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NUCLEAR STRUCTURE STUDIES IN THE LEAD REGION WITH (d, p) REACTIONS

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Abstract: Energy distributions of protons from (d, p) reactions in the isotopes of Pb and Bi were measured with ≈ 100 keV resolution and angular distributions of several of the groups were studied. The angular distribution from $Pb^{207}(d, p)Pb^{208}$(g.s.) agrees very well with the distorted wave Born approximation calculation of Tobocman and with measured angular distributions of $l_n = 1$ transitions in $Pb^{206}(d, p)Pb^{207}$. The difference from a measured $l_n = 3$ transition is quite large. From studies of the excitation of the various single hole states of Pb^{207} in $Pb^{206}(d, p)Pb^{207}$, the wave function of the ground state of Pb^{206} is determined to be approximately

$$\Psi_{206} = \sqrt{0.56}(p_{\frac{1}{2}})^{-2}+\sqrt{0.25}(f_{\frac{5}{2}})^{-2}+\sqrt{0.12}(p_{\frac{3}{2}})^{-2}+\sqrt{0.04}(i_{\frac{13}{2}})^{-2}+\sqrt{0.03}(f_{\frac{7}{2}})^{-2},$$

which has less $(p_{\frac{1}{2}})^{-2}$, and more $(f_{\frac{5}{2}})^{-2}$, $(i_{\frac{13}{2}})^{-2}$ and $(f_{\frac{7}{2}})^{-2}$ than the theoretical wave functions of True and Ford, and of Kearsley. A new low lying non-hole state of Pb^{207} is found. Extensive efforts to understand the spectrum from $Pb^{208}(d, p)Pb^{209}$ are reported. The relative intensities of the various peaks are very difficult to explain.

1. Introduction

Deuteron stripping reactions have been used to study nuclear structure in the region of Pb^{208} by Harvey [1]) and by McEllistrem, Martin, Miller and Sampson [2]). While the first is an early low resolution survey at a single angle, the second is a reasonably detailed, good resolution experiment. However, the study reported here represents several improvements relative to it, namely: (1) it is carried out at 15 MeV bombarding energy as compared to 10 MeV in ref. [2]) so that Coulomb effects are less dominating; (2) the intensity relative to background is much higher so that several peaks are observed which were missed there; (3) this work includes Bi^{209} and large parts of the spectra from Pb^{207} and Pb^{208} which were not included in that study; and (4) the resolution is somewhat better.

In addition, the analysis is carried considerably further, and the conclusions are somewhat different.

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2. Experimental

The method of obtaining energy spectra with photographic plate detection was reviewed in the previous paper on (d,t) reactions [3], so that it will not be repeated here. The only difference is that the reaction product analysing magnet is run at lower magnetic fields to pass protons, and since such fields also pass deuterons, tritons and alpha particles, an absorber is placed in front of the photographic plates to remove all particles except protons. Some typical data are shown in fig. 1. In some of the angular distribution studies where the best resolution is not needed, the photographic plates are replaced by a 0.32 cm wide × 2.54 cm high scintillation detector; data are then obtained by measuring the counts per μC of the incident beam for various settings of the magnetic field.

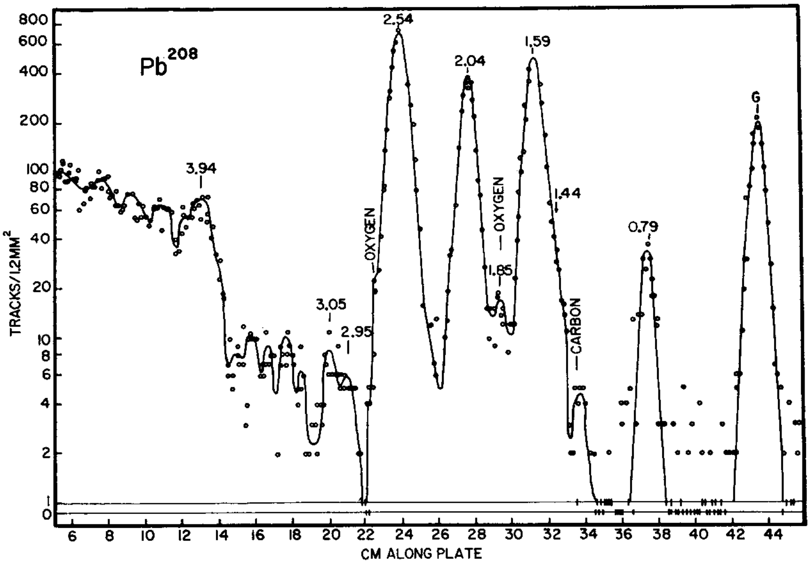

Fig. 1. Energy distribution of protons from Pb^{208}(d, p)Pb^{209}. Angle of observation is 60°. Different symbols for ordinates equal to 0 and 1 are only to facilitate clarity. Numbers are excitation energies of final states in MeV.

3. Results and Conclusions ($N \leqq 126$ Shell)

It is natural to divide the results into two parts, those bearing on the $N \leqq 126$ shell (N is the neutron number) which will be discussed in this section, and those bearing on the $N > 126$ shell which will be taken up in the following section. The first group consists of data pertaining to the excitation of the low lying single hole states of Pb^{207} by Pb^{206}(d, p) and the excitation of the ground state of Pb^{208} by Pb^{207}(d, p). Two aspects of this work which are particularly interesting are (1) the study of angular distributions of proton groups from these levels; since the neutron transfer angular momentum l_n is known, they can be compared with theoretical calculations from stripping theory and used as a test of the feasibility of using angular distributions for nuclear spectroscopy in this mass and energy region; and (2) the study of the True and Ford [4]) and Kearsley [5]) wave functions for the ground state of Pb^{206}.

3.1. ANGULAR DISTRIBUTIONS

The angular distribution of the ground state proton group from Pb^{207} (d, p)Pb^{208} is shown in fig. 2 together with the theoretical curve from the distorted wave Born approximation calculation of Tobocman [6]), and a previous measurement at this energy by Wall [7]). In so far as there is disagreement with the previous experimental results, it may be presumed that the results reported here are the more reliable as the detection method was more elaborate. Ref. [7]) does not give an absolute cross section so that the normalization to it is arbitrary.

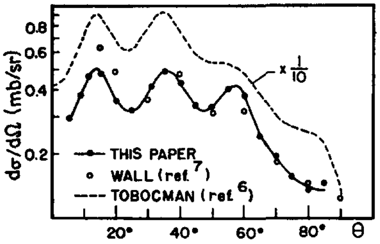

Fig. 2. Angular distribution of ground state proton group from Pb^{207}(d, p)Pb^{208}. Data from ref. [7]) are arbitrarily normalized to data from this paper.

The shape of the angular distribution is in very good agreement with the theory; the angular positions and relative heights of the two principle peaks agree almost perfectly, the angular position of the third peak is well reproduced, and there are even indications of agreement in the fourth peak. On the basis of shape, therefore, the fit of theory to experiment in fig. 2 is as good as almost any ever achieved in deuteron stripping investigations. A rather bad disagreement arises, however, in considering absolute cross sections; the theoretical cross section is larger than the experimental by about a factor of eighteen. It might be pointed out that the theoretical calculations were done with $j = \frac{3}{2}$ whereas the actual value of j here is $\frac{1}{2}$; this would reduce the absolute cross sections, but not nearly by a factor of 18; it would not affect the angular distribution.

Fig. 3 shows angular distributions of the three highest energy proton groups from Pb^{206}(d, p) which lead to the $p_{\frac{1}{2}}$, $f_{\frac{5}{2}}$ and $p_{\frac{3}{2}}$ states of Pb^{207}. These data were obtained from photographic plates so that the accuracy is not as good as in fig. 2, but it is apparent that the principle features of the angular distribution in that figure ($l_n = 1$, $Q = 5.4$ MeV) are retained here for the transition to the

$p_{\frac{1}{2}}$ state ($l_n = 1$, $Q = 4.6$ MeV) and are still recognizable for the transition to the $p_{\frac{3}{2}}$ state ($l_n = 1$, $Q = 3.7$ MeV), whereas the angular distribution for the transition to the $f_{\frac{5}{2}}$ state ($l_n = 3$, $Q = 4.0$ MeV) is quite different. This gives reasonable encouragement for attempting to do nuclear spectroscopy with (d, p) angular distribution studies in this region of mass and energy. It should be noted, however, that the angular distributions depend much more sensitively on Q than in usual stripping theory; a very noticeable feature of this type is the decrease of intensity of small angle peaks relative to large angle peaks as Q decreases.

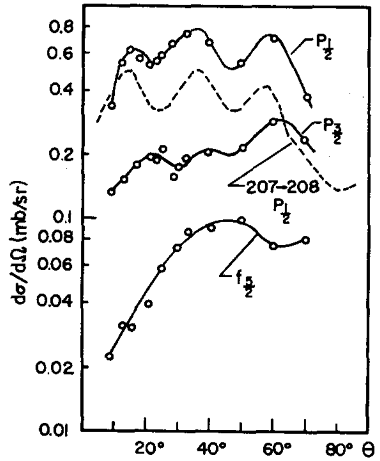

Fig. 3. Angular distributions of three highest energy proton groups from Pb^{206}(f, p)Pb^{207}. The curve through the data from fig. 2 is included for reference.

3.2. WAVE FUNCTIONS FOR GROUND STATES OF Pb^{206}

The cross section for a (d, p) reaction may be written [8])

$$\sigma \propto \frac{2I_f+1}{2I_i+1} f(l_n, Q, \theta)\, Sc^2, \tag{1}$$

where I_f and I_i are the spins of the initial and final states respectively, S is the coefficient of fractional parentage, c is the overlap integral between the initial and final states, and f is a quantity derived from nuclear reaction theory. In usual theories, f depends on l_n, the orbital angular momentum with which the "stripped" neutron enters the nucleus, the Q of the reaction, and the angle of observation. It also depends on the radius and charge of the target nucleus, but these are essentially the same for all cases considered in this paper. Since reaction theories in their present state cannot predict f and we have no direct way of determining it, we simplify (1) by assuming that the dependence of f on l_n and Q are separable as

$$f(l_n, Q, \theta) = F(l_n, \theta) g(Q). \tag{2}$$

The function $g(Q)$ can be very crudely estimated by comparing the intensities of the ground state group and the most intense groups from Pb^{207}(d, p). The former is due to the filling of the $p_{\frac{1}{2}}$ single hole state, while the latter must be due to the excitation of the single particle $s_{\frac{1}{2}}$, $d_{\frac{5}{2}}$ and $d_{\frac{3}{2}}$ states (see below), so that c^2 is unity for all cases, S is always calculable [8]) and $F(l_n, \theta)$ should be about the average of $F(0, \theta)$ and $F(2, \theta)$. The result of this calculation is that $g(Q)$ increases by about a factor of 5 as Q decreases by 5 MeV. If we assume that $g(Q)$ varies logarithmically with Q, it increases by a factor of 1.35 per MeV decrease in Q.

This procedure is, of course, quite crude but there is little alternative except to ignore the dependence on Q (as was done in ref. [2])); it seems reasonably justifiable because it will be used only over relatively short intervals of Q so that it is never a very large correction. Furthermore, it agrees in direction, and roughly in magnitude with the prediction of Butler theory which is a factor of about 1.2 per MeV.

The function $F(l_n, \theta)$ is also not known. The dependence on θ is handled by determining all interesting quantities at each of several angles, and averaging over angles. The uncertainty introduced by this procedure may be judged by the straggling of the values which are used to compute the average; while this is far from insignificant, it is perhaps not unduly large. The dependence of F on l_n will be considered as a parameter in analysing results; however, the direction of the variation is certainly known, and its magnitude is known at least roughly.

The wave function of the ground state of Pb^{206} may be written

$$\Psi_{206} = a(p_{\frac{1}{2}})^{-2}+b(f_{\frac{5}{2}})^{-2}+d(p_{\frac{3}{2}})^{-2}+e(i_{\frac{13}{2}})^{-2}+f(f_{\frac{7}{2}})^{-2}. \tag{3}$$

According to (1) and (2), the coefficient a may be determined from the ratio of the cross sections for the ground state transitions in Pb^{206}(d, p) and Pb^{207}(d, p) as

$$\frac{\sigma(206)}{\sigma(207)} = 2.43\ a^2. \tag{4}$$

The values of this ratio at various angles is shown in table 1. A straight average of the values listed there is $\sigma(206)/\sigma(207) = 1.44$ which corresponds to $a^2 = 0.59$; any other method of taking an average would not give an appreciably different result. The True and Ford prediction [4]) is $a^2 = 0.73$, whereas the experimental result from our study of (d, t) reactions [3]) was $a^2 \approx 0.56$.

TABLE 1

Ratio of cross sections for ground state transitions in Pb^{206}(d, p) and Pb^{207}(d, p) reactions at various angles

θ	8°	11°	23°	30°	33°	40°	60°	70°
$\frac{\sigma(206)}{\sigma(207)}$	1.30	1.30	1.49	1.71	1.20	1.54	1.49	1.54

By measuring the excitation of the various single hole states of Pb^{207} by Pb^{206}(d, p) all of the coefficients in (3) may be estimated. A typical spectrum of protons from this reaction is shown in fig. 4. It is seen that each of the single hole states in Pb^{207} is clearly resolved. Besides these, an additional state is observed at an excitation of 1.99 MeV; this will be discussed below. Spectra of this type were obtained at 30°, 45°, 60° and 90°, and the relative intensities of

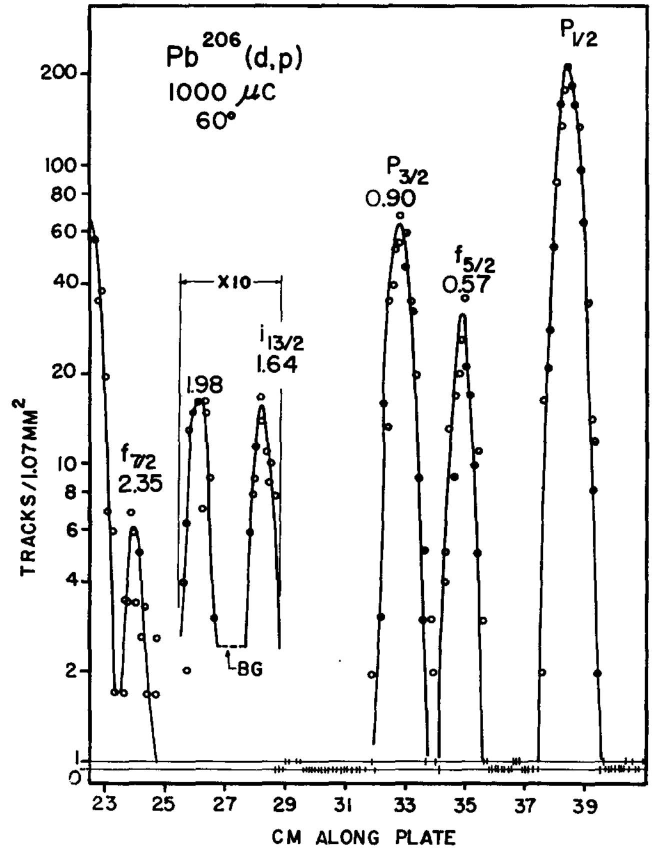

Fig. 4. Energy distribution of protons from Pb^{206}(d, p)Pb^{207} at 60°. This is the high energy end of the spectrum; the low energy part is very similar to that shown in fig. 5.

the various peaks are listed in table 2. From these, the quantity Fc^2 may be calculated from (1) and (2); it turns out that $\{(2I_f+1)/(2I_i+1)\}S$ is unity in cases, so that the only correction is for $g(Q)$. The results are listed in table 2 for each angle, and a weighted average over angles, $\bar{F}(l_n)$, is shown in the last column. In obtaining this average, angles where the intensity is highest are weighted most heavily.

TABLE 2

Relative intensities σ of transitions to various states of Pb^{207} from Pb^{206}(d, p) reactions and relative values of Fc^2 calculated from them using eqs. (1) and (2). The last column is $\bar{F}c^2$, obtained by averaging Fc^2 over angles as discussed in text.

State in Pb^{207}	30°		45°		60°		90°		$\bar{F}c^2$
	σ	Fc^2	σ	Fc^2	σ	Fc^2	σ	Fc^2	
$p_{\frac{1}{2}}$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$f_{\frac{5}{2}}$	0.12	0.10	0.23	0.19	0.15	0.13	0.31	0.26	0.18
$p_{\frac{3}{2}}$	0.26	0.20	0.28	0.21	0.29	0.22	0.47	0.36	0.22
$i_{\frac{13}{2}}$	0.009	0.006			0.009	0.006	0.018	0.011	0.008
$f_{\frac{7}{2}}$			0.05	0.025	0.03	0.015	0.09	0.045	0.025

To obtain c^2, some estimate must be made of $\bar{F}(l_n)$. For this purpose, we assume

$$\bar{F}(l_n) = 2^{-kl_n}, \tag{5}$$

which is an expression that has been used successfully in other mass regions. The values of the coefficient in (3) obtained for various values of k are listed in table 3, where in each case they have been normalized to sum to unity. To assume $k = 0$, i.e. that the average cross section is independent of l_n, is contradictory to all experience in stripping reactions. In lighter nuclei, $k \approx 1$ is approximately correct and this is approximately the value found in (d, t) reactions in the Pb region. In (d, p) reactions in the Pb isotopes, an estimate of k may be obtained by comparing the $g_{\frac{9}{2}}$ ground state peaks from Pb^{208}(d, p) with the peaks which lead to the s and d single particle states; this gives $k \approx 0.8$. There is thus every reason to believe that k is considerably larger than zero, and very probably larger than $\frac{1}{2}$. From table 3, then, a^2 must be definitely less than 0.70, and very probably less than 0.60, which is in agreement with results discussed above.

TABLE 3

Values of coefficients in Pb^{206} ground state wave function of eq. (3) for various assumptions on the value of k in eq. (5). The "Best estimates" are obtained by choosing the value of k as discussed in text. The last two columns give values of these coefficients from theoretical calculations cited.

$(\text{coeff.})^2$	Hole pair state	Dependence of F on l_n			Best estimate	Calculated	
		$k = 0$	$k = \frac{1}{2}$	$k = 1$		True and Ford	Kearsley
a^2	$(p_{\frac{1}{2}})^{-2}$	0.70	0.60	0.43	0.56	0.73	0.70
b^2	$(f_{\frac{5}{2}})^{-2}$	0.13	0.22	0.32	0.25	0.13	0.11
d^2	$(p_{\frac{3}{2}})^{-2}$	0.15	0.13	0.09	0.12	0.13	0.14
e^2	$(i_{\frac{13}{2}})^{-2}$	0.006	0.02	0.12	0.04	0.009	0.02
f^2	$(f_{\frac{7}{2}})^{-2}$	0.017	0.03	0.04	0.03	0	0.02

In order to obtain the other coefficients in (3), it seems reasonable to choose the value of k to obtain agreement with the other determinations of a^2. This

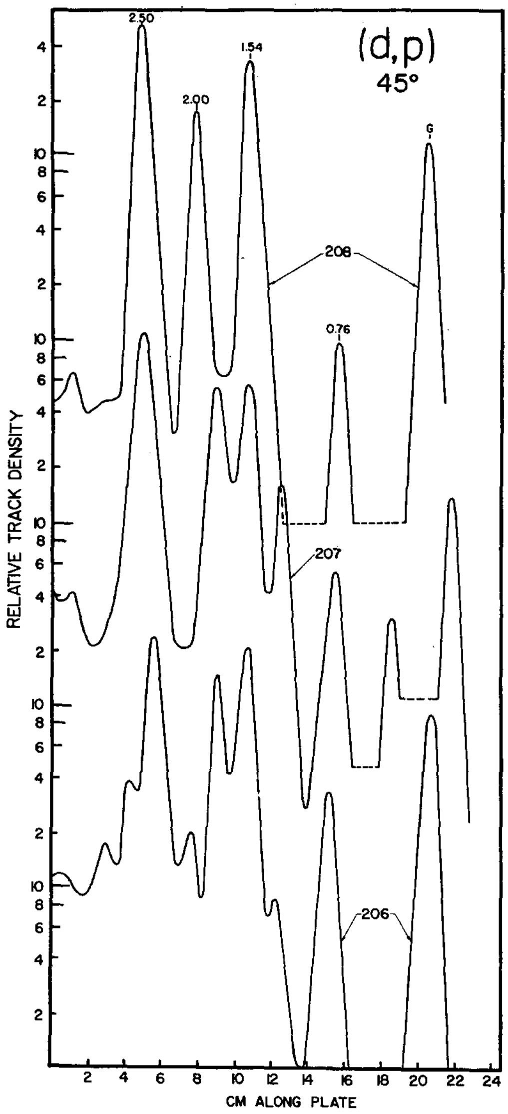

Fig. 5. Survey of energy distributions of protons from (d, p) reactions on Pb^{206}, Pb^{207} and Pb^{208} at 45°. Energies on peaks in spectrum from Pb^{208} are in MeV above the ground state (G). Upper end of spectrum from Pb^{207} contains only a single peak, and upper end of spectrum from Pb^{206} is as shown in fig. 4.

leads to the results listed in the "Best estimate" column of table 3. Also listed in table 3 are the values of these coefficients obtained from the calculated wave functions of True and Ford [4]), and Kearsley [5]). The principal discrepancy seems to be that their wave functions have too much $(p_{\frac{1}{2}})^{-2}$.

Some comment is perhaps in order concerning the peak in fig. 5 at 1.99 MeV. It was observed at the same energy at 30°, 60° and 90°; it therefore cannot be due to an impurity of an element of mass less than 150 since centre of mass effects in elements lighter than this would cause noticeable shifts in the energy.

It would be very difficult to explain the intensity if it were due to an impurity with less then 0.2 % abundance, and no heavy element impurity of this abundance is known or expected to be present. It thus seems quite likely that this is a previously unknown level of Pb^{207}. Since it is not observed in Pb^{208}(d, t) reactions, it cannot be a hole level; it is thus the lowest lying non-hole level in Pb^{207}.

4. Results and Conclusions ($N > 126$ Shell)

The low energy part of the energy spectra of protons from (d, p) reactions on Pb^{206} and Pb^{207}, and the entire spectrum from Pb^{208} and Bi^{209} is due to excitation of particle states in the $N > 126$ shell. Some typical measured spectra are shown in figs. 5 and 6.

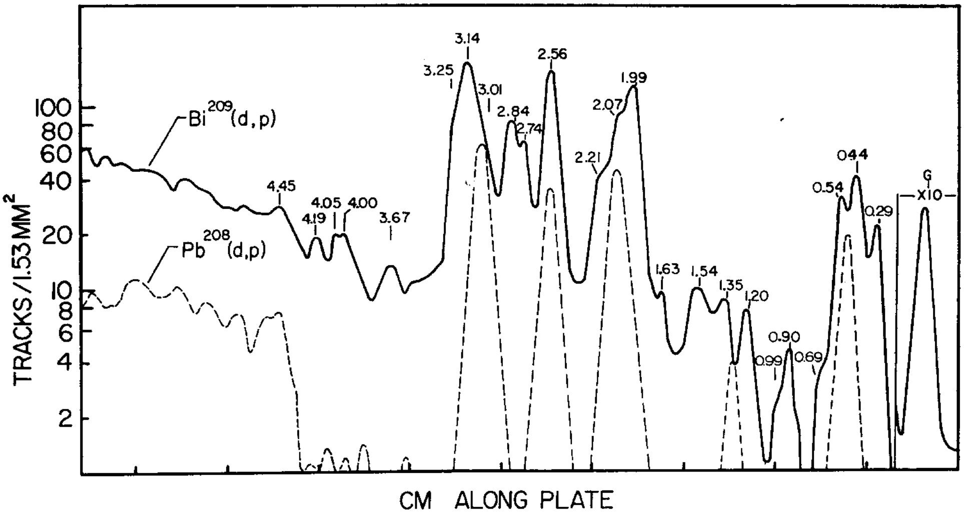

Fig. 6. Energy distribution of protons from Bi^{209}(d, p)Bi^{210} at 90°. Numbers attached to peaks are energies above the ground state in MeV; values in parentheses are from ref. [11]). The spectrum from Pb^{208}(d, p)Pb^{209} is shown for comparison.

The simplest spectrum to understand theoretically should be that from Pb^{208}(d, p)Pb^{209}, as in this case each single particle state should be a single nuclear level. Seven single particle levels are expected in this shell, namely those listed in the first column of table 4. This immediately raises a problem, as there are only five peaks in the spectrum (cf. figs. 1 and 5); the only simple explanation is that two of the observed peaks are unresolved doublets. To test whether any of the peaks are wider than can be explained by the experimental resolution, the ratios of peak height to area were determined; they were found to be about 5 % less for the two lowest energy peaks (2.00 MeV and 2.50 MeV) than for the other three, which might indicate that the former are double. However, this small a difference could probably be explained by experimental error and small corrections, so that the evidence is far from conclusive.

TABLE 4

States of Pb^{209} expected to be excited by Pb^{208}(d, p) reactions. The last three columns are obtained under the following assumptions: (A) energies are as in column 2 of this table; (B) observed peaks in spectrum are G-$g_{\frac{9}{2}}$, 0.76-$i_{\frac{11}{2}}+j_{\frac{15}{2}}$, 1.54-$d_{\frac{5}{2}}+s_{\frac{1}{2}}$, 2.00-$g_{\frac{7}{2}}$, 250-$d_{\frac{3}{2}}$; (C) same as (B) except 1.54-$d_{\frac{5}{2}}$ and 2.50-$d_{\frac{3}{2}}+s_{\frac{1}{2}}$.

State	Expected energy (MeV)	Expected intensity (relative)		
		(A)	(B)	(C)
$s_{\frac{1}{2}}$	2.0	10	10	10
$d_{\frac{5}{2}}$	≈ 1.3	9	11	7
$d_{\frac{3}{2}}$	≈ 2.7	10	11	8
$g_{\frac{9}{2}}$	0	4	4	3
$g_{\frac{7}{2}}$	2.5	7	7	5
$i_{\frac{11}{2}}$	≈ 0.4	2	2	2
$j_{\frac{15}{2}}$	≈ 1.0	2	2	2

A more important, and somewhat related problem is the identification of the various single particle states. First estimates of the expected positions of these are listed in table 4; they were arrived at as follows:

(1) The location of the $s_{\frac{1}{2}}$ was estimated from the position of the neutron giant resonance at $A \approx 160$ and the rate of shift of this state with A as calculated from optical model and experimentally verified in lighter mass regions [9]).

(2) The location of the "centre of gravity" of the d-states was taken to be coincident with the s-state. This is the situation in the previous s-state (Zr^{91}) but there is no reason to expect it to be more than very roughly true here.

(3) The spin orbit splitting ΔE_{so} should be roughly given by

$$\Delta E_{\text{so}} = K(2l+1). \tag{6}$$

In Pb^{207}, eq. (6) is valid for the p-states and f-states with $K = 0.28$ MeV. This determines the location of the $g_{\frac{7}{2}}$ state if we accept the evidence [2]) that the ground state is $g_{\frac{9}{2}}$, and determines the location of the d-states if we accept the above crude arguments on the location of their centre of gravity.

(4) The $i_{\frac{11}{2}}$ and $j_{\frac{15}{2}}$ are located relative to the other states from Nillson's calculation [10]).

Given the expected energy, the expected relative intensity may be calculated from eqs. (1), (2) and (5) with $g(Q)$ and k taken as above. The results are listed in table 4 as "Expected intensities (A)".

In order to throw more light on the problem, angular distributions of the five peaks in the spectrum were measured. The results are shown in fig. 7; they are somewhat disappointing in that there are very few differences among angular distributions for various groups. The most noticeable feature is that the 2.00 MeV group is different from any of the others, and falls off very rapidly in the forward direction. The latter property plus the intensity discrepancy would indicate that it is not the $s_{\frac{1}{2}}$ state. Since angular distributions from the two d-states should be similar, it is very probably not a d-state. If it were, eq. (6) would also be badly violated, and the intensity fit would be poor. The only alternative at all consistent with its intensity would be to assign it as $g_{\frac{7}{2}}$. The difference between its angular distribution and that of the ground state group might be explained by the large difference in Q-value.

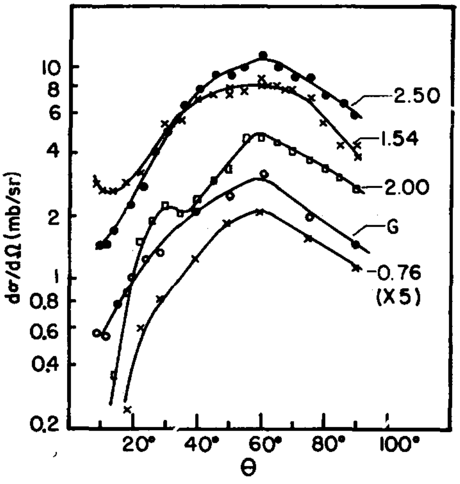

Fig. 7. Angular distributions of various peaks from $Pb^{208}(d, p)Pb^{209}$. All data shown are consistently normalized except those for the 0.76 MeV group which have been multiplied by 5.

If we accept this, it seems obvious to assign the 1.54 and 2.50 MeV states as $d_{\frac{5}{2}}$ and $d_{\frac{3}{2}}$ respectively and assume that the $s_{\frac{1}{2}}$ is degenerate with one of them. In view of the angular distributions, it seems most natural to assume it to be degenerate with the 1.54 MeV state; the intensities calculated from this assumption are listed in table 4 as "Expected Intensities (B)". The discrepancy with measured intensities is quite bad; for example the ratio of the 1.54 to 2.50 MeV state intensities is expected to be ≈ 2 as compared to an average value of 0.8 at large angles. While the intensity calculations are indeed crude, such a large discrepancy over such a short range of Q and l_n would be most unexpected.

The other alternative, that the $s_{\frac{1}{2}}$ is nearly coincident with the $d_{\frac{5}{2}}$ at 2.50 MeV gives the relative intensities listed in the last column of table 4. The expected intensity ratio of the two peaks is now about 0.4 which disagrees as badly with the experimental value (0.8) as the other assumption. In addition, the angular distributions can only be interpreted as indicating that the $l_n = 0$ angular distribution decreases rapidly at forward angles, which is most distasteful. Thus, there are important difficulties with any assignments we may make, and the resolution of this problem must await further experimental information.

The spectra from the other reactions studied show similarities to that from Pb^{208}, except that each level is split into two or more as expected. However, any detailed analysis is difficult without the solution to the Pb^{208} problem. It might be noted that the data show the ground state of Bi^{210} and Bi^{209}(d, p); this was only recently observed for the first time. From its low intensity, it has very little single particle component.

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