AN ELECTROTHERMAL INSTABILITY : THE INFLUENCE OF *ALBEDO* BOUNDARY CONDITIONS ON THE STATIONARY STATES OF AN EXACTLY SOLVABLE MODEL

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ABSTRACT

In order to study the effect of partially reflective (*albedo*) boundary conditions on pattern formation and stability in reaction-diffusion systems, we have analyzed an exactly soluble model of an electrothermal instability: *the Ballast resistor*. The present results allow to interpolate continuously between those corresponding to Neumann and Dirichlet boundary conditions, and to follow the qualitative changes in the patterns.

I. INTRODUCTION

One of the most charming and striking natural phenomena is the capability of some systems to undergo a nonequilibrium phase transition towards a qualitatively different kind of behaviour when subject to the variation of a control parameter. Under certain conditions, it is possible to reach more ordered states where temporal and/or spatial structures arise. Such self-organization phenomena originate from the nonlinear dynamics as well as in the existence of constraints that keep the system far from equilibrium. An important feature of these systems is the possibility of wave propagation, e.g. the spiral waves in the Belousov-Zhabotinsky reaction [1a], and impulses propagating in the nerve axon [1b]. In both cases, and despite the different typical spatial and/or temporal scales the nonlinear reaction-diffusion models have proved to provide a useful and adequate picture, supporting its wide use in describing several far from equilibrium phenomena [2,3]. Moreover, the reaction-diffusion approach, besides describing propagating structures is also adequate to analyze the existence of localized structures both in spatially finite or infinite systems [4]. In the particular case of finite size systems, the boundary conditions affect the formation and stability of patterns. Due to the complexity of the reaction-diffusion equations governing most of these systems, one usually needs to resort to numerical or singular perturbational methods [5]. On the other hand, and in order to gain insight, one try to analytically study some simplified models. One of those simple cases that has recently attracted some attention, is a model of an electrothermal instability based on the Ballast resistor [6,7a,b,c,8]. The studies so far done on this finite model involve the analysis of spatial structures and their stability for Neumann and Dirichlet boundary conditions [7b,c]. The choice of Neumann or Dirichlet boundary conditions physically corresponds to zero current or absorption at the boundary respectively. Also, and in order to interpret experimental results related with superconducting microbridges, the description of more realistic thermal contacts was done by increasing the complexity of the description [8]. However, there are intermediate situations between those corresponding to Dirichlet or Neumann boundary conditions, that could represent the realistic thermal contacts needed in the former situation, describable in terms of simple boundary conditions that essentially do not increase the complexity of the analysis.

The above indicated goals can be achieved by considering the so called *albedo* boundary conditions, which have been used in neutron and radiation transport problems [9]. These intermediate boundary conditions seem to have passed almost unnoticed in reaction-diffusion descriptions [10], despite the fact that they model the more realistic situation of partial reflectiveness at the boundary.

In this paper we study the effect of imposing *albedo* boundary conditions in the reaction-diffusion problem. In order to keep the study at an analytical level, we adopt the same simple piecewise linear model of the Ballast resistor previously described [7b,c,8]. It must also be remarked that such a model can be thought as a *mimic* of the Schlögl model [11], in the same spirit of McKean's treatment of the FitzHugh-Nagumo model [12], or as a simplified representation of more general bistability problems [4c].

II MODEL AND ANALYSIS

We start directly with the model we will work on [7b,c]. For the physical description of the Ballast resistor we refer the interested reader to Refs.[6, 7a].

We consider a wire of length L along which a current I may flow and which is surrounded by a heat bath at constant temperature T_B . The state of the system is characterized by the time dependent temperature distribution $T(x,t)$ which satisfies :

$$c \frac{\partial}{\partial t} T(x,t) = \frac{\partial}{\partial x} \lambda(x) \frac{\partial}{\partial x} T(x,t) - q(T(x,t) - T_B) + I^2 R(T) \quad (1)$$

$$-L/2 < x < L/2$$

where c is the specific heat per unit length, $\lambda(x)$ the heat conductivity, q the heat transfer coefficient to the heat bath, and $R(T)$ the resistance of the wire per unit length. We take c, λ, q as constants [7b,c,8]. The non linear term will be given by $R(T) = R_o\, \theta(T-T_c)$, θ being the Heaviside step function, and T_c is the critical temperature. Taking $T_B = 0$, and introducing dimensionless variables ($y = x\sqrt{q/\lambda}$, $\tau = t\, q/c$, with $-y_L < y < y_L$), Eq.(1) reduces to :

$$\frac{\partial}{\partial \tau} T(y,\tau) = \frac{\partial}{\partial y^2} T(y,\tau) - T(y,\tau) + T_h\, \theta(T(y,\tau)-T_c) \quad (2)$$

with $T_h = I^2 R_o/q$. For the stationary states we have :

$$\frac{d^2}{dy^2} T(y) + \frac{d}{dT} \phi(T) = 0 \quad (3)$$

with

$$\phi(T) = \begin{cases} -1/2\ T^2 & T < T_c \\ \phi_h - 1/2\ (T-T_h)^2 & T > T_c \end{cases} \tag{4}$$

and $\phi_h = 1/2\ T_h^2 Z$, $Z = (1 - 2T_c/T_h)$. It is possible to exploit a mechanical analogy, identifying $T \to r$ (position) and $y \to t$ (time) [6,7a]. Then Eq.(3) becomes the equation of motion and

$$H = 1/2 \left(\frac{dT}{dy}\right)^2 + \phi\ (T) \tag{5}$$

is a constant of motion. In Fig.1 the form of the *potential* $\phi(T)$ is depicted.

We consider now the general *albedo* boundary conditions (ABC), that imply a partial reflectiveness at the boundary. For completeness, a simplified derivation of this boundary condition is included in appendix A. The ABC conditions, in the case of the diffusion approximation, takes the form [9]:

$$\begin{aligned} \left.\frac{dT}{dy}\right|_{y=-y_L} &= k\ T(-y_L) \\ \left.\frac{dT}{dy}\right|_{y=y_L} &= -k\ T(y_L) \end{aligned} \tag{6}$$

with $k > 0$. For $k=0$ we have Neumann b.c. and for $k \to +\infty$ Dirichlet b.c. results. These two limiting cases were studied in Ref.[7b,c]. There it was found that :

a) Neumann boundary conditions (NBC) : There exist two homogeneous solutions

$$T^s(y) = 0$$

and

$$T^s(y) = T_h$$

If $T_h > T_c$ (two maxima in the potential) there also exist periodic solutions which consist of a sequence of hot $(T(y)>T_c)$ and cold $(T(y) < T_c)$ sections. See Fig.2.

b) Dirichlet boundary conditions (DBC) : There is only one homogeneous solution

$$T^s(y) = 0$$

If $T_h > 2\ T_c$ then there exist two inhomogeneous solutions of the form cold-hot-cold. See Fig.3.

Regarding the stability of the different structures for the case of fixed current, the results obtained in [7a,b,c] are summarized as follows:

For NBC the homogeneous stationary states are stable, while the inhomogeneous ones are unstable, even though for all practical purposes, due to the long lifetime of the latter, such instability will be unobservable. For DBC the homogeneous stationary state is stable. The inhomogeneous state is stable if the length of the hot section is longer than the one corresponding to the sum of the cold sections, otherwise it is unstable. However, in the latter case, as the decay time is again very long, also in this case the instability will be unobservable. For fixed voltage, the situation is a little more involved and there are some changes, but we will not discuss this situation here.

By using the *albedo* boundary conditions (ABC) Eq.(6), we may follow the change of the patterns going from Neumann to Dirichlet B.C.. It is clear that the only, trivial, homogeneous solution will correspond to $T^s(x) = 0$. The solution, starting from the *hot* homogenenous Neumann case, without cold-hot transitions, and for a given value of the parameter k, takes the following form:

$$T^s(y) = T_h \left\{ 1 - \frac{k \cosh y}{\sinh y_L + k \cosh y_L} \right\} \tag{7}$$

After k reaches the value

$$k^* = (T_h/T_c - 1) \tanh (y_L) \tag{8}$$

we have a qualitative change in the solution, with a transition from cold to hot regions and back. See for instance the situation depicted in Fig. 4. In the limit of $k \to \infty$, this pattern coincides with the stable inhomogeneous solution for DBC.

On the other hand, starting from the inhomogeneous *cold-hot-cold* NBC solution, we have a family of patterns given by:

$$T^s(y) = T_h \begin{cases} \sinh y_c \ \gamma(k,\ y_L+y)\ \gamma(k,y_L)^{-1} & -y_L \le y \le -y_c \\ 1 - \cosh y\ \gamma(k,y_L - y_c)\ \gamma(k,y_L)^{-1} & -y_c \le y \le y_c \\ \sinh y_c \ \gamma(k,\ y_L - y)\ \gamma(k,\ y_L)^{-1} & y_c \le y \le y_L \end{cases} \tag{9}$$

with $\gamma(k,x) = \sinh (x) + k \cosh (x)$, and the (double valued) coordinate point y_c, at which we have the matching conditions

$$T^{s}(\pm y_c + 0^-) = T^{s}(\pm y_c + 0^+)$$

$$\frac{d}{dy} T^{s} (\pm y_c + 0^-) = \frac{d}{dy} T^{s'}(\pm y_c + 0^+) \tag{10}$$

is given by:

$$y_c^{(\pm)} = \frac{y_L}{2} - \frac{1}{2} \ln \left\{ \frac{Z\, \gamma(k, y_L) \pm \sqrt{Z^2 \gamma(k, y_L)^2 + 1 - k^2}}{1 + k} \right\} \tag{11}$$

This cold-hot-cold pattern, that in the limit $k \to \infty$ coincides with the unstable branch of the inhomogeneous DBC solutions, shows a kind of *hard bifurcation* at the value given by (8), when a new cold-hot-cold pattern emerges. This new structure is coincident with the one coming from the homogeneous *hot* NBC solution. However, there are other solutions arising from the corresponding periodic solution, but these ones, with the exception of the cold-hot-cold just mentioned, ceases to exist at a critical value of k which depends on the periodicity of the pattern.

The mechanical analogy based on Eqs. (3,4), allows us to classify the solutions, and to prove some simple properties in a straightforward graphical way. The intersection of the parabola

$$H - 1/2\, (k\, T)^2 \tag{12}$$

with the potential $\phi(T)$, fixes the possible points where the ABC is satisfied. For $k < 1$ it is easy to see that the only inhomogeneous solutions that can exist are of type *cold-hot-cold* and *hot* (the *energy* has to be positive). Further, it is also easy to see that solutions which start and end with the same kind of section (cold or hot), have to be symmetric. Moreover, due to the requirement of matching a fixed *period* (which is given by "L"), it is possible to see that the periodic solutions given above cease to exist for certain values of k, that depend on the periodicity of the pattern. However, our interest lies in the structures given by Eqs. (7) and (9).

III. STABILITY OF THE STRUCTURES FOR ABC

Here we will discuss the stability of the structures shown in the previous section. We restrict ourselves to a linear stability analysis, looking for local equilibrium, instead of global stability. We will also restrict our analysis to the case of constant current, without considering the case of constant voltage.

As usual we propose a perturbation around the stationary solution of the form

$$T(y,\tau) = T^s(y) + \psi(y,\tau) \tag{13}$$

Replacing this expression in Eq.(2) and keeping up to terms of first order in ψ, we get

$$\frac{\partial\psi}{\partial\tau} = \left\{\frac{\partial^2}{\partial y^2} T^s(y) - T^s(y) + T_h\,\theta(T^s - T_c)\right\} + \frac{\partial^2}{\partial y^2}\psi - \psi - \kappa_o\sum_i \delta(y-y_i)\,\psi \tag{14}$$

The term within brackets is zero, as it corresponds to the stationary condition, then Eq. (13) reduces to an equation for ψ. The parameter κ_o comes out from the discontinuities at $y = \pm y_c$ in the case of the *cold-hot-cold* structures given by Eq. (9), but will not arise when looking at the solutions corresponding to Eq. (7). This parameter is given by

$$\kappa_o = T_h \left| \frac{dT^s_{(y)}}{dy} \right|^{-1}_{y=y_1} \tag{15}$$

As usual we propose $\psi = \psi_o(y)\, e^{-\alpha\tau}$, leading in each region to solutions of the form

$$\psi_o(y) = a_\eta\, e^{\kappa y} + b_\eta\, e^{-\kappa y} \tag{16}$$

where the coefficients a_η, b_η depend on the region.

For the situation corresponding to the solution (7), it is easy to prove that the α eigenvalue is always positive (=1), indicating stability.

For the situation described by the solution (9), we found

$$\kappa_o^{(\pm)} = T_h \left| \frac{dT^s}{dy} \right|^{-1}_{y=y_c^{(\pm)}} = \frac{\gamma(k, y_L)}{\sinh y_c^{(\pm)} \gamma(k, y_L - y_c^{(\pm)})} \tag{17}$$

The solution of Eq. (14), writing the eigenvalue as $\alpha = 1 - \kappa^2$, have, in each region, the form (16). The values of a_η and b_η are fixed applying the ABC, Eq. (6), and the matching conditions at $y = \pm\, y_c$, Eq. (10). After some algebra we arrive at the following equation for κ:

$$\kappa \left\{ \tanh\left[\kappa\, y_c^{(\pm)}\right] + \frac{\kappa \sinh\,[\kappa(y_L - y_c^{(\pm)})] + k \cosh\,[\kappa(y_L - y_c^{(\pm)})]}{\kappa \cosh\,[\kappa(y_L - y_c^{(\pm)})] + k \sinh\,[\kappa(y_L - y_c^{(\pm)})]} \right\} = \kappa_o^{(\pm)}$$

where κ_o is given by Eq.(15). In the limits k=0 and $k \to \infty$, this equation reduces to those corresponding to NBC and DBC respectively [7c].

Indicating by $f(\kappa)$ the l.h.s. of the Eq. (18) and making an analytical and numerical analysis of this function it is simply to prove that, for $\kappa > 0$, it is a monotonous increasing function of κ. Then, calling $\bar{\kappa}$ to the value of κ such that $f(\kappa) = \kappa_0$, it follows that

$f(\bar{\kappa}) - f(1) > 0$ implies $\bar{\kappa} > 1$, and $\alpha = 1-\bar{\kappa}^2 < 0$: unstability

$f(\bar{\kappa}) - f(1) < 0$ implies $\bar{\kappa} < 1$, and $\bar{\alpha} = 1-\bar{\kappa}^2 > 0$: stability

For $\kappa_o^{(+)}$, corresponding to the choice $y_c^{(+)}$, we meet the first situation. This corresponds to a structure with a hot region shorter than the cold one, and indicates an unstable branch. On the other hand, for $\kappa_o^{(-)}$, with the choice $y_c^{(-)}$, corresponding to a structure with a hot region longer than the cold one we meet the second situation and have a stable branch.

IV DISCUSSION

The results shown in the previous sections indicate that the ABC allows us to interpolate smoothly between those results corresponding to NBC (k=0) and DBC ($k \to \infty$) limits. This could be seen in Fig.4 for the particular case of the structure that originates in the stable homogeneous NBC solution ($T^s(y) = T_h$). This point is even more transparent in Figs. 5a,b,c, where we present the dependence of the two branches of y_c as a function of k, for different values of the current (or $T_h/2\ T_c$), and different values of the length y_L. In such graphics the asymptotic approach of the DBC values for $k \to \infty$ is clearly seen. The branch that starts at $y_c = y_L$ for a value of k given by k^* (Eq.(8)) corresponds to the stable one. It is also possible to see the same behaviour in the characteristic curve, that is the current-voltage curve. These results are depicted in Figs. 6a,b,c, for different values of k and of y_L . The current and voltage have been normalized according to

$$I_{eff} = I\ \sqrt{R_o/qTc} \qquad ; \qquad V_{eff} = V/\left[2L\ \sqrt{qR_o Tc}\right]$$

The branches on the right part of those graphs correspond to the stable ones. Similar curves are shown in Figs. 7a,b, where the quantitative changes for different wire lengths are more clearly seen, and the non existence of a general scaling of these curves is also apparent. An interesting point that arises in Figs. 5a,b,c as well as in Figs. 6a,b,c, is the existence of *gaps* in these curves. The origin of these cuts could be understood from the expresion of $y_c^{(\pm)}$ given by Eq.(11), where the argument of the square root on the r.h.s. could become negative for certain values of k, y_L ,or Z (or $T_h/2\ T_c$), so that $y_c^{(\pm)}$ ceases to be real. An important consequence, is the cut in the characteristic curve for short wires, even though this effect could be due to the crudeness of our model (i.e. a more physically sound functional representation of the potential $\phi(T)$ in Eq.(4), would probably wash out this effect).

The results discussed so far allows us to argue about the connection between the different stable and unstable patterns in

NBC and DBC cases. The stationary homogeneous (constant) structure $T^s(y) = T_h$, for NBC, goes over the stable inhomogeneous branch in DBC (corresponding to the structure where the hot section of the wire is longer than the cold sections). Meanwhile, the unstable inhomogeneous structure in NBC, goes over the unstable branch in DBC (where in opposition to the previous case the hot section is shorter than the cold one). The qualitative comparison of the present results for the characteristic curves with those obtained for superconducting microbridges [8], indicate that the curves obtained for values of k intermediate among those corresponding to the NBC and DBC limits, compares favorably with the experimental data of short microbridges.

We have then shown that the consideration of the *albedo* boundary conditions are not only more realistic and adequate in the study of some physical problems, but also open new possibilities in the study of formation and stability of patterns in physical, chemical and biological finite systems. For instance, the analysis of multicomponent and higher dimensional systems, with the boundaries having different *reflectivities* for each component, or *albedo* parameters changing in each boundary, would offer a wealth of possibilities on the arising structures and its stability. Also, front propagation in finite systems could be strongly affected by the *albedo* properties at the boundaries. All these topics will be the subject of further work.

APPENDIX A : ALBEDO BOUNDARY CONDITIONS

In order to clarify the meaning of the *albedo* boundary conditions (ABC) given by Eq. (6), we will briefly reproduce here some results that are very well known in neutron transport theory [9]. As the analysis is more transparent in the case of particle diffusion, we will adopt such a framework.

We start defining three related quantities :

$\phi(\bar{r},\bar{\Omega},t)$ = the angular flux of particles: the number of particles crossing a given surface per unit time flying with direction $\bar{\Omega}$.

$N(\bar{r},\bar{\Omega},t)$ = angular density of particles: number of particles contained in a given volume and flying with direction $\bar{\Omega}$

$\bar{J}(\bar{r},\bar{v},t)$ = angular current density vector

The relations among them came from:

$$\phi(\bar{r},\ \bar{\Omega},\ t) = v\ N(\bar{r},\ \bar{\Omega},\ t) \tag{A.1}$$

$$\bar{J}(\bar{r},\bar{v},t) = \bar{\Omega}\ \phi(\bar{r},\bar{\Omega},t) = v\ \bar{\Omega}\ N(\bar{r},\ \bar{\Omega},\ t) \tag{A.2}$$

They can also be related with the net current density vector:

$$\bar{J}(\bar{r},t) = \int \bar{\Omega}\ .\ \phi(\bar{r},\ \bar{\Omega},\ t)\ d\bar{\Omega} = \int v\ \bar{\Omega}\ N(\bar{r},\ \bar{\Omega},\ t)d\bar{\Omega} \tag{A.3}$$

$$= \int \bar{J}\ (\bar{r},\bar{v},t)\ d\bar{v}$$

In the diffusion approximation, the validity of Fick's law is assumed:

$$\bar{J}(\bar{r}, t) = - D \nabla \phi (\bar{r}, t) \tag{A.4}$$

with D playing the role of a diffusion coefficient , and $\phi(\bar{r},t)$ is the integral of $\phi(\bar{r}, \bar{\Omega}, t)$ over all directions. Defining the positive direction of the normal surface versor $\hat{n}$ as the one pointing outwards the boundary located at r_s, we define the partial in- and out- currents as:

$$j^{\pm} (t) = \pm \int \hat{n}, \bar{J}(\bar{r}_s, \bar{v}, t) \, d\bar{v}$$

$$= \pm \int \hat{n}. \bar{\Omega} \, \phi (\bar{r}_s, \bar{\Omega}, t) \, d\bar{\Omega} \tag{A.5}$$

In order to fix ideas we restrict ourselves to plane geometry and consider the stationary case (independent of t). Making a Legendre expansion of the angular quantities and keeping only the lowest order (the usual P_1 approximation [9]), we get

$$\phi(\bar{r}, \mu) = \frac{1}{4\pi} \left\{ \phi(\bar{r}) + 3 \mu J(\bar{r}) \right\} \tag{A.6}$$

where $\mu = \cos \theta = \hat{n}.\hat{\Omega}$. Then, for the partial in-current we found:

$$j^{-} = - \int_{-1}^{0} d\mu \, \mu \, \phi(\bar{r}_s, \mu) \tag{A.7a}$$

and similarly for the partial out-current

$$j^{+} = \int_{0}^{1} d\mu \, \mu \, \phi (\bar{r}_s, \mu) \tag{A.7b}$$

Replacing (A.6) in (A.7a) we obtain:

$$j^{-} = \frac{1}{4}\,\phi\,(\bar{r}_s) - \frac{1}{2}\,J(\bar{r}_s) \qquad \text{(A.8a)}$$

and similarly

$$j^{+} = \frac{1}{4}\,\phi\,(\bar{r}_s) + \frac{1}{2}\,J\,(\bar{r}_s) \qquad \text{(A.8b)}$$

We now introduce the ABC in the form:

$$j^{-} = a\,j^{+} \qquad \text{(A.9)}$$

where the *albedo* parameter is: $0 < a < 1$, and this condition indicates that the in-current is a function of the out-current. Clearly $a = 1$ implies total reflection and then corresponds to NBC; meanwhile $a = 0$ implies no reflection at all (or absorbing boundary conditions), and in certain limit corresponds to DBC. Eventually an extra external current could be included in the condition (A.9), but we will not consider it here. Operating with (A.8a,b) and (A.9), we obtain:

$$J \fallingdotseq (1-a)/(1+a)\ \phi/2 \qquad \text{(A.10)}$$

and through Fick's law (A.4) and (A.1), we get:

$$\frac{d}{dx}\,N\,(x_s) = -k\,N\,(x_s) \qquad \text{(A.11)}$$

where

$$k = \frac{vD\,(1-a)}{2\,(1+a)}$$

Equation (A.11) has the form of the ABC we have used in the analysis of the present work.

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FIGURE CAPTIONS

Figure 1 :

Representation of the *potential* $\phi(T)$ as a function of the temperature, for diferent values of T_h.

Figure 2 :

An example of a periodic structure for NBC, for $y_L = 3$, $T_h = 2T_c$

Figure 3 :

The two inhomogeneous structure for DBC, for $T_h > 2\,T_c$ and $y_L = 3$.

Figure 4 :

Change in the homogeneous NBC stable solution due to the increase on the value of k, for $y_L = 3$, and $T_h = 2T_c$.

Figure 5 :

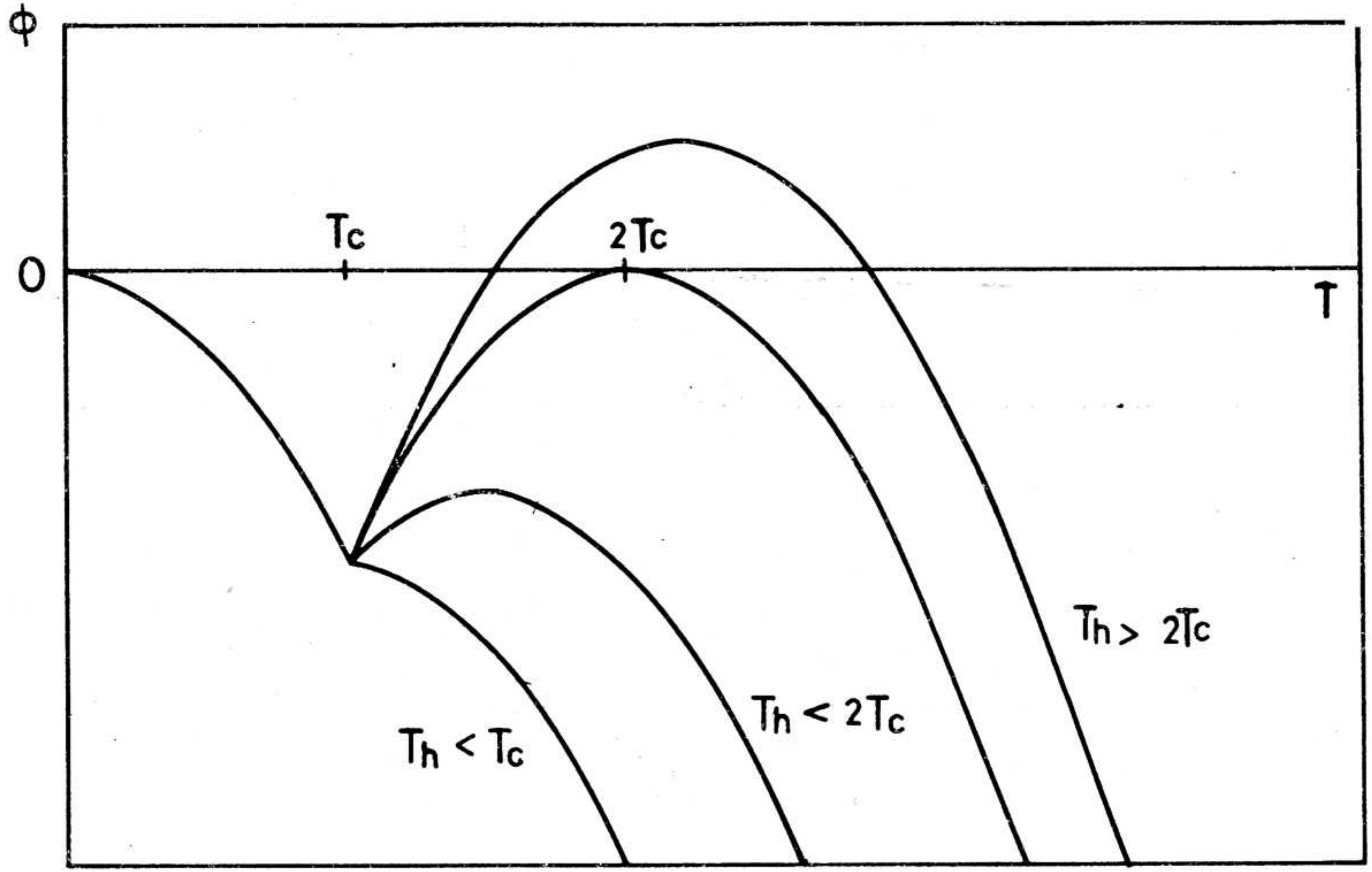
Matching distance y_c, given by Eq.(11), as a fuction of k, for different values of $T_h\,/\,2\,T_c$, and for : a) $y_L = 2$; b) $y_L = 3$; c) $y_L = 5$.

Figure 6 :

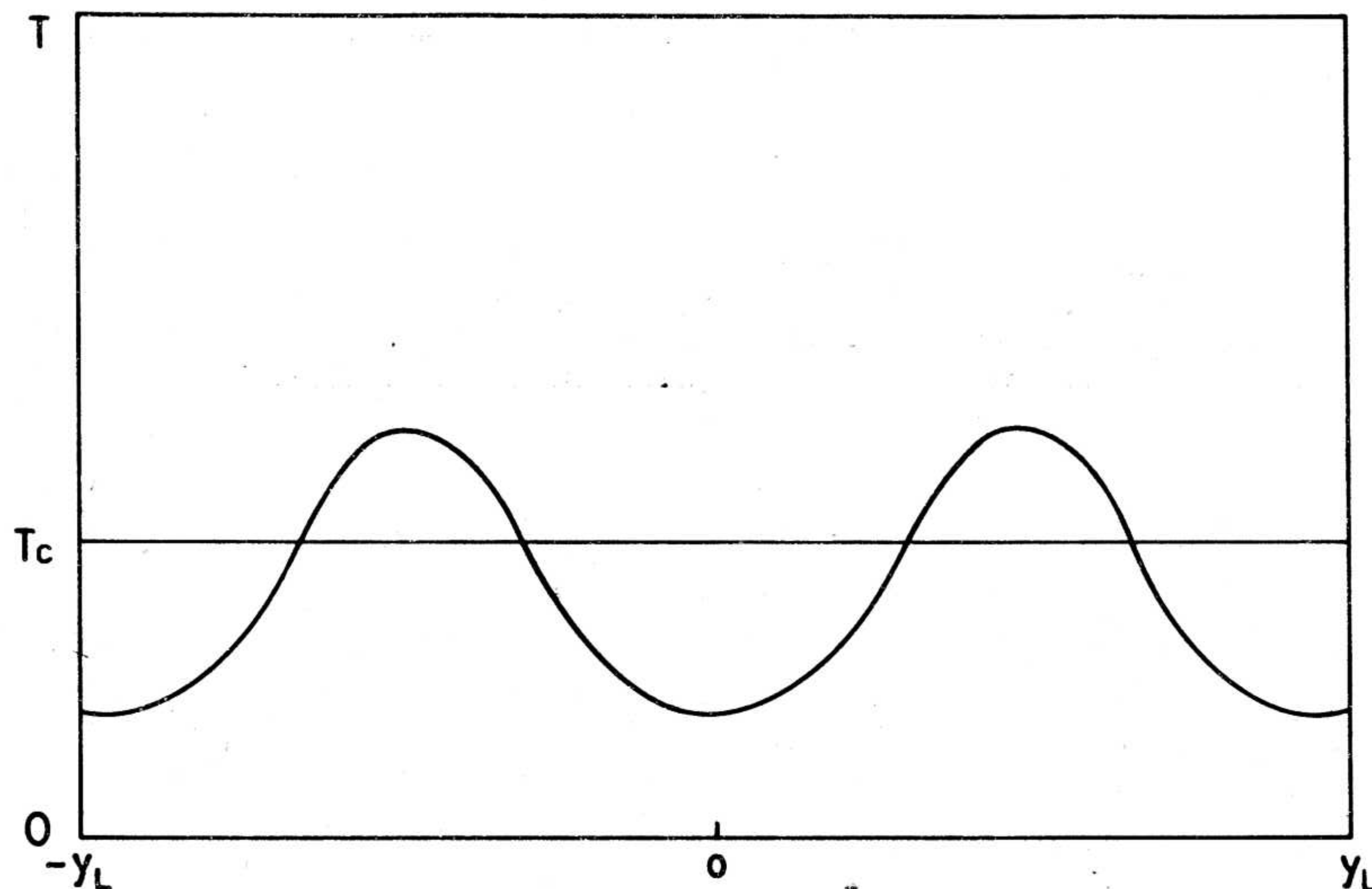
Characteristc curve I vs. V, for different values of k, and for : a) $y_L = 1$; b) $y_L = 3$; c) $y_L = 5$.

Figure 7 :

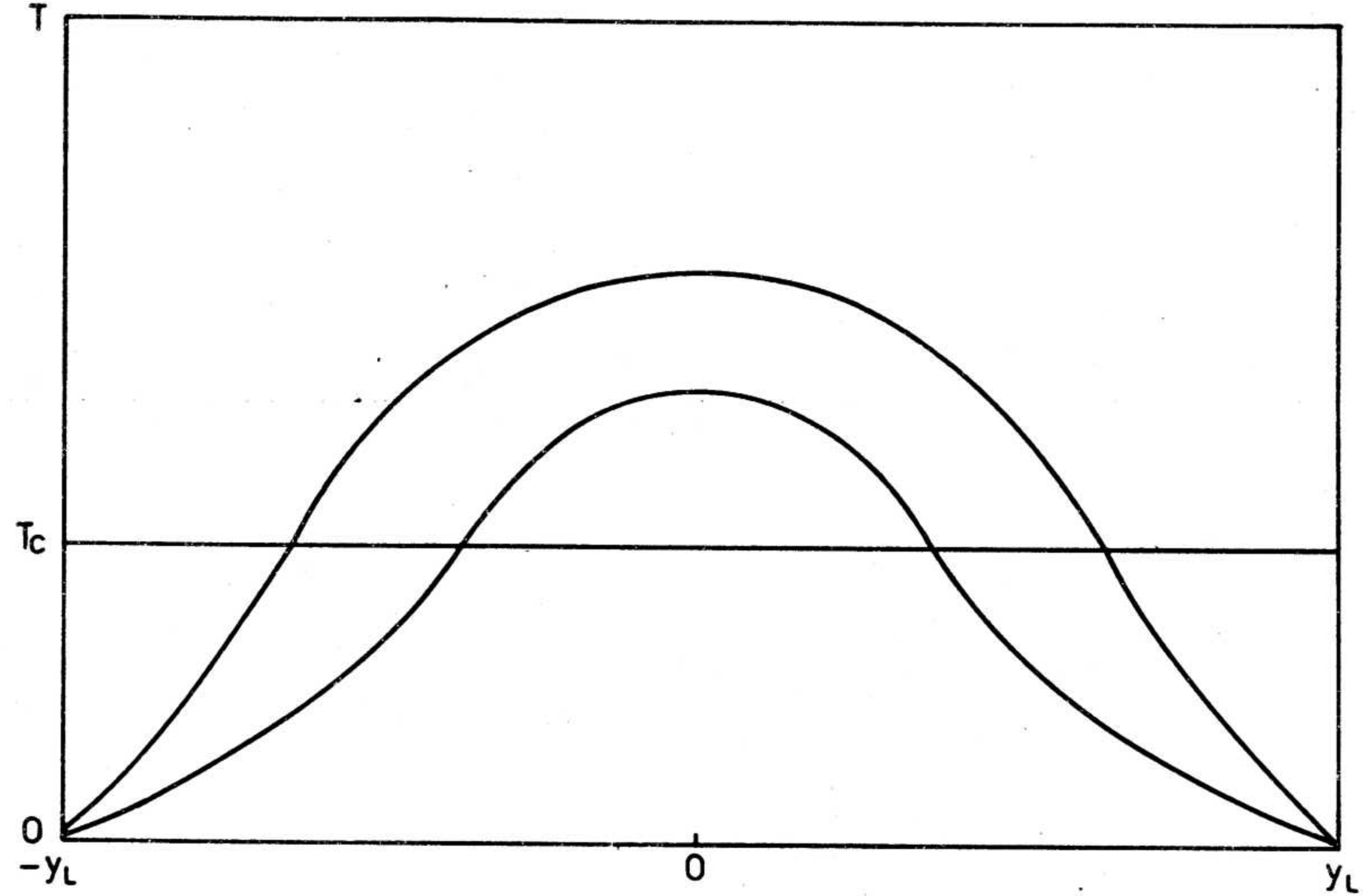
Characteristic curve I vs. V showing the difference of the curves for NBC and DBC from those of ABC for long and short wires, and for different values of k :a) k = 1.5;b) k=5. The values of y_L are 1 and 6.

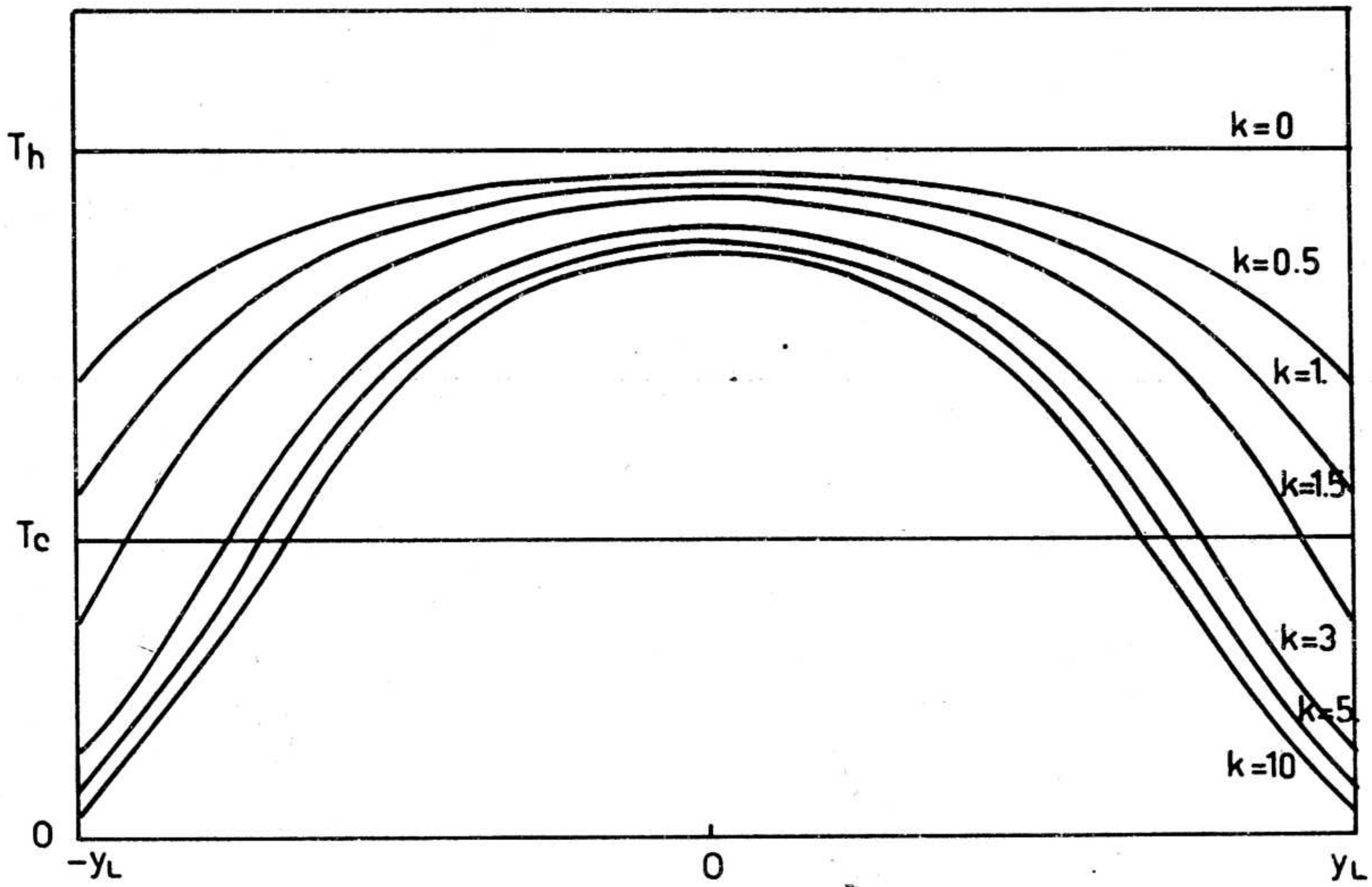


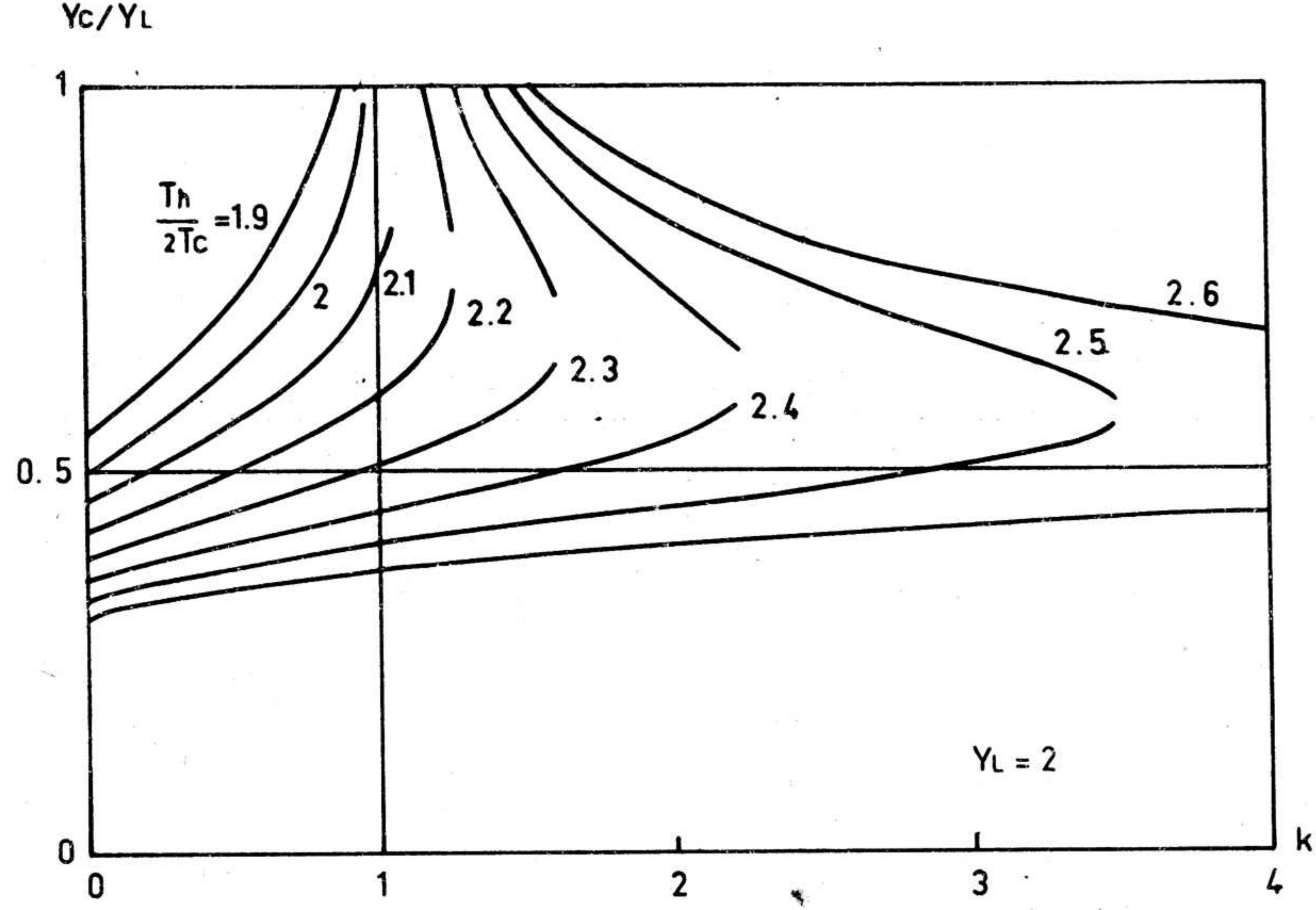
Φ
0
Tc
2Tc
T
Th > 2Tc
Th < 2Tc
Th < Tc



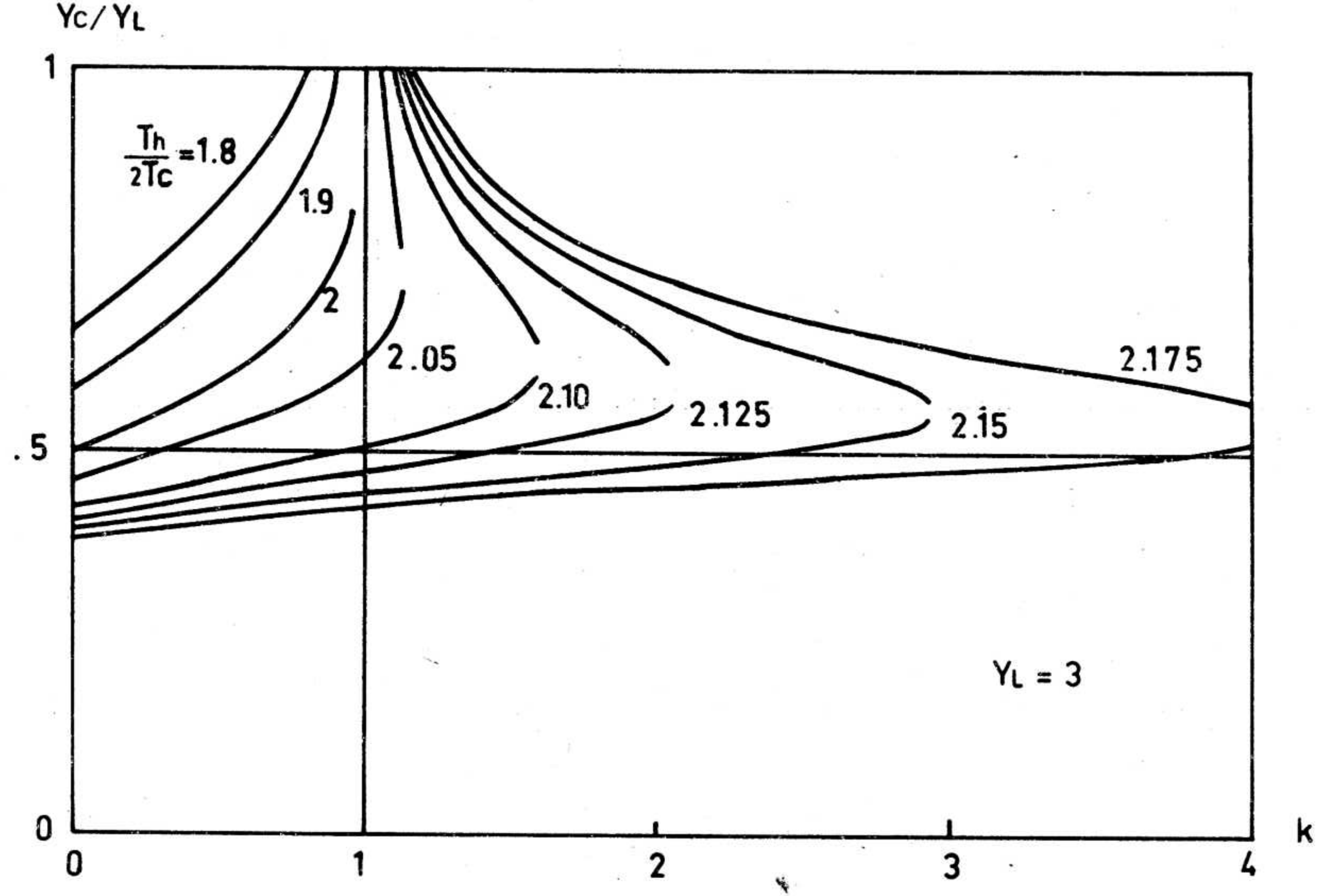
T
Tc
0
-yL
0
yL


T
T_c
0
$-y_L$
0
y_L

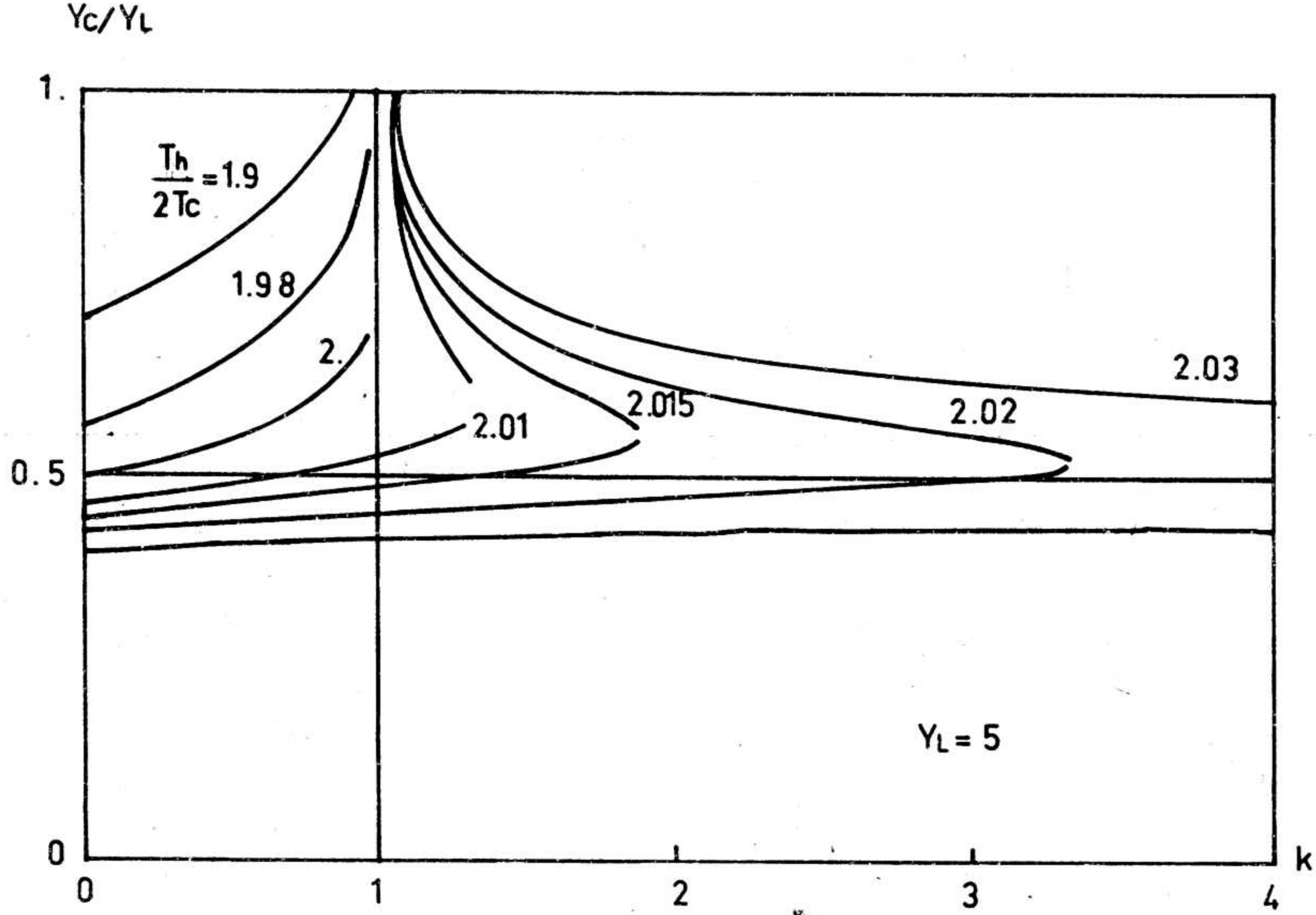

k=0
k=0.5
k=1
k=1.5
k=3
k=5
k=10
Th
Tc
0
-yL
0
yL



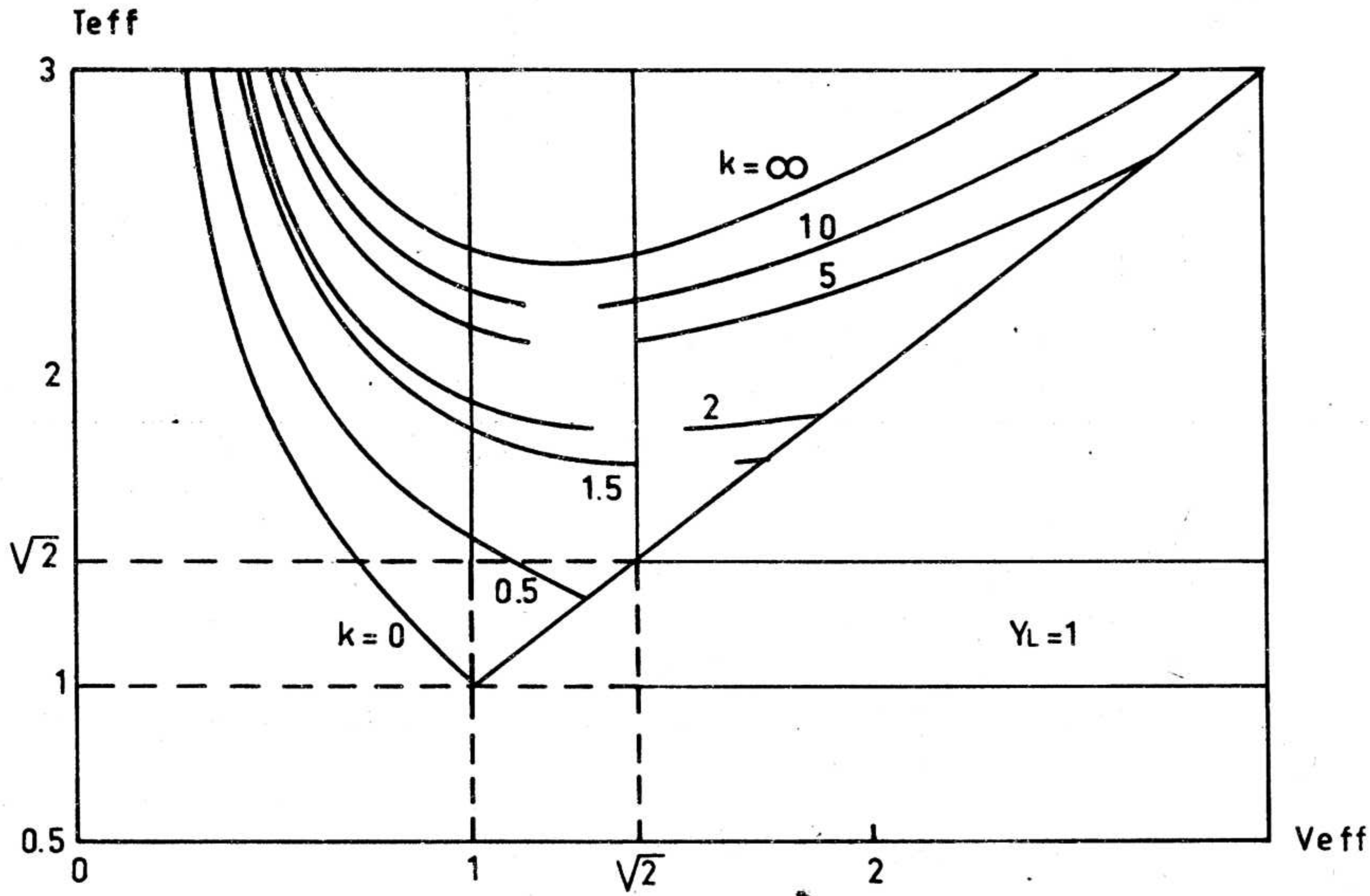
Yc/YL
1
0.5
0
0
1
2
3
4
k
Th/2Tc =1.9
2
2.1
2.2
2.3
2.4
2.5
2.6
YL = 2

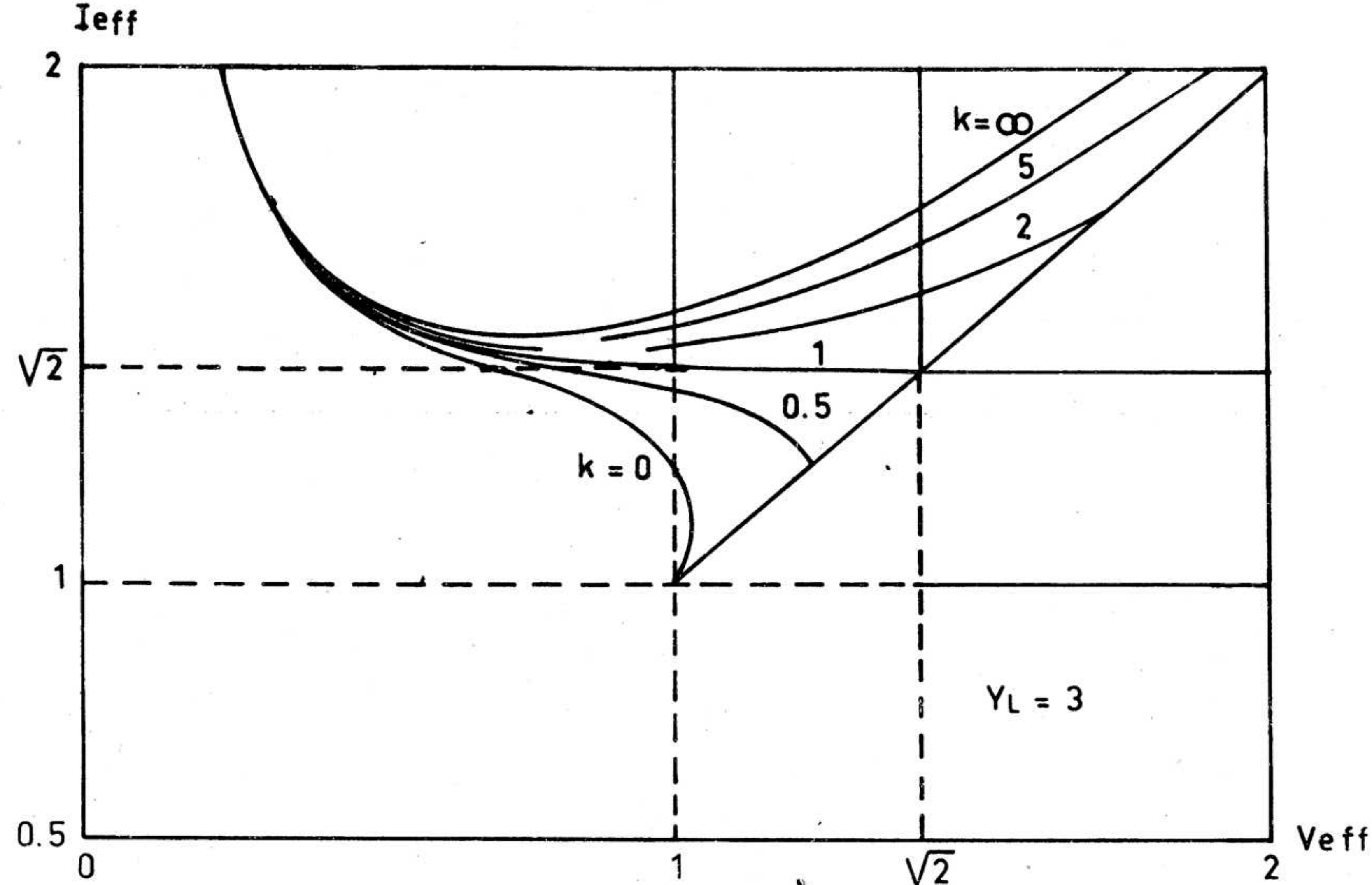


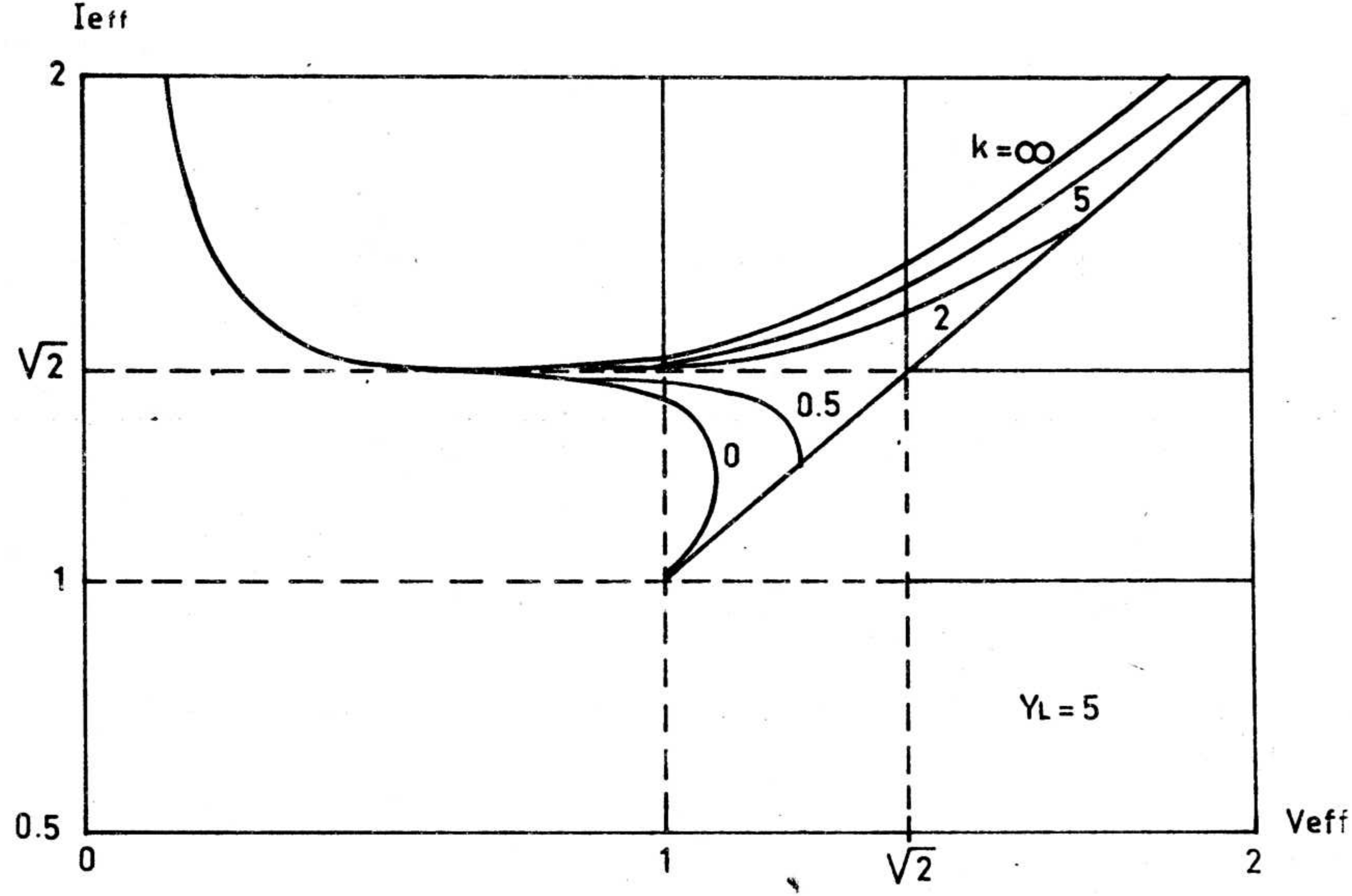
Yc/YL
1
.5
0
Th/2Tc =1.8
1.9
2
2.05
2.10
2.125
2.15
2.175
YL = 3
0
1
2
3
4
k

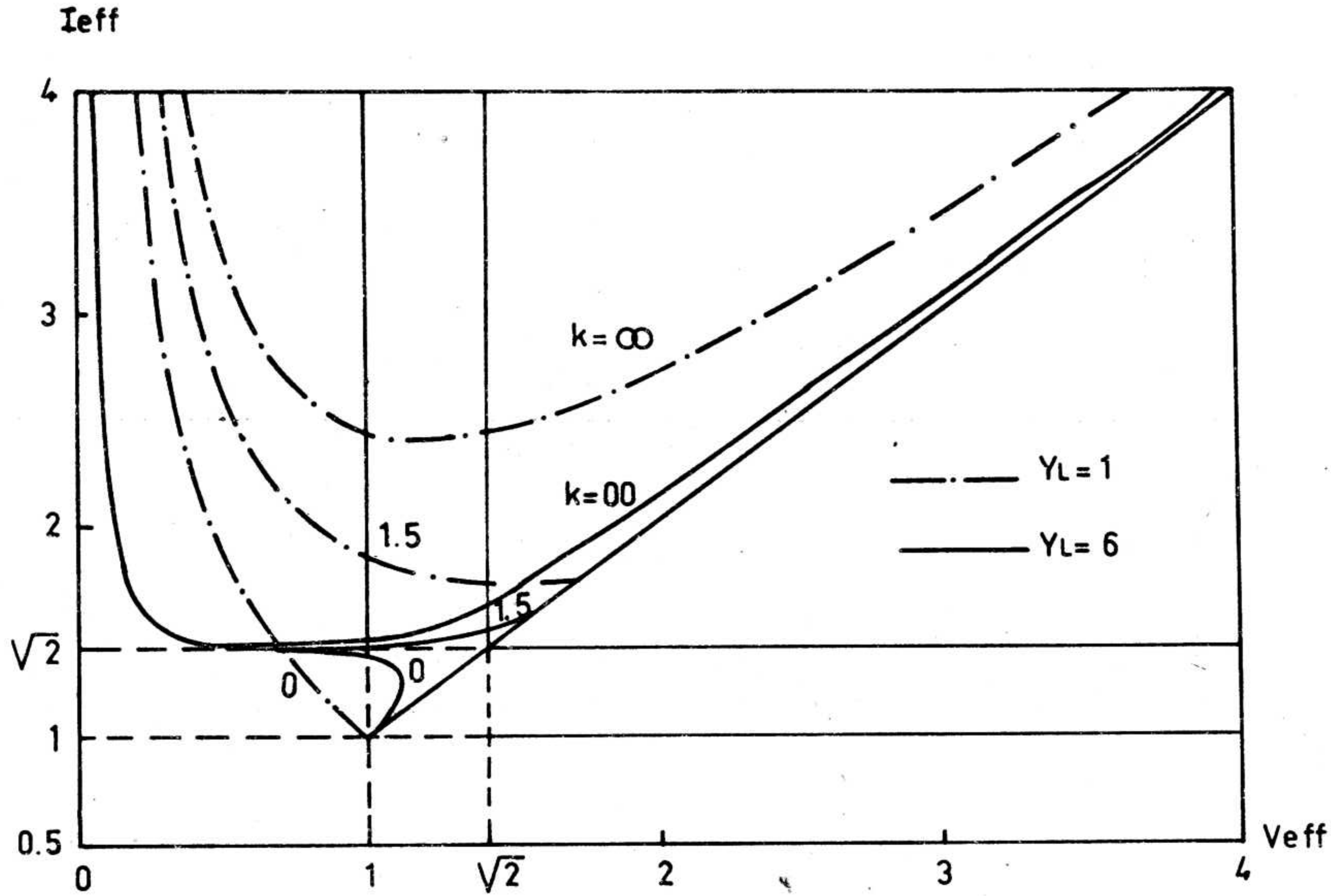


Yc/YL
1.
0.5
0
0
1
2
3
4
k
Th/2Tc = 1.9
1.98
2.
2.01
2.015
2.02
2.03
YL = 5

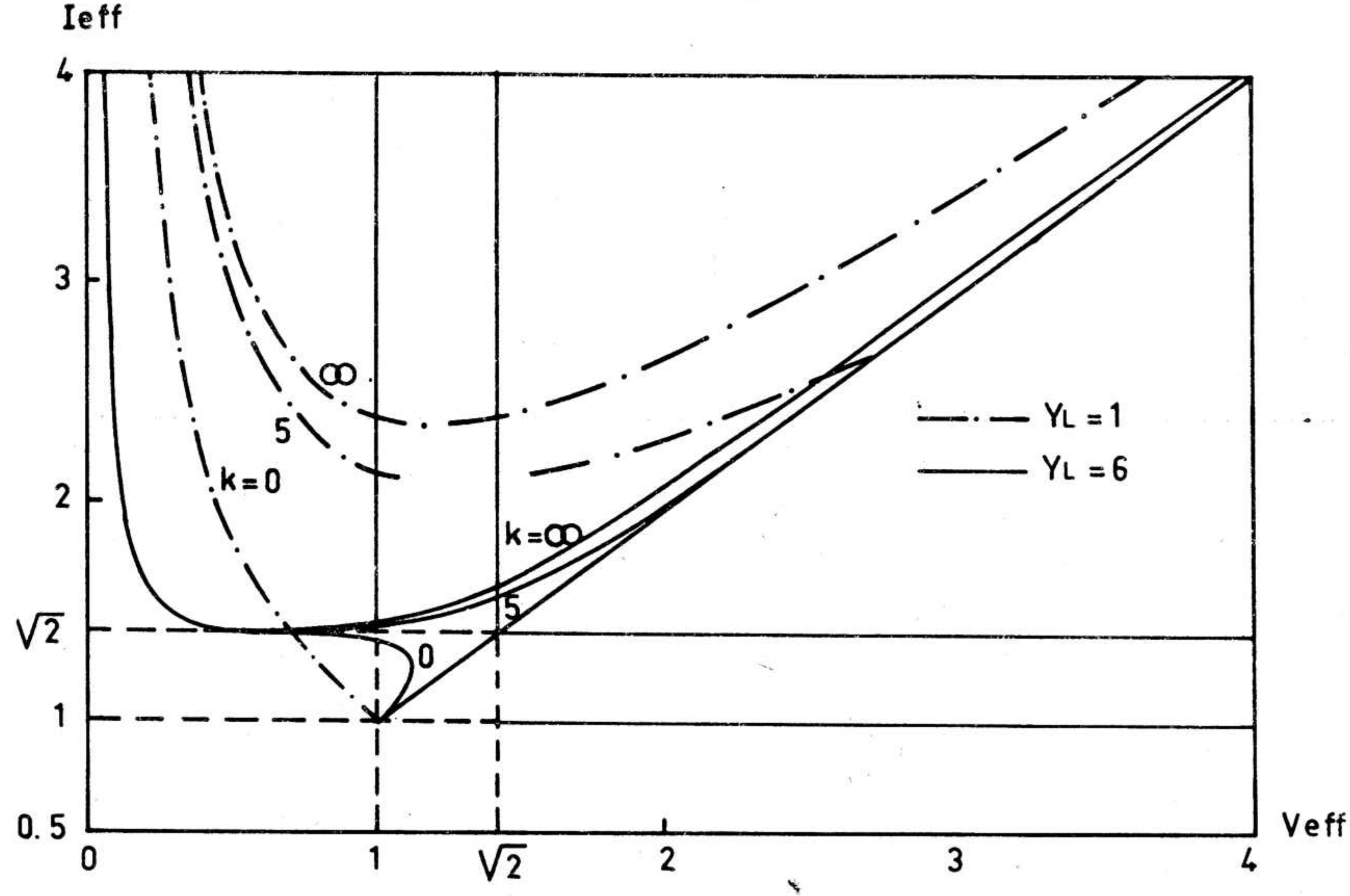

Teff
3
2
$\sqrt{2}$
1
0.5
0
1
$\sqrt{2}$
2
Veff
k = ∞
10
5
2
1.5
0.5
k = 0
$Y_L = 1$


Ieff
2
√2
1
0.5
0
1
√2
2
Veff
k=∞
5
2
1
0.5
k = 0
Y_L = 3


Ieff
2
$\sqrt{2}$
1
0.5
0
1
$\sqrt{2}$
2
Veff
k = ∞
5
2
0.5
0
$Y_L = 5$



Ieff
4
3
2
√2
1
0.5
0
1
√2
2
3
4
Veff
k = ∞
k = ∞
1.5
1.5
0
0
YL = 1
YL = 6


Ieff
Veff
4
3
2
√2
1
0.5
0
1
√2
2
3
4
k = 0
5
∞
k = ∞
5
0
Y_L = 1
Y_L = 6