

ENERGY LEVELS OF EVEN NUCLEI ON THE HYDRODYNAMICAL MODEL WITH VIBRATION-ROTATION INTERACTION

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Abstract: The Davydov and Chaban model for even nuclei, in which the beta-vibration-rotation interaction is taken into account exactly, has been used to calculate the excited states of these nuclei. The calculated energies have been compared with experimental data, special attention being paid to nuclei with known 0^+ excited levels. Where possible, comparison has been made also with the correlation of experimental data by means of the perturbation approach of Mallmann and Kerman and of the Bohr-Mottelson model.

1. Introduction

In a recent paper, Davydov and Chaban ¹⁾ have extended the work of Davydov and Filippov ²⁾ and Davydov and Rostovsky ³⁾ to include in an exact manner beta-vibration-rotation interactions of non-axially symmetric even nuclei. This interaction had been taken into account as a perturbation by Mallmann and Kerman ⁴⁾.

In the present work, machine calculations have been made of the energies of a large number of excited states according to the model of Davydov and Chaban. The detailed results of the calculations have been made available in an ANL report ⁵⁾ to allow others to compare experimental data easily with the model.

The results of the calculations with the model have been compared with presently available data on the measured energies of excited states of even nuclei. Special attention has been paid to those nuclei which have measured 0^+ excited levels, since these levels are the lowest ones arising from the β -vibrational modes.

2. Calculations

The first three roots ν_l ($l = 1, 2, 3$) of the transcendental equation derived in ref. ¹⁾ (eq. (2.12)) were obtained numerically by means of a digital computer, and energies were calculated for values of the "non-axiality" parameter γ of

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30° , 25° , 20° , 15° , 12° , 10° , 9° and 8° and of the "non-adiabaticity" parameter μ of 1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.35, 0.3, 0.25, 0.2, 0.15 and 0.1. The energies of the following 36 levels were obtained: 10 , 12 , 14 , 16 , 18 , 110 , 112 , 22 , 13 , 24 , 15 and 26 for ν_1 , ν_2 and ν_3 , where the superscript is the ordering parameter K of Davydov and Filippov ²⁾ and the main number is the angular momentum J of the level.

In each case the series evaluation of the solution of the transcendental equation was carried out to a sufficient number of terms so that the last term contributed $< 10^{-4}$ to the previous result. The calculations were monitored by printing out in each case the number of terms required to satisfy the above condition. Checks on the present calculations were also obtained by making sure that the results agreed with those given previously by Mallmann and Kerman ⁴⁾, which are identical for the case of small μ . The results of the calculations are presented as ratios of the energy of the state in question to that of the first excited state of spin 2 for easy comparison of the energy predictions with experimental results.

3. Comparison with Data

The results of the calculations relevant to the present work are given graphically in figs. 1—8. In each case the first subscript of the R gives the ordering number K of the level of angular momentum given by the number in the parenthesis, and the second subscript l gives the number of the root of the transcendental equation which gives rise to the level. The straight lines represent the perturbation theory results of Mallmann and Kerman ⁴⁾, except for the $R_{21}(2)$ lines on figs. 4 and 5. The $R_{21}(2)$ lines have been drawn to allow the data to be compared with the relationship $R_{11}(3) = R_{21}(2) + 1$, which is valid for any rotor ⁴⁾. The values of the parameters γ and μ are indicated at the ends of the respective curves. As discussed previously ⁴⁾, all ratios are plotted as functions of $R_{21}(2)$, the ratio of the energies of the second spin 2 and the first spin 2 states (arising from the root ν_1). The experimental data are given as open circles for the indicated nuclei.

Each experimental point on the graphs corresponds to a value of γ and a value of μ . These values, determined by interpolation on the graphs, are collected in table 1. The uncertainties $\Delta\gamma_e$ and $\Delta\mu_e$ correspond to the uncertainties of the experimentally determined ratios of energies. The uncertainties $\Delta\gamma_1$ and $\Delta\mu_1$ correspond to the uncertainties due to the interpolation procedure for the determination of γ and μ . In each case the identification of the energy level, the ratio of its energy to that of the first spin 2 state, and the corresponding values of γ and μ with their uncertainties are given, with K giving the ordering number of Davydov and Filippov of the level of angular momentum J . The ordering number of the roots is given by the parameter l . Particular care has been taken to include all cases of known spin 0 excited states.

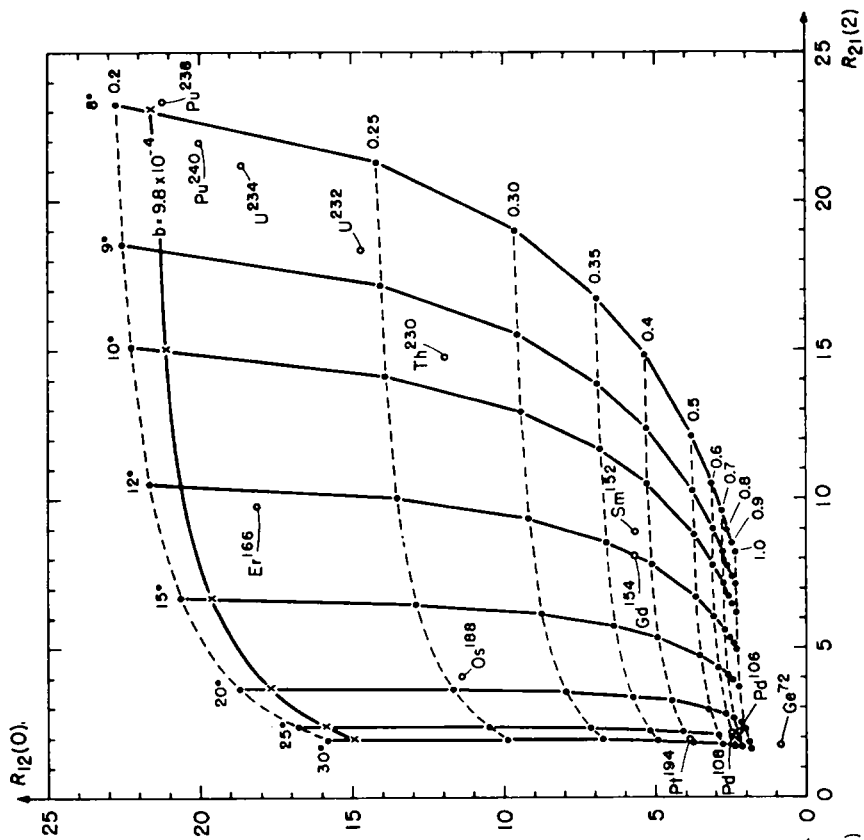


Fig. 2. See caption of fig. 1.

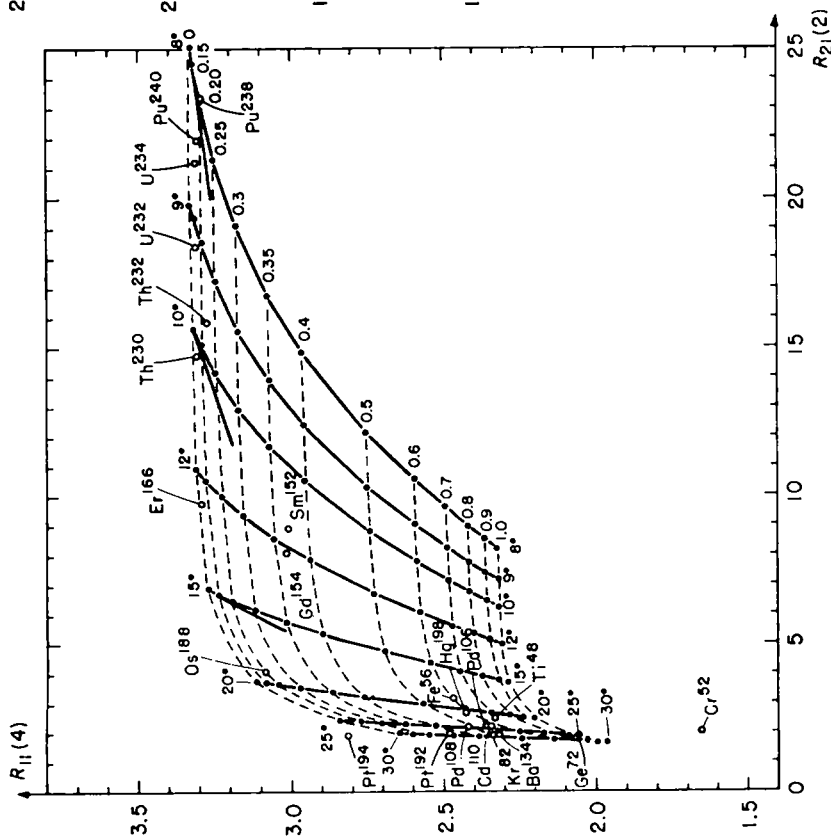


Fig. 1. Values of $R_{KI}(I)$ as a function of $R_{21}(2)$. The open circles represent the experimental data for the indicated nuclei. The solid circles are computed points; the solid lines connect equal values of γ (non-axiality parameter) and the dashed lines connect equal values of μ (non-adiabaticity parameter). The straight solid lines on figs. 1, 3, 4 and 6, except for those marked $R_{21}(2)$ on fig. 4, represent the perturbation theory results of Mallmann and Kerman. The $R_{21}(2)$ lines on figs. 4 and 5 are shown to allow comparison of the data with the relationship $R_{11}(3) = R_{21}(2) + 1$, which is valid for any rotor. The curve joining the crosses is for the b value of Mallmann and Kerman corresponding to $\mu = 0.2$

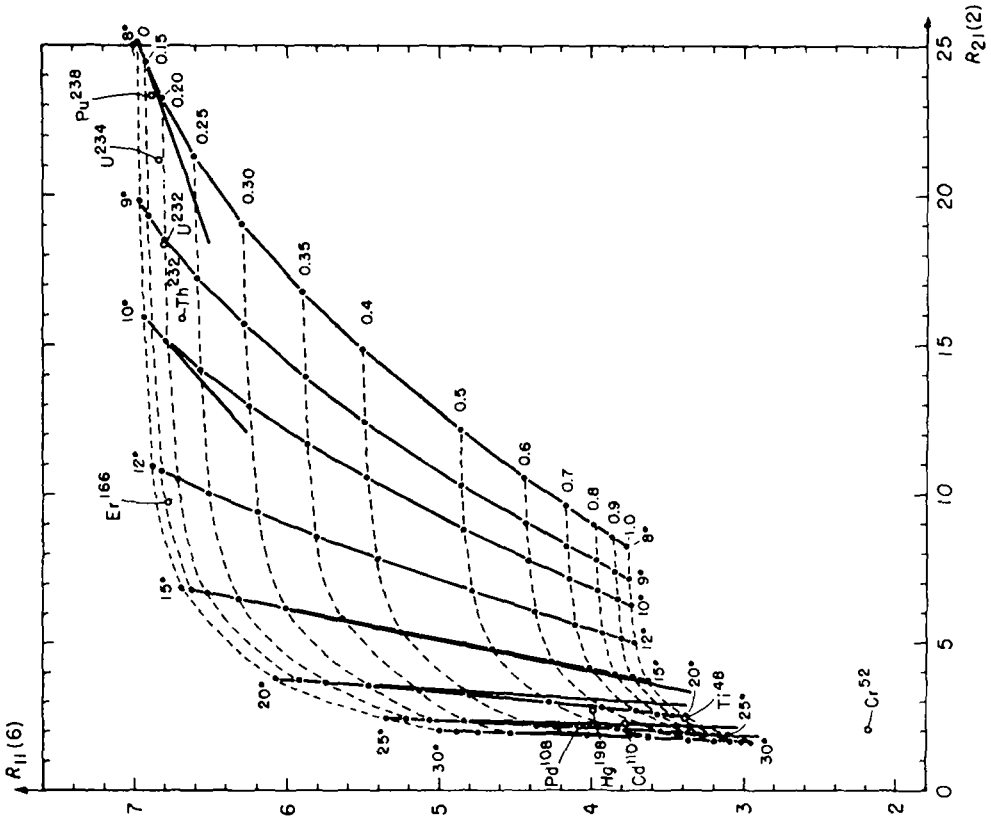


Fig. 3. See caption of fig. 1.

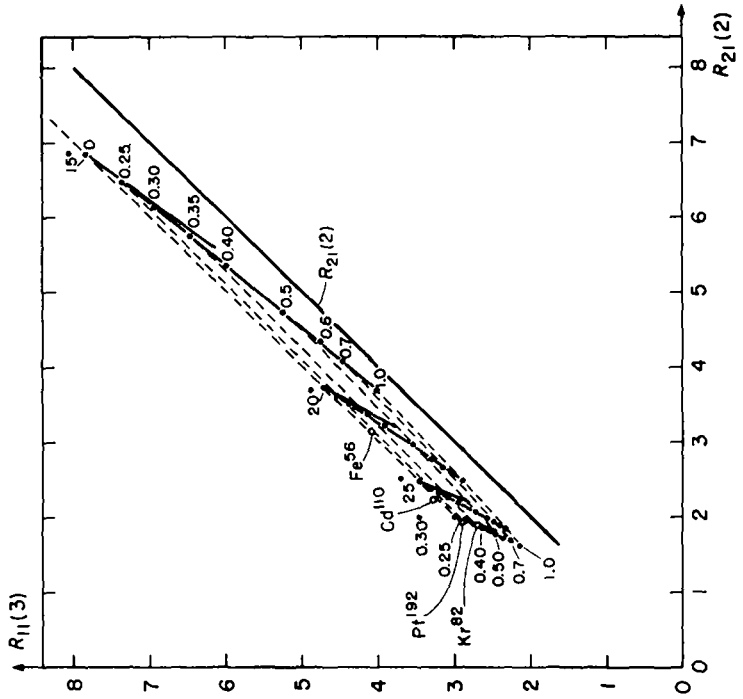


Fig. 4. See caption of fig. 1.

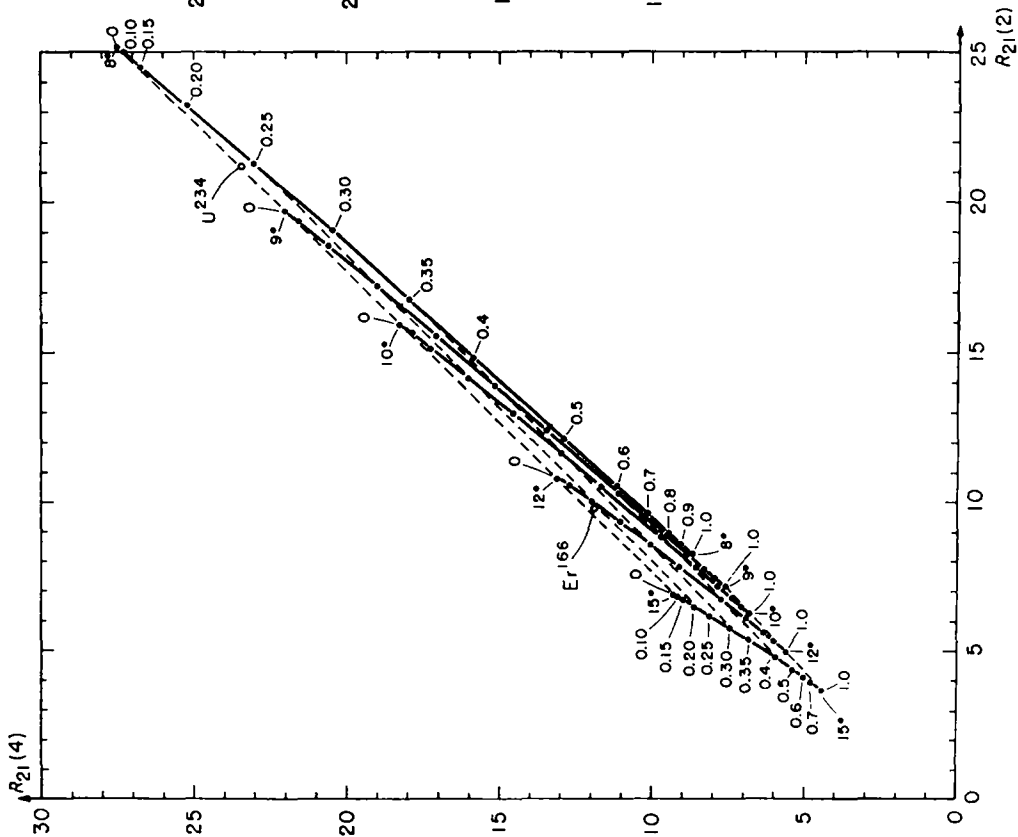


Fig. 7. See caption of fig. 1.

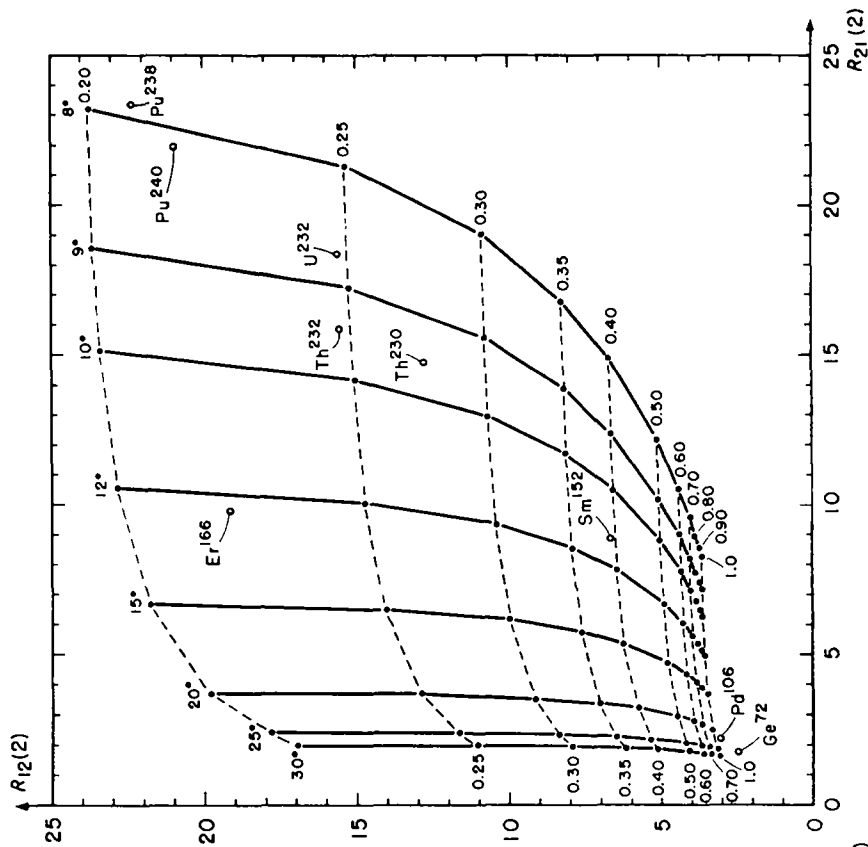


Fig. 8. See caption of fig. 1.

TABLE 1

Values of the "non-axiality" parameter γ and the "non-adiabaticity" parameter μ

The table gives the values of γ and μ as determined from the measured energies of the various excited states of the indicated even nuclei. The measured energies are given as ratios to that of the first spin 2 state in terms of the parameters K and l , defined in the text, and the angular momentum J . The indicated uncertainties $\Delta\gamma_e$ and $\Delta\mu_e$ correspond to the uncertainties of the experimental measurements, and the uncertainties $\Delta\gamma_1$ and $\Delta\mu_1$ correspond to the uncertainties due to the interpolation procedure for the determination of γ and μ . The uncertainties are given as follows: $1.754 \pm 0.20 = 1.754 \pm 0.020$.

Nucleus	$KJ \nu_1$	$R_{Ki}(J)$	γ	$\Delta\gamma_1$	$\Delta\gamma_e$	μ	$\Delta\mu_1$	$\Delta\mu_e$	Ref.
Ti ⁴⁸	²² , ν_1	2.469 <u>26</u>	21.2 20.05	<u>2</u> <u>5</u>	<u>3</u> <u>30</u>	0.66 1.00	<u>1</u> <u>1</u>	<u>2</u> <u>4</u>	*)
	¹⁴ , ν_1	2.333 <u>13</u>							
	¹⁶ , ν_1	3.387 <u>17</u>							
Cr ⁵²	²² , ν_1	2.069 <u>13</u>	(out of range) (out of range)						*)
	¹⁴ , ν_1	1.654 <u>9</u>							
	¹⁶ , ν_1	2.178 <u>13</u>							
Fe ⁵⁶	²² , ν_1	3.153 <u>35</u>	18.4 21.4	<u>2</u> <u>2</u>	<u>2</u> <u>5</u>	0.61 0.25	<u>1</u> <u>5</u>	<u>6</u> <u>+7</u> <u>-25</u>	*)
	¹⁴ , ν_1	2.467 <u>50</u>							
	¹³ , ν_1	4.079 <u>65</u>							
Ge ⁷²	²² , ν_1	1.754 <u>20</u>	26.9 (out of range) (out of range)	<u>2</u>	<u>9</u>	0.78	<u>2</u>	<u>10</u>	b)
	¹⁴ , ν_1	2.071 <u>25</u>							
	¹⁰ , ν_2	0.826 <u>11</u>							
	¹² , ν_2	2.467 <u>33</u>							
Kr ⁸²	²² , ν_1	1.899 <u>1</u>	27.70 29.0	<u>20</u> <u>10</u>	<u>2</u> <u>1</u>	0.450 0.380	<u>2</u> <u>20</u>	<u>1</u> <u>2</u>	b)
	¹⁴ , ν_1	2.344 <u>1</u>							
	¹³ , ν_1	2.695 <u>2</u>							
Pd ¹⁰⁶	²² , ν_1	2.205 <u>37</u>	23.6 22.5 (out of range)	<u>2</u> <u>2</u>	<u>5</u> <u>2</u>	0.54 0.75	<u>1</u>	<u>4</u> <u>5</u>	c)
	¹⁴ , ν_1	2.364 <u>23</u>							
	¹⁰ , ν_2	2.216 <u>42</u>							
	¹² , ν_2	3.047 <u>53</u>							
Pd ¹⁰⁸	²² , ν_1	2.170 <u>24</u>	24.4 24.8 23.3	<u>2</u> <u>1</u> <u>2</u>	<u>4</u> <u>3</u> <u>1</u>	0.480 0.444 0.627	<u>10</u> <u>5</u> <u>10</u>	<u>30</u> <u>20</u> <u>20</u>	d)
	¹⁴ , ν_1	2.417 <u>27</u>							
	¹⁶ , ν_1	4.083 <u>65</u>							
	¹⁰ , ν_2	2.419 <u>35</u>							
Cd ¹¹⁰	²² , ν_1	2.245 <u>4</u>	23.10 23.20 (out of range)	<u>10</u> <u>10</u>	<u>5</u> <u>6</u>	0.573 0.562	<u>10</u> <u>5</u>	<u>6</u> <u>6</u>	*)
	¹⁴ , ν_1	2.348 <u>4</u>							
	¹⁶ , ν_1	3.776 <u>6</u>							
	¹³ , ν_1	3.293 <u>5</u>							
Ba ¹³⁴	²² , ν_1	1.931 <u>3</u>	26.7 24.5	<u>10</u> <u>5</u>	<u>1</u> <u>1</u>	0.489 0.900	<u>2</u> <u>30</u>	<u>2</u> <u>3</u>	*)
	¹⁴ , ν_1	2.317 <u>1</u>							
	²⁴ , ν_1	3.259 <u>3</u>							

TABLE I (continued)

Nucleus	$KJ \nu_i$	$R_{KI}(J)$	γ	$\Delta\gamma_1$	$\Delta\gamma_0$	μ	$\Delta\mu_1$	$\Delta\mu_0$	Ref.					
Sm ¹⁵²	² ₂ , ν_1	8.914 <u>10</u>	11.50 (out of range)	<u>10</u>	<u>1</u>	0.374	<u>3</u>	<u>1</u>	*)					
	¹⁴ ₂ , ν_1	3.009 <u>2</u>												
	¹³ ₁ , ν_1	10.138 <u>10</u>												
	¹⁰ ₂ , ν_2	5.624 <u>6</u>								11.28 <u>3</u>	<u>1</u>	0.383	<u>1</u>	<u>1</u>
	¹² ₂ , ν_2	6.658 <u>20</u>								11.30 <u>20</u>	<u>2</u>	0.390	<u>10</u>	<u>1</u>
Gd ¹⁵⁴	² ₂ , ν_1	8.090 <u>11</u>	12.22 (out of range)	<u>2</u>	<u>1</u>	0.366	<u>3</u>	<u>1</u>	*)					
	¹⁴ ₂ , ν_1	3.013 <u>2</u>												
	¹³ ₁ , ν_1	9.162 <u>11</u>												
	¹⁰ ₂ , ν_2	5.681								12.02 <u>3</u>	<u>1</u>	0.379	<u>1</u>	
Er ¹⁶⁶	² ₂ , ν_1	9.770 <u>40</u>	12.67 12.58 12.50 12.40 12.33 12.30	<u>2</u>	<u>5</u>	0.140	<u>30</u>	+ <u>20</u> - <u>140</u> <u>30</u> + <u>50</u> - <u>200</u> <u>20</u>	*)					
	¹⁴ ₂ , ν_1	3.294 <u>6</u>												
	¹⁶ ₂ , ν_1	6.774 <u>30</u>												
	¹³ ₁ , ν_1	10.686 <u>60</u>												
	¹⁴ ₂ , ν_1	11.889 <u>50</u>								12.40 <u>10</u>	<u>10</u>	0.220	<u>10</u>	<u>20</u>
	¹⁰ ₂ , ν_2	18.12								12.33 <u>10</u>	<u>3</u>	0.217	<u>1</u>	<u>1</u>
	¹² ₂ , ν_2	19.13								12.30 <u>10</u>	<u>2</u>	0.220	<u>2</u>	<u>2</u>
Os ¹⁸⁸	² ₂ , ν_1	4.085 <u>21</u>	18.80 19.20	<u>4</u>	<u>10</u>	0.259	<u>2</u>	<u>20</u>	b)					
	¹⁴ ₂ , ν_1	3.084 <u>18</u>												
	¹⁰ ₂ , ν_2	11.37 <u>3</u>								19.20 <u>14</u>	<u>7</u>	0.255	<u>2</u>	<u>1</u>
Pt ¹⁹²	² ₂ , ν_1	1.936 <u>1</u>	28.80 (out of range)	<u>20</u>	<u>3</u>	0.346	<u>2</u>	<u>2</u>	b)					
	¹⁴ ₂ , ν_1	2.490 <u>4</u>												
	¹³ ₁ , ν_1	2.910 <u>3</u>												
Pt ¹⁹⁴	² ₂ , ν_1	1.897 <u>15</u>	(out of range) 29.20	<u>50</u>	<u>20</u>	0.401	<u>1</u>	<u>3</u>	e)					
	¹⁴ ₂ , ν_1	2.809 <u>17</u>												
	¹⁰ ₂ , ν_2	3.853 <u>36</u>												
Hg ¹⁹⁸	² ₂ , ν_1	2.644 <u>3</u>	20.80 20.90	<u>5</u>	<u>3</u>	0.580	<u>2</u>	<u>20</u>	*)					
	¹⁴ ₂ , ν_1	2.44 <u>2</u>												
	¹⁶ ₂ , ν_1	3.97 <u>3</u>								20.90 <u>10</u>	<u>10</u>	0.561	<u>2</u>	<u>20</u>
Th ²³⁰	² ₂ , ν_1	14.77	10.20 9.50 9.50	<u>5</u>	<u>10</u>	0.175	<u>5</u>	<u>5</u>	f)					
	¹⁴ ₂ , ν_1	3.302												
	¹⁰ ₂ , ν_2	11.96								9.50 <u>10</u>	<u>10</u>	0.270	<u>5</u>	<u>5</u>
	¹² ₂ , ν_2	12.75								9.50 <u>10</u>	<u>10</u>	0.276	<u>5</u>	<u>5</u>
Th ²³²	² ₂ , ν_1	15.88 <u>30</u>	9.60 9.48 9.45	<u>5</u>	<u>10</u>	0.225	<u>5</u>	+ <u>50</u> - <u>225</u> <u>25</u> <u>2</u>	*)					
	¹⁴ ₂ , ν_1	3.271 <u>60</u>												
	¹⁶ ₂ , ν_1	6.69 <u>10</u>								9.48 <u>5</u>	<u>10</u>	0.225	<u>5</u>	<u>25</u>
	¹² ₂ , ν_2	15.54 <u>30</u>								9.45 <u>10</u>	<u>10</u>	0.248	<u>1</u>	<u>2</u>

TABLE 1 (continued)

Nucleus	KJ, ν_1	$R_{KI}(J)$	γ	$\Delta\gamma_1$	$\Delta\gamma_0$	μ	$\Delta\mu_1$	$\Delta\mu_0$	Ref.
U ²³³	² 2, ν_1	18.39 <u>12</u>							*)
	¹⁴ 4, ν_1	3.311 <u>22</u>	9.20	<u>5</u>	<u>3</u>	0.170	<u>5</u>	+ <u>40</u> - <u>170</u>	
	¹⁶ 6, ν_1	6.801 <u>50</u>	9.02	<u>5</u>	<u>2</u>	0.202	<u>5</u>	<u>10</u>	
	¹³ 3, ν_1	19.34 <u>12</u>	9.26	<u>10</u>	<u>5</u>	0.150	<u>80</u>	+ <u>130</u> - <u>150</u>	
	¹⁰ 0, ν_2	14.68	8.75	<u>5</u>	<u>3</u>	0.244	<u>1</u>		
	¹² 2, ν_2	15.57	8.75	<u>5</u>	<u>2</u>	0.248	<u>1</u>		
U ²³⁴	² 2, ν_1	21.20 <u>9</u>							*)
	¹⁴ 4, ν_1	3.31 <u>2</u>	8.55	<u>5</u>	<u>2</u>	0.170	<u>5</u>	+ <u>50</u> - <u>170</u>	
	¹⁶ 6, ν_1	6.83 <u>4</u>	8.43	<u>5</u>	<u>2</u>	0.196	<u>5</u>	<u>30</u>	
	¹³ 3, ν_1	22.18 <u>9</u>	8.61	<u>10</u>	<u>2</u>	0.100	<u>100</u>	+ <u>130</u> - <u>100</u>	
	²⁴ 4, ν_1	23.45 <u>11</u>	8.58	<u>5</u>	<u>2</u>	0.145	<u>10</u>	+ <u>45</u> - <u>145</u>	
	¹⁰ 0, ν_2	18.6	8.23	<u>5</u>	<u>3</u>	0.219	<u>1</u>		
Pu ²³⁸	² 2, ν_1	23.349 <u>70</u>							*)
	¹⁴ 4, ν_1	3.292 <u>6</u>	7.90	<u>5</u>	<u>2</u>	0.208	<u>5</u>	<u>10</u>	
	¹⁶ 6, ν_1	6.889 <u>14</u>	8.12	<u>5</u>	<u>2</u>	0.170	<u>5</u>	<u>10</u>	
	¹³ 3, ν_1	24.302 <u>80</u>	8.19	<u>10</u>	<u>2</u>	0.150	<u>80</u>	+ <u>80</u> - <u>150</u>	
	¹⁰ 0, ν_2	21.216 <u>70</u>	7.90	<u>3</u>	<u>2</u>	0.206	<u>1</u>	<u>1</u>	
	¹² 2, ν_2	22.351 <u>70</u>	7.90	<u>5</u>	<u>2</u>	0.208	<u>1</u>	<u>1</u>	
Pu ²⁴⁰	² 2, ν_1	21.96							*)
	¹⁴ 4, ν_1	3.305	8.30	<u>5</u>		0.183	<u>5</u>		
	¹⁰ 0, ν_2	20.00	8.14	<u>5</u>		0.214	<u>1</u>		
	¹² 2, ν_2	20.98	8.20	<u>10</u>		0.215	<u>1</u>		

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4. Discussion

The present work was undertaken to provide a detailed comparison between the experimental data and the predictions of the Davydov and Chaban model in view of the work of Mallmann on the correlation of measured energies by means

of the quasi-adiabatic general asymmetric rotor model ⁷⁾ and that of Mallmann and Kerman ⁴⁾.

Initially considering energy levels arising from the first root ν_1 for those cases ⁴⁾ for which $\mu \leq 0.25$ ($b \leq 5.3 \times 10^{-3}$), there is no distinction to be made between the predictions of the present model and those of the work of Mallmann and Kerman ⁴⁾. This point is illustrated in table 2, in which we give the limiting

TABLE 2

Comparison of Davydov-Chaban model with that of Mallmann and Kerman.

The table gives the values of the parameter μ at which the predictions of $R_{KI}(J)$ as a function of $R_{21}(2)$ of the Davydov and Chaban model and the Mallmann and Kerman calculations are in agreement within 1% for the levels with $(KJ, \nu_1) = (14, \nu_1), (16, \nu_1), (13, \nu_1), (24, \nu_1), (10, \nu_2)$ and different values of the parameter γ

γ \diagdown KJ, ν_i	$14, \nu_1$	$16, \nu_1$	$13, \nu_1$	$24, \nu_1$	$10, \nu_2$
8°	0.25	0.23	0.33	0.28	0.09
10°	0.27	0.26	0.35	0.27	0.09
15°	0.31	0.55	0.32	0.28	0.09
20°	0.46	0.31	0.36	0.31	0.09
25°	0.62	0.21	0.36	0.31	0.09
30°	0.70	0.19	0.36	0.31	0.09

values of μ at which the ratios of the energies of the indicated levels to those of the first spin 2 states as given by Mallmann and Kerman agree with those of the present model within 1%. For smaller values of μ , of course, the agreement is better than 1%. Thus the comparison of the present model with experimental data is essentially that given in table 2 for $\frac{8}{3} \leq R(14) \leq \frac{10}{3}$ of the paper of Mallmann and Kerman ⁴⁾. The quasi-adiabatic general asymmetric rotor model ⁷⁾ is in better agreement with these data than the present model. For completeness, the relationship between the parameters of Davydov and Chaban (DC) and those of Mallmann and Kerman (MK) for $\mu \ll 1$ is given by

$$A(\beta_0) = \frac{\hbar^2}{4B\beta_0^3} \left(1 + \frac{3}{2}\mu^2\right), \quad b = \frac{1}{2}\mu^4 \left(1 + \frac{45}{8}\mu^2\right).$$

From these one obtains

$$(\hbar\omega_0)_{DC} = (\hbar\omega_0)_{MK} \left(1 + \frac{21}{16}\mu^2\right).$$

We next consider the cases with $\mu > 0.25$ ($b > 5.3 \times 10^{-3}$). For these values of μ the predictions of Davydov and Chaban differ from those of Mallmann and Kerman, which were in disagreement with the experimental results. The predictions of the present model are in agreement within experimental errors with the data for two nuclei, Pd¹⁰⁸ and Hg¹⁹⁸. The data for Cd¹¹⁰ are correlated within experimental errors except for the $KJ = 13$ level, and the data for Kr⁹² are in good agreement, although not within experimental errors. The data for

Ti⁴⁸, Cr⁵², Fe⁵⁶, Ba¹³⁴, Sm¹⁵², Gd¹⁵⁴, Pt¹⁹² and Pt¹⁹⁴ are not in agreement with the present model. Again, the quasi-adiabatic general asymmetric rotor model gives better agreement with the data.

We now consider the levels arising from the root ν_2 and compare their γ and μ values with those arising from the root ν_1 . Of these levels the only ones for which there are enough data for comparison with experiment are the ¹⁰ and ¹² states. Here it is of interest to compare the present model with the Bohr-Mottelson model also, since the predictions are different enough to make possible a clear-cut distinction. For those nuclei with $\mu \leq 0.25$ ($b \leq 5.3 \times 10^{-3}$), in the case of the Davydov and Chaban model, the data for Th²³² agree within experimental errors. The values of γ and μ are in agreement for Er¹⁶⁶ except for the ¹⁴ ν_1 level and for Pu²³⁸ except for the ¹⁶ ν_1 level. Good correlation is found, although outside experimental errors in the cases of U²³² and U²³⁴. In the cases of Th²³⁰ and Pu²⁴⁰, for which no experimental errors are given, good correlation is also found. It is well known⁸⁾ that in the case of the Bohr-Mottelson model the discrepancy between theory and experiment is of the order of a factor 2. The predictions of Mallmann and Kerman are a first approximation to the exact results of Davydov and Chaban, and the two are in agreement only for cases with $\mu \leq 0.09$ ($b \leq 3 \times 10^{-4}$) (see table 2). There are no nuclei with such small values of b .

Finally, for those nuclei with $\mu > 0.25$ ($b > 5.3 \times 10^{-3}$), the data for Os¹⁸⁸ are correlated by the present model. The data for Sm¹⁵² and Gd¹⁵⁴ are in quite good agreement, although outside of experimental errors, with the model if one uses the energies of the first spin 2 and first spin 4 states to determine γ and μ and then calculates the energies of the second spin 0 states; however, the model predicts incorrect energies for the first spin 3 states. The data for the other 4 nuclei, Ge⁷², Pd¹⁰⁶, Pd¹⁰⁸ and Pt¹⁹⁴, are not correlated by the present model.

In summary, the present comparison of the model of Davydov and Chaban with experimental data indicates that it is quite successful for nuclei far away from closed shells ($\mu \approx 0.35$), although the correlation between theory and experiment is not always within experimental error. For nuclei with $\mu \geq 0.35$, the theory is not in agreement with experimental results; however, if one considers only levels arising from the ν_1 root, fair agreement is found (4 cases out of 10). This fact seems to indicate that taking into account the interaction of the hydrodynamical rotations with the β -vibrations is insufficient to explain the observed spectra, and that it is necessary to take into account additional degrees of freedom, such as γ -vibrations.

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