PHYSICAL REVIEW C VOLUME 17, NUMBER 6 JUNE 1978

Nuclear field theory and the inelastic scattering of structureless particles by nuclei

G. G. Dussel,* E. E. Maqueda,* and R. P. J. Perazzo*
Física Nuclear, Comision Nacional de Energia Atomica, Avenida del Libertador 8250, 1429 Buenos Aires, Argentina
(Received 23 March 1977; revised manuscript received 9 September 1977)

The nuclear field theory is applied to the inelastic scattering of structureless particles by nuclei. It is found that the coupling Hamiltonian between scattering and collective variables is linear. Therefore for one-step processes the only anharmonicities present correspond to the description of the bound state.

[NUCLEAR REACTIONS. Coupling with collective variables in inelastic scattering.]

I. INTRODUCTION

The nuclear field theory[1] (NFT) has established a way of solving a normal (nondeformed) many fermion system by means of a graphical perturbative expansion. In this method single particle (fermionic) and collective (bosonic) building blocks are used and the exact results are reproduced to any given order of perturbation theory. The inclusion of the same footing of fermionic and bosonic [e.g., Hartree-Fock and random-phase approximation (RPA)] elementary excitations to describe the nuclear spectrum, is both conceptually and practically advantageous, since it constitutes a practical way of performing the bookkeeping of the processes that one wants to include, avoiding problems such as double counting, Pauli principle violations, etc.

We present here as an extended version of Ref. 2 an application of the NFT, hitherto used for bound systems, to the direct inelastic scattering of structureless particles by nuclei. This specific example sheds light on the connections between a microscopic theory of inelastic scattering[3] and the elementary nuclear excitations.

II. FORMULATION OF THE SCATTERING PROBLEM IN THE NFT LANGUAGE

We first review the basie arguments underlying the field approach.[4,5] A general (fermionic) Hamiltonian,

$$H_F = H_{sp} + V = \sum_r \epsilon_r a_r^\dagger a_r + \tfrac{1}{4} \sum_{ijkl} V_{ij,kl}\, a_i^\dagger a_j^\dagger a_l a_k \,, \tag{1}$$

is replaced by

$$\begin{aligned} H_{\text{field}} &= H_F + H_B + H_{FB} \\ &= H_F + \sum_n \omega_n \Gamma_n^\dagger \Gamma_n + \sum_{nj_1j_2} [\Lambda^*(j_1 j_2 n)\Gamma_n^\dagger a_{j_1}^\dagger a_{j_2} \\ &\qquad + \Lambda(j_1 j_2 n)\Gamma_n a_{j_2}^\dagger a_{j_1}] \,. \end{aligned} \tag{2}$$

All the Hartree-Fock contributions of V are assumed to be already included in the single particle energies ϵ_r. The collective boson excitations $\Gamma_n^\dagger$ are independent of the fermionic variables. Its frequencies ω_n can be taken to be those of the normal modes arising from an RPA treatment.[6] The vertex functions $\Lambda(j_1 j_2 n)$, coupling linearly the $(j_1 j_2)$ fermion variables with the nth boson, are defined in terms of the forward (λ) and backward (μ) RPA amplitudes,

$$\Lambda(j_1 j_2 n) = \sum_{\substack{k>\epsilon_F \\ i<\epsilon_F}} (V_{j_1 k, j_2 i}\lambda_{ki}^n + V_{j_1 k, j_2 i}\mu_{ki}^n) \,. \tag{3}$$

If the Hamiltonian (2) is treated with specific perturbative diagrammatic rules, then (2) becomes completely equivalent to (1) but acting on a product (fermion-boson) rather than only a fermion space. The one body external operators,

$$Q_F = \sum_{jj'} q_{jj'} a_j^\dagger a_{j'} \,, \tag{4}$$

when acting on the fermion-boson product space should also have a collective part,

$$\begin{aligned} Q_{\text{field}} &= Q_F + Q_{\text{coll}} \\ &= Q_F + \sum_{nki} (q_{ki}\lambda_{ki}^n + q_{ik}\,\mu_{ki}^n)\Gamma_n^\dagger + \text{H.c.} \end{aligned} \tag{5}$$

Each term of the NFT perturbative expansion of any physical operator [such as (2) or (5)] is linked to a subset of Feynmann-Goldstone diagrams. When studying a schematic model,[7] consisting of two equally degenerate levels with degeneracy Ω and a monopole particle-hole interaction, the contributions of each subset correspond to a given power of $1/\Omega$. In the discussion that follows, in which a more general type of problem is considered, Ω can be taken to be a sort of effective degeneracy of the valence orbitals.

We now turn our attention to the inelastic scattering problem. In doing so we may assume that the Hamiltonian is given by an expression such as (1) in which summations are understood to run over the bound orbitals $(i, j, k, \ldots)$ of the A-nucleon target and over the continuum $(\alpha, \beta, \gamma, \ldots)$ projectile states satisfying the scattering boundary conditions.

In the NFT the single particle excitations include the Hartree-Fock (HF) corrections. In the same way we may consider that within a scattering problem both bound and continuum wave functions fulfill this requirement. Usually the unbound (scattering) wave functions are instead taken as solutions of an optical potential. In such a case discrete and continuum states are no longer orthogonal; nevertheless, the equivalence between the optical and HF potentials has been shown to be valid at least in the lowest orders.[8]

With regard to the two body potential V, several types of vertices are involved. Firstly there is one in which two fermions are scattered within bound states. These are the responsible of the structure of the bound system. Secondly, there are vertices that do not conserve the flux of unbound particles, for instance those that involve three fermion lines corresponding to continuum (or bound) states and the remaining line in a bound (or continuum) orbital. These terms will contribute primarily to the imaginary part of the optical potential and are therefore already accounted for in the continuum wave function. The vertices of V that involve four continuum fermion lines are not active on wave functions meeting the required asymptotic boundary conditions of a single incoming (and outgoing) structureless particle. Finally, there are those vertices that involve one continuum fermion line in both the initial and the final states. This is the unique part of the projectile-target interaction responsible for the inelastic scattering. The effect of these terms, as seen from either the target or the projectile system is that of an external one body operator, namely,

$$\sum_{\alpha\beta} \sum_{ij}{}' V_{i\alpha, j\beta}\, a_i^\dagger a_j a_\alpha^\dagger a_\beta = \sum_{\alpha\beta} \nu_{\alpha\beta}\, a_\alpha^\dagger a_\beta = \sum{}' \omega_{ij} a_i^\dagger a_j \,,$$
$$\nu_{\alpha\beta} = \sum_{ij} V_{i\alpha, j\beta}\, a_i^\dagger a_j \,, \tag{6}$$

where $\nu_{\alpha\beta}(\omega_{ij})$ is a one body operator acting on the target (projectile) degrees of freedom.

Thus using (4) and (5) we may write the term, coupling the continuum and the discrete degrees of freedom, in the field language as

$$H_{\mathrm{CD}} = \sum_{\alpha\beta} a_\alpha^\dagger a_\beta \Big(\nu_{\alpha\beta} + \sum_{\substack{i<\epsilon_F \\ k>\epsilon_F \\ n}} (V_{\alpha i, \beta k} \lambda_{ki}^n + V_{\alpha k, \beta i}\, \mu_{ki}^n) \Gamma_n^\dagger + \mathrm{H.c.} \Big) \,. \tag{7}$$

The field Hamiltonian suitable to describe the scattering problem under consideration will then involve the following terms:

$$H_{\mathrm{field}} = H_{\mathrm{sp}} + V^{(b)} + H_{\mathrm{B}} + H_{\mathrm{FB}}^{(b)} + H_{\mathrm{CD}} \,. \tag{8}$$

In (8) H_{sp} is the one nucleon term having contribution from both the discrete and continuum parts, $V^{(b)}$ is the part of the two body interaction effective between bound states, and the two terms H_{B} and $H_{\mathrm{FB}}^{(b)}$ correspond to the free boson and fermion-boson coupling terms that are introduced in the usual NFT procedure as we outlined in the discussion following Eq. (2). The index (b) denotes that summation indices are restricted to discrete (bound) orbitals.

We notice that the NFT prescriptions lead to a linear coupling between the scattering and collective nuclear degrees of freedom. This result is obtained in spite of the fact that no assumption of any kind has been made concerning the two body interaction. Furthermore, as we know from the application of the NFT to bound systems,[5] this coupling term, treated with the proper perturbative diagrammatic techniques, will reproduce the exact result to any given order in perturbation theory.

For definiteness we will focus our attention on examples in which a collective J^π state of an even-even target is excited. The lowest-order contribution for populating the one-phonon state is given in Fig. 1(a). The corresponding form factor can be obtained from the nuclear matrix element, following from (7) as in Eq. (5.43) of Ref. 9,

$$\sum_{ki}{}' V_{\alpha k, \beta j}(\lambda_{ki}^{J_n} + \mu_{ik}^{J_n}) = \sum_{ls} i^{-l} A_{lsJ_n} Y_j^{m*}(\hat{r}) F_{lsJ_n}(r) (-1)^{1/2 - m_\alpha} \times \langle \tfrac{1}{2}, \tfrac{1}{2}, m_\beta, -m_\alpha \,|\, s, m_\beta - m_\alpha \rangle \langle l, s, M - m_\beta + m_\alpha, m_\beta - m_\alpha \,|\, J_n M \rangle \,. \tag{9}$$

Higher-order processes, such as those shown in Figs. 1(b) and 1(c), also lead to the same final state. For instance, the correction to the nuclear matrix element of (9), associated to the diagram of Fig. 1(b), is

$$\sum_{ijkm} V_{\alpha i, \beta k} V_{kj, mi}\, \Lambda(mj, n)' (\epsilon_k - \epsilon_i - \omega_n)(\epsilon_m - \epsilon_j - \omega_n) \,. \tag{10a}$$

The remaining time permutations of the diagram 1(b) as well as the other $(1/\Omega)$ corrections can be worked out in a similar fashion. As far as two-phonon states are concerned, some of the lowest-order one-step processes that populate for example a 4^+ state are shown in Figs. 1(d) and 1(e). The contribution to the nuclear matrix element of a diagram such as Fig. 1(d) is

$$\sum_{\substack{ijk\\ \mu_1\mu_2}} \frac{V_{\alpha i,\beta k}\Lambda(ji,2\mu_1)\Lambda(kj,2\mu_2)}{(\epsilon_k-\epsilon_i-2\omega_2)(\epsilon_k-\epsilon_j-\omega_2)} \times \frac{\langle 2,2,\mu_1,\mu_2|4M\rangle}{\sqrt{2}} . \tag{10b}$$

We see that from a microscopic point of view the possibility of populating the two-phonon state is due to its anharmonicities. Within the NFT the structure of such anharmonicities is accounted for by the bound part of the diagrams of Fig. 1 (i.e., the right-hand side of each drawing).

The same state is attanined via two-step processes such as that shown in Fig. 1(f). These can be included for instance in a usual coupled channel calculation where only a linear coupling with collective modes [Fig. 1(a) or a matrix element (9)] is included.

More complicated processes will involve two (or more) interactions of the projectile with the bound system. Some of the relevant diagrams are Figs. 1(g) and 1(h). One can estimate the relative importance of these diagrams in a plane wave Born approximation.[10] They result to be at least of order $(1/kR)^*(1/\Omega)$ with respect to 1(a) for the case of diagram 1(g) and of order $(1/kR)$ with respect to diagrams 1(d) or 1(e) for the case of diagram 1(h). The factor $(1/\Omega)$ accounts for the part acting on the bound system.

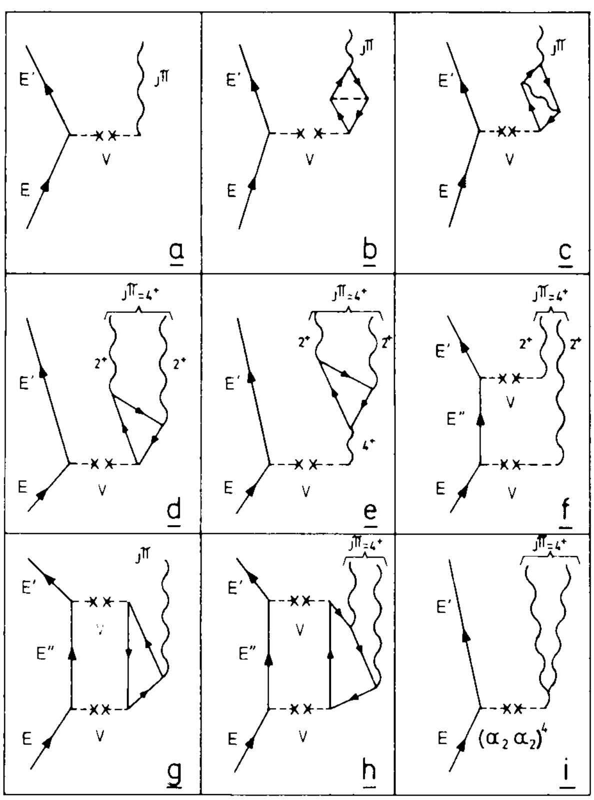

FIG. 1. Several scattering processes are displayed, V stands for the two body interaction. Throughout it has been stressed which part of the diagram acts on the coordinates of the bound system and which part acts on the scattering variables.

III. COMPARISON WITH A MACROSCOPIC APPROACH

Within a phenomenological framework[11] the coupling of scattering and nuclear collective degrees of freedom is derived from the dependence of the nuclear radius R on the dynamical deformation parameters $\alpha_{\lambda\mu}$. The Taylor expansion of the optical potential is performed,

$$\begin{aligned} V_{\text{opt}}(r,R) &= V_{\text{opt}}(r,R_0) + \left.\frac{\partial V_{\text{opt}}(r,R)}{\partial R}\right|_{R_0} \times R_0 \sum_{\lambda\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu} \\ &+ \frac{R_0^2}{2}\left.\frac{\partial^2 V_{\text{opt}}(r,R)}{\partial R^2}\right|_{R_0} \times \left(\sum \alpha_{\lambda\mu} Y^*_{\lambda\mu}{}^2 + \cdots\right), \end{aligned} \tag{11}$$

and the succesive derivatives of the optical potential provide the radial dependence of the form factors.

In order to make possible the comparison of the treatment explained in Sec. II with this phenomenological approach, it seems reasonable to choose in (7) the two body interaction V such that in the lowest order [Fig. 1(a)] one has the same form factor in both formulations. We then write

$$V(r_1,r_2) = R_0 \left.\frac{\partial V_{\text{opt}}(r_1,R)}{\partial R}\right|_{R_0} \sum q_{\lambda\mu}(\vec{r}_2) Y^*_{\lambda\mu}(\Omega_1) ,$$

$$q_{\lambda\mu}(r) = r^\lambda Y_{\lambda\mu}(\Omega) \tag{12}$$

and neglect exchange contributions. In fact, we built (12) such that, using (9), we get a form factor for the excitation of the one-phonon state that is identical in lowest order to the one arising from (11).

Since (12) factorizes into parts acting on nuclear and scattering variables the use of (12) in (10a) and (10b) shows that the form factor for one step processes in the NFT has always the same radial dependence (i.e., the first derivative of the optical potential), regardless of the structure of the final state attanined. This feature is not found if (11) is used instead, to compare for instance the excitation of a one- and a two-phonon state. For the former the linear term of (11) is active, while for the latter the form factor has contributions proportional to the second derivative of the optical potential. We see that the α^2 term gives rise to processes that correspond, for instance, to the one sketched in Fig. 1(i). In the NFT approach the excitation of a two-phonon state is instead achieved, as discussed above, via the anharmonicities of the nuclear spectrum that allow a nonvanishing matrix element of $q_{\lambda\mu}$ between the zero- and the two-phonon states [see Fig. 1(d)].

IV. CONCLUSIONS

The present treatment of the inelastic scattering allows a microscopic description that takes properly into account the collective degrees of freedom of the target nucleus. The term of the field Hamiltonian that couples continuum and discrete states has been found to be linear in the collective coordinates. Nevertheless, the diagrammatic rules that must be used allow the valuation of a nonvanishing matrix element of $q_{\lambda\mu}$ between the ground and a many boson state. The fact that a microscopic description can be developed containing only linear terms casts some doubts on the physical meaning of the multipole shape deformation parameters obtained via a collective phenomenological analysis of the reaction data that use higher-order coupling terms. There are two sources of anharmonicities within the NFT approach. As shown in Figs. 1(d) and 1(e), one corresponds to the anharmonic description of the bound states; the other stems from two-or-more-step processes that may act as a sort of nonlinear effective coupling. The method here presented is unable, however, to avoid the standard convergence problems of the rescattering series[12] in accounting for multistep processes that are not included in a coupled channel calculation. An additional advantage of the NFT procedure is that no assumption has to be made upon the quantification of classical dynamical variables, as required by the macroscopic theory.

---

*Fellow of the Consejo Nacional de Investigaciones Científicas y Técnicas.

[1]D. R. Bes, G. G. Dussel, R. A. Broglia, R. J. Liotta, and B. R. Mottelson, Phys. Lett. 52B, 253 (1974).

[2]G. G. Dussel, E. E. Maqueda, and R. P. J. Perazzo, Nucl. Instrum. Methods (unpublished).

[3]R. Satchler, Comments Nucl. Part. Phys. 5, 39 (1972).

[4]H. Reinhardt, Nucl. Phys. A251, 317 (1975).

[5]D. R. Bes, R. A. Broglia, G. G. Dussel, R. J. Liotta, and R. P. J. Perazzo, Nucl. Phys. A260, 77 (1976).

[6]In Ref. 5 it was shown that it is possible to include other types of processes in the definition of the boson.

[7]D. R. Bes, R. A. Broglia, G. G. Dussel, R. J. Liotta, and H. M. Sofia, Nucl. Phys. A260, 1 (1976).

[8]N. Vinh-Mau, *Proceedings of the International Course on Nuclear Theory, Trieste, 1969* (International Atomic Energy Agency, Vienna, Austria, 1970).

[9]N. Austern, *Direct Nuclear Reaction Theories* (Wiley, New York, 1970).

[10]Estimates based in this approximation should be cautiously considered as indicated by R. H. Lemmer, A. de Shalit, and N. S. Wall, Phys. Rev. 124, 1155 (1961); B. Buck, Phys. Rev. 127, 940 (1962).

[11]P. E. Hodgson, *Nuclear Reactions and Nuclear Structure* (Clarendon, Oxford, 1971), Chap. 13 and references therein.

[12]R. D. Amado and M. H. Rubin, Phys. Rev. C 7, 2151 (1973).