IL NUOVO CIMENTO Vol. XXVI, N. 4 16 Novembre 1962

Spurious State in Connection with β-Vibration of Nuclei.

D. Bès

Facultad de Ciencias Exactas y Naturales, Universitad de Buenos Aires - Buenos Aires

Z. Szymański

Institute for Nuclear Research - Warsaw

(ricevuto il 2 Luglio 1962)

Summary. — An equation for the frequency of the quadrupole axially symmetric oscillations (γ-vibrations) of the atomic nucleus is derived. The derivation includes pairing interaction between nucleons. The equation contains one solution, which corresponds to the spurious zero-frequency state, which is a mixture of the ground-states of the neighbouring nuclei.

The β-vibrations of the deformed nuclei consist in the quadrupole axially-symmetric oscillations of the deformed nuclear potential. The rotational band associated with β-vibration is characterized by the $K=0$ value of a projection of the total angular momentum on the symmetry-axis of the nuclear intrinsic motion (1). In order to describe the mechanism of a β-vibration (as well as the other vibrational modes in nuclei) one can introduce a Hamiltonian as a sum of an independent-particle term plus pairing plus quadrupole forces (2). The appropriate method of treating such a Hamiltonian is the quasi-particle formalism developped in a close analogy with the calculations in the theory

(1) A. Bohr and B. Mottelson: *Mat. Fys. Medd. Dan. Vid. Selsk.*, **27**, no. 16 (1953).
(2) B. Mottelson: *The many-body problem*, in *Lectures given at the «Ecole d'Eté de Physique Théorique»* (Paris, 1959).

of superconductivity [3]. It turns out that the formalism accounts properly for the most important part of the pairing interaction [4]. The quadrupole forces contribute mostly to the selfconsistent field acting on the quasi-particles the other part of the force appearing as an important quadrupole interaction between quasi-particles. A suitable nonperturbative method of treating the quadrupole interaction, called sometimes the « boson approximation » leads to the equation [5].

$$\frac{1}{2\chi} = \sum_{\nu\nu'} \frac{(u_\nu v_{\nu'} + v_\nu u_{\nu'})^2 q_{\nu\nu'}^2}{(E_\nu + E_{\nu'})^2 - \Omega^2} \tag{1}$$

determining the excitation energy of a vibration, Ω. Here E_ν denotes quasi-particles energy, v_ν^2 is the probability that the single-particle state ν is occupied, $u_\nu = \sqrt{1 - v_\nu^2}$ and $q_{\nu\nu'}$ is the matrix element of the operator $4\sqrt{(\pi/5)}\, r^2 Y_{20}$ calculated in the single-particle representation.

However the application of eq. (1) to the β-vibration phenomenon seems somewhat dubious. It is known [2] that the quasi-particle formalism brings into the consideration one spurious state with $K=0$. This fact is connected with the nonconservation of the number of particles and the existence of the spurious state is associated with the presence of the admixtures of the ground-states of the neighbouring nuclei. In general case the spurious state is mixed over all the $K=0$ two-quasi-particle excited states. Now from the point of view of the boson approximation, the β-vibrational state, whose energy is given by (1) appears just as a certain mixture of the two-quasi-particle $K=0$ states. Therefore there seems very likely that the state determined as a solution of eq. (1) will contain a large admixture of a spurious state. If this is the case the solution of eq. (1) will not be able to describe properly a β-vibration.

Keeping this disadvantage of eq. (1) in mind we have tried to find a way of separating out the spurious state from the rest of the states and then find the energy of a β-vibration. The aim can be achieved if one includes into the consideration the part of the quasi-particle interaction which comes from the pairing force [6]. In this way we obtain a Hamiltonian which contains the quasi-particle interaction coming both from pairing and quadrupole forces. If we treat the problem again within the boson approximation we get an equation determining the energy Ω of a β-vibration which has to replace (1).

---

(3) I. BARDEEN, L. N. COOPER and J. R. SCHRIEFFER: *Phys. Rev.*, **108**, 1175 (1957).

(4) S. T. BELYAEV: *Mat. Fys. Medd. Dan. Vid. Selsk.*, **31**, no. 11 (1959).

(5) A. BOHR and B. MOTTELSON: unpublished; M. BARANGER: *Phys. Rev.*, **120**, 957 (1960); R. ARVIEU and M. VENERONI: *Compt. Rend.*, **250**, 2155 (1960); T. MARUMORI: *Progr. Theor. Phys.*, **24**, 331 (1960).

(6) J. HOGAASEN-FELDMAN: *Nucl. Phys.*, **28**, 258 (1961).

4620

This equation reads:

$$(2)\qquad \begin{vmatrix} 2\varkappa \sum\limits_{\nu\nu'} \dfrac{q^2_{\nu\nu'}(u_\nu v_{\nu'} + v_\nu u_{\nu'})^2(E_\nu + E_{\nu'})}{(E_\nu + E_{\nu'})^2 - \Omega^2} - 1 & G \sum\limits_{\nu\nu'} \dfrac{q_{\nu\nu'}\delta_{\nu\nu'}(u_\nu v_{\nu'} + v_\nu u_{\nu'})(u^2_\nu - v^2_\nu)(E_\nu + E_{\nu'})}{(E_\nu + E_{\nu'})^2 - \Omega^2} & \Omega G \sum\limits_{\nu\nu'} \dfrac{q_{\nu\nu'}(u_\nu v_{\nu'} + v_\nu u_{\nu'})\delta_{\nu\nu'}}{(E_\nu + E_{\nu'})^2 - \Omega^2} \\ 2\varkappa \sum\limits_{\nu\nu'} \dfrac{q_{\nu\nu'}\delta_{\nu\nu'}(u_\nu v_{\nu'} + v_\nu u_{\nu'})(u^2_\nu - v^2_\nu)(E_\nu + E_{\nu'})}{(E_\nu + E_{\nu'})^2 - \Omega^2} & G \sum\limits_{\nu\nu'} \dfrac{\delta_{\nu\nu'}(u^2_\nu - v^2_\nu)^2(E_\nu + E_{\nu'})}{(E_\nu + E_{\nu'})^2 - \Omega^2} - 1 & \Omega G \sum\limits_{\nu\nu'} \dfrac{\delta_{\nu\nu'}(u^2_\nu - v^2_\nu)}{(E_\nu + E_{\nu'})^2 - \Omega^2} \\ 2\varkappa\Omega \sum\limits_{\nu\nu'} \dfrac{q_{\nu\nu'}(u_\nu v_{\nu'} + v_\nu u_{\nu'})\delta_{\nu\nu'}}{(E_\nu + E_{\nu'})^2 - \Omega^2} & G\Omega \sum\limits_{\nu\nu'} \dfrac{(u^2_\nu - v^2_\nu)\delta_{\nu\nu'}}{(E_\nu + E_{\nu'})^2 - \Omega^2} & G \sum\limits_{\nu\nu'} \dfrac{(E_\nu + E_{\nu'})\delta_{\nu\nu'}}{(E_\nu + E_{\nu'})^2 - \Omega^2} - 1 \end{vmatrix} = 0\,,$$

where G denotes the strength of a pairing force. One can easily see that (2) contains a solution $\Omega = 0$ which follows from the fact that the quasi-particle energies E_ν fulfil the equation.

$$(3)\qquad 2/G = \sum_\nu 1/E_\nu$$

following from the variational principle of Bardeen, Cooper, Schrieffer. It is found that the degree of freedom associated with the solution $\Omega = 0$ is described by a co-ordinate proportional to the fluctuation of the number of particles, so that the solution really corresponds to the spurious state.

Expanding eq. (2) in powers of Ω^2 in the case of low values of Ω^2, we get adiabatic approximation according to which the vibrational problem is reduced to the slow oscillations of the nuclear average potential. Then we get for the energy the usual vibrational formula:

$$(4)\qquad \Omega = \sqrt{C/B}\,,$$

where C and B are the rigidity and inertial parameters respectively. In this way we are led to the usual expressions for C and B (comp ([4-7])) plus some corrections, which have been found ([7]) in the adiabatic case as coming from the separation of the spurious admixture in the vibrational state.

([7]) D. Bès: *Mat. Fys. Medd. Dan. Vid. Selsk.*, **33**, no. 2 (1961).

RIASSUNTO (*)

Si deduce un'equazione per la frequenza delle oscillazioni di quadrupolo (vibrazioni γ) del nucleo atomico. La derivazione comprende l'interazione di accoppiamento fra i nucleoni. L'equazione ha una soluzione, che corrisponde allo stato spurio di frequenza nulla, che è una mescolanza degli stati fondamentali dei nuclei vicini.

(*) *Traduzione a cura della Redazione.*