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"AUTOMATIC SOLUTION OF HEAT RADIATION PROBLEMS IN TWO-DIMENSIONAL DIFFUSE GRAY ENCLOSURES"

Sergio Pissanetzky

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Abstract

A computer program is presented for the numerical treatment of two-dimensional heat transfer problems where heat radiation is present. The heat radiation problem is solved in diffuse-gray enclosures of any shape, simply or multiply connected, bounded by hot surfaces. Each enclosure is approximated by a set of short, straight, isothermal line segments. A matrix of view-factors is automatically determined and the net radiant heat flux entering each segment is calculated. There heat fluxes are then used as boundary fluxes for the bodies involved in the system. The bodies are in turn approximated by finite elements and the entire coupled problem is solved in time steps.

RESUMEN

Se presenta un programa de computadora para el tratamiento numérico de problemas de transferencia de calor en dos dimensiones, en los que exista radiación de calor. El problema de radiación del calor se resuelve en cavidades de cualquier forma, simplemente o múltiplemente conexas, con paredes calientes que sean difusoras grises. Cada cavidad es aproximada por un conjunto de segmentos cortos, rectos e isotérmicos. La matriz de coeficientes de vista entre los segmentos es determinado automáticamente. El flujo neto de calor radiante que entra en cada segmento es calculado. Estos flujos de calor son usados como flujos de borde para los cuerpos que componen el sistema. Los cuerpos son aproximados a su vez por elementos finitos y el problema acoplado completo se resuelve en pasos de tiempo.

I Introduction

The Finite Element method is being used widely in Engineering and Physics for the numerical solution of heat transfer problems. In systems which contain surfaces at high temperatures, the effect of heat radiation may be considerable. This report presents a program for the quantitative analysis of heat radiation within the framework of a finite element heat transfer model.

A general body may have both internal and external boundaries. Heat radiated from points of an internal closed boundary impinges upon other points of the same boundary and is retained inside the body. Heat radiated from the external boundary may reach other portions of the same boundary (in a concave region), or otherwise be effectively lost. Also, a radiation field normally exists at the surroundings of the body as a consequence of the presence of other material bodies at temperatures higher than absolute zero. Some radiation reaches the outer surface of the body of interest and is partially absorbed, resulting in an effective gain of energy. The radiation emitted from the outer surface is also partially reflected back by the other bodies and reabsorbed at the outer surface.

The gains and losses of energy balance out only when the outer surface and all the surrounding bodies are at the same temperature. Except for this simple case, the radiation phenomena of a body may never be examined independently of the surrounding bodies. In other words, we should enclose the given body by a surface representing the other bodies, defining in this way an enclosure in which the heat transfer problem should be solved taking into account the emission, absorption and reflection of radiation. Therefore, we confine our attention only to enclosures . The theory will be briefly outlined here.

II The radiation model in two dimensions

Consider a body in which a two-dimensional heat transfer problem is being solved by the Finite Element method. The body has a "hole", which is a connected region surrounded by one or several curves of any shape, and is empty or filled with some transparent and non-absorbant material such as a gas. The body has been divided into finite elements, and the boundary of the enclosure is approximated by the sides of the neighbouring finite elements. The radiation model requires the boundary to be divided into short straight segments, which we call "sides". Each side is isothermal and its temperature is taken to be equal to the mean temperature of the corresponding portion of boundary. The subdivision of the boundary into sides is such that the temperatures of the sides can be defined in terms of the nodal temperatures of the body, which are the basic unknowns.

Each side radiates heat into the enclosure. The radiation reaches other sides and part is absorbed and the rest is reflected back into the enclosure. This process is repeated until each ray of radiant energy is extinguished. The assumption is now made that the time required for the complete extinction of a ray is very short compared with the characteristic time in which the temperatures of the sides change appreciably. A quasi-stationary state is thus reached in the sense that the instantaneous rates of heat emission, absorption and reflection at each side become constant in time and depend only on the temperatures and material properties of the sides at that particular instant.

The radiation problem to be solved can now be stated as follows: given the temperatures of the sides, determine the instantaneous heat flux entering each side as a result of the difference between the energy absorbed and that radiated by the side. The results are then incorporated in the finite element problem as nonlinear surface flows or internal heat sources associated with the elements bordering the enclosure, and the problem is solved by any of the usual procedures. Now we focus our attention on the heat radiation problem itself.

III Automatic calculation of viewfactors

The basic radiation problem will now be stated as follows. A given side AB of finite size radiates energy uniformly from all its points towards all directions in space. It is wanted to calculate which fraction of the total energy radiated by AB reaches another given side DE. Two given sides may have different relative positions. Figure 1 illustrates two such positions in which no energy is transferred from AB to DE. Our convention is that the face of a side AB is at the left when going from A to B. In figure 2 seven different situations are depicted in which AB and DE face each other to-

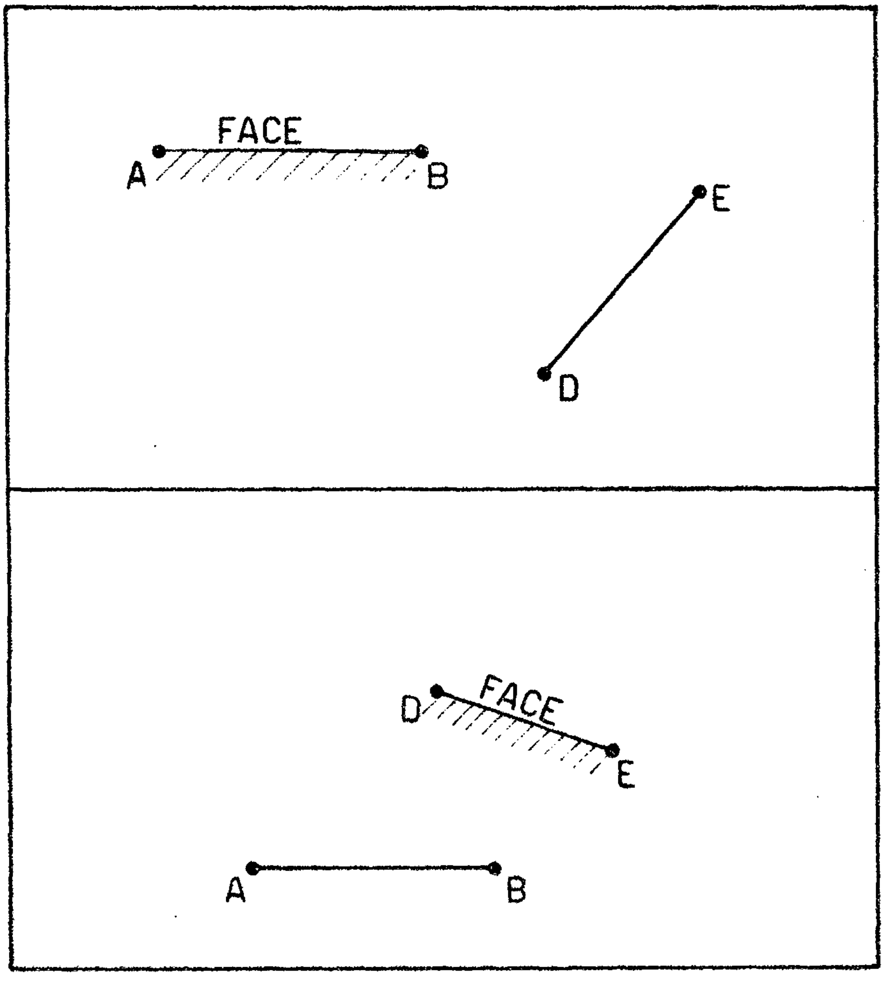

Figure 1: Relative positions of two radiation sides AB and DE, in which no radiation is transferred between the sides.

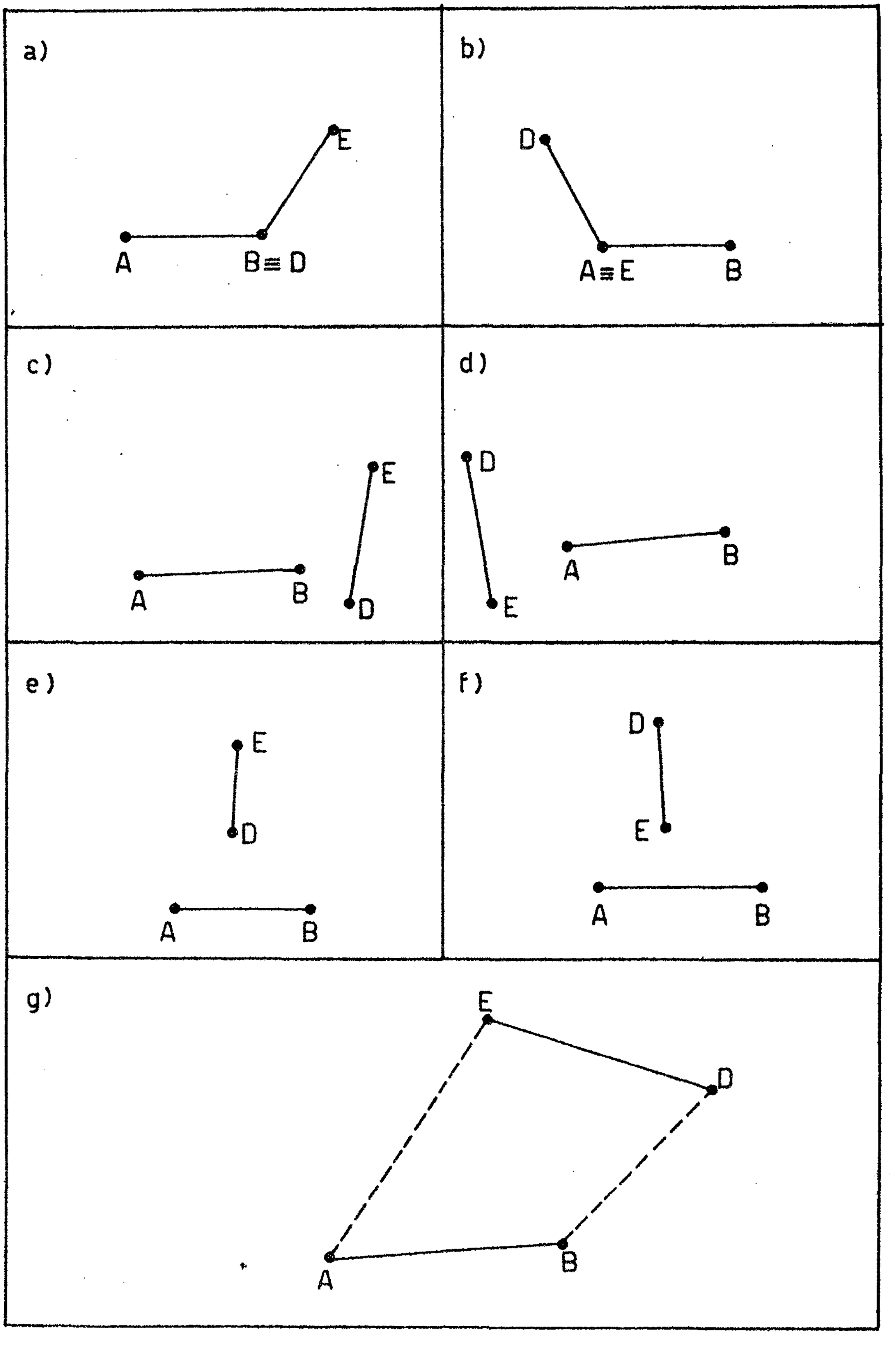

Figure 2: Different relative positions of two sides AB and DE in which heat exchange by radiation can take place between the two sides.

tally or partially. When they face totally as in Fig. 2-g the fraction of the total energy emitted in unit time by AB, which reaches DE, per unit length of the dimension perpendicular to the plane, is proportional to [1] :

$$f=AD+BE-AE-BD \qquad (1)$$

where AD means "distance between A and D", etc. In any of the seven situations of fig. 2 there may be other bodies interposed between AB and DE, producing complicated shade effects. Figure 3 illustrates this case. Of course a body can be completely interposed between AB and DE, thus allowing no energy flow from one of the sides to the other. The simple formula (1) cannot be used for most of the cases and a more general relation is needed.

The required general relation will now be stated for a fundamental situation, and then it will be shown how the different situations of figures 2 and 3 can be reduced to the fundamental situation. The fundamental situation is shown in Fig. 4. Two convex bodies are interposed between AB and DE leaving an opening which will be called a radiation channel. Some of the radiation emitted by AB goes through the channel and reaches DE. Two limiting visual lines are also shown in Fig. 4: they are the two common tangents to both boundaries, and determine the points of contact P,Q,R and S. Then, the fraction of the energy emited by AB in unit time, which reaches DE after passing through the channel, by unit of length of the dimension perpendicular to the plane, is proportional to [1] :

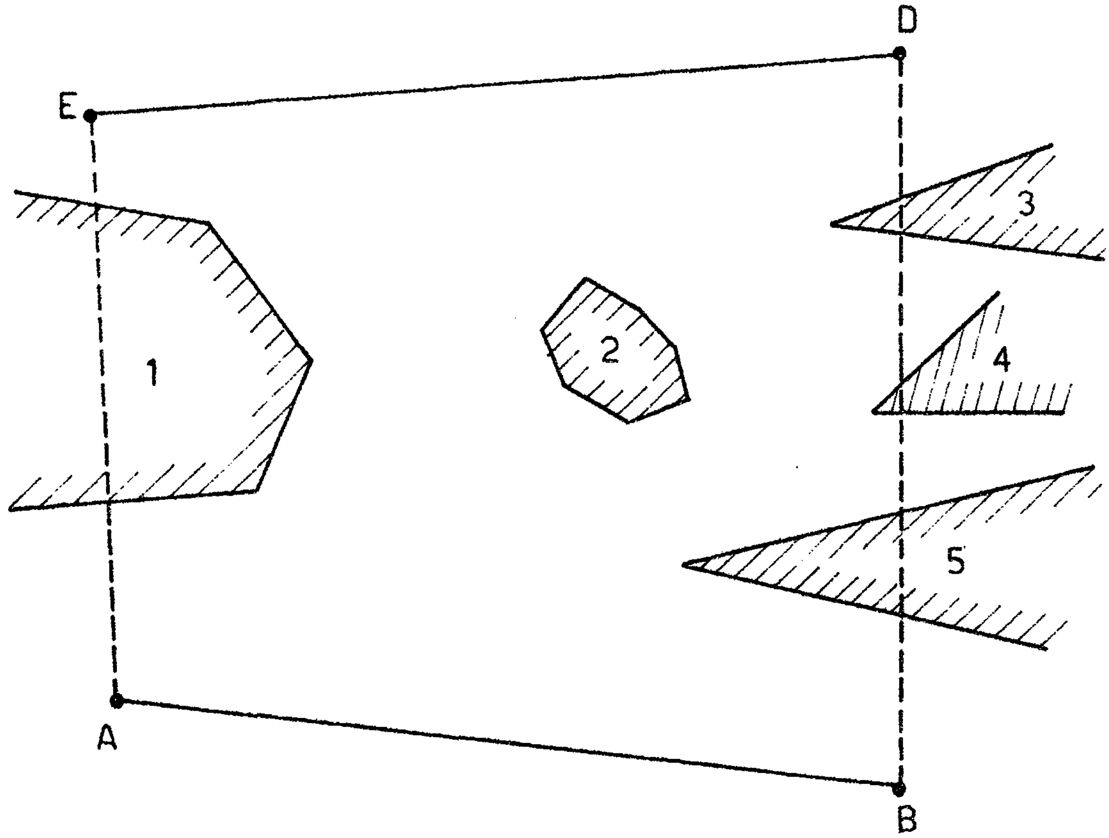

Figure 3: The heat radiation stream between sides AB and DE is split into two channels by partially interposing bodies.

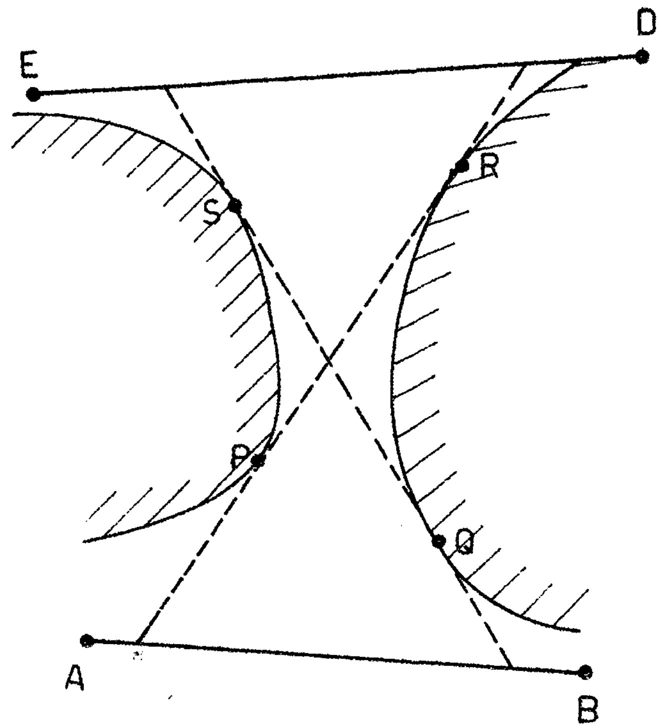

Figure 4: A radiation channel between sides AB and DE determined by two partially interposed bodies.

$$f=PR+QS-PS-QR \qquad (2)$$

where PR and QS are the lengths of the two straight lines PR and QS while PS and QR are the lengths of the curves PS and QR, respectively and f will be called the transmittance of the channel. This relation is remarkable for its simplicity and generality. It is the basis for all the developments which follow.

The proposed two-dimensional heat radiation model involves approximating the boundaries of the bodies by short line segments. This will lead us to deal with convex polygonal boundaries, which are a particular case of convex curves. The relation (2) is thus valid for convex polygonals. In fact, since proving relation (2) is somewhat easier for polygonals than for curves, and we are only interested in polygonals, we have proved it only for polygonals (see Apendix I). The proof for convex curves then follows simply by considering the given curves as polygonals of very short sides.

Now consider how the fundamental situation, consisting of a radiation channel between the sides AB and DE bounded by two convex polygonals, may be obtained from any of the different situations encountered in practice. The two situations of Fig. 1 require no consideration since there is no energy flow from AB to DE. For the situation in Fig. 2-g, the left polygonal is AE, the right one is BD, and the two limiting visuals are AD and BE. The general relation (2) immediately reduces to the particular relation (1), as it should. For the situation of Fig.2-a, the left polygonal is AE and the right one reduces to a single point, B. The two visuals are AB and BE, respectively. The general relation (2) is directly applicable and the transmittance results:

$$f=AB+BE-AE$$

The case of Fig. 2-c has AE and BD as the polygonals, and AB and BE as the visuals. Note that the points of contact of both visuals with the right polygonal coincide at B. Relation (2) immediately gives the correct transmittance:

$$f=AB+BE-AE$$

By a similar reasoning for the case 2-e it is obtained:

$$f=AD+DE-AE$$

The cases of Fig.2-b, 2-d and 2-e are treated in a similar way. A more general situation is illustrated in Fig. 3: four bodies are interposed between AB and DE, producing complicated shade effects. Two radiation channels exist in this case, and their contruction is shown in Fig. 5 and 6. The total transmittance is the sum of the transmittances of the channels, which are calculated by means of the general relation (2). This case also illustrates our use of convex polygonals throughout. Consider a body such as body 4 in Fig. 3. Clearly, such a body does not play any role with regard to the energy transfer from AB to DE, and can be eliminated from any further consideration. The same will happen to any body which is entirely <u>inside</u> the convex polygonal which contains the set of bodies of interest. Therefore, given the set of bodies, it is sufficient to construct the smallest convex polygonal with the property of containing them all.

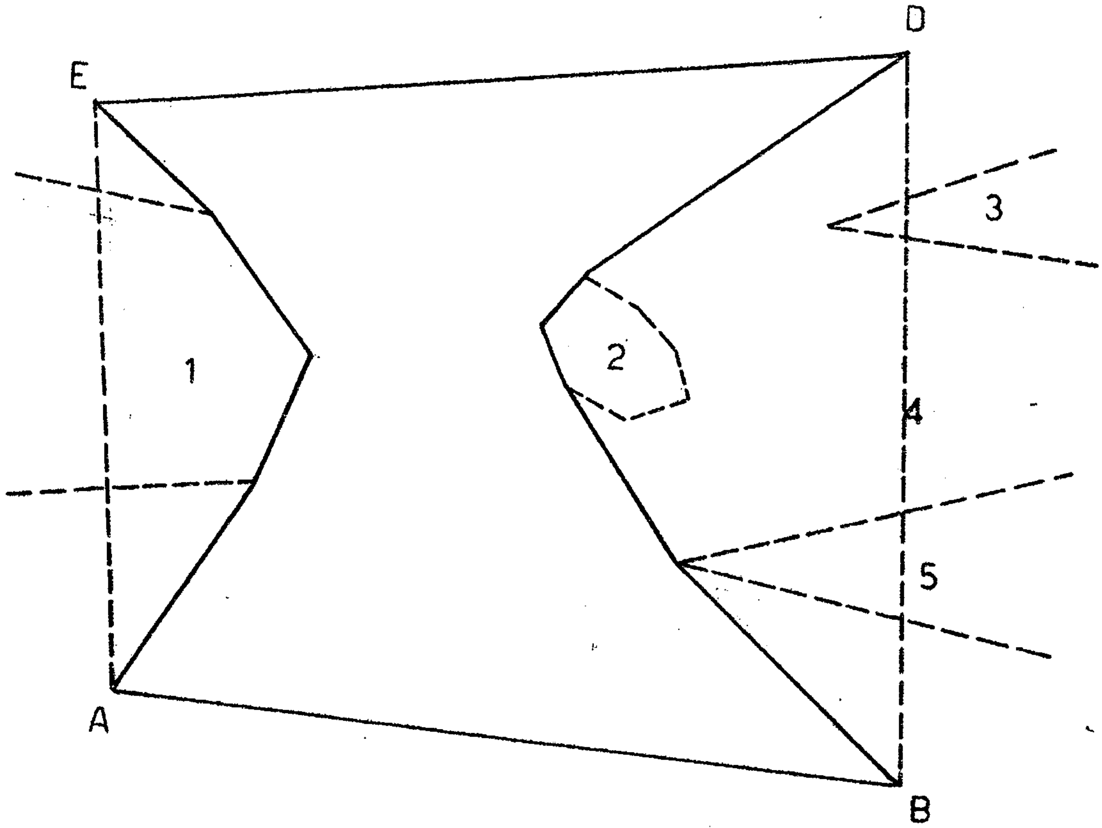

Figure 5: Left radiation channel for the system of Fig. 3.

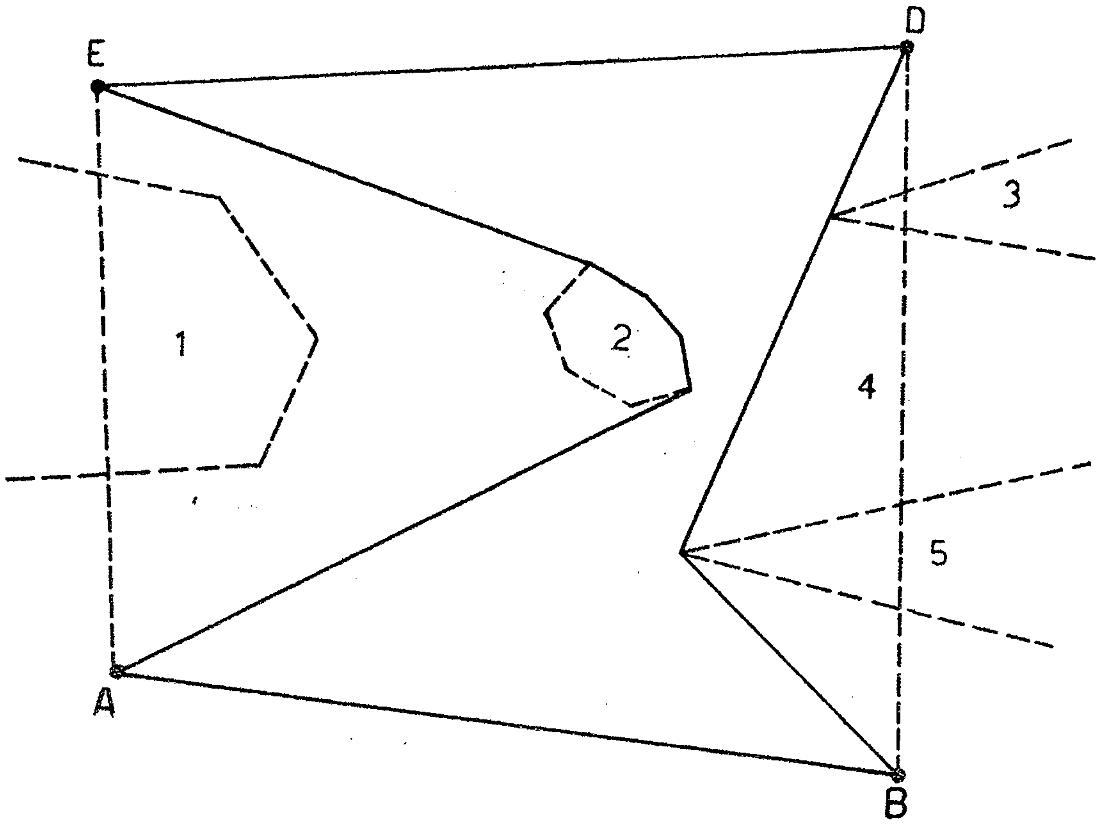

Figure 6: Right radiation channel for the system of Fig.3.

The case of Fig. 3 may be combined with any of the situations in Fig. 2, giving rise to many different situations which are all treated by the same general procedure. One or more radiation channels are generated for each pair of sides AB and DE. Some of these channels may be occasionally closed, when the left and the right polygonals intersect each other. For the open channels, the relation (2) is always used. When several bodies are involved in the calculation for some particular pair of sides, the number of radiation channels to be constructed may become very large. However, their description and generation by the computer is very simple. A channel is a path between some point of AB and some point of DE. Such a path has the property that some bodies are at its left while others are at its right. Therefore, it suffices to sort the bodies into two sets, a left one and a right one, and to construct the corresponding pair of convex polygonals. Generating all possible combinations of left and right sets is equivalent to the generation of all possible radiation channels. A more sophisticated logic could presumably detect which combinations will give rise to closed channels, but such a logic was not found to be necesarry in practice. Once the transmittance of all the open channels have been calculated, they are added up in order to obtain the total transmittance from AB to DE. For each pair of sides, the program performs all the necessary steps automatically.

IV Solution of the radiation problem

The rest of the formulation follows closely reference [2], except for some significant improvements. The transmittance calculated as discussed above for the pair of sides i (line segment AB) and j (DE), becomes the element i,j of the <u>matrix of viewfactors</u> G, where $G_{ii}=0$ since a side is flat by definition. This matrix is symmetric because the transmittance of any radiation channel is exactly the same in both directions, as the construction of Fig. 4 shows. The sum of all the elements in row i of G, which is the sum of the transmittances of all the channels starting at AB, is proportional to the total energy emitted by AB in all directions, and is therefore equal to the transmittance of a channel starting at AB and ending at an imaginary side BA, which is $2l_i$, where l_i is the length of the side AB. Therefore:

$$Gu=2Lu \tag{3}$$

where u is a column vector of all ones and L is a diagonal matrix of elements l_i. This relation is used by the program in order to verify that G has been correctly calculated.

Let n be the total number of sides of the enclosure. Let the sides be numbered sequentially in the counter-clockwise sense. We define the matrix E of elements E_{ij}, where E_{ij} is the heat flux emitted by side i toward side j. We have:

$$E=KG \tag{4}$$

where K is a diagonal matrix of constant elements K_i to be determined below.

The energy emitted in unit time by the side i is $\sigma a_i l_i T_i^4$ [3] according to Stefan law, where σ is Stefan's constant, a_i is the emissivity of the surface and T_i is the absolute temperature of side i. In order to preserve the fundamental law of energy conservation we require that the total energy emitted in unit time by side i be equal to the sum of the energy fluxes E_{ij}. We thus write:

$$Eu=\sigma ALt \tag{5}$$

where A is a diagonal matrix whose elements are the emissivities a_i and t is a column vector of elements T_i^4. Using equations (3) and (4) we obtain

$$2KLu=\sigma ALt$$

and, since all the matrices are diagonal, we can write:

$$2Ku=\sigma At \tag{6}$$

Expression (6) is used to calculate the constants K_i, which are then used in equation (4) in order to obtain the energy fluxes E_{ij}.

V <u>Reflection and absorption of radiation</u>

A given side i receives radiant energy from other sides, absorbs part of it and reflects the rest. Walls of transparent materials are excluded from the present treatment, therefore no part of the radiation can be transmitted

The spectral composition of the heat radiation depends on the temperature of the radiator. Each pencil of rays coming from some side j and falling on the side i is composed of the energy emitted by side j and of that reflected by side j. This last energy is in turn composed of the reflections of all the rays which fall on the side j. The spectral composition of a pencil of rays is thus complicated, and the absorption coefficient of some materials may well be a function of the wavelength of the radiation. Also, pencils of radiation are highly directional since each pencil falls on side i at a definite angle with the normal. This situation can be handled with the help of the following simplifying hypothesis:

i) The absorption coefficient of side i is assumed to depend only on the temperature of the side i and is taken equal to the emissivity a_i. This assumption is certainly valid when the temperature of the sides are not excessively different, since then all the radiation will have a similar spectral composition. From the practical point of view it should also be remembered that data on wavelength-dependent absorption coefficients are scarce.

ii) The material is assumed to reflect the radiation uniformly in all directions, independently of the angle of incidence. Thus mirrors and highly polished materials are excluded from our treatment, and the angular distribution of the reflected radiation is taken to be exactly similar to that of the emitted radiation.

We define the matrix R of elements R_{ij}, where R_{ij} is the total energy flow reflected by the side i towards the side j. In accordance with our assumptions we can write an expression for R similar to eq. (4):

$$R=K'G \tag{7}$$

where K' is a diagonal matrix of elements K'_i. As before we will determine K' by requiring that the energy be conserved. The total radiant energy which falls on a side i in unit time is given by the i-th component of the vector $(E^T+R^T)u$, where T means transpose. The fraction a_i of this energy is absorbed, and the fraction $(1-a_i)$ is reflected. By energy conservation, this last fraction must be equal to the i-th component of the vector Ru. We thus have a relation similar to equation (5):

$$Ru=(I-A)(E^T+R^T)u \tag{8}$$

Using equations (7), (4) and (3), equation (8) gives:

$$2LK'u=(I-A)G(K+K')u \tag{9}$$

This expression is a system of n linear equations with the n unknowns K_i'. After some transformations, (9) is written as follows:

$$K'u=[2L+(A-I)G]^{-1}(I-A)GKu \tag{10}$$

VI <u>Heat flux over the surface</u>

We define the column vector of components q_i, where q_i is the heat flux over the side i, taken as positive when the heat enters the side. q_i is the difference between the rates of absorption and emission of heat by the side i, or:

$$q=A(E^T+R^T)u-Eu \tag{11}$$

Equation (11) is transformed by the successive use of equations (4), (7) and (3) with the following result:

$$q=AG(K+K')u-2LKu \tag{12}$$

At this point, it is important to note that G may not have an inverse. The diagonal matrix (I-A) has an inverse only if none of the sides is a black-body ($\varepsilon_i \neq 1$ for all i). Thus, if the final expression should be general, we have to avoid the use of the inverses of G and (I-A). Using eq. (10) in eq. (12) we obtain:

$$q=AGKu-2LKu+AG[2L+(A-I)G]^{-1}(I-A)GKu \tag{13}$$

and using eq. (6):

$$q=Pt \tag{14}$$

where the symmetric matrix P is defined by:

$$P=\{AG-2L+AG[\,2L+(A-I)G\,]^{-1}\,(I-A)G\}\frac{1}{2}\sigma A \tag{15}$$

The following identity is now used to simplify the expression for P:

$$[\,2L+(A-I)G\,]^{-1}\;[\,2L+(A-I)G\,]=I \tag{16}$$

or:

$$[\,2L+(A-I)G\,]^{-1}\,(I-A)G=[\,2L+(A-I)G\,]^{-1}\,2L-I \tag{17}$$

Using eq. (17) in (15) we obtain:

$$P=\{AG[\,2L+(A-I)G\,]^{-1}-I\}\sigma AL \tag{18}$$

Eq. (14) is the general solution to the heat transfer problem in the enclosure. The matrix P is calculated by the general expression (18). For a black enclosure A=I and eq. (18) reduces to

$$P=\frac{1}{2}\sigma(G-2L) \tag{19}$$

When the enclosure has sides which are not black eq. (18) has a computational disadvantage. All the matrices involved are symmetric, but the matrix in square brackets is not symmetric For an enclosure with many sides, it may be computationally more convenient to calculate the inverse of a symmetric matrix. This can be easily achieved for enclosures for which none of the sides is black, which is the most common case. In this case, (A-I) has an inverse, and we can easily obtain:

$$P=[\,2ALZ^{-1}+(A-I)^{-1}\,]\;\sigma\;LA \tag{20}$$

where the symmetric matrix Z is given by:

$$Z=2L(I-A)-(I-A)G(I-A) \tag{21}$$

As a final check of the theory we apply the first and second laws of Thermodynamics. Since there is no heat generation inside the closed region under consideration, the first law requires that the sum of all the heat fluxes over the sides be equal to zero:

$$u^T q=0$$

or, using equation (14):

$$u^T Pt=0 \tag{22}$$

In order that equation (22) be satisfied for any value of the side temperatures we require that:

$$u^T P=0 \tag{23}$$

This is in fact the case, as can be verified using the general definition (15) and eq. (3).

The application of the second law is best understood by imagining a thermodynamic system henceforth called the isolated system, with the same geometry as the real system under consideration, and composed of material sides at the same temperatures and with the same emissivities, but perfectly isolated from the exterior and from each other. All radiant heat fluxes will be exactly the same in the two systems, the only difference being a conceptual one: in the isolated system the temperature of a side can change only as a consequence of that side emitting or absorbing radiant energy.

Consider first a situation where the temperature of the side i is T_i, while all other sides are at absolute zero. In this case, for the isolated system, the second law requires that $q_i<0$ and $q_j\geq 0$ for $j\neq i$. From equation (14) we have, for any 1:

$$q_1= \sum_k P_{1k}t_k=P_{1i}t_i$$

The requirement of the second law is therefore:

$$\begin{aligned} &P_{ii}<0 \\ &P_{ji}\geq 0 \qquad \text{for } j\neq i \end{aligned} \tag{24}$$

Consider now a situation where all the sides are at the same temperature, say t=u. In this case, for the isolated system, the second law requires that $q_i=0$ for all i or, using equations (14) and (23), an identify is obtained.

Since the state of radiation in the real system is exactly the same as in the isolated system, we conclude that the same requirements, expressions (23) and (24), are valid for the real system. Our formulation of the real system is expected to comply with the second law because the absorption coefficient was taken equal to the emissivity as a consequence of Kirchhoff's law, which is in turn based on the second law of Thermodynamics. During the numerical solution of a variety of problems, [4,5,6] the computer program automatically checked that the relations (23) and (24) were satisfied in all cases This check was also found to be excellent for detection of human errors, which generally resulted in an incorrect matrix P which did not satisfy the relations (23) or (24). The converse is not true, however. P may satisfy the relations (24) even when G is incorrect.

VII Description of the program HORNO

A preprocessor determines all the information associated with each type of enclosure and mesh. Input to the preprocessor is the topology and geometry of each type and the material properties. Output is the matrix of viewfactors G, the radiation matrix of eq.(15) when the emissivities are assumed to be constant, the mesh/enclosure connectivity, the mesh connectivity and geometry, definitions of types of elements and material properties of each, etc. This information constitutes an interface to the main processor. It is temperature and time independent, and thus it is used for the solution of the problem under any initial conditions.

A main processor then actually solves the problem. Input to the processor is the interface just described, the initial temperature distribution in the bodies, and any given temperatures or heat fluxes as functions of the time. Information concerning size of time steps and a stopping condition has also to be given. The processor outputs the nodal values of the temperatures and desired heat fluxes at the end of each time step.

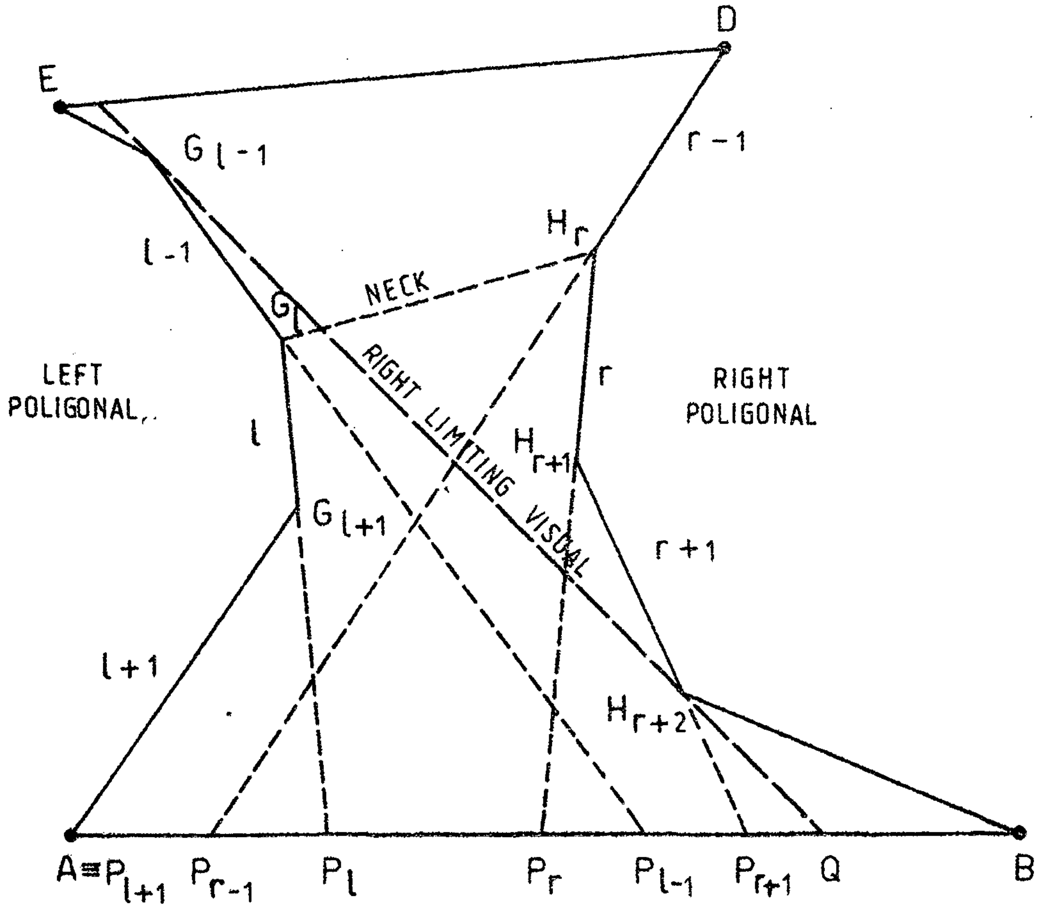

Figure 7: Construction used for the proof of Hottel's law.

IX Appendix

Calculation of the energy flux through a two-dimensional radiation channel

*Consider a radiation channel defined by the radiation sides AB and DE and by two convex polygonals, which interpose partially between AB and DE (see Fig. 7) AB radiates heat uniformly from all its points and towards all directions in space. It is desired to calculate what fraction of this energy reaches the side DE directly, that is, without undergoing any intermediate reflection. In this order the side AB will be divided into suitable intervals and the desired energy flow will be calculated as the sum of the energy flows from each interval on AB to the side DE.

The prolongations of the sides of the two polygonals intersect AB and determine two sets of points. The sets will be referred to as left-type points or ℓ-points and right-type points or r-points, in accordance with the relative position of the corresponding polygonal with respect to AB. The points divide AB into intervals. Each interval is defined by two consecutive points, each of which may be an ℓ-point, an r-point, or coincident r and ℓ-points. In order to make the proof general it is necessary to assume that the points A and E belong to the left polygonal, and B and D to the right one, as is shown in the example of Fig. 7. This makes A an ℓ-point and B an r-point. Consider now the interval determined by the ℓ-point P_ℓ and the r-point P_r of fig. 7. The line segment $G_\ell H_r$ is called the <u>neck</u> associated with the interval $P_\ell P_r$. It has the following property:it is at sight from any point of $P_\ell P_r$ and every ray emitted from some point of the interval $P_\ell P_r$ and intersecting the neck $G_\ell H_r$ will fall on the side DE, and no ray emitted from $P_\ell P_r$ may reach DE without having intersected $G_\ell H_r$. Therefore the energy flow from $P_\ell P_r$ to DE is equal to the energy flow from $P_\ell P_r$ to the neck $G_\ell H_r$, which is proportional to

$$f=P_\ell H_r+P_r G_\ell-P_\ell G_\ell-P_r H_r \tag{A1}$$

accordingly with eq. (1). Similar expressions are obtained for all the intervals in AB, and then added up in order to obtain the total energy flow from AB to DE. The terms of the summation are now grouped accordingly to whether they correspond to an ℓ-point or to an r-point. For example, the contribution of the expression (A-1) to the ℓ-sum is:

$$f_\ell=P_\ell H_r-P_\ell G_\ell$$

while that to the r-sum is:

$$f_r=P_r G_\ell-P_r H_r$$

Now, consider a point such as P_r, which is at the same time the right end of an interval and the left end of the next interval. The next interval is in this case $P_r P_{\ell-1}$, the corresponding neck is $G_\ell H_{r+1}$, and the new contribution of P_r to the r-sum is :

$$f'_r=P_r H_{r+1}-P_r G_\ell$$

Therefore:

$$f_r+f'_r=-H_r H_{r+1} \tag{A2}$$

This result states that the contribution of P_r to the right sum is the negative value of the length of the side of the right polygonal corresponding to P_r. This conclusion remains valid even in the case where the r-point P_r coincides with an ℓ-point. This situation is illustrated in Fig. 8. For the interval $P_{\ell+1}P_r$ the contribution of P_r to the r-sum is $P_rG_{\ell+1}-P_rH_r$. For the interval P_rP_{r+1} the neck is $G_\ell H_{r+1}$ and the contribution of P_r to the r-sum is $P_rH_{r+1}-P_rG_\ell$. The total contribution is thus

$$-H_rH_{r+1}-G_\ell G_{\ell+1}$$

The term $-G_\ell G_{\ell+1}$ is considered to be the contribution of P_ℓ to the left sum, and $-H_rH_{r+1}$ remains as the contribution of P_r to the right sum.

Now we consider points having a single contribution to the sums, such as A and Q in Fig. 7. Q is determined by the right limiting visual line, which in this case is $G_{\ell-1}H_{r+2}$. Q is the right end of the interval $P_{r+1}Q$ and the left end of QB, but QB does not contribute to the enrgy flow from AB to DE. A is not the right end of any interval, but for the sake of uniformity in the definition, is considered to be determined by the left limiting visual line, which in this case is AD. The neck for the interval AP_{r-1} is $G_{\ell+1}D$, and the total contribution of A is

$$AD-AG_{\ell+1}$$

which is the length of the limiting visual AD minus the length of the side $\ell+1$ of the polygonal, this last term correspon- ding to the fact that A coincides with the ℓ-point $P_{\ell+1}$.

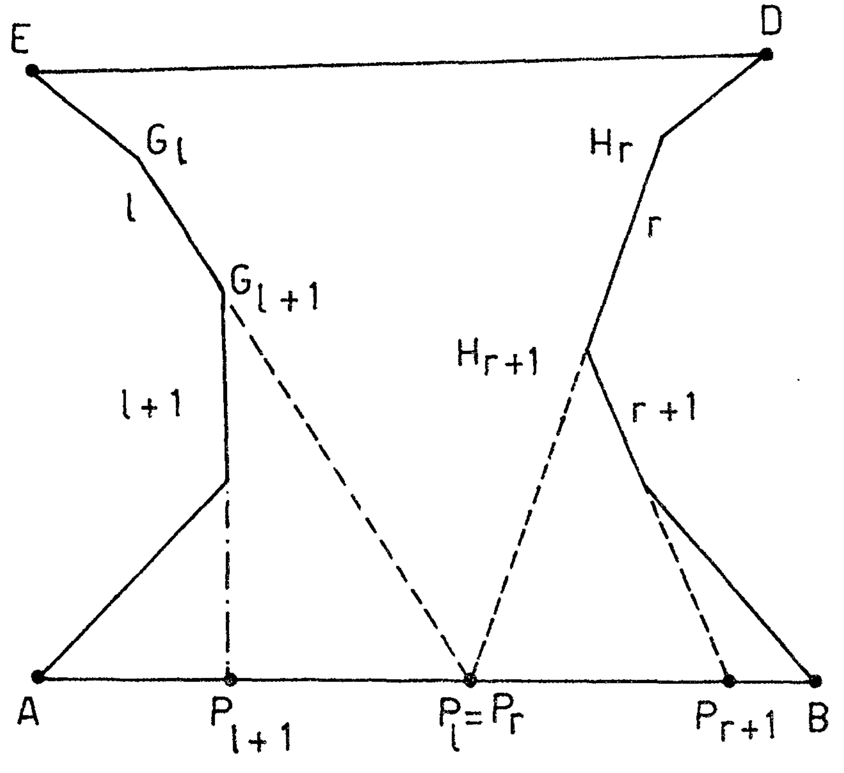

Figure 8: Situation in which a left-point P_ℓ *and a right point* P_r *are coincident.*

We note that the side $\ell+1$ belongs to the portion of polygonal enclosed by the points of contact of the two limiting visuals, in this case A and $G_{\ell-1}$. For the interval $P_{r+1}Q$ the neck is $G_{\ell-1}H_{r+2}$, and the contribution of Q is

$$QG_{\ell-1}-QH_{r+2}=H_{r+2}G_{\ell-1}$$

which is the length of the right limiting visual between the points which determine it.

In summary, we have proved the following:

1) Every intermediate point, which is determined by the intersection with AB of the prolongation of a side of one of the polygonals, and has the property to be both the right end of an interval and the left end of the next interval, contributes to the energy sum with an amount equal to minus the length of the corresponding polygonal side. The sides involved are only those which belong to that portion of each polygonal enclosed by the two points of contact of the two limiting visuals on that polygonal. Therefore, the total contribution of such points will be equal to the negative of the sum of the total lengths of the polygonals between the points of contact of the limiting visuals.

2) Two other points exist which are determined by the intersection of the two limiting visuals with AB. Each of them contributes to the energy sum by an amount equal to the length of the corresponding visual line measured between its points of contact with each of the polygonals. This completes the desired calculation.

VIII REFERENCES

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