1.D.2. *Nuclear Physics* A153 (1970) 469—480; © *North-Holland Publishing Co., Amsterdam*

THE $T = 1$ PAIRING HAMILTONIAN IN A TWO-LEVEL MODEL

G. G. DUSSEL, E. MAQUEDA† and R. P. J. PERAZZO†
C.N.E.A., Sincrociclotrón, Buenos Aires, Argentina

Received 27 April 1970

Abstract: The $T = 1$ pairing Hamiltonain is exactly solved in a two-level model and both energy eigenvalues and transtion matrix elements are obtained. The system undergoes a phase transition for a value G_c of the coupling constant. In both limits, $G \ll G_c$ and $G \gg G_c$ vibrational and rotational patterns are observed. The exact results are compared to the RPA and to a Nilsson-like treatment.

1. Introduction

The purpose of the present paper is to study the additions and changes of the $|T_z| = 1$ pairing vibrational and rotational patterns when the isospin degree of freedom is taken into account [1,2]). In the framework of the two-level model one can make an exact calculation and check the accuracy of some approximate treatments.

In the vibrational limit the pattern obtained is the same as the one corresponding to two three-dimensional harmonic oscillators, i.e., the addition and removal modes[1]). The selection rules for the two-particle transfer operator are checked. In the superconducting limit the system presents a permanent deformation in isospace as suggested by the $T(T+1)$ splitting of the levels, and performs rotations also in gauge space. Within the adiabatic approximation for a deformed system [3]) one can factorize the wave function into the collective and intrinsic parts. The first will be responsible for the selection rules for the two-particle transfers, while the intrinsic part will give enhancements of the matrix elements. The intrinsic ground states are obtained through a BCS calculation and the Δ- and Γ-vibrations are obtained by means of the RPA.

2. Exact treatment of the $T = 1$ pairing Hamiltonian

In this section we follow ref. [4]) closely. In order to take into account the fact that the $J = 0$ pairs carry unit isospin one can extend the pair creation operators introduced in the treatment of the $|T_z| = 1$ pairing Hamiltonian defining

$$A_1^+(j) = \sum_m \tfrac{1}{2}(-)^{j-m} n_{jm}^+ n_{j-m}^+, \qquad A_{-1}^+(j) = \sum_m \tfrac{1}{2}(-)^{j-m} p_{jm}^+ p_{j-m}^+,$$

$$A_0^+(j) = \sum_m \frac{(-)^{j-m}}{2\sqrt{2}} (n_{jm}^+ p_{j-m}^+ + p_{jm}^+ n_{j-m}^+), \tag{1}$$

† Fellow of the Consejo Nacional de Investigaciones Cientificas y Técnicas, Argentina.

where n^+_{jm}, (p^+_{jm}) are the ordinary fermion operators which create neutrons (protons) in states with angular momentum j and projection m. The operators $A^+_\mu(j)$ can be regarded as the three components of a vector operator $A^+(j)$ in isospace. One can further introduce

$$\hat{N}(j) = \sum_m n^+_{jm} n_{jm} + p^+_{jm} p_{jm},$$

$$T_+(j) = \sum_m n^+_{jm} p_{jm}, \qquad T_-(j) = \sum_m p^+_{jm} n_{jm}, \tag{2}$$

$$T_0(j) = \tfrac{1}{2} \sum_m (n^+_{jm} n_{jm} - p^+_{jm} p_{jm}).$$

These operators together with A and A^+ are related to the infinitesimal generators of R_5 written in the standard form by

$$H_1 = \tfrac{1}{2}\hat{N}(j) - \Omega_j, \quad E_{11}(j) = A^+_1(j), \quad E_{10}(j) = A^+_0(j), \quad E_{1-1}(j) = -A^+_{-1}(j),$$

$$H_2 = T_0(j), \quad E_{-1-1}(j) = A_1(j), \quad E_{-10}(j) = A_0(j), \quad E_{-11}(j) = -A_{-1}(j),$$

$$E_{01}(j) = T_+(j)/\sqrt{2}, \quad E_{0-1}(j) = T_-(j)/\sqrt{2},$$

$$\Omega_j = j + \tfrac{1}{2}. \tag{3}$$

The irreducible representations of R_5 can be labelled by the seniority v, the reduced isospin t (t is the total isospin of the v nucleons which are not coupled in pairs of angular momentum zero), the total number of pairs M (only even nuclei are going to be considered), the total isospin T and its projection T_z and an extra quantum number that is not needed for the values of v and t in which we are interested. Since only $J = 0$ states are going to be considered, the angular momentum quantum numbers are omitted throughout.

2.1. DEGENERATE CASE

It must be pointed out that if in the definitions (1) and (2), a summation over j is performed and a total degeneracy $\Omega = \sum \Omega_j$ is introduced, the new operators so defined have the same commutation relations and therefore they are also related to the infinitesimal generators of R_5 by eq. (3). An overall seniority and overall reduced isospin can be introduced.

As in the $|T_z| = 1$ pairing case, all the states with overall seniority zero are a subspace of the one spanned by the states with individual shell seniority zero. In what follows, whenever the index j is dropped, a summation over j is understood. With this convention, the pairing Hamiltonian can be written as

$$\mathscr{H}_{\mathrm{p}} = -G \sum_\mu A^+_\mu A_\mu \tag{4}$$

and the two-particle transfer operator is A^+_μ. The Hamiltonian (4) commutes with the total number of particles operator and with T^2, and can be written in terms of the

R_5 Casimir operator

$$C = H_1^2 + H_2^2 + \sum_{a,b} E_{a,b} E_{-a,-b} \tag{5}$$

as

$$\mathscr{H}_p = -\tfrac{1}{2}G\{C - \mathbf{T}^2 - H_1^2 + 3H_1\} \tag{6}$$

being therefore diagonal in the basis $|v, t, M, T, T_z\rangle$. The eigenvalues are:

$$E_{v,t,M,T} = -\tfrac{1}{2}G\{(M - \tfrac{1}{2}v)(2\Omega + 3 - M - \tfrac{1}{2}v) - T(T+1) + t(t+1)\}. \tag{7}$$

For a given M, v and t a "rotational band" appears following the $T(T+1)$ rule, with an associated moment of inertia $\mathscr{I} = 1/G$. If the states with $v = 0$ are considered, the band has only even (odd) T states if M is even (odd).

The splitting of two states with the same T but belonging to the bands $v = 0$ and $v = 2$ is $G\Omega$, which is the same result as the one found in the $|T_z| = 1$ pairing force problem. There are other families of states that can be grouped into bands, those are the states having the same v, t, $M = 2\Omega + m$ and $T = |m - 2s|$ for s fixed and $m = 0, 1, \ldots$. In all the cases the energies follow a parabola and are the equivalent of the superconducting bands of the pairing among identical particles. This grouping into bands is particularly meaningful if transition probabilities are considered. Since the two-particle transfer operator with $J = 0$ and $T = 1$ is one of the infinitesimal generators of R_5, it will not connect states labelled by different v and t. Its reduced matrix elements can be obtained from the R_5/R_3 Wigner coefficients and are tabulated for several values of v in ref. [4]). In the $v = 0$ case the matrix elements are given by

$$\begin{aligned} \langle M+1, T+1 || A^+ || M, T\rangle &= [\tfrac{1}{2}(T+1)(2\Omega - M - T)(T+M+3)]^{\frac{1}{2}}, \\ \langle M+1, T-1 || A^+ || MT\rangle &= [\tfrac{1}{2}T(2\Omega + 1 - M + T)(M - T + 2)]^{\frac{1}{2}}. \end{aligned} \tag{8}$$

It must be pointed out that any state with given T should be considered as a $2T+1$ isobaric multiplet, therefore the $T_z = T$ member of the lowest multiplet for a given T is the ground state of a nucleus having $A = 2M$, $N = T + M$, $Z = M - T$. Since only doubly even nuclei are considered, transitions starting from any state of the $v = 0$ band should be take into account. Due to the $\Delta v = 0$ selection rule, members of the $v = 2$ band are not going to be populated by any of these reactions. All the transitions among members of the same s-band have essentially the same relative values, independent of s and depending basically on the initial and final T.

2.2. TWO-LEVEL CASE

Now the Hamiltonian is

$$\mathscr{H} = \sum_j \varepsilon_j \hat{N}(j) - \mathscr{H}_p, \tag{9}$$

where ε_j are the single-particle energies. Since $\mathscr{H}$ is a scalar in isospace, a suitable intermediate coupling is the one in which the irreducible representations of R_5 are left uncoupled while the isospin quantum numbers associated with each level are

coupled to a total isospin T. We are going to restrict the treatment to those states with shell seniority $v_j = 0$, due to the fact that the states with overall seniority $v = 0$, contained in this basis, are possible ground states of doubly even nuclei. If the v_j and t_j quantum numbers are dropped, the basis for the two-level case can be written as $|M_1, T_1, M_2, T_2, T, T_z\rangle$; where M_1 (M_2), T_1 (T_2) are the total number of pairs and isospin of the level 1 (2).

The pairing part of the Hamiltonian (8) can be split into:

$$-G\sum_{\mu} A_{\mu}^{+}A_{\mu} = \mathscr{H}_{\mathrm{p}} = -G[\mathscr{H}_{\mathrm{p}}(1)+\mathscr{H}_{\mathrm{p}}(2)+\mathscr{H}_{\mathrm{p}}(1,2)],$$

$$\mathscr{H}_{\mathrm{p}}(j) = \sum_{\mu} A_{\mu}^{+}(j)A_{\mu}(j), \qquad \mathscr{H}_{\mathrm{p}}(1,2) = \sum_{\mu}(A_{\mu}^{+}(1)A_{\mu}(2)+A_{\mu}^{+}(2)A_{\mu}(1)). \tag{10}$$

Both $\mathscr{H}_{\mathrm{p}}(1)$ and $\mathscr{H}_{\mathrm{p}}(2)$ are diagonal in the above mentioned basis, the matrix elements can be evaluated from eq. (6) setting $v = t = 0$

$$\langle M_1\, T_1\, M_2\, T_2\, TT_z|\mathscr{H}_{\mathrm{p}}(1)|M_1\, T_1\, M_2\, T_2\, TT_z\rangle = \tfrac{1}{2}[M_1(2\Omega_1+3-M_1)-T_1(T_1+1)]. \tag{11}$$

The off-diagonal terms are due to $\mathscr{H}_{\mathrm{p}}(1,2)$

$$\langle M_1\, T_1\, M_2\, T_2\, TT_z|\mathscr{H}_{\mathrm{p}}(1,2)|M_1'\, T_1'\, M_2'\, T_2'\, TT_z\rangle = \begin{Bmatrix} T_2 & T_1 & T \\ T_1' & T_2' & 1 \end{Bmatrix}$$
$$\times[(-)^{T_1'+T_2+T}\langle M_1\, T_1||A^{+}(1)||M_1'\, T_1'\rangle\langle M_2\, T_2||A(2)||M_2'\, T_2'\rangle + (-)^{T_2'+T_1+T}$$
$$\times\langle M_2\, T_2||A^{+}(2)||M_2'\, T_2'\rangle\langle M_1\, T_1||A(1)||M_1'\, T_1'\rangle]. \tag{12}$$

The reduced matrix elements are given in eq. (8). The eigenvectors of the full Hamiltonian will be expressed as

$$|\alpha MTT_z\rangle = \sum_{T_1,\, T_2,\, M_1+M_2=M} C^{MT\alpha}_{M_1T_1M_2T_2}|M_1\, T_1\, M_2\, T_2\, TT_z\rangle. \tag{13}$$

If both levels have a similar radial dependence, the two particle transfer operator can still be defined by A_{μ}^{+}, its matrix elements are

$$\langle M_1\, T_1\, M_2\, T_2\, T||A^{+}||M_1'\, T_1'\, M_2'\, T_2'\, T'\rangle = (-)^{T_1+T_2'+1}\sqrt{(2T+1)(2T'+1)}$$
$$\times\left[\delta_{T_2T_2'}\delta_{M_2M_2'}(-)^{T'}\begin{Bmatrix} T_1 & T_1' & 1 \\ T' & T & T_2 \end{Bmatrix}\langle M_1\, T_1||A^{+}(1)||M_1'\, T_1'\rangle\right.$$
$$\left.+\delta_{T_1T_1'}\delta_{M_1M_1'}(-)^{T}\begin{Bmatrix} T_2 & T_2' & 1 \\ T' & T & T_1 \end{Bmatrix}\langle M_2\, T_2||A^{+}(2)||M_2'\, T_2'\rangle\right]. \tag{14}$$

Within this approximation the cross section will be related to

$$|\langle \alpha MT||A^{+}||\alpha' M'T'\rangle|^2$$
$$= |\sum C^{MT_\alpha}_{M_1T_1M_2T_2}C^{M'T'\alpha'}_{M_1'T_1'M_2'T_2'}\langle M_1\, T_1\, M_2\, T_2\, T||A^{+}||M_1'\, T_1'\, M_2'\, T_2'\, T'\rangle|^2. \tag{15}$$

3. Approximate treatments

In the following two equally degenerate levels are considered, and the number of particles is such that for $G = 0$, the lowest level is completely filled.

3.1. SMALL G-LIMIT

When $\Omega G/D \ll 1$, D being the distance between the levels, the collective excitations associated with the pairing force have a phonon-like spectrum. The RPA in this model gives a frequency W such that

$$\hbar W = D(1-x)^{\frac{1}{2}}, \qquad x = \frac{G}{G_c} = \frac{2G\Omega}{D}. \tag{16}$$

The phonon creation operator with $T = 1$, $T_z = \mu$ are

$$\begin{aligned} \Gamma^+_{a,\mu} &= \frac{G\sqrt{\Omega}}{(1-x)^{\frac{1}{2}}}\left[\frac{A^+_\mu(j_2)}{D-W} + \frac{A^+_\mu(j_1)}{D+W}\right], \\ \Gamma_{r,\mu} &= \frac{G\sqrt{\Omega}}{(1-x)^{\frac{1}{2}}}\left[\frac{A_\mu(j_2)}{D+W} + \frac{A_\mu(j_1)}{D-W}\right] \end{aligned} \tag{17}$$

and obey the usual quasi-boson commutation relations †. Within this approximation, the different states of the system are those of two uncoupled three-dimensional harmonic oscillators, therefore, the wave functions can be written as:

$$|N_a T_a, N_r T_r; TT_z\rangle = \frac{1}{\mathcal{N}_{a,r}}\{\{\Gamma^{+N_a}_a\}^{T_a}\{\Gamma^{+N_r}_r\}^{T_r}\}^T_{T_z}|0\rangle, \tag{18}$$

where N_a (N_r) are the total number of addition (removal)-type phonons coupled to isospin T_a (T_r); T, T_z stand for the total isospin and its projection and $\mathcal{N}_{ar}$ is a normalization constant.

The two-particle transfer operator can be written in terms of the phonon operators as:

$$P_\mu = \Lambda_a \Gamma^+_{a,\mu} + \Lambda_r \Gamma_{r,\mu} \qquad \Lambda_a = \Lambda_r = \Omega^{\frac{1}{2}}(1-x)^{-\frac{1}{4}} \tag{19}$$

and its reduced matrix elements are

$$\begin{aligned} \langle N_a T_a N_r T_r; T||P^+||N'_a T'_a N'_r T'_r; T'\rangle &= (-)^{T_a+T'_r+1}[\Omega(1-x)^{-\frac{1}{2}}(2T+1)(2T'+1)]^{\frac{1}{2}} \\ &\times \left[(-)^{T'}\delta_{N_r N'_r}\delta_{T_r T'_r}\begin{Bmatrix} T_a & T'_a & 1 \\ T' & T & T_r \end{Bmatrix}\langle N_a T_a||\Gamma^+_a||N'_a T'_a\rangle\right. \\ &\left. +(-)^{T}\delta_{N_a N'_a}\delta_{T_a T'_a}\begin{Bmatrix} T_r & T'_r & 1 \\ T' & T & T_a \end{Bmatrix}\langle N_r T_r||\Gamma_r||N'_r T'_r\rangle\right]. \end{aligned} \tag{20}$$

† In performing the RPA one has to retain terms in N/Ω for the levels below the Fermi surface; since here it is assumed that the level j_1 is full, one has

$$[A^+_\mu(j_1), A_{\mu'}(j_1)] \approx \delta_{\mu\mu'}.$$

The reduced matrix elements of Γ^+ can be evaluated using the well-known results of the three-dimensional harmonic oscillator

$$\langle N+1, T-1||\Gamma^+||NT\rangle = [T(N-T+2)]^{\frac{1}{2}},$$
$$\langle N+1, T+1||\Gamma^+||NT\rangle = [(T+1)(N+T+3)]^{\frac{1}{2}},$$
$$\langle NT||\Gamma||N'T'\rangle = (-)^{T-T'}\langle N'T'||\Gamma^+||NT\rangle. \tag{21}$$

3.2. LARGE G-LIMIT

For large values of G/G_c, the results for the two-level case will resemble those of the degenerate model in which $G_c = 0$. As suggested by the latter the system will present permanent deformations in isospace as well as in gauge space. If both, intrinsic and collective degrees of freedom are split, one can think in a Nilsson-like picture in which a given intrinsic structure performs rotations in both spaces.

The intrinsic wave functions can be evaluated using the BCS formalism for the $T = 1$ pairing force [6]). The total normalized wave function for the system will be [3])

$$|A, n_\Delta, n_\Gamma, K, T_1, T_z\rangle = \left[\frac{2T+1}{16\pi^3}\right]^{\frac{1}{2}} \exp[i(A-A_0)\phi] D^T_{T_z K}(\alpha_i)|n_\Delta, n_\Gamma, K\rangle, \tag{22}$$

where ϕ is the gauge angle, and α_i are the Euler angles specifying the orientation of the intrinsic system in isospace. Within this picture the two-particle transfer operator is

$$\mathscr{P}_\mu = e^{2i\phi} \sum_k \Delta_k D^1_{\mu k}(\alpha_i), \tag{23}$$

where Δ_k are the expectation values of the two-particle transfer operator in the intrinsic system. In a symmetric model ($j_1 = j_2$, the level 1 is full) one has $\Delta_1 = \Delta_{-1} = 0$, therefore the operator $\mathscr{P}_\mu$ will only connect states having the same K. For ground state to ground state transitions one finds (both initial and final states have $n_\Delta = n_\Gamma = K = 0$),

$$|\langle M+1, T'||\mathscr{P}||MT\rangle| = \Delta_0 \sqrt{2T+1}\langle 1T00|T'0\rangle = \sqrt{2T_>}\, \Omega \left(\frac{x^2-1}{x^2}\right)^{\frac{1}{2}}, \tag{24}$$

where $T_>$ stands for the larger of the two T values that are involved and Δ_0 is related [6]) to the gap parameter Δ through $\Delta_0 = \Delta\sqrt{2/G}$.

The energy splitting can be calculated with the cranking model approximation of the inertial parameters [7]). It is found

$$E^N_T = \tfrac{1}{2}G\left[\frac{x^2}{1-x^2}\right][T(T+1)+(M-M_0)^2] - 6GA. \tag{25}$$

The $6GA$ term is a self-energy Hartree-type contribution of the pairing field that should be included since it is not supposed to be taken into account in the cranking-model approach. Using the RPA it is found that the energy of the pairing vibration with $T = 0$ is 2Δ as in the ordinary pairing force problem. It must be pointed out that the $T = 1$ state will have in addition a rotation energy $T(T+1)/2\mathscr{I}$.

4. Discussion of the results

All the calculations, except when otherwise stated, where performed for two equally degenerate levels with pair degeneracy $\Omega = 10$, separated by a distance D. The Hamiltonian was diagonalized for several values of M ranging from $M = 2\Omega$ up to $M = 2\Omega+4$; in all the calculations x was allowed to vary from 0.25 up to 4.0.

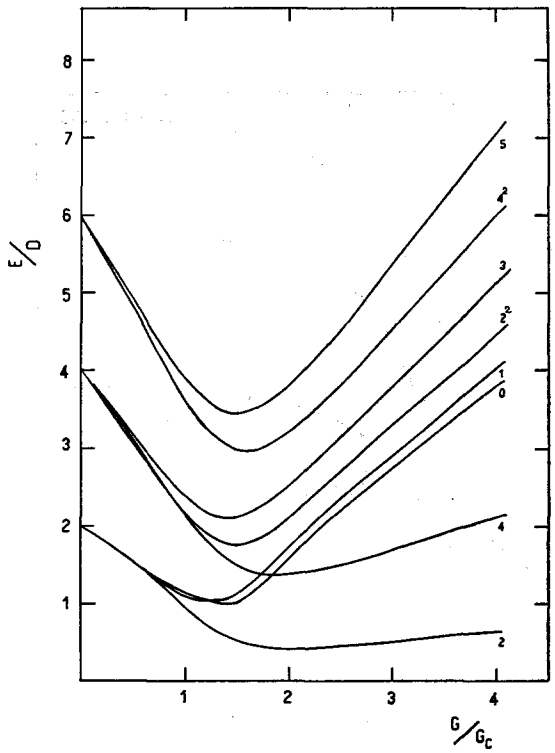

Fig. 1. The energy of the states belonging to the first two rotational bands (with $T \leqq 5$) are plotted as a function of G/G_c for $\Omega = 10$ and $M = 20$.

In fig. 1 a vibrational like spectrum can be observed in the limit $x \ll 1$. The frequency (16) given by the RPA is compared to the exact result in fig. 5. Anharmonic effects cause the levels with different T to split. A phase transition takes place for $x = 1$ and for $x > 1$ the levels tend to group into rotational bands. All the spectrum for $x \approx 4$ is already accurately given by formula (7) of the degenerate case. In the intermediate region, when $x \approx 1$, the RPA frequency tends to zero and neither description is good.

In fig. 2, states have been labeled by (T, m, α), α denotes if it is the first, second etc. time that a state with isospin T and $M = 2\Omega+m$ appears. In the limit $x \ll 1$ the $\Delta N = 1$ selection rule is observed to be satisfied, N being the number of phonons. Different selection rules are valid for the superconducting region. In general, three

types of transitions can be considered, namely those that are allowed or forbidden in both limits and those that are allowed in the vibrational and forbidden in the superconducting descriptions. This latter type of transitions populate the so called pairing vibration states [8]) of the system with a given M. In the case that is plotted the two states with $T = 0$ and 1 will show enhanced transition probabilities for $x = 1$.

There are not forbidden-allowed transitions that could be compared to the quadrupole moment operator in the electromagnetic case. This is connected with the fact that the specific operator here involved has not diagonal matrix elements. As

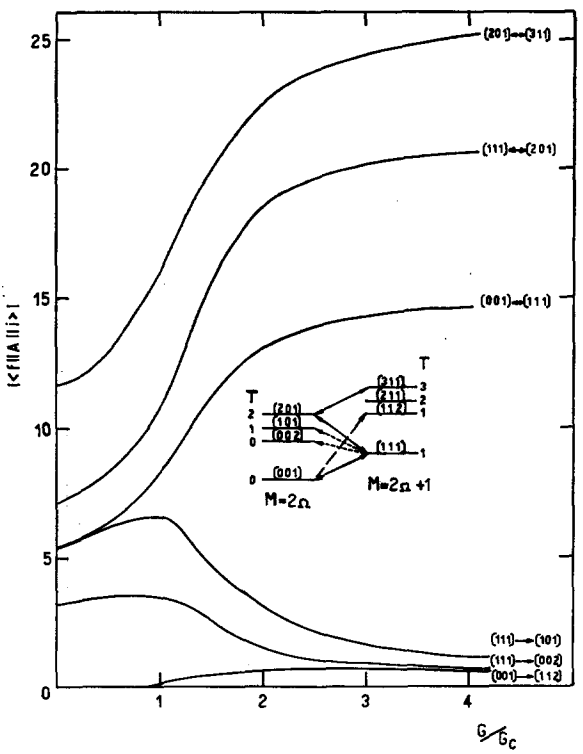

Fig. 2. The absolute value of the reduced transition matrix elements between $M = 20$ and $M = 21$ as a function of G/G_c and $\Omega = 10$. A level scheme clarifies the transitions that are considered. The full double arrows are allowed-allowed, the dashed one is forbidden-forbidden connecting a ground state of $M = 20$ with an excited state of $M = 21$; the dotted double arrows are the allowed-forbidden transitions populating the pairing vibration of the $M = 20$ system.

can be seen, all the general features shown by both energy levels and transition probabilities observed in the $|T_z| = 1$ pairing problem [9]) are obtained here, and will give the same results in the limit of large T and Ω.

The $x \gg 1$ limit is displayed in fig. 3. The lowest part of the spectrum is structured in two bands that correspond to the $v = 0$ and $v = 2$ of the degenerate model. Within the picture suggested in ref. [3]) the $v = 0$ band has $n_\Delta = n_\Gamma = 0 = K$. The

sequence of possible isospin of the $v = 2$ bands is $0, 1, 2^2, 3, 4^2, \ldots$ etc. if M is even and $1, 2^2, 3, 4^2, 5, \ldots$ if M is odd. For the symmetric model here considered, the doublets are degenerate for any value of G if $M = 2\Omega$, this fact can be checked if a perturbative calculation is performed. If $M \neq 2\Omega$ the states are no longer degenerate but their splitting is very small (less than 1 %). If the condition $\Omega_1 = \Omega_2$ is relaxed, the degeneracy is removed, for the tested case ($\Omega_1 = 5$, $\Omega_2 = 6$) the splitting is also very small (less than 1 %). In the limit of large x, the splitting completely disappears as predicted by the degenerate model [4]). The $v = 2$ bands correspond to rotations

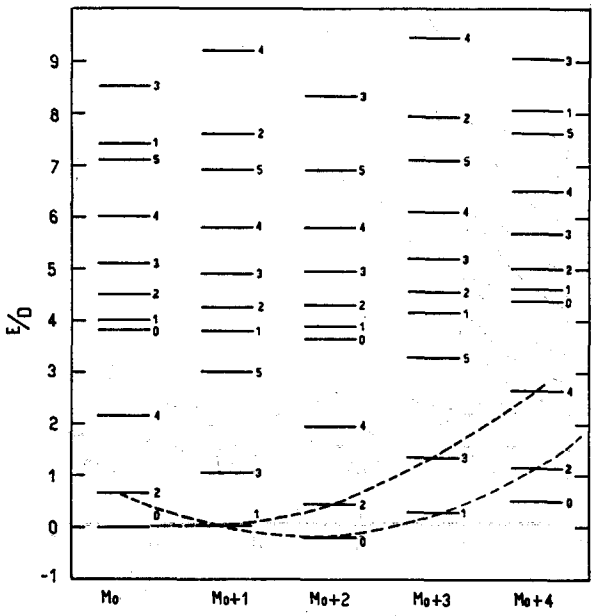

Fig. 3. The two lowest states for each isospin are displayed for $G = 4G_c$; the energy scale is given in units of the single-particle splitting of the two levels.

on top of the Δ-vibrations ($n_\Delta = 1, n_\Gamma = 0, K = 0$) and on top of the Γ-vibration ($n_\Delta = 0, n_\Gamma = 1, K = 1$) of ref. [3]). The possible isospins of each band are $T = K$, $K+1$, $K+2, \ldots$ with the additional restriction $|T-M|$ even if $K = 0$. Each of the members of the above mentioned doublets belongs to different bands, therefore the fact that they are essentially degenerate indicates that $\omega_\Delta = \omega_\Gamma$, as can be further checked by the RPA frequency 2Δ. Furthermore from the exact calculation it is obtained that both frequencies are independent of M if $G \gg G_c$.

If $G \gg G_c$ rotations will also appear in gauge space. All the states belonging to the $s = 0$ and $s = 1$ bands are joined by a dotted line in fig. 3. In fig. 4 the transition matrix elements among members of these bands are plotted as a function of x. In this figure the last number of the label (T, m, α) was dropped. In the small x-limit all the matrix elements are related to each other by phonon-type geometric coeffi-

cients. In the large x-limit the transitions involving the same initial and final T tend essentially to the same value regardless of the s-value for each band. This can be seen comparing one-to-one the transitions $(0, 0) \leftrightarrow (1, 1) \leftrightarrow (2, 2)$ to the $(0, 2) \leftrightarrow (1, 3) \leftrightarrow (2, 4)$. The grouping into bands is already noticeable for $x \approx 1.5$.

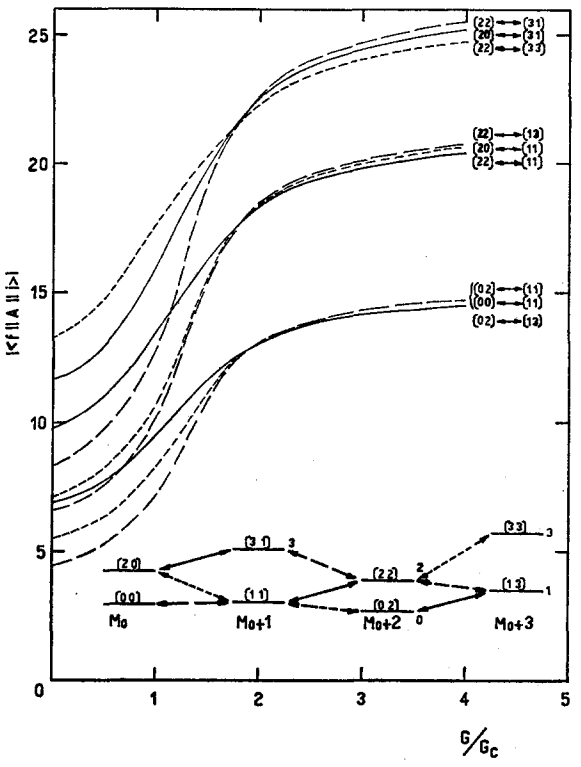

Fig. 4. The absolute value of the transition matrix elements connecting the ground states of $M = 2\Omega+m$, $m = 0, 1, 2, 3$. A level scheme clarifies the transitions that are considered. Transitions involving the same initial and final T have essentially the same value for $G/G_c \gg 1$.

The small differences for $x \gg 1$ between transitions within different s-bands tend to zero with increasing Ω, formula (8) yields

$$\left| \frac{(T, m) \leftrightarrow (T+1, m+1)}{(T, m+2) \leftrightarrow (T+1, m+1)} \right| = \left[\frac{(\Omega-m-T)(\Omega+T+m+3)}{(\Omega-m+T+1)(\Omega+m-T+2)} \right]^{\frac{1}{2}} \underset{\Omega\to\infty}{\to} 1. \quad (26)$$

Furthermore, the ratio between any two transitions among states belonging to the same s-band is

$$\left| \frac{(T, m) \leftrightarrow (T+1, m+1)}{(T+1, m+1) \leftrightarrow (T+2, m+2)} \right| = \left[\frac{(T+1)(\Omega-m-T)(\Omega+T+m+3)}{(T+2)(\Omega-m-T-2)(\Omega+T+m+5)} \right]^{\frac{1}{2}} \underset{\Omega\to\infty}{\to} \left[\frac{T+1}{T+2} \right]^{\frac{1}{2}}. \quad (27)$$

The limiting ratio (27) is the one predicted by the Nilsson-like approach (23). Out of all the transitions that are involved in this calculation, for those having T, $T_z = T$ and $T+1$, $T_z = T+1$ as initial and final states, the same result as that of the ordinary pairing force problem is obtained. One further relates here these to all other possible transitions by geometric coefficients.

The approximate treatments mentioned in the preceding section were checked against the exact results in both energies and transition probabilities. The comparison

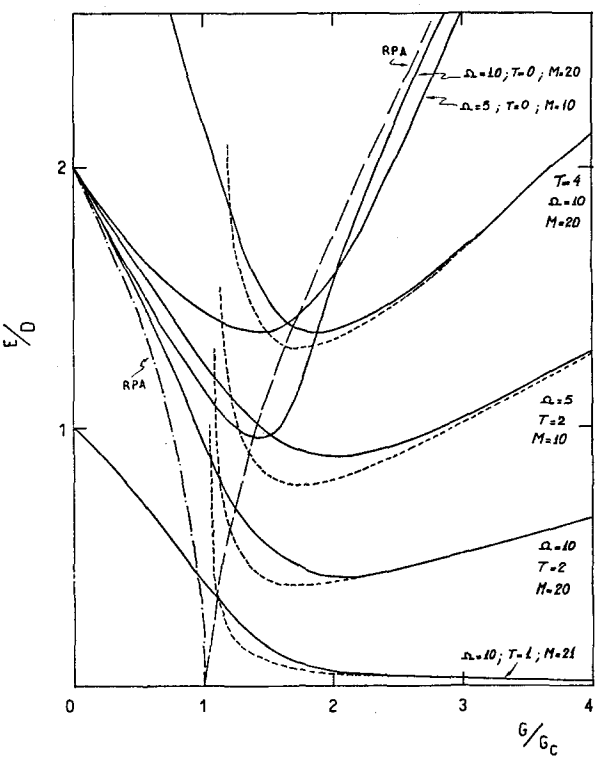

Fig. 5. The energy levels for $\Omega = 10$, with $T = 0, 2, 4$ when $M = 20$, and $T = 1$ when $M = 21$, and $T = 0, 2$ for $\Omega = 5$ and $M = 10$ as a function of G/G_c. The dotted lines are the cranking model results, the dashed line is the RPA energy for the $T = 0$ excited states, and the dot and dash line is the two-phonon energy.

is shown in figs. 5 and 6. The Nilsson-like approach was performed for the $T = 2$ and 4 states of the $v = 0$ band with $M = 2\Omega$ and for the $T = 1$ state with $M = 2\Omega+1$; the superconducting RPA result was checked with the first excited $T = 0$ state with $M = 2\Omega$. For the sake of simplicity the non-superconducting RPA is plotted only for the two phonon states. In fig. 5 also the $T = 2$ and $T = 0$ states for $\Omega = 5$ are plotted to check the Ω-dependence of the approximations. As expected all of them improve with increasing Ω, anyhow a good accuracy of the cranking calculations for low Ω and values of x close to the phase transition region is found. Transition matrix

elements show departures of the order of $1/\Omega$ (10%) although ratios are better predicted. In the region were $x \approx 1$ and the phase transition takes place all approximations fail to give the right answer as shown by the fact that the cranking parameters and the RPA frequency tend to zero.

For an α-like transfer the specific operator in a symmetric model will be $\{A^+A^+\}^{T=0}$. It must be pointed out that matrix elements are Ω times larger in the deformed region than in the normal one.

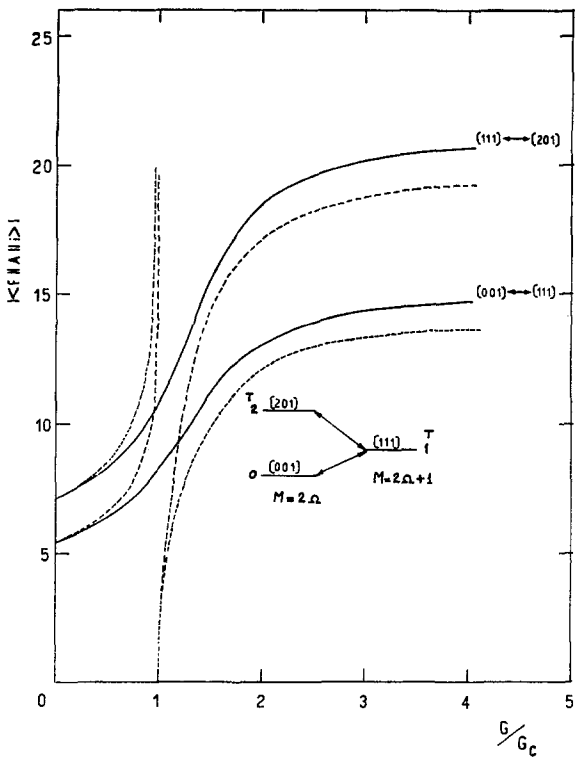

Fig. 6. Approximate and exact results as a function of G/G_c of two allowed-allowed transitions for $M = 20$ and $\Omega = 10$. The two transitions that are considered are displayed in a level scheme.

The authors wish to thank Prof. D. R. Bès and Dr. R. A. Broglia for helpful comments and discussions.

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