THE AMPLITUDES OF s AND d BOSONS IN DEFORMED NUCLEI

J. DUKELSKY, G.G. DUSSEL [1] and H.M. SOFÍA
Departamento de Física, Comisión Nacional de Energía Atómica,
1429 Buenos Aires, Argentina

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It is shown that the ground state of deformed nuclei can be considered as a condensate of bosons that do not have a well defined angular momentum. The projection on well defined angular momentum states shows that the s and d bosons take care of nearly 90% of the boson wave function.

The Interacting Boson Model [1] (IBM) provides a phenomenological description of a large variety of collective nuclear states, including states which are spherical, deformed or transitional. Using group theory techniques and a few parameters it is able to describe quite well energy level systematics.

Microscopic approaches to the IBM have been attempted, in particular the interpretation of s and d bosons in terms of the shell model assuming good seniority [2] or the discussion of the boson structures in terms of the usual quasiparticle formalism [3]. This latter method has the advantage of a wider range of applicability. In this formalism a natural definition for the s boson appears [3]:

$$S^+ = N_0 \sum_{\alpha, m>0} \frac{V_{\alpha m}}{U_{\alpha m}} b^+_{\alpha m} b^+_{\alpha \bar{m}}, \tag{1}$$

where b^+_m creates a particle in the state (m), and m is the eigenvalue of the third component of the angular momentum and the remaining quantum numbers necessary to specify the state. By $(\bar{m})$ we have denoted the time reversed state of (m). V^2_m (U^2_m) is the probability that the state (m) is occupied (unoccupied) and N is a normalization constant.

The structure for the s boson given by (1) has nevertheless the unwanted property that S^+ depends strongly on what are the states included in the core. If the core is decreased, including in the active space states which have U_m very close to zero, then this new set of states will dominate and determine almost completely the structure of the s boson.

The purpose of the present note is to give a microscopic description of deformed nuclei in terms of the Principal Series Approximation [4] (PSA). This treatment will yield a different structure than in ref. [3] for the elementary boson and will provide a microscopic justification for the use of the IBM. In order to obtain a quantitative (and also qualitative) understanding of the ground state structure for deformed nuclei, we consider the usual pairing plus quadrupole hamiltonian using for the strength of the interactions as well as for the single particle energies the values given by Baranger and Kumar [5].

The underlying physical picture of the PSA is that the ground state of a system of $2M$ fermions can be treated as M identical bosons. One then considers the diagrams describing the excitations (both fermionic and bosonic) of this boson condensate. It is possible [4] to isolate the set of diagrams that do not vanish when the number of fermions $(2M)$ is very large. It is found that the fermionic excitations (quasiparticles) are defined in terms of the bosonic ones and viceversa. In particular, the bosons of the condensate are formed by the successive interaction of a particle and a quasiparticle and they, in principle, will not have a well-defined angular momentum nor its projection on the 3-axis. In order to simplify the treatment we have as

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[1] Fellow of the Consejo Nacional de Investigaciones Científicas y Técnicas, Buenos Aires, Argentina.

sumed that the bosons have $I_3 = 0$ (this will be shown later to correspond to the assumption of axial symmetry). Therefore, the creation operator of a boson can be written as:

$$\Gamma^+ = \sum_{\alpha,m>0} \eta_{\alpha m} b^+_{\alpha m} \beta^+_{\alpha \bar{m}}, \tag{2}$$

where the evaluation of $\eta_{\alpha m}$ will be discussed later on.

The fermionic excitations depend on the boson structure through the interactions shown in figs. 1a and 1b while fig. 1c shows the boson dependence on the fermionic structure. The set of non-linear equations obtained diagrammatically are identical to the equations that one gets in a self consistent Nilsson–BCS treatment.

The scattering terms of the quadrupole–quadrupole force (fig. 1a) yields the Nilsson orbitals and the part acting on the external line will be equivalent to $-\chi Q_{20}\langle \hat{Q}_{20}\rangle$ (we represent by $\hat{Q}_{2\mu}$ the quadrupole operator).

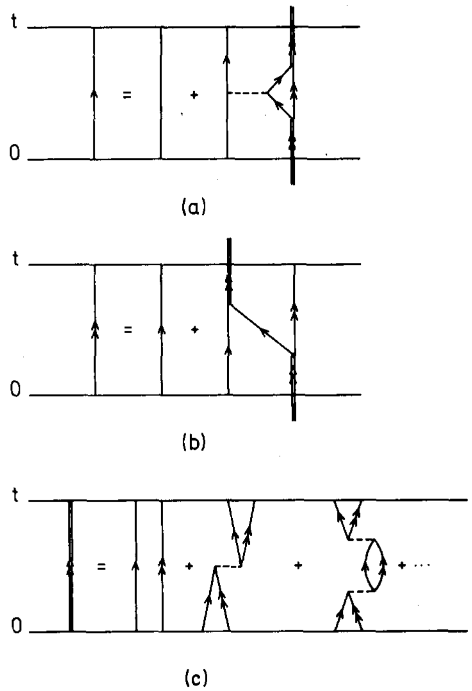

Fig. 1. Processes considered in the PSA. Diagrams (a) and (b) correspond to fermionic excitations, while (c) defines the bosonic ones. Diagram (a) yields the usual Nilsson states while (b) and (c) correspond to the BCS treatment.

The mean value $\langle \hat{Q}_{20}\rangle$ comes from the effect of Q_{20} on the ground state of the even system and must be evaluated selfconsistently. Therefore, one must diagonalize the one-body hamiltonian:

$$H_{\text{ob}} = H_{\text{sp}} - \chi \hat{Q}_{20}\langle \hat{Q}_{20}\rangle, \tag{3}$$

where H_{sp} is the initial spherical one-body hamiltonian while (3) is the axially symmetric Nilsson hamiltonian.

The equations associated with figs. (1b) and (1c) are discussed in ref. [4] where it is shown that they yield the gap and number equations as well as the expression for the quasiparticle energies. One may choose the zero of the single particle energies such that the energy of the "dressed" boson is zero. In that case, the one-body Green's function equation, displayed in fig. (1b), will be written as

$$G(\alpha m, t) = G_0(\alpha m, t) - M\Lambda^2_{\alpha m} \iint \mathrm{d}\tau_1\, \mathrm{d}\tau_2\, G_0(\alpha m, \tau_1) \times G_0(\alpha \bar{m}, \tau_2 - \tau_1) G(\alpha m, t - \tau_2), \tag{4}$$

where $G_0(\alpha m, t)$ is the one-body Green's function associated with the hamiltonian (3) and $\Lambda_{\alpha m}$ is the coupling constant between the "dressed" boson and the pair of fermions $(\alpha m, \alpha \bar{m})$. This coupling constant is evaluated in terms of the amplitudes that appears in the two-body Green's function of fig. (1c) as:

$$\Lambda_{\alpha m} = \sum_{\beta, m'>0} \langle \alpha m, \alpha \bar{m}|H|\beta m', \beta \bar{m}'\rangle \langle 0|b_{\beta m'} b_{\beta \bar{m}'}|n\rangle. \tag{5}$$

The quasiparticle energy and residue are:

$$\begin{aligned} E_{\alpha m} &= [(\epsilon_{\alpha m} - \lambda)^2 + M\Lambda^2_{\alpha m}]^{1/2} \\ &= [(\epsilon_{\alpha m} - \lambda)^2 + \Delta^2_{\alpha m}]^{1/2}, \end{aligned} \tag{6}$$

$$U_{\alpha m} = \{\tfrac{1}{2}[1 + (\epsilon_{\alpha m} - \lambda)/E_{\alpha m}]\}^{1/2},$$

where λ is the chemical potential whose value has been chosen to make the energy of the dressed boson zero. This last condition yields for the residues of the boson:

$$\langle 0|b_{\beta m'} b_{\beta \bar{m}'}|n\rangle = \Lambda_{\beta m'}/2E_{\beta m'} = \Delta_{\beta m'}/2E_{\beta m'}\sqrt{M}. \tag{7}$$

Eq. (5) yields therefore the usual gap equation, i.e.

$$\Lambda_{\alpha m} = -\sum_{\beta, m'>0} \frac{\Lambda_{\beta m'}}{2E_{\beta m'}} \langle \alpha m, \alpha \bar{m}|H|\beta m', \beta \bar{m}'\rangle. \tag{8}$$

The amplitudes $\eta_{\alpha m}$ of eq. (2) can be evaluated within the framework of the nuclear field theory [6]

$$\eta_{\alpha m} = \Lambda_{\alpha m} U_{\alpha m} / [E_{\alpha \bar{m}} + (\epsilon_{\alpha m} - \lambda)] , \tag{9}$$

i.e. it is proportional to the coupling constant between the boson and the fermion pair times the quasiparticle residue divided by the sum of the energies of the particle and the quasiparticle.

From eqs. (5) and (6) we can write

$$\Lambda_{\alpha m} = 2 E_{\alpha m} U_{\alpha m} V_{\alpha m} / \sqrt{M}, \tag{10}$$

and, therefore, one obtains for $\eta_{\alpha m}$:

$$\eta_{\alpha m} = V_{\alpha m} / \sqrt{M}. \tag{11}$$

This wave function for the boson is normalized and does not have the unwanted characteristic that the one defined in eq. (1) has.

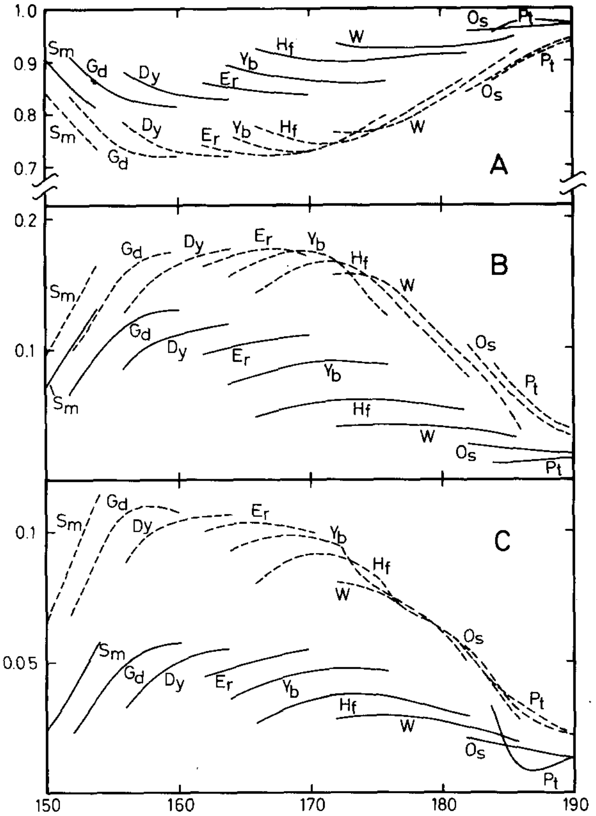

Fig. 2. The probability that the boson of the condensate has angular momentum 0, 2 or any other is plotted in a, b and c, respectively, for the rare earth nuclei. The full (dashed) line corresponds to protons (neutrons).

Although the boson does not have a well-defined angular momentum, it is possible to project it on states in which it does. Towards that goal, we will need the usual Nilsson transformation

$$b^+_{\alpha m} = \sum_j R^\alpha_{jm} a^+_{jm}, \tag{12}$$

where a^+_{jm} creates particles in the spherical potential associated with $H_{\rm sp}$ and R^α_{jm} is formed by the eigenfunctions of (4). In a similar way one may relate the quasiparticle operator β^+ to the spherical one γ^+. Therefore,

$$b^+_{\alpha m} \beta^+_{\alpha \bar{m}} = \sum_{jj'J} \langle jj'm - m | J0 \rangle \times (-)^{j-m} R^\alpha_{jm} R^\alpha_{j'\bar{m}} [a^+_j \gamma^+_{j'}]^J_0 . \tag{13}$$

Hence, the probability that the boson Γ^+ has angular momentum J will be:

$$P_J = \sum_{jj'} \left(\sum_{\alpha, m>0} \frac{V_{\alpha m}}{\sqrt{M}} R^\alpha_{jm} R^\alpha_{j'-m} \times \langle jj'm - m | J0 \rangle (-)^{j-m} \right)^2 . \tag{14}$$

In fig. (2a) we show P_0 for the rare earth nuclei, with the dashed (full) line corresponding to neutrons (protons). In fig. (2b) we plot P_2 with the same con-

Table 1
The probability of different angular momenta inside the boson of the condensate is given for ^{154}Sm and ^{170}Hf.

J	^{154}Sm		^{170}Hf	
	Neutrons	Protons	Neutrons	Protons
0	0.7206	0.8111	0.7420	0.9062
1	0.0075	0.0068	0.0065	0.0061
2	0.1644	0.1308	0.1652	0.0579
3	0.0413	0.0300	0.0412	0.0156
4	0.0240	0.0070	0.0114	0.0091
5	0.0223	0.0082	0.0138	0.0028
6	0.0062	0.0022	0.0090	0.0005
7	0.0074	0.0018	0.0052	0.0009
8	0.0015	0.0008	0.0025	0.0001
9	0.0027	0.0013	0.0013	0.0007
10	0.0005	0	0.0003	0
11	0.0015	0	0.0014	0
12	0.0001	0	0.0002	0

vention and in (2c) the sum for all angular momenta different from 0 or 2.

Table 1 gives P_J as a function of J for ^{154}Sm (which has the smallest $P_0 + P_2$) and for ^{170}Hf (which can be considered a typical deformed nucleus).

The present results provide a microscopic justification for the IBM in the sense that the truncation of the space only into the s and d bosons, seems to be reasonable.

References

[1] A. Arima and F. Iachello, Ann. Phys. (NY) 99 (1976) 253; 111 (1978) 201.

[2] A. Arima, T. Otsuka, F. Iachello and I. Talmi, Phys. Lett. 66B (1977) 205;
T. Otsuka, A. Arima, F. Iachello and I. Talmi, Phys. Lett. 76B (1978) 139;
T. Otsuka, A. Arima and F. Iachello, Nucl. Phys. A309 (1978) 1;
R.A. Broglia, K. Matsunyamagi, H.M. Sofia and A. Vittori, to be published.

[3] M. Berard and N. de Takacsy, Phys. Rev. C20 (1979) 2439.

[4] G.G. Dussel and D.R. Bes, Nucl. Phys. A323 (1979) 392.

[5] M. Baranger and K. Kumar, Nucl. Phys. A110 (1968) 490.

[6] D.R. Bes, R.A. Broglia, G.G. Dussel, R. Liotta and H.M. Sofía, Nucl. Phys. A260 (1976) 1.