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Indications on the ϱ-Like Singularities in the Complex J-Plane.

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We present a sum rule which is satisfied by the amplitude of the forward π-$\mathcal{N}$ charge-exchange scattering. It relates the relevant high-energy parameters to lower-energy data. In contrast with other dispersion sum rules, the very-low-energy data contribute here with much more weight.

This sum rule provides a new insight into the structure of the singularities in the complex J-plane. We find that, besides the ϱ-pole, other singularities must be present to obtain consistency, and a rough indication of their structure is obtained.

Let us indicate the π-p forward charge-exchange amplitude by $F^{(-)}(\nu)$ (1). We assume at first that the ϱ-pole is the only Regge singularity with $\operatorname{Re} J > -1$. Therefore, at large $|\nu|$, $F^{(-)}(\nu)$ has the form

$$F^{(-)}(\nu) \simeq \beta \frac{1-\exp[-i\pi\alpha]}{\sin \pi\alpha} \nu^{\alpha} . \tag{1}$$

By integrating $F^{(-)}(\nu)$ along the closed curve C indicated in Fig. 1, one gets the sum rule

$$\int_0^{\nu_0} d\nu \operatorname{Im} F^{(-)}(\nu) = \beta \frac{\nu_0^{\alpha+1}}{\alpha+1} = \operatorname{Im} F^{(-)}(\nu_0) \frac{\nu_0}{\alpha+1} , \tag{2}$$

quoted by the authors of ref. (2).

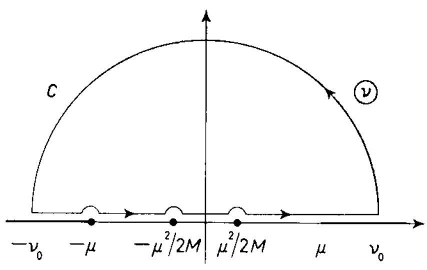

Fig. 1.

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(1) ν is the π laboratory energy.

(2) A. A. LOGUNOV, L. D. SOLOVIEV and A. N. TAVKHELIDZE: *Phys. Lett.*, **24** B, 181 (1967); K. IGI and S. MATSUDA: *Phys. Rev. Lett.*, **18**, 625 (1967); R. GATTO: *Phys. Rev. Lett.*, **18**, 803 (1967); D. HORN and C. SCHMID: CALTECH report 68-127 (1967).

It is interesting to consider

$$f(\nu) = \frac{F^{(-)}(\nu)}{\nu(\nu+\mu)^\alpha}\,. \tag{3}$$

Since $f(\nu)$ is regular in the upper half-plane of ν [3], we have

$$\int_C \mathrm{d}\nu\, f(\nu) = 0\,, \tag{4}$$

where C is the closed curve.

At large $|\nu|$, $f(\nu)$ has the form

$$f(\nu) \simeq \beta\,\frac{1-\exp[-i\pi\alpha]}{\sin\pi\alpha}\,\frac{1}{\nu}\,. \tag{5}$$

If ν_0 is sufficiently large as in Fig. 1, eq. (4) becomes

$$\int_{-\nu_0}^{\nu_0} \mathrm{d}\nu\, f(\nu) = \pi\beta\left(1 - i\,\mathrm{tg}\frac{\pi}{2}\,\alpha\right). \tag{6}$$

The imaginary part of this equation takes the following form:

$$\begin{aligned} &f^2\,\frac{4\pi M}{\mu^2}\left[\left(\frac{\nu_0}{\mu+\mu^2/2M}\right)^\alpha - \left(\frac{\nu_0}{\mu-\mu^2/2M}\right)^\alpha\right] + \\ &+\int_\mu^\infty \frac{\mathrm{d}\nu}{\nu}\left\{\sin\pi\alpha\left(\frac{\nu_0}{\nu-\mu}\right)^\alpha \mathrm{Re}\,F^{(-)}(\nu) - \left[\left(\frac{\nu_0}{\nu_0+\mu}\right)^\alpha - \left(\frac{\nu_0}{\nu-\mu}\right)^\alpha \cos\pi\alpha\right]\mathrm{Im}\,F^{(-)}(\nu)\right\} = \\ &= \pi\,\mathrm{tg}\frac{\pi}{2}\,\alpha\beta\nu_0^\alpha = \pi\,\mathrm{tg}\frac{\pi}{2}\,\alpha\,\mathrm{Im}\,F^{(-)}(\nu_0)\,, \end{aligned} \tag{7}$$

where μ and M are the mass of the pion and nucleon, respectively. The number α is determined by eq. (7), when the experimental results are used to evaluate $F^{(-)}(\nu)$ up to ν_0.

To evaluate the sum rule, one has to introduce the data for the real part of the amplitude. This data have errors considerably higher than those for the imaginary part.

However, sum rule (7) is much more sensitive to variations in α than sum rule (2) [4]. This fact allows meaningful conclusions to be drawn about the existence and structure of other singularities, in addition to the ϱ-pole, in the complex J-plane with $\mathrm{Re}\,J > -1$.

Another characteristic feature of the sum rule (7) is that the dominant contribution is given in the low-energy region up to 1.2 GeV, in contrast to sum rule (2) in which the contribution of this data is relatively small in comparison with the high-energy tail.

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[3] Because $F^{(-)}(\nu)$ is odd, $f(\nu)$ is continuous at $\nu = 0$.

[4] In the latter sum rule a variation of α as big as 0.2 does not cause disagreement with experimental data within the errors.

We observe in this connection that the ϱ-pole does not contribute to the integral of eq. (7) as is apparent from eqs. (5) and (6).

In the evaluation (5) of the sum rule we have taken $\nu_0 = 6$ GeV. $\text{Im}\, F^{(-)}(\nu_0)$ has been calculated for α ranging from 0.30 to 0.80. The results are reported in Table I.

TABLE I. – $\text{Im}\, F^{(-)}(\nu_0)$ *for* $\nu_0 = 6$ GeV *calculated from the sum rule* (7) *for various values of* α. In column *a*) the calculated value is reported, in columns *b*) and *c*) the maximum and minimum figures compatible with the errors of the input data are listed. Experimentally $\text{Im}\, F^{(-)}(\nu_0) = 0.415 \pm 2\% \mu^{-1}$.

α	*a*) Im $F^{(-)}(\nu^0)$ (μ^{-1})	*b*) Max (μ^{-1})	*c*) Min (μ^{-1})
0.30	0.40	0.68	0.17
0.40	0.26	0.55	0.02
0.50	0.06	0.32	−0.18
0.60	−0.21	0.02	−0.42
0.70	−0.52	−0.35	−0.69
0.80	−0.79	−0.69	−0.89

Because of the large uncertainties in the data for the real part, we have reported in two distinct columns the maximum and minimum values of $\text{Im}\, F^{(-)}(\nu_0)$ compatible with errors. These calculated values are to be compared with the experimental value $\text{Im}\, F^{(-)}(\nu_0) = 0.415 \pm 2\% \mu^{-1}$.

From the Table, α appears to be too low by about 0.2 when compared with the value $\alpha = 0.58$ which results from the high-energy fits. This is in agreement with what has been found by the authors of ref. (8) in their semi-quantitative analysis of non-forward π-$\mathcal{N}$ charge-exchange scattering. The low value of α indicates that other singularities in the complex J-plane with $\text{Re}\, J < \alpha$ must occur with an appreciable weight. The sum rule (7) does not distinguish well between the various kinds of singularities (cuts, poles, etc.); it is sensitive only to the J at which the singularity occurs and to its weight in the high-energy fit.

In order to get consistency with the experimental data and the ϱ parameters as determined by the high-energy fits, we may tentatively introduce a ϱ'-pole together with the ϱ-pole. However, with the parameters given for the ϱ' by LOGAN *et al.* (9), the disagreement is not noticeably reduced. To get better agreement, a stronger weight for the ϱ' in the high-energy fit, the same sign for the residues of the ϱ and ϱ', and

(5) We have taken $f^2 = 0.081 \pm 0.002$. For the real part we have used the data quoted in ref. (6) and for the total cross-sections the data quoted in ref. (7).

(6) G. HÖHLER, J. BAACKE and R. STRAUSS: *Phys. Lett.*, 21, 223 (1966).

(7) B. AMBLARD, P. BORGEAUD, Y. DUCROS, P. FALK-VAIRANT, O. GUISAN, W. LASKAR, P. SONDEREGGER, A. STIRLING, M. YVERT, A. TRAN HA and S. D. WARSHAW: *Phys. Lett.*, 10, 138 (1964); A. CITRON, W. GALBRAITH, T. F. KYCIA, B. A. LEONTIĆ, R. H. PHILLIPS, A. ROUSSET and P. H. SHARP: *Phys. Rev.*, 144, 1101 (1966); G. GIACOMELLI: *Proc. S.I.F.* Course XL (to appear),

(8) R. DOLEN, D. HORN and C. SCHMID: *Phys. Rev. Lett.*, 19, 402 (1967).

(9) R. K. LOGAN, J. BEAUPRE and L. SERTORIO: *Phys. Rev. Lett.*, 18, 259 (1967).

a value of the ϱ' intercept α' nearer to that quoted by HÖGAASEN *et al.* (10) ($\alpha' = -0.63$) rather than to that of LOGAN *et al.* (9) ($\alpha' = 0.17$) is required. Instead of the ϱ' one may introduce a cut. With the parameters given by CHIU *et al.* (11) the situation does not improve. In order to go in the right direction, the discontinuity $D(J, t=0)$ along the cut has to be of the same sign as the ϱ residue and be particularly large for low values of J.

The contribution of the resonances is of negligible weight in the above considerations; indeed, with our choice of ν_0, it is roughly 15 times less than that of the ϱ (12).

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(10) H. HÖGAASEN and W. FISCHER: *Phys. Lett.*, 22, 516 (1966).

(11) C. B. CHIU and J. FINKELSTEIN: *Nuovo Cimento*, 48 A, 820 (1967).

(12) The lower value $f^2 = 0.070 \pm 0.004$ proposed by HÖHLER (13) would lead to higher values of α, however still considerably smaller than $\alpha = 0.58$.

(13) G. HÖHLER, R. STRAUSS and F. GLEISBERG: unpublished.

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