AN INTERACTING QUARTET-BOSON MODEL

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A new boson model is proposed, in which the bosons represent quartets of two neutrons and two protons rather than pairs of identical nucleons. The model is applied to calculate the energy spectra of ^{168}Er and other even–even rare earth nuclei containing equal number of valence neutrons and protons, with very satisfactory results.

The microscopic description of nuclei with many valence nucleons has been an important problem in nuclear physics for many years. Such nuclei are not amenable to shell model treatments due to the astronomical dimensions of the basis involved [1]. The collective model takes a view complementary to that of the shell model, concentrating on the degrees of freedom associated with the nucleus as a whole rather than with the individual nucleons. In between these two extreme outlooks, cluster models consider the degrees of freedom associated with groups of nucleons [2]. Perhaps the most sophisticated of them is the "roton model", which includes both quartets of two neutrons and two protons and pairs of identical nucleons [3].

More recently, the interacting boson approximation (IBA or IBA-1) considers pairs of identical valence-nucleons, representing them by bosons with angular momenta $L = 0$ (s-bosons) and $L = 2$ (d-bosons) [4]. The six s- and d-bosons give rise to a U(6) group structure which leads to an appealingly simple description for the collective spectra of even–even heavy nuclei [5]. The IBA does not distinguish between neutrons and protons. When this distinction is made, leading to the microscopic interacting boson model (IBM or IBA-2) [6], some of the simplicity is lost. In this case there are twelve bosons, giving rise to a more complicated $U(6) \otimes U(6)$ group structure. The dimensions of the basis involved grow to the order of the few thousand, and moreover the strongest part of the residual nucleon–nucleon interaction, namely the neutron–proton (n–p) component, has to be explicitly diagonalized [7].

In this letter we present an alternative boson model which focusses on the degrees of freedom associated with clusters that include most of the n–p interaction, giving rise to a framework simpler than that of the IBM. This is achieved using as building blocks quartet-bosons that represent clusters of two neutrons and two protons.

The most bound cluster observed in nature is ^{4}He or the alpha-particle. In heavy nuclei neutrons and protons occupy different shells. Nevertheless, the strong interaction between them, particularly when they occupy spin–orbit "partner" orbits [8,9], energetically still favours correlated states of four nucleons with total isospin $T = 0$ [10]. We therefore propose to group the valence nucleons of heavy even–even nuclei into as many quartets as possible and the remaining identical particles into pairs. We then describe both types of clusters in terms of s- and d-bosons, in analogy to the IBA. Clearly, both of the boson spaces considered above formally contain the three types of collective spectra that are associated with the extensively discussed IBA group chains [5]:

I: $U(6) \supset U(5) \supset O(5) \supset O(3)$.
II: $U(6) \supset U(3) \supset O(3)$.
III: $U(6) \supset O(6) \supset O(5) \supset O(3)$.

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However, it is an experimental fact that nuclear deformation arises only when *both* valence neutrons and protons are present. We are therefore lead to include the SU(3) group describing rotations only for the quartet-bosons (Q-bosons) hamiltonian and exclude it from the alike-nucleon pair-bosons (P-bosons) part. This picture suggests that only chain I may be necessary to describe the P-boson hamiltonian, as generally assumed in the IBM [7].

The most general two-body hamiltonian [5,7] for the Q-bosons involves at most nine parameters in addition to the double magic core energy E_c. The ϵ_s and ϵ_d represent the single boson energies and the seven two-body matrix elements V_{ssss}, V_{sddd}, V_{sdsd}, V_{ddss} and $V_{dddd}(J = 0, 2, 4)$ represent the boson–boson interaction, in a self-explanatory notation.

If we restrict our analysis to excitation energies, the only relevant single particle energy parameter is the difference $\epsilon = \epsilon_s - \epsilon_d$ and moreover not all the two-body matrix elements are linearly independent. When dealing simultaneously with the excitation spectra of several nuclei the hamiltonian contains at most seven linearly independent parameters [11]. We therefore set $V_{ssss} = 0$ in all our calculations. When dealing with a single nucleus the number of independent parameters further reduces to six. The hamiltonian matrices can easily be constructed using conventional shell models [12] techniques and fractional parentage coefficients [13].

We present here the first tests of this model, for the case of "pure-quartet" heavy even–even nuclei in the rare earth region. We first consider the case of ^{168}Er. This deformed nucleus has been recently studied in great detail both from the experimental and theoretical points of view. Experimentally, its spectrum is claimed to be completely known up to an excitation energy of 2.2 MeV, including nine positive parity K-bands [14]. Theoretically, an IBA calculation including 18 bosons accurately reproduces the energies of 26 levels belonging to the six $K = 0^+$ and 2^+ bands [15].

In the present model, ^{168}Er is described in terms of only nine Q-bosons ($N_Q = 9$) with no P-bosons present. The largest matrix involved is of dimension 25 which makes possible a least-square search of the effective hamiltonian parameters. The search is performed including also the levels belonging to the experimental $K = 3^+$ and 4^+ bands and setting $\epsilon = 0$, thus leaving only six free parameters in the hamiltonian. The above procedure is the same as the one used in the determination of shell model effective interactions.

The agreement with the experimental data is quite remarkable as indicated by an rms deviation σ of only 58 keV for 33 energy levels. The results are presented in table 1, together with the experimental data. Essentially the same results are obtained when in addition $V_{dddd}(J = 4)$ is set identically zero, thus leaving only 5 free parameters. If a further reduction to four parameters is attempted by also setting $V_{sdsd} = 0$, still a quite satisfactory $\sigma = 76$ keV is obtained. However any other choice significantly deteriorates the agreement between theoretical and experimental energies. The values obtained for the six two-body hamiltonian matrix elements are shown in table 2, and are in general quite small, as expected since most of the residual nucleon–nucleon interaction is already absorbed in the quartet-structures. A few excited states that were not included in the least square fit are nevertheless successfully reproduced (see table 1). Within the present model a cutoff is expected for ^{168}Er at $J = 18^+$. This actually means that at such values of J the rotational motion must increasingly involve additional degrees of freedom. It is therefore of interest to investigate experimentally any possible structure changes that may occur at such values of the angular momentum.

It is worth mentioning that the present results do not imply a perturbed SU(3) scheme as assumed in ref. [15]. Consequently no K-quantum numbers can be assigned to the theoretical eigenstates.

In the second test of the present model we extend the calculation to encompass all nuclei in the rare earth region that can be described in terms of only Q-bosons, with $2 \leqslant N_Q \leqslant 9$. Greater values of N_Q are not included since the protons would fill more than half of the shell. The experimental data [16] show in this case a quite definite transition to deformed shape as N_Q increases, with the onset of deformation occurring around $N_Q = 5$. In addition, the rotational features observed for $N_Q \geqslant 5$ exhibit a very uniform behaviour. The moments of inertia are almost constant and tend to become smaller for neighbouring even–even nuclei. This "quartet-valley" for the excitation energy of the first 2^+ state provides support to the present model, in which the presence of P-bosons is considered to be responsible for the departure from an extreme rotational picture. The quasi-vibrational structure observed for $N_Q < 5$ is a manifestation of the reduced sizes of the spaces generated by such small numbers of bosons.

In this second calculation also ϵ is taken as a free parameter. The least-square search yields an excellent

Table 1
Experimental and calculated positive-parity energy levels of ^{168}Er. Levels are grouped by the experimentally [14] assigned K-quantum numbers (1st and 5th columns).

K_n	J	E_{exp} [keV]	E_{cal} [keV]	K_n	J	E_{exp} [keV]	E_{cal} [keV]
0_1^+	0	0	0	0_3^+	0	1422	1412
	2	79	77		2	1493	1526
	4	264	255		4	1656	1584
	6	548	536		6	1902	1792
	8	928	919				
	10	1396	1404	3_1^+	3	1653	1722
	12	1942 a,b)	1994		4	1736	1650
	14	(2564) b,d)	2694		5	1839	1715
	16	–	3520		6	1961	1904
	18	–	4491		7	–	1988
2_1^+	2	821	868	2_2^+	2	1848	1775
	3	895	937		(3)	1915	1961
	4	994	1033		(4)	2002	1927
	5	1117	1150		5	2108	2035
	6	1263	1288				
	7	1432	1452	2_3^+	2	1930	1942
	8	1624	1642		(3)	1994 b)	2398
	9	(1836) b,d)	1851		4	2080	2077
	10	(2071) b,d)	2081	4_1^+	4	2030	2087
0_2^+	0	1217	1185	4_2^+	4	2055 b)	2272
	2	1276	1262		5	2169	2172
	4	1411	1426	c)	0	–	1892
	6	1616	1692	c)	0	–	2216
	(8)	(1890) b)	2034	c)	2	2137	2119

a) Experimental energy from ref. [16]. b) Level not included in the least-square search.
c) K not assigned. d) Experimental energy from footnote on page 524 of ref. [14].

Table 2
Effective hamiltonian parameters obtained with the least-square procedure for the two calculations mentioned in the text.

N_Q	ϵ [keV]	V_{sdsd} [keV]	$V_{dddd}(0)$ [keV]	$V_{dddd}(2)$ [keV]	$V_{dddd}(4)$ [keV]	V_{ssdd} [keV]	V_{sddd} [keV]	σ [keV]
9	0	−61	−153	−120	−8	−75	−116	58
2–8	699	−434	−812	−740	−681	−115	−357	85

overall fit of 33 levels with an rms deviation of 85 keV, as shown in table 3. The parameters obtained are listed in table 2. The single-quartet-boson splitting turns out to have a reasonable value for the region, similar to the one expected for ^{136}Te.

The results obtained in both calculations lend support to the present model. In particular, the description of the ^{168}Er energy spectrum is probably the most detailed ever obtained for such a heavy nucleus. A more extensive analysis of the implications of this model, particularly when both P- and Q-bosons are present is in progress.

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Table 3
Experimental and calculated positive parity energy levels of even–even rare earth nuclei with $2 \leq N_Q \leq 8$.

Nucleus	N_Q	J	E_{exp} [keV]	E_{cal} [keV]
^{164}Dy	8	0	0	0
		2	73	77
		4	242	237
		6	501	495
		8	844	827
		10	1261	1245
		12	1745	1711
		14	2290	2258
		2	761	837
		3	828	989
		4	916	972
		5	1025	1065
		6	1155	1255
^{160}Gd	7	0	0	0
		2	75	65
		4	249	245
		6	514	503
		8	868	860
		2	990	815
		4	1150	959
^{156}Sm	6	0	0	0
		2	76	98
		4	250	272
		6	518	567
		0	1070	991
^{152}Nd	5	0	0	0
		2	76	67
		4	241	294
		6	488	580
		8	810	996
^{148}Ce	4	0	0	0
		2	159	146
		4	454	370
		6	841	758
^{144}Ba	3	0	0	0
		2	199	169
		4	530	540
		6	962	900
^{142}Xe	2	0	0	0
		2	377	403
		4	835	887

lication, ref. [17] came to the attention of the authors. In this paper Q-bosons are introduced in order to describe negative parity states.

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