# COUPLING BETWEEN INELASTIC AND PAIR TRANSFER DEGREES OF FREEDOM. THE $^{208}$Pb(t, p)$^{210}$Pb($3^-$) REACTION ANALYZED IN THE COUPLED CHANNEL BORN APPROXIMATION

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The reaction $^{208}$Pb(t, p$^{210}$Pb($3^-$) is analyzed allowing for indirect processes to take place. Use is made of two-body transfer spectroscopic amplitudes which include transitions proceeding through ground state correlations induced by pairing and particle-hole collective states. Both effects are found to be important.

Elementary modes of excitation (e.g. rotations, vibrations, independent quasi-particles) can be considered as the building blocks of the nuclear spectrum [1]. Yet these modes are not completely uncoupled and how they interact (anharmonic effects) is one of the open questions in nuclear spectroscopy [2]. Another rapidly developing area of study is concerned with channel coupling (two-step processes) in nuclear reactions [3]. These two fields of research are intimately related. The importance of channel coupling depends directly on the structure of the states connected by the reaction and thus on any coupling between elementary modes of excitation which might describe such states. Bearing this in mind we discuss the $^{208}$Pb(t, p)$^{210}$Pb($3^-$) reaction [4], calculating the coupling between the 2p and 3p-1h $J^\pi = 3^-$ states of $^{210}$Pb in perturbation theory** and the reaction cross section in the coupled channel Born approximation (CCBA). A complete analysis of the $^{208}$Pb(t, p)$^{210}$Pb reaction will be published elsewhere [4].

We consider the following to be the two lowest unperturbed $3^-$ states in $^{210}$Pb:

$$|1\rangle = |3^-(^{208}\text{Pb} \otimes 0^+(^{210}\text{Pb(gs)});3^-\rangle ,$$

$$|2\rangle = |(j_{15/2} \otimes g_{9/2})3^-\rangle \tag{1}$$

The corresponding energies are equal to 2.62 MeV and 3.07 MeV respectively. The bare energy of the $j_{15/2}$ single particle state ‡ was set equal to 1.85 MeV according to ref. [5].

The collectivity of the pairing phonon, $|^{210}\text{Pb(gs)}\rangle$, is evidenced by an enhanced (t, p) cross section †. We describe this state by two neutrons moving in the seven single-particle orbits above the $N = 126$ shell closure (see table 1)

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** A systematic investigation of octupole states around $^{208}$Pb has been carried out by Hamamoto [11].

‡ According to this reference and to ref. [10] the $15/2^-$ state observed at 1.4 MeV of excitation is not a pure single-particle state, but has an appreciable admixture of the $|(g_{9/2} \otimes 3^-); 15/2^-\rangle$ configuration. This feature is borne out by experiment. In order to obtain a $15/2^-$ state at the observed energy one has to assume that the unperturbed energy of the $j_{15/2}$ state is equal to 1.85 MeV.

† The reaction $^{208}$Pb(t, p)$^{210}$Pb(gs) proceeds with an intensity of about $6 \times \sigma_{2\text{pu}}^{\text{av}}$ where $\sigma_{2\text{pu}}^{\text{av}} = \sum_j \sigma(j^2(0))/7$ [6] and $j = 1, 7$ refers to the neutron particle states in table 1.

Table 1
Single-neutron and proton orbits above (p) and below (h) the $N = 126$ and $Z = 82$ shell closures respectively.

| Neutrons | | Protons | |
|---|---|---|---|
| Orbit | | | |
| $d_{3/2}$ | 5.93 | $p_{1/2}$ | 0.53 |
| $g_{7/2}$ | 5.88 | $f_{5/2}$ | -0.47 |
| $s_{1/2}$ | 5.44 | $p_{3/2}$ | -0.95 |
| $d_{5/2}$ | 4.97 | $i_{13/2}$ | -2.16 |
| $j_{15/2}$ | 4.82 | $f_{7/2}$ | -2.87 |
| $i_{11/2}$ | 4.18 | $h_{9/2}$ | -3.77 |
| $g_{9/2}$ | 3.41 | | |
| $p_{1/2}$ | 0.0 | $s_{1/2}$ | -8.03 |
| $f_{5/2}$ | -0.57 | $d_{3/2}$ | -8.38 |
| $p_{3/2}$ | -0.90 | $h_{11/2}$ | -9.37 |
| $i_{13/2}$ | -1.64 | $d_{5/2}$ | -9.70 |
| $f_{7/2}$ | -2.35 | $g_{7/2}$ | -11.43 |
| $g_{9/2}$ | -3.47 | $g_{9/2}$ | -15.43 |

and interacting via the pairing force. A coupling constant of $G = 21.6$ MeV/A was determined [9] by fitting the extra binding energy (= 1.237 MeV) of the ground state of $^{210}$Pb as compared to the binding energy of $g_{9/2}(0)$. This description, and the use of the universal normalization factor [6, 8], yields an absolute (t, p) cross section which agrees with experiment [4, 9].

The surface phonon, $|^{208}\mathrm{Pb}(3^-)\rangle$ is also rather collective, showing a large [7] (p, p') cross-section*. Its wave function is taken to be a superposition of particle-hole excitations whose amplitudes are determined by diagonalizing (RPA) an octupole-octupole residual interaction. The particles and holes are allowed to move in the corresponding proton and neutron orbits listed in table 1. The octupole coupling constant $\chi_3 = 113\, A^{-3}(M\omega_0/\hbar)^3$ MeV was obtained [9] by fitting the excitation energy (= 2.617 MeV) of the lowest $3^-$ state in $^{208}$Pb. The resulting wave function, with a polarization charge of 0.13, predicts a $B$(E3) value in agreement with experiment [9].

The coupling of the pairing and particle-hole vibrations to the particle and hole degrees of freedom is the basic mechanism that generates the collective modes themselves, making the problem of calculating the microscopic structure of the phonons a selfconsistent one [1]. The strengths of these couplings determine the collectivity of the modes. We label these strengths by

* The $B$(E3) value is about [7, 20] $32\, B_{\mathrm{sp}}$ corresponding to $\beta_3 \approx 0.11$.

Table 2
The energies and ratios of the inelastic scattering to the lowest $3^-$ state in $^{208}$Pb are given for the two lowest $3^-$ states in $^{210}$Pb.

| $E$(MeV) | | $\sigma/\sigma\,(^{208}\mathrm{Pb}(3_1^-))$ | |
|---|---|---|---|
| Expt [24] | Theory | Expt [24] | Theory |
| 1.85 | 2.28 | 0.66 | 0.70 |
| 2.81 | 3.41 | 0.33 | 0.30 |

$\Lambda(210(\mathrm{gs}))$ and $\Lambda(208(3_1^-))$. They are determined by $G$ and $\chi_3$ respectively (for details see ref. [9]). The elementary vertices [10] of the particle (hole)-vibration coupling are shown in figs. 1(a-f). For example the matrix elements corresponding to graphs 1(a) and 1(d) are

$$\langle ^{210}\mathrm{Pb(gs)}|\, H_{\mathrm{c}}\, |(\mathrm{k}_1\mathrm{k}_2)0^+\rangle = -\Lambda(210(\mathrm{gs}))\langle \mathrm{k}_1 \| r^0 Y_0 \| \mathrm{k}_2\rangle$$

and

$$\langle ^{208}\mathrm{Pb}(3_1^-)|\, H_{\mathrm{c}}\, |(\mathrm{ki})\, 3^-\rangle = -\Lambda(208(3_1^-))\langle \mathrm{k} \| r^3 Y_3 \| \mathrm{i}\rangle, \tag{2}$$

respectively, where $H_{\mathrm{c}}$ is the coupling Hamiltonian and $Y_\lambda$ a spherical harmonic**.

The particle-vibration coupling is also responsible for the coupling among the modes of the even Pb-isotopes (phonon-phonon interaction). In perturbation theory the states $|1\rangle$ and $|2\rangle$ interact to third and higher order in the coupling strengths $\Lambda$. We calculated all possible third order contributions to this interaction. The corresponding graphs are displayed in figs. 1(g-k). The sum of these graphs yield a matrix element coupling $|1\rangle$ and $|2\rangle$ of -520 keV. The main contribution to the total matrix elements comes from graph 1(g) which contributes with -400 keV and -100 keV for the configurations $k_1 = g_{9/2}$ $k_2 = j_{15/2}$ and $k_1 = j_{15/2}$ $k_2 = g_{9/2}$ respectively. Thus we obtain the states

$$\begin{aligned} |\mathrm{I}\rangle &= 0.826|1\rangle + 0.549|2\rangle, \\ |\mathrm{II}\rangle &= -0.549|1\rangle + 0.836|2\rangle, \end{aligned} \tag{3}$$

with energies of 2.28 MeV and 3.41 MeV respectively. The centroid of these levels is 510 keV above that of the two $3^-$ levels observed in $^{210}$Pb. However the observed splitting (960 keV) is almost reproduced. We predict that states $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$ will respectively carry 70% and 30% of the inelastic cross section to the $^{208}\mathrm{Pb}(3_1^-)$ state in agreement with the data*** (table 2).

** The relation of $\Lambda$ to the usual coupling strength [10] $K = \langle r\,\partial V/\partial r\rangle$ is given by $\Lambda(\lambda) = (K/\langle r^\lambda\rangle 2\lambda+1)\sqrt{\hbar\omega_\lambda/2C_\lambda}$ where $\sqrt{\hbar\omega_\lambda/2C_\lambda}$ gives a measure of the zeropoint motion associated with the mode of multipolarity $\lambda$.

*** The correct splitting would be obtained with a matrix element of -400 keV. This would imply a change in the structure of $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$ of about 8%.

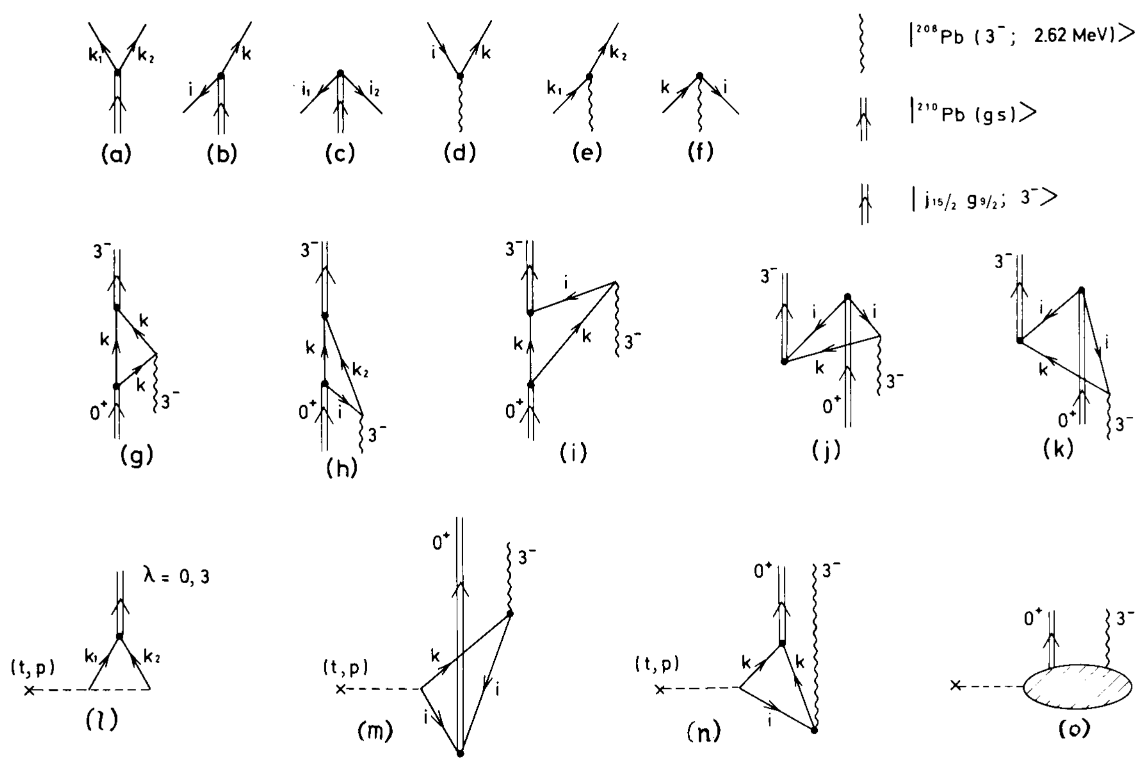


Fig. 1. The graphs (a) - (f) are vertices corresponding to the coupling of a particle (hole) to a particle-hole or to a pairing vibration. Graphs (g) - (k) represent the matrix elements connecting the state $|1\rangle$ with the state $|2\rangle$ (eq. (1)). The spectroscopic amplitudes for the transfer of two neutrons are displayed in graphs (l) - (m).

In the calculations of ref. [5] more basis states than in the present case were used to calculate the $J^{\pi} = 3^-$ states in $^{210}$Pb. This has little effect on the structure of the low-lying levels though it shifts down its centroid by 300 keV, bringing the result to agree with experiment in 200 keV.

Let us consider the (t, p) reaction populating these states $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$ in $^{210}$Pb. The bare two-particle operator illustrated in fig. 1(l) only allows for transfer processes which connect the ground state of $^{208}$Pb to the ground state of $^{210}$Pb ($\lambda = 0$) or to the component $|2\rangle$ of the $3^-$ states* ($\lambda = 3$). We refer to the corresponding form factors [13, 14] as FF(0) and FF(2). In this picture then, the form factors for direct transfer to the states $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$ are 0.549FF(2) and 0.836FF(2) respectively. Two-step processes may be included by which we inelastically excite the 2.62 MeV state $|^{208}\mathrm{Pb}(3_1^-)\rangle$ ($\beta_3(208) = 0.11$) and then make $\lambda = 0$ fransfers to the states $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$, with form factors 0.836FF(0) and -0.549FF(0) respectively; or, we transfer to the ground state of $^{210}$Pb and inelastically scatter to these states ($\beta_3(\mathrm{I}) = 0.836\beta_3(208)$, $\beta_3(\mathrm{II}) = 0.549\beta_3(208)$). The results for bare two-particle transfer are

$$R_1 = \frac{\sigma_{\mathrm{theor}}(3_1^-)}{\sigma_{\mathrm{exp}}(3_1^-)} = \begin{cases} 0.16 \text{ (DWBA)} \\ 0.27 \text{ (CCBA)} \end{cases} \tag{4}$$

and

$$R_2 = \frac{\sum_{\theta=5,25,30} \mathrm{d}\sigma_{\mathrm{theor}}(3_2^-;\theta)/\mathrm{d}\Omega}{\sum_{\theta=5,25,30} \mathrm{d}\sigma_{\mathrm{theor}}(3_1^-;\theta)/\mathrm{d}\Omega} = \begin{cases} 2.3 \text{ (DWBA)} \\ 1.0 \text{ (CCBA)} \end{cases} \tag{5}$$

where $\sigma$ stands for a sum of the angular distribu-

* The spectroscopic amplitude associated with graph 1(l) is equal to the amplitude with which the configuration $(k_1, k_2)$ enters in the pairing phonon. In the present case the $(j_{15/2}, g_{9/2})_{3^-}$ configuration is supposed to have amplitude equal to unity (see eq. (1)). For the spectroscopic amplitudes associated with the transition $^{208}$Pb(t, p)$^{210}$Pb(gs) ($\lambda = 0$) the amplitudes $\alpha(k^2(0))$ of the pair addition mode were used [9].

tion over the measured points*. Thus the intensity to the $3_1^-$ state cannot be accounted for by thinking only in terms of bare two-particle transfer. Moreover $R_2$ is in complete disagreement with experiment which places an upper limit** of $R_2 \approx 0.5 - 0.2$.

There is, however, another contribution to the $J^{\pi} = 3^-$ form factors which completely modifies the previous results. Because of the presence of the pairing or particle-hole modes in $^{208}$Pb, there are admixtures in the ground state of this system of the type $(j^{-2}(0) \otimes {}^{210}\mathrm{Pb(gs)})_{0+}$ or $((j_1 j_2^{-1})3^- \otimes {}^{208}\mathrm{Pb}(3_1^-))_{0+}$. The reaction can then proceed from the ground state of $^{208}$Pb to the component $|1\rangle$ of $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$ by filling one hole and transfering the other neutron into an empty orbit above the Fermi level. Such processes are illustrated by graphs $1(m-n)$ which, taken together, may be regarded as an effective two-particle transfer operator [9] which directly populates the $|1\rangle$ component of the states (3) (fig. 1(o)). We call the corresponding form factor FF(1).

It is important to realize that this effect is not related to the competition between inelastic scattering and transfer reaction channels, but has to do with the fact that the configurations not included in our basis would renormalize the different matrix elements calculated with wave functions built in this restricted subspace. This is the same concept as the one introduced by Talmi in relation to the matrix elements but generalized to transfer operators [15].

Although all twenty-seven contributions to FF(1) are small, they combine with the same sign to produce a form factor of the same order as FF(2). This effect was already found in the analysis of the $^{206}$Pb(t, p) $^{208}$Pb($3^-$; 2.62 MeV) reaction [14]. The particle-hole residual interaction which is responsible for the large $B$(E3) values observed in the decay of the $3^-$ state*** produces also very strong spatial correlations of the particle and the hole [23]. Because the two holes in the $^{206}$Pb(gs) and the two neutrons in the triton are also highly correlated in space, the above effect is responsible for the large (t, p)

* The theoretical cross section to the ground state of $^{210}$Pb has been normalized to the data.

** Only three points were measured in the (t, p) reaction at forward angles for a state at 2.8 MeV which is where the $3_2^-$ state was observed in inelastic scattering [12]. No definite assignment can be made for this state from the (t, p) data.

*** Recently [22] the dynamical basis for the observed correlation between large inelastic scatteering and large two-nucleon transfer reaction cross sections has been discussed.

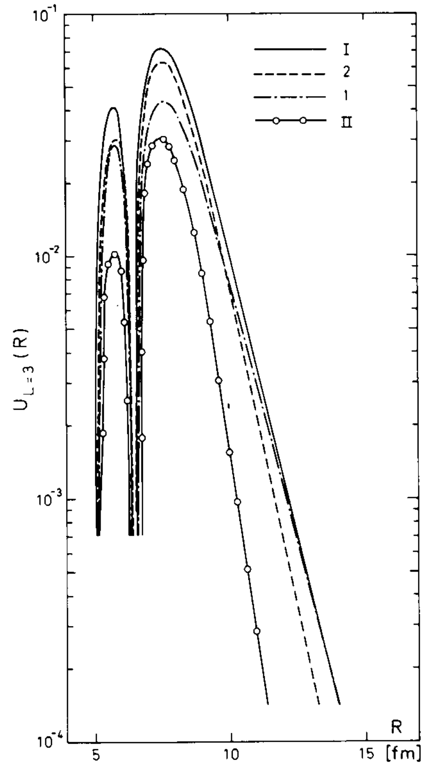


Fig. 2. Form factors associated with two lowest $J^{\pi} = 3^-$ states excited in the $^{208}$Pb(t, p)$^{210}$Pb reaction:
1) using the spectroscopic amplitudes displayed in fig. 1(n) and 1 (m) corresponding to transfer events proceeding via ground state correlations.
2) corresponding to the pure $|2\rangle = (j_{15/2}, g_{9/2})\,3^-$ configuration.
I) corresponding to the state $|\mathrm{I}\rangle = 0.836|1\rangle + 0.549|2\rangle$
II) corresponding to the state $|\mathrm{II}\rangle = -0.549|1\rangle + 0.836|2\rangle$
The form factors 1), 2) and I) were calculated using a $Q$-value of -1.209 MeV corresponding to an excitation energy of 1.85 MeV, while for II), $Q = -2.169$ corresponding to an excitation of 2.81 MeV. The calculations were done with the code TWOPAR [18] and were checked against the results of the code DWUCK [19].

cross sections, with which the $|^{208}\mathrm{Pb}(3^-)\rangle$ is excited.

The effective direct transfer from the ground state of $^{208}$Pb to the state $|\mathrm{I}\rangle$ is governed by an enhanced FF(I) = 0.836FF(1) + 0.549FF(2) while FF(II) = -0.549FF(1) + 0.836FF(2) is reduced by strong cancellation (see fig. 2). The results corresponding to these form factors are

$$R_1 = \begin{cases} 0.35 & \text{(DWBA)} \\ 0.64 & \text{(CCBA)} \end{cases} \tag{6}$$

and

$$R_2 = \begin{cases} 0.1 & \text{(DWBA)} \\ 0.3 & \text{(CCBA)} \end{cases} \tag{7}$$

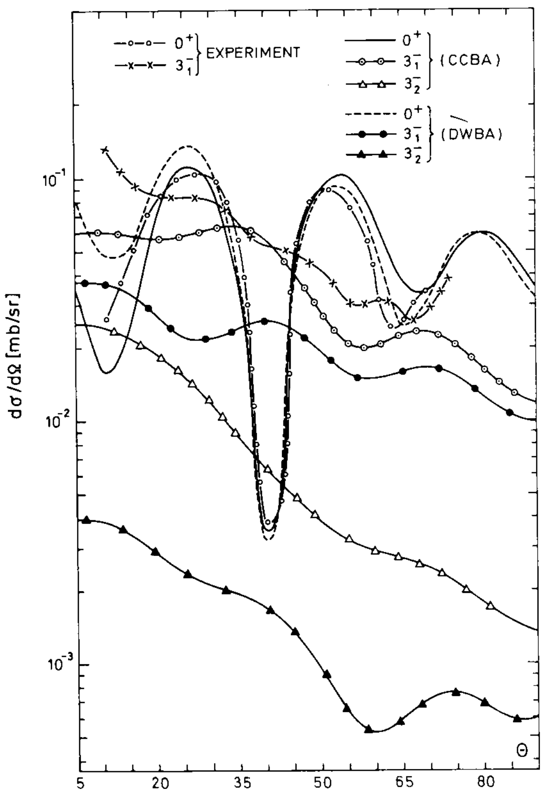


Fig. 3. Angular distributions for $^{208}Pb(t,p)^{210}Pb$ reactions: The ground state, $3_1^-$ (1.85 MeV) and $3_2^-$ (2.81 MeV) distributions were calculated using the form factors I) and II) of fig. 2 for the $L = 3$ transfers. The ground state $L = 0$ form factor was obtained using the $^{210}Pb$(gs)-wave function calculated by diagonalizing a pairing force in the two-particle subspace (see text). The absolute normalization factor was determined to be $D_0^2 = 30\ \mathrm{MeV^2 fm^3}$ by fitting the CCBA ground state angular distribution to the experimental data. (A value of $D_0^2 = 25\ \mathrm{MeV^2 fm^3}$ is obtained using the DWBA results [6, 8]. The DWBA calculations were done with the code TWOPAR [18] and checked with the code DWUCK [19]. The CCBA calculations were done with the code MARS [21]. The experimental data are from ref. [4].

The improvement over using bare two particle transfer amplitudes (eqs. (4) and (5)) is apparent.

The improved calculations and the data are shown in fig. 3. All the calculations use Perey's optical parameters for the proton channel [16] and the average parameters of ref. [17] for the triton channel. The absolute normalization factor used in plotting both the CCBA and DWBA calculations was determined by fitting the predicted CCBA ground state angular distribution to the experimental one (fig. 3). The resulting value is equal to $D_0^2 = 30 \times 10^4\ \mathrm{MeV^2\ fm^3}$ which is 20%

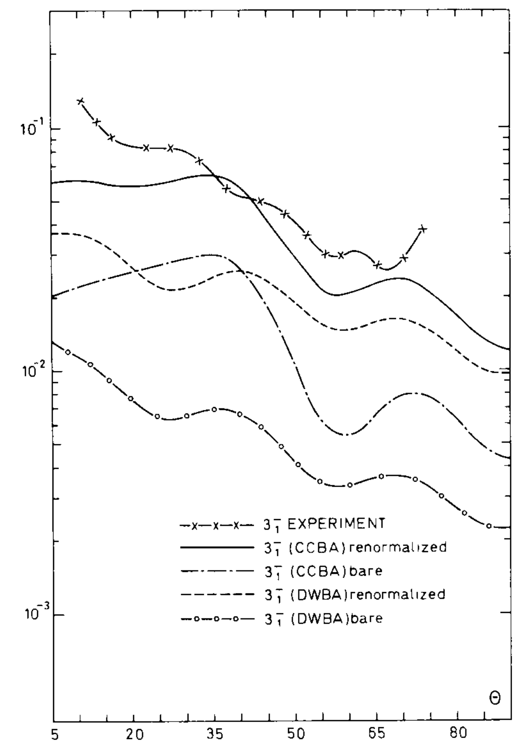


Fig. 4. Angular distributions for the $^{208}Pb(t,p)^{210}Pb(3_1^-)$ reaction using the full $L = 3$ form factor I) of fig. 2 (renormalized) compared to the direct two-particle transfer neglecting ground state correlations (bare).
See comments in the caption to fig. 3.

larger than the universal value of $D_0^2$ (=$25 \times 10^4$ $\mathrm{MeV^2 fm^3}$) empirically determined [6, 8] in the framework of the DWBA. It is then clear that the indirect amplitude interferes destructively with the direct amplitude for the ground state transition. In fig. 4 we show separately how the two-step processes and ground state correlations influence the distribution of the $3_1^-$ state. Each effect increases the intensity by about a factor of two.

The final numbers display a general agreement with the experimental magnitudes of the $3^-$ cross sections. This is not the case for the shape of the $3_1^-$ angular distribution. No parameter fitting has been attempted. We conclude that

a) the (t, p) cross section of the two lowest $3^-$ states of $^{210}Pb$ cannot be explained without accounting for the contributions due to the correlations induced in the $^{208}Pb$ ground state by the $J^\pi = 0^+$ pairing vibration and by the $J^\pi = 3^-$ particle-mole mode.

b) the inclusion of indirect reaction channels which interfere with the direct one brings the improved results obtained by a) in closer agreement with experiment.

c) both a) and b) are needed in a consistent discussion of the $^{208}$Pb (t, p)$^{210}$Pb(3$^-$) reaction. The importance of these two effects is due to the collectivity of the pairing and particle-hole modes in $^{208}$Pb and their strong coupling in $^{210}$Pb.

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