

the neighboring He^3 appear necessary. If these functions assume the form anticipated in Sec. II, then significant relaxation about the He^4 will not be necessary. To get J near the observed value an improved form for the correlation function $f(r_{ij})$ is also needed, and this represents a difficult task. One of the chief difficulties is that those parts of $|O\rangle$ which are important to J have little influence on E_0 and so are not well determined by an energy variational method.

Simply evaluating the change in the self-consistent harmonic force constants around the He^4 does not explain the large observed τ_{pt}^{-1} suggesting that this model is too restrictive. On the other

hand, although τ_{pt}^{-1} can be predicted directly using a continuum model, this model is so crude that it tells us little in detail about the defect.¹⁴ It does suggest, however, that division of defect properties into mass and force constant changes may be artificial and that large relaxation around the He^4 is not necessarily needed to explain τ_{pt}^{-1} .

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¹M. G. Richards, J. Hatton, and R. P. Giffard, *Phys. Rev.* **139**, 91 (1965); R. P. Giffard and J. Hatton, *Phys. Rev. Letters* **18**, 1106 (1967).

²B. Bertman, H. A. Fairbank, R. A. Guyer, and C. W. White, *Phys. Rev.* **142**, 79 (1966).

³Bal Krishna Agrawal, *Phys. Rev.* **162**, 731 (1967).

⁴J. Callaway, *Phys. Rev.* **113**, 1046 (1959).

⁵Lord Rayleigh, *Theory of Sound* (MacMillan and Co., London, 1896), Vol. II, p. 149.

⁶P. G. Klemens, *Proc. Phys. Soc. (London)* **A68**, 1113 (1955).

⁷P. G. Klemens and A. A. Maradudin, *Phys. Rev.* **123**, 804 (1961).

⁸L. H. Nosanow, *Phys. Rev.* **146**, 120 (1966).

⁹T. R. Koehler, *Phys. Rev. Letters* **18**, 654 (1967).

¹⁰L. H. Nosanow and W. J. Mullin, *Phys. Rev. Letters* **14**, 133 (1965). [Also the $T=0^\circ\text{K}$ limit in L. H. Nosanow and C. M. Varma, *Phys. Rev. Letters* **20**, 912 (1968)].

¹¹J. H. Hetherington, W. J. Mullin, and L. H. Nosanow, *Phys. Rev.* **154**, 175 (1967).

¹²R. E. Peierls, *Quantum Theory of Solids* (Oxford-Clarendon Press, Oxford, England, 1955), p. 121.

¹³J. H. Vignos and H. A. Fairbank, *Proceedings of the Eighth International Conference on Low-Temperature Physics, London, 1962* edited by R. O. Davies (Butterworths Scientific Publications Ltd., London, 1962).

¹⁴The model is reasonable in this case as $T \lesssim \theta/10$ and long wave phonons are unaffected by changes in detail.

Experimental Heat Capacity of Pure Liquid He^3 †

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Measurements of the heat capacity of pure He^3 at 0.24 atm obtained by a differencing method are presented for the temperature range from 20–150 m°K. The temperature scale T^* used is that valid for powdered cerium magnesium nitrate in the form of a right circular cylinder with diameter equal to height. There is excellent agreement between the present and earlier measurements. Considering all low-pressure difference data we find that from 6–125 m°K the ratio of heat capacity to magnetic temperature decreases linearly with increasing temperature. The relationship of the measurements to the temperature scale and to theories of spin fluctuations in He^3 is also discussed.

I. INTRODUCTION

In recent years the heat capacity of pure He^3 has been of considerable interest. It is related to the Landau theory of Fermi liquids.¹ It bears directly on the question of superfluidity in He^3 .^{2,3} What is known of its temperature dependence was

not predicted by theory and has been the subject of theoretical speculation.^{4,5} Owing to the possibility of a superfluid transition, most recent measurements have emphasized the properties of He^3 at the lowest achievable temperatures.^{3,6} As a result, in no single experiment have accurate measurements been made in the temperature range,

20–150 m°K, of the present work. Recently Doniach and Engelsberg⁷ devised a qualitatively successful theory to explain the heat capacity of pure He³. In their theory, liquid He³ is conceived as a nearly ferromagnetic Fermi fluid in which spin fluctuations play a major role. Following this idea through, Rice⁸ worked out a theory of transport in He³ which predicted remarkably well the results of thermal conductivity measurements.^{9–11} The importance of spin fluctuations in understanding the heat capacity of He³ has been given further emphasis by calculations of Brenig and co-workers¹² and of Amit, Kane, and Wagner,¹³ though it has not been universally accepted.¹⁴ One characteristic of the above theories is that they predict not only the limiting low-temperature behavior of He³, as in the Landau theory, but also the finite temperature properties. In particular they predict that

$$C/n_3RT = \gamma + \Gamma T^2 \ln(T/\theta_c) + \dots, \quad (1)$$

where C is the heat capacity, n_3 is the number of moles of He³, R is the gas constant, T is the absolute temperature, γ and Γ are constants, and θ_c is a characteristic temperature. According to recent work of Brinkman and Engelsberg,¹⁵ however, the limiting temperature dependence of Eq. (1) should not be achieved except at temperatures below a few millidegrees. They suggest that in the present temperature range many terms must be considered in the series development of C/nRT .

With the exception of the results of Ref. 6, which are based on a nuclear magnetic resonance temperature scale in copper and are of relatively poor precision, all very low temperature measurements of the heat capacity of He³ have been made on a magnetic temperature scale T^* valid for powdered cerium magnesium nitrate (CMN) in the form of a right circular cylinder with diameter equal to height. This magnetic temperature scale was interpreted by one of us¹⁶ in terms of properties of Fermi liquids with the conclusion that T^* , subject to a correction estimated to be less than a few tenths of a millidegree, is the same as the Kelvin temperature T down to about 3m°K. This interpretation was called in question recently by Abraham and Eckstein¹⁷ who based their analysis on an assumed T^{-2} dependence¹⁸ of the heat capacity of CMN between 6 and 15 m°K. They concluded that the above powder thermometer temperature T^* should be corrected upward by 1.7 m°K to obtain the magnetic temperature T_S^* indicated by a spherical single crystal. According to Ref. 18 the temperature T_S^* is the same as the Kelvin temperature above 6 m°K. Abraham and Eckstein's proposed correction to the powder temperature scale has a profound effect on the interpretation of the experiments in terms of the Fermi-liquid theory. This effect has not yet been discussed quantitatively. Such a discussion will be undertaken in this paper since there seems to be some question²⁰ whether the experimental precision of the He³ heat-capacity measurements is high enough to demonstrate that in fact Abraham and Eckstein's correction does lead to any real

difficulties.

Recent direct measurements of Abel and Wheatley¹⁹ intercomparing the magnetic temperatures indicated by a powder and a single-crystal sphere do not support Abraham and Eckstein's conclusions but do favor the original interpretation.¹⁶ However, Abel and Wheatley find that, as one might expect, the shape correction for a powder thermometer is not precisely the same for all such thermometers but in fact varies by an amount probably of order 0.1–0.2 m°K. In the present work we shall therefore use the uncorrected T^* scale with the word of caution that the Kelvin temperature differs from T^* by a rather uncertain amount which is estimated to be a few tenths of a millidegree. This uncertain correction will not have a large effect on the interpretation of the present results, whose lower temperature limit is 20 m°K.

Measurements of the heat capacity of liquid He³ are reviewed in Ref. 3. In no single experiment were sufficiently accurate measurements obtained over a wide enough temperature range to allow quantitative comparison with the new theoretical work. Systematic errors in the measurement of T and n_3 , Eq. (1), inhibit the quantitative comparison of different sets of data. Moreover the lowest-temperature results of Anderson, Reese, and Wheatley,²¹ for example, were marred by the defect that the calorimeter "background" heat capacity was not known. This defect was rectified in the work described in Ref. 3 by using a "difference" method described in Sec. II to eliminate the background heat capacity. However, in the work reported there the heat capacity at low pressure was measured only to 50 m°K.

In the present experiments we have measured the heat capacity of pure He³ (less than 10 ppm He⁴) at a pressure of 0.24 atm over a temperature range from 20–150 m°K. These measurements are important not only as they determine the temperature dependence of the heat capacity of pure He³ but also as they relate to the confidence which may be placed in the earlier measurements³ which extend to lower temperatures.

II. EXPERIMENTAL

Both our general experimental method and our methods of data analysis are very similar to those described in Ref. 3, to which the reader is referred for a detailed discussion of these matters. The He³ was contained in an epoxy cell in intimate contact with a magnetic thermometer of powdered cerium magnesium nitrate in the form of a right circular cylinder with diameter equal to height and of mass 1.042 g. Heat was added electrically. Susceptibilities were measured by means of a 17-Hz mutual inductance bridge. In the first series of measurements the heat-capacity cell contained 0.0183 moles of He³. The cell was then modified without changing the thermometer in any way to increase the volume available for He³. The modified cell contained 0.0344 moles of He³. Measurements of heat capacity were then repeated. Above 30 m°K the cell background heat capacity was negligible.

Aside from its advantage of eliminating the calorimeter "background" heat capacity the differencing method which we use has other advantages. Each new experiment requires a new calibration of the magnetic thermometer and a new measurement of the number of moles of He³ in the cell. Hence errors in these quantities are readily detected as systematic inconsistencies in the values of $(C_1 - C_2)/(n_1 - n_2)RT^*$, C_1/n_1RT^* , and C_2/n_2RT^* at higher temperatures where the "background" heat capacity is negligible compared with that of the He³.

III. RESULTS AND DISCUSSION

The results of our measurements at 0.24 atm with C/nRT^* plotted linearly against T^* are shown by closed circles in Fig. 1. Also shown are both the results of Ref. 3 for pure He³ at 0.28 atm (open circles) and at 27.0 atm (open squares), and the results^{22,23} for a 5.0% solution of He³ in superfluid He⁴ (closed squares). Even without numerical analysis it is clear that there is excellent agreement between the present measurements and those of Ref. 3 in the temperature region from 20–50 m°K where the two sets of data overlap. This increases our confidence in both measurements. It is also clear that within the experimental scatter the quantity C/nRT^* depends linearly on T^* from the lowest temperatures to 125 m°K. This temperature dependence was suggested in Ref. 21, but both experimental imprecision and ignorance of the calorimeter "background" heat capacity prevented a strong case from being made. Owing either to experimental imprecision or to inaccuracy resulting from ignorance of the $T^* - T$ correction it is quite possible that C/nRT really should be described by some other temperature dependence below approximately 20 m°K.

We have used the method of least squares to treat the data on low-pressure He³ in Fig. 1. Since there appears to be some upward curvature at the highest temperatures we have used only data below 125 m°K to make the straight-line fits described below. Data were fitted to

$$C/nRT^* = \gamma - \beta T^* \quad (2)$$

We considered four sets of data: (1) present results between 50 and 125 m°K; (2) all present results below 125 m°K; (3) all results of Ref. 3 at 0.28 atm; and (4) all present results below 125 m°K and all results of Ref. 3 at 0.28 atm. All points are equally weighted. The results of these analyses are given in Table I. The errors indicated are the standard deviations. The present experi-

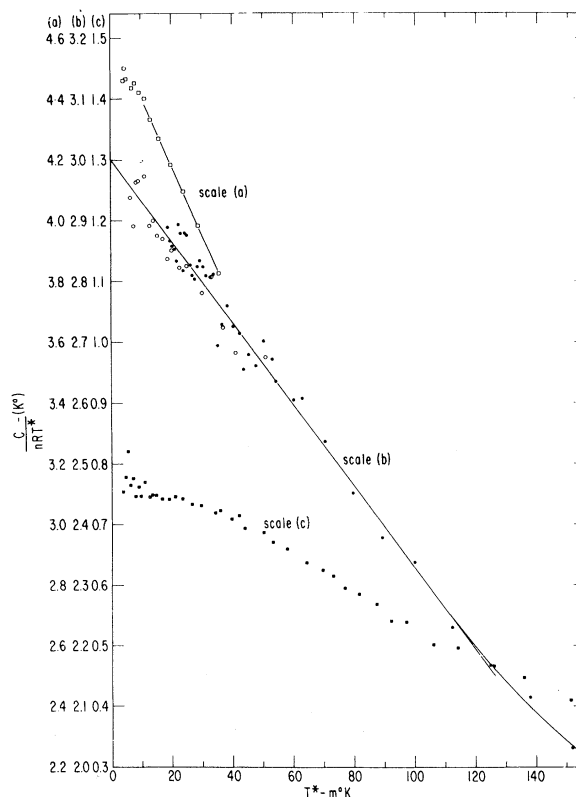


FIG. 1. Ratio of normalized specific heat C/nR to magnetic temperature T^* of pure He³ at 0.24 atm (●●●) from the present work, of pure He³ at 0.28 atm (○○○) from Ref. 3, of pure He³ at 27.0 atm (□□□) from Ref. 3, and of a 5% solution of He³ in superfluid He⁴ (■ ■ ■) from Refs. 22 and 23.

mental results are remarkably consistent with the 0.28-atm results of Ref. 3. There seems to be little reason not to combine the two sets of data. The values of γ given in Table I correspond to $m^*/m = 3.03$, which probably is an upper limit. For the purpose of allowing other interpretations of our data we list them explicitly in Table II. The data of Ref. 3 are listed in Table I of that paper. Above 10 m°K the 27.0-atm data of Ref. 3 also are an excellent fit to a linear relationship between C/nRT^* and T^* . A least-squares fit to the experimental data for $T^* > 10$ m°K is

$$C/nRT^* = (4.63 \pm 0.01)(K^\circ)^{-1} - (22.5 \pm 1.1)T^*(K^\circ)^{-2}, \quad (27.0 \text{ atm}).$$

TABLE I. Least-squares determined parameters for Eq. (2).

| Experimental data processed | No. of points | Temperature range (m°K) | γ (K°) ⁻¹ | β (K°) ⁻² |
|--|---------------|-------------------------|-----------------------------|----------------------------|
| Present experiment: 0.24 atm | 11 | 50–125 | 3.04 ± 0.02 | 7.2 ± 0.2 |
| Present experiment: 0.24 atm | 38 | 19–125 | 3.01 ± 0.01 | 6.9 ± 0.2 |
| Reference 3: 0.28 atm | 40 | 6–51 | 3.00 ± 0.01 | 7.1 ± 0.6 |
| Present experiment: 0.24 atm and Reference 3: 0.28 atm | 78 | 6–125 | 3.00 ± 0.01 | 6.74 ± 0.15 |

TABLE II. C/nR and C/nRT^* for pure He³ at 0.24 atm.

| T^* (m°K) | $10^2 C/nR$ | C/nRT^* [(K°) ⁻¹] |
|-------------|-------------|---------------------------------|
| 18.93 | 5.47 | 2.888 |
| 19.62 | 5.62 | 2.866 |
| 20.27 | 5.79 | 2.858 |
| 21.03 | 6.00 | 2.853 |
| 21.71 | 6.15 | 2.833 |
| 22.40 | 6.48 | 2.894 |
| 23.09 | 6.65 | 2.879 |
| 23.80 | 6.70 | 2.817 |
| 24.48 | 7.05 | 2.880 |
| 25.13 | 7.23 | 2.876 |
| 26.11 | 7.38 | 2.826 |
| 26.80 | 7.53 | 2.809 |
| 27.64 | 7.75 | 2.803 |
| 28.39 | 8.02 | 2.824 |
| 29.32 | 8.31 | 2.834 |
| 30.25 | 8.54 | 2.824 |
| 31.37 | 8.81 | 2.809 |
| 32.71 | 9.18 | 2.806 |
| 33.82 | 9.51 | 2.811 |
| 35.07 | 9.45 | 2.694 |
| 36.55 | 9.97 | 2.729 |
| 38.35 | 10.58 | 2.759 |
| 40.20 | 10.96 | 2.725 |
| 42.28 | 11.48 | 2.714 |
| 43.56 | 11.57 | 2.655 |
| 45.29 | 12.13 | 2.679 |
| 47.74 | 12.70 | 2.660 |
| 50.20 | 13.56 | 2.702 |
| 53.06 | 14.17 | 2.671 |
| 56.24 | 14.82 | 2.635 |
| 60.13 | 15.66 | 2.605 |
| 63.54 | 16.57 | 2.607 |
| 70.28 | 17.82 | 2.536 |
| 79.53 | 19.49 | 2.450 |
| 89.14 | 21.19 | 2.377 |
| 99.89 | 23.34 | 2.337 |
| 112.2 | 25.01 | 2.229 |
| 124.8 | 27.05 | 2.167 |
| 138.0 | 29.17 | 2.114 |
| 152.0 | 30.88 | 2.031 |

It is also possible to fit the experimental data to a relation of the form of Eq. (1). In this case there are three parameters to fit: γ , Γ , and θ_C . These are obtained by obtaining the best straight-line fit to

$$(\gamma - C/nRT^*)/T^{*2} = \Gamma \ln(T/\theta_C).$$

Such a fit is demonstrated in Fig. 2 and corresponds to

$$\gamma = 2.93(\text{K}^\circ)^{-1}, \quad \Gamma = 57.4(\text{K}^\circ)^{-3}, \quad \theta_C = 0.29_3^\circ\text{K}.$$

The low-temperature scatter is extremely sensitive to γ . Even for the best fit the scatter below 50 m°K is so large as to make impossible any meaningful comparison with Eq. (1). Between 50 and 150 m°K the data are reasonably represented by Eq. (1). There appears to be no compelling reason, particularly considering the arguments of Brinkman and Engelsberg,¹⁵ to prefer Eq. (1) over

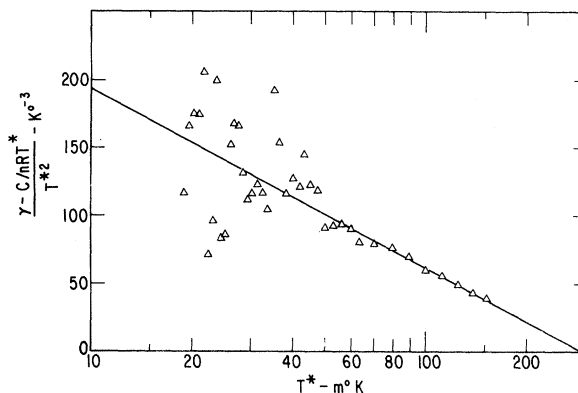


FIG. 2. Fit of the present specific heat data to the temperature dependence of Eq. (1) in the text.

Eq. (2) in the present temperature region. Nevertheless, it is interesting to compare the characteristic temperature $\theta_C = 0.29^\circ\text{K}$ derived from the heat-capacity data with the characteristic temperature θ_K which may be derived from thermal conductivity data. Using the most probable corrections to the temperature scale, Abel and Wheatley¹⁹ reworked the thermal conductivity data.⁹ From their results and the numbers worked out in Ref. 10 we find

$$\theta_K = (0.68 \pm 0.12)^\circ\text{K}.$$

Following Doniach and Rice θ_K and θ_C should be related by $\theta_K/\theta_C = \bar{Q}$, where \bar{Q} is a dimensionless number equal¹⁰ to 1.24 for low-pressure He³. There is not quantitative agreement.

At higher temperatures the present measurements overlap those of a number of other workers. At 125 m°K we find $C/nR = 0.270$. Estimating graphically, we find that at this temperature Abraham *et al.*²⁵ measured $C/nR = 0.273$; Anderson, Reese, and Wheatley²¹ measured 0.256; Brewer, Daunt, and Sreedhar²⁶ measured 0.247; and Strongin, Zimmermann, and Fairbank²⁷ measured 0.249. Our results are thus on the high side of the scatter, though rather close to those of Ref. 25. They are 5% different from those of Ref. 21 and hence within the estimated error of that experiment.

We now wish to turn our attention to the effect on the heat-capacity results represented in Fig. 1 of a correction to the temperature scale based on the arguments given by Abraham and Eckstein.¹⁷ These authors start with the assertion, which they assumed had been proved quantitatively by Hudson and Kaeser,¹⁸ that the heat capacity of CMN obeys the T^{-2} law (T is Kelvin temperature) above 6 m°K. Actually Hudson and Kaeser claimed the T^{-2} law to be valid only between 6 and 15 m°K. According to Ref. 18 above $T = 6$ m°K the magnetic temperature $T_S^*(SC)$ indicated by a single crystal sphere is the same as T . Abraham and Eckstein recognize that since the powder thermometers do not have a spherical shape there should be some correction Δ such that

$$T_S^*(P) = T^* + \Delta, \quad (3)$$

where $T_S^*(P)$ is the magnetic temperature indicated by a powder having a spherical shape. They assume that $T_S^*(P)$ is the same as $T_S^*(SC)$. This is probably a good assumption at sufficiently high temperatures though it is not at the lowest temperatures.¹⁹ The quantity Δ can be evaluated empirically by plotting $(C_{CMN}/n_{CMN}R)^{-1/2}$ versus T^* . A straight line should be found with Δ as intercept.

In order to make the above analysis we need C_{CMN} versus T^* . The difference measurements made in Ref. 3 were needed to eliminate the calorimeter "background" heat capacity, which is probably almost entirely that of the CMN thermometer. If we assume that the "background" heat capacity is that of the CMN, then we can proceed with the analysis. It is important to recognize that in the actual measurements one measures the sum of CMN and He³ heat capacity, the differencing method allowing the individual parts to be determined. Above $T^*=6$ m°K the majority of the heat capacity was in the He³, so C_3 can be expected to have greater precision than C_{CMN} . This is important to keep in mind when considering the credibility of the He³ data with respect to those for the CMN. A final point is that in Ref. 3 care was taken not to refer to the background heat capacity as the CMN heat capacity since it could not be assumed that there were no other appreciable sources of heat capacity. The recent results of Ref. 17 for the CMN heat capacity are sufficiently similar to those of Ref. 3 to make reasonable the assumption that it was indeed C_{CMN} measured in Ref. 3.

In Fig. 3 we plot $(C_{CMN}/n_{CMN}R)^{-1/2}$ versus T^* with data taken from Ref. 3. The straight line shown was fitted by the method of least squares with equal point weighting using data for $T^* > 6$ m°K. We find

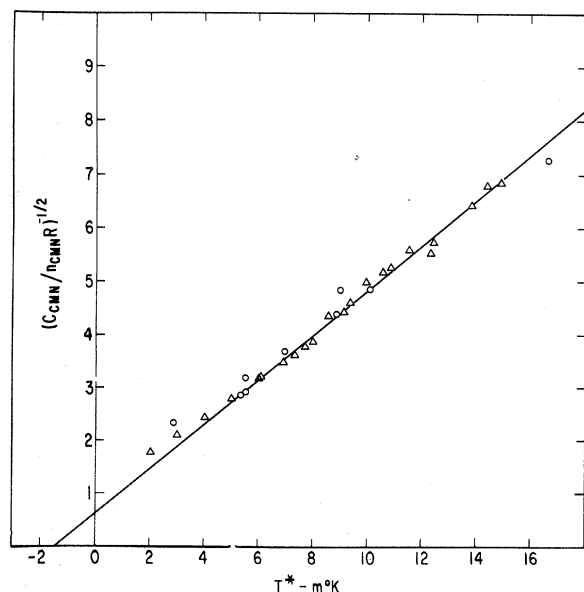


FIG. 3. Plot of $C_{CMN}/n_{CMN}R)^{-1/2}$ for the data of Ref. 3. ($\Delta \Delta \Delta$) and of Ref. 17 ($\circ \circ \circ$). Assuming a T^{-2} law for the normalized specific heat, this plot can be used to determine Δ (see text).

$$\Delta = +(1.48 \pm 0.24) \text{ m}^\circ\text{K}$$

and

$$C_{CMN}(T^* + \Delta)^2/n_{CMN}R = (5.67 \pm 0.14) (\text{m}^\circ\text{K})^2.$$

Inspection of the figure shows that if magnetic temperatures lower than 6 m°K are accepted in making the least-squares fit, then the derived value of Δ will increase. If data are used to 5 m°K, the mean value of Δ increases from 1.48 to 1.54 m°K. The value of

$$C_{CMN}(T^* + \Delta)^2/n_{CMN}R$$

is essentially the same as that, 5.76 (m°K)², given in Ref. 18. Whether or not this agreement is coincidental or is a clue to the source of the discrepancy is not known. We also show in Fig. 3 the data of Abraham and Eckstein¹⁷ for CMN heat capacity. Their results are rather similar to ours, though when fitted by least squares using data from 5.3 to 16.6 m°K they yield

$$\Delta = +(2.5 \pm 0.5) \text{ m}^\circ\text{K}$$

and

$$C_{CMN}(T^* + \Delta)^2/n_{CMN}R = (6.7 \pm 0.3) (\text{m}^\circ\text{K})^2.$$

The CMN heat capacity data treated to obtain $\Delta = 1.48$ m°K were obtained simultaneously with the pure He³ data of Fig. 1 at 0.28 and 27.0 atm. Moreover the results for the heat capacity of a 5% solution of He³ in He⁴ plotted in Fig. 1 were obtained²² by subtracting from a total measured heat capacity that of CMN as measured in Ref. 3, appropriately adjusted for the actual mass of CMN. Hence a consistent analysis of these data using Abraham and Eckstein's suggestions would adjust T^* upward by 1.48 m°K to find T . We have therefore replotted in Fig. 4 the data shown on

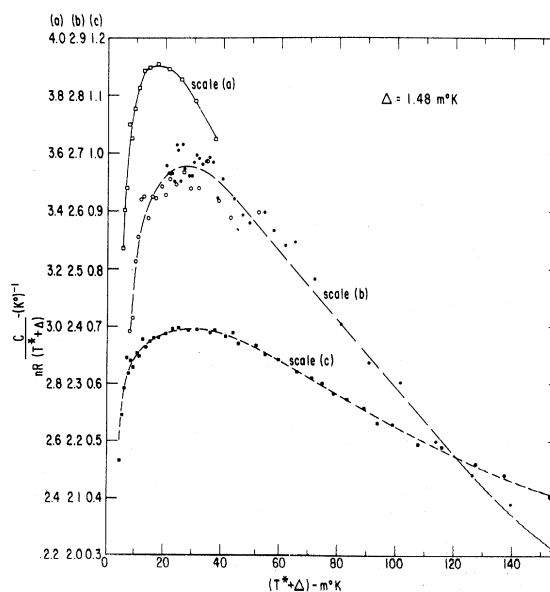


FIG. 4. Replot of the data shown in Fig. 1 after making the substitution of $T^* + \Delta$ for T^* , with $\Delta = 1.48$ m°K as derived from the results shown on Fig. 3.

Fig. 1 using this correction. Relative to the precision of the measurements it is clear that such a correction does indeed have a profound effect on the temperature dependence of the He³ heat capacity, both pure He³ and the dilute solution being affected in a qualitatively similar manner. There is a clear-cut discrepancy. Either pure He³ and dilute solutions have characteristics which are not predicted by theories of the Fermi liquid or the heat capacity of CMN does not obey quantitatively the T^{-2} law in the range 6–15 m°K. The direct magnetic thermometry experiments of Abel and Wheatley¹⁹ suggest that in powder thermometers Δ is only a few tenths of a millidegree. Hence one concludes that in fact

$$C_{\text{CMN}}(T^* + \Delta)^2/n_{\text{CMN}}R$$

is not constant, but increases with temperature in the range of measurement.

IV. CONCLUSIONS

The ratio C/nRT^* for pure He³ at low pressure,

where T^* is the magnetic temperature indicated by a powder thermometer, decreases linearly with T^* from 6–125 m°K within an rms experimental scatter of about $\pm 1\%$. A similar linear dependence between 10 and 35 m°K is found for pure He³ at 27.0 atm. Above 50 m°K the data also fit a law

$$C/nRT = \gamma + \Gamma \ln T/\theta.$$

The above results are affected by corrections to T^* needed to convert it to Kelvin temperature. It is demonstrated that such a correction, based on the assumptions of Ref. 17 but unwarranted according to the direct measurements of Ref. 19, qualitatively affects the temperature dependence of the heat capacity of both pure He³ and a 5% dilute solution. The actual dependence of C/nRT on T , where T is Kelvin temperature, below 20 m°K will depend on the magnitude of the adjustment of T^* to obtain T . These corrections are expected to be no more than a few tenths of a millidegree down to about 3 m°K.^{16,28}

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¹L. D. Landau, Zh. Eksperim. i Teor. Fiz. **30**, 1058 (1956) [English transl.: Soviet Phys. - JETP **3**, 920 (1957)].

²V. P. Peshkov, Zh. Eksperim. i Teor. Fiz. **48**, 997 (1965) [English transl.: Soviet Phys. - JETP **21**, 663 (1965)].

³W. R. Abel, A. C. Anderson, W. C. Black, and J. C. Wheatley, Phys. Rev. **147**, 111 (1966).

⁴P. W. Anderson, Physics **2**, 1 (1965).

⁵R. Balian and D. R. Fredkin, Phys. Rev. Letters **15**, 480 (1965).

⁶E. B. Osgood and J. M. Goodkind, Phys. Rev. Letters **18**, 894 (1967).

⁷S. Doniach and S. Engelsberg, Phys. Rev. Letters **17**, 750 (1966).

⁸M. J. Rice, Phys. Rev. **159**, 153 (1967).

⁹W. R. Abel, R. T. Johnson, J. C. Wheatley, and W. Zimmermann, Phys. Rev. Letters **18**, 737 (1967).

¹⁰J. C. Wheatley, Phys. Rev. **165**, 304 (1968).

¹¹A. C. Anderson, J. I. Connolly, O. E. Vilches, and J. C. Wheatley, Phys. Rev. **147**, 86 (1966).

¹²W. Brenig and H. J. Mikeska, Phys. Letters **24A**, 332 (1967); W. Brenig, H. J. Mikeska, and E. Riedel, Z. Physik **206**, 439 (1967); E. Riedel, Z. Physik **210**, 403 (1968).

¹³D. J. Amit, J. W. Kane, and H. Wagner, Phys. Rev.

Letters **19**, 425 (1967).

¹⁴S. Misawa, Progr. Theoret. Phys. (Kyoto) **38**, 1207 (1967).

¹⁵W. F. Brinkman and S. Engelsberg, Phys. Rev. **169**, 417 (1968).

¹⁶J. C. Wheatley, Ann. Acad. Sci. Fennicae Ser. A VI, No. 210, 15 (1966).

¹⁷B. M. Abraham and Y. Eckstein, Phys. Rev. Letters **20**, 649 (1968).

¹⁸R. P. Hudson and R. S. Kaeser, Physics **3**, 95 (1967).

¹⁹W. R. Abel and J. C. Wheatley, Phys. Rev. Letters **21**, 597 (1968).

²⁰R. P. Hudson, private communication to J. C. Wheatley.

²¹A. C. Anderson, W. Reese, and J. C. Wheatley, Phys. Rev. **130**, 495 (1963).

²²W. R. Roach, Ph. D. thesis, University of Illinois, 1966 (unpublished).

²³A. C. Anderson, D. O. Edwards, W. R. Roach, R. E. Sarwinski, and J. C. Wheatley, Phys. Rev. Letters **17**, 367 (1966).

²⁴S. Doniach and M. J. Rice, Contribution to the Conference on Theoretical Physics, Birmingham, England, July, 1967 (unpublished).

²⁵B. M. Abraham, M. Durieux, C. J. N. van den Meijdenberg, and D. W. Osborne in Proceedings of the Ninth International Conference on Low Temperature Physics, edited by J. G. Daunt, D. O. Edwards, F. J. Milford, and M. Yaquib (Plenum Press, New York, 1965), Pt. A, p. 133.

²⁶D. F. Brewer, J. G. Daunt, and A. K. Sreedhar, Phys. Rev. **115**, 836 (1959).

²⁷M. Strongin, G. O. Zimmermann, and H. A. Fairbank, Phys. Rev. **128**, 1983 (1962).

²⁸W. C. Black, Ph. D. thesis, University of Illinois, 1967 (unpublished).