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HEAT FLOW SHAPE FACTORS FOR CIRCULAR RODS WITH REGULAR POLYGONAL CONCENTRIC INNER BORE

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The title problem is solved using an approximate conformal mapping approach. The results are in good agreement with those obtained using a finite element code.

1. Introduction

The study of diffusion and conduction through the walls of long, prismatic bars with concentric cylindrical holes is of interest in nuclear, mechanical and civil engineering, and several papers have been written on the subject [1–6]. The problem of heat flow through the walls of long, hollow cylinders with concentric inner, regular polygonal perforations is also of practical importance, but a very limited amount of information on it is available in the open technical literature.

It is shown that a very simple solution of the problem is obtained when one uses a conformal mapping technique. The results agree rather well with values calculated using a finite element approach. Isothermal walls are presumed.

2. The approximate conformal mapping approach

The mapping function which transforms the infinite plane with a hole of regular polygonal shape (z-plane) onto the infinite plane with a circular perforation of unit radius (ξ-plane) is given by the infinite series:

$$z = a_p A_s' \left(\xi + \sum_{n=1}^{\infty} a_{1-ns}\xi^{1-ns}\right), \tag{1}$$

where a_p is the apothem of the regular polygon, A_s' the coefficient and s the order of the polygon.

Expressing ξ in the form

$$\xi = r\,e^{i\theta}, \tag{2}$$

it is clear from (1) that when one takes $r >> 1$, the first term of (1) will predominate. Accordingly, an annular shape of outer radius λ and inner unit radius will transform onto an approximate circular domain with a polygonal hole in the z-plane (fig. 1). From the examination of fig. 1 and using eq. (1), one obtains

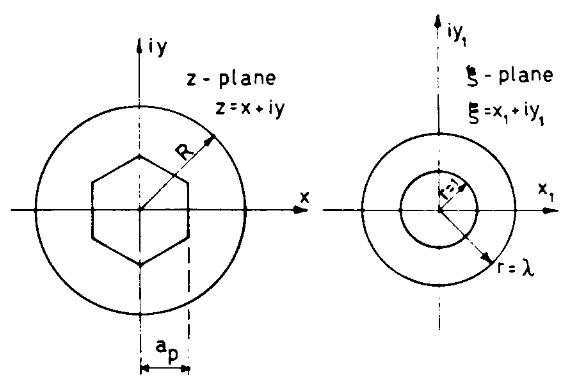

Fig. 1. Doubly connected region under study and its image in the ξ-plane.

immediately

$$R \simeq a_p A_s' \lambda. \quad (3)$$

It is convenient to express now the total heat flux per unit length of the hollow circular cylinder Q, in the form

$$Q = Sk(t_i - t_o), \quad (4)$$

where S is the shape factor, t_i and t_o are the inner and outer wall temperatures of the given configuration, respectively ($t_i > t_o$), and k the thermal conductivity.

In the ξ-plane one has

$$Q = \frac{2\pi}{\ln \lambda} k(t_i - t_o) \quad (5)$$

and obviously:

$$S = 2\pi / \ln \lambda. \quad (6)$$

From (3) and (6) one obtains

$$S \simeq \frac{2\pi}{\ln(R/a_p) - \ln A_s'} \quad (7)$$

The values of A_s' have been calculated using the approach explained in [7] (see table 1).

3. The finite element solution

The results were obtained using a finite element code *. The domain is subdivided into triangular elements and a linear variation of the temperature field is assumed inside the element.

It was decided, in view of the symmetry, to consider the subdomains defined by

$$0 \leqslant \phi \leqslant \pi/s, \quad (8)$$

where s is the order of the polygon with the conditions (see fig. 2):

$$T = t_i \quad \text{on } c \quad \text{and} \quad T = t_o \quad \text{on } d, \quad (9a, b)$$

and

$$\partial T/\partial n = 0 \quad \text{on } a \text{ and } b, \quad (9c)$$

where n denotes the outer normal to the subdomain and T is the temperature.

* Developed at Centro Atómic Bariloche, Comisión Nacional de Energía Atómica.

Table 1
Values of A_s'

s	A_s'
4	1.1812
5	1.0993
6	1.06319
7	1.0438
8	1.0323

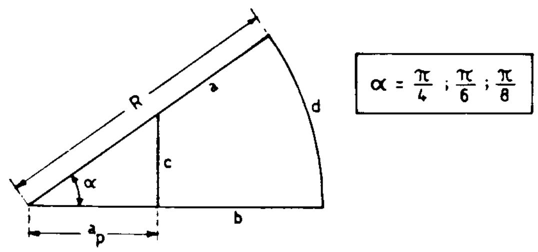

Fig. 2. Portion of the domain under study.

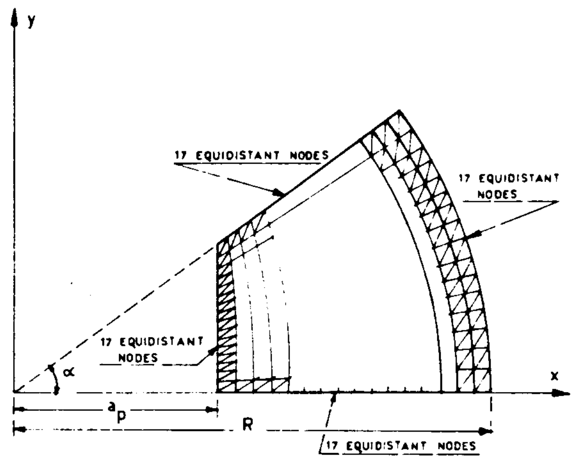

Fig. 3. Element distribution.

Fig. 3 shows the element distribution using in the present analysis (512 triangular elements and 289 nodal points). In accordance with (9a, b) the values of the temperature are known at 34 nodal points (fig. 3).

4. Numerical results

Fig. 4 depicts the variation of S for square, pentagonal, hexagonal and octagonal holes as a function of

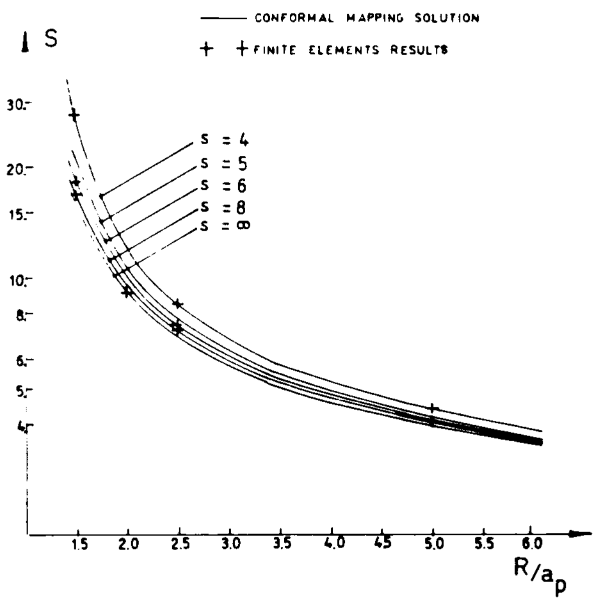

Fig. 4. Shape factors for circular cylinders with concentric perforations of regular polygonal shape.

R/a_p. The case of an annular domain is also included for the sake of comparison.

Fig. 4 also displays values obtained using the finite element approach for $\alpha = \pi/4, \pi/6$ and $\pi/8$ for R/a_p = 1.50, 2.50 and 5.00.

In the case of an annular shape the shape factor has been computed by means of the finite element approach for R/a_p = 2.00.

From the inspection of fig. 4 one concludes that the agreement between analytical and numerical values of the shape factor is excellent from an engineering viewpoint.

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