$

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Using the transformation $y = \chi(1-r)/\sqrt{2}\mu$, where y is proportional to the distance measured from the surface of the cylinder in units of ξ, eqs. (2) and (3) become

$$\frac{\mathrm{d}^2F}{\mathrm{d}y^2} - \frac{\sqrt{2}\mu}{\chi}\left(1 - \frac{\sqrt{2}\mu}{\chi}y\right)^{-1}\frac{\mathrm{d}F}{\mathrm{d}y} = 2\mu^2\left[Q^2 - 1\right]F\,, \tag{4}$$

$$Q = \frac{\chi}{2\sqrt{2}\kappa}\left[\frac{H_0}{H_c}\left(1 - \frac{\sqrt{2}\mu}{\chi}y\right) - \frac{2\sqrt{2}\kappa}{\chi^2}\,n\left(1 - \frac{\sqrt{2}\mu}{\chi}y\right)^{-1}\right] \tag{5}$$

Because $F(y)$ is finite only near the surface of the cylinder, where y is of order unity, and zero far away from it inside the cylinder, we expand $(1 - \sqrt{2}\mu y/\chi)^{-1}$ in a series for $y/\chi \ll 1$. Substituting eq. (5) into eq. (4), with $\zeta = \sqrt{2}y$, one gets

$$-\frac{\mathrm{d}^2F}{\mathrm{d}\zeta^2} + \frac{\mu}{\chi}\,\frac{\mathrm{d}F}{\mathrm{d}\zeta} + \left[\zeta a + \zeta^2\frac{\mu}{\chi}b - c\right]^2 F = \mu^2 F\,, \tag{6}$$

where

$$a = (1 + 2\mu^2 n/\chi^2)/2\,,$$

$$b = \mu^2 n/\chi^2\,,\quad c = \chi/2\mu - n\mu/\chi\,.$$

Since n is of order of χ^2 the values a, b and c are of order unity. For $\chi \gg 1$, eq. (6) becomes

$$-\mathrm{d}^2F/\mathrm{d}\zeta^2 + (\zeta a - c)^2 F = \mu^2 F\,. \tag{7}$$

Comparing eq. (6) with the differential equation of the surface sheath near H_{c3} for a semi-infinite half-space (ref. [6] eq. (5′)):

$$-\mathrm{d}^2F/\mathrm{d}\zeta^2 + (\zeta - \Gamma)^2 F = \mu^2 F ,$$

(with $\Gamma^2 = \mu^2 = 0.5901$ for the lowest eigenvalue) we find that $c^2 = \mu^2 = 0.5901$ or (for $\kappa > 0.4172$)

$$n = (1.695/2)(R/\xi)^2 - R/\xi . \qquad (8)$$

Also $a \to 1$, so that in the limit $\chi \gg 1$

$$\mu^2 = H_{c2}/H_{c3} = 0.5901 . \qquad (9)$$

Eq. (9) implies that the nucleation field of a cylinder with $R \gg \xi$ and that of the semi-infinite half-space are identical [3]. At H_{c3} the value of Q at a distance $\delta(= 0.5901\ \xi)$ from the surface is zero [7]. Because R/ξ in eq. (8) is equal to $2\pi R\delta H_{c3}/\phi_0$, the value R/ξ is equal to the number of flux quanta within a distance δ from the surface (ϕ_0 = fluxoid quantum). The term $1.695(R/\xi)^2/2$ in eq. (8) is the total number of applied flux quanta in the cylinder. Thus for $R \gg \xi$ the quantum number n at H_{c3} is determined by the total number of flux quanta minus the number of flux quanta contained in the annular area $2\pi R\delta$ of the cylinder.

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