PAIR EXCHANGE REACTIONS

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The possibility of using the pair (or double charge) exchange reactions to populate interesting – and sometimes elusive – collective excitation modes is studied. Two estimates of the cross sections are made: one is an approximation of the reaction amplitudes built upon the experimental two-nucleon transfer intensities and the other is based on a more detailed DWBA calculation. Both confirm the fact that this type of processes may turn out to be a useful spectroscopic tool.

Experiments involving the exchange of a pair of neutrons and a pair of protons between the target and the projectile have been recently performed [1]. These reactions may complement the information obtained, for instance, in pion double charge exchange reactions (π^+, π^-) which also depend on the two-particle correlations [2]. They can also populate the double analogue states ‡1. The present note intends to further stress the advantages of using pair exchange reactions ‡2 like (^{14}C, ^{14}O) as a new tool in nuclear spectroscopy, in order to excite, for instance, two-particle–two-hole states, and thus obtain specific (and, in many cases, so far elusive) information about two-phonon states. In these reactions, the protons may be affected in three different ways: (i) the reaction may annihilate a proton addition pairing phonon if the two protons are in the valence shell; (ii) it creates a particle–hole state if one to be removed proton is in the valence shell and the other below or (iii) it creates a pairing removal phonon if both particles are in the shell below. The reaction presents similar three options for the addition of two neutrons.

Thus, if we consider a target nucleus described as a proton addition and a neutron removal pairing phonon, we may find enhanced transitions leading to the following states in the closed shell system: ground state, proton and neutron pairing two-phonon states, proton and neutron pairing four-phonon states, particle–hole phonon states and two particle–hole phonon states. Moreover, the respective cross sections may be related to the corresponding two-particle transfer processes, in a similar way as has been done for the alfa-transfer process [3].

Using the phonons labelled by (ν, α, τ, λ) as building blocks of the spectrum of even nuclei we construct the states around doubly magic nuclei. Here α denotes pair addition (a) or pair removal (r); τ labels neutron (n) or proton (p) pairing bosons; both α and τ equal zero for particle–hole bosons; λ is the angular momentum quantum number and ν is any extra label that may be needed in order to specify the states. If k (h) denotes single-particle states above (below) the Fermi energy, the TDA boson creation operators are expressed as linear combination of pairs of particles, pairs of holes or particle–holes with amplitudes $X(\nu, \mathrm{a}, \tau, \lambda; k, k')$, $X(\nu, \mathrm{r}, \tau, \lambda; h, h')$ and $X(\nu, 0, \tau, \lambda; k, h)$, respectively.

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‡1 This fact has been pointed to us by F. Bertrand.
‡2 We prefer the name pair exchange reactions to double charge exchange reactions since the last one concerns the electromagnetic interaction which is not very relevant in nuclear structure calculations.

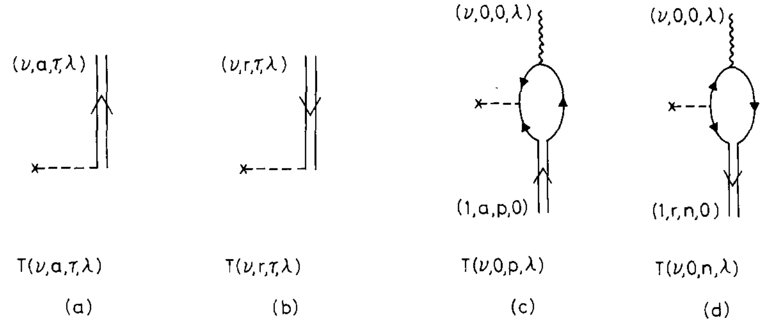

Fig. 1. Two-particle transfer processes.

The matrix elements $T(\nu, \alpha, \tau, \lambda)$ corresponding to the two-body transfer processes of fig. 1 which excite the modes $(\nu, \alpha, \tau, \lambda)$ can be written as

$$T(\nu, \alpha, \tau, \lambda) = \sum_{j \geqslant j'} X(\nu, \alpha, \tau, \lambda; j, j') g(j, j'; \tau, \lambda), \tag{1}$$

where

$$X(\nu, \alpha, \tau, \lambda; j, j') = X(\nu, \alpha, \tau, \lambda; j, j'), \quad \alpha = \text{a or r},$$
$$X(\nu, 0, \mathrm{p}, \lambda; k, h) = -\sqrt{2}\, X(1, \mathrm{a}, \mathrm{p}, 0; k, k) X(\nu, 0, \mathrm{p}, \lambda; k, h)/\hat{k},$$
$$X(\nu, 0, \mathrm{n}, \lambda; k, h) = -\sqrt{2}\, X(1, \mathrm{r}, \mathrm{n}, 0; h, h) X(\nu, 0, \mathrm{n}, \lambda; k, h)/\hat{h}, \tag{2}$$

and the coefficients $g(j, j'; \tau, \lambda)$ act as weighting factors that characterize each particular reaction process. In the amplitudes X and matrix elements T, τ = n or p for all modes (α = a, r, 0). These processes have been observed throughout the periodic table with enhanced cross sections in case of reactions populating collective modes. Since these enhancements can be ultimately related to the short-range character of the residual nuclear interaction [4] one may also expect similar coherences for the pair exhange reactions. Thus a rough prediction of the results can be made using the operator

$$Q_I \approx \sum_{\substack{j_\mathrm{n} \geqslant j'_\mathrm{n}, j_\mathrm{p} \geqslant j'_\mathrm{p} \\ \lambda_\mathrm{n}, \lambda_\mathrm{p}}} g(j_\mathrm{n}, j'_\mathrm{n}; \mathrm{n}, \lambda_\mathrm{n}) g(j_\mathrm{p}, j'_\mathrm{p}; \mathrm{p}, \lambda_\mathrm{p}) \frac{[(C^+_{j_\mathrm{n}} C^+_{j'_\mathrm{n}})^{\lambda_\mathrm{n}} (C_{j_\mathrm{p}} C_{j'_\mathrm{p}})^{\lambda_\mathrm{p}}]^I}{[(1+\delta_{j_\mathrm{n} j_\mathrm{n}'})(1+\delta_{j_\mathrm{p} j'_\mathrm{p}})]^{1/2}}. \tag{3}$$

Under this assumption, the amplitudes for the pair exchange processes may be simply related to the amplitudes (1) of the two-particle transfer processes. In particular, if the target nucleus A is represented by a proton addition and a neutron removal phonon ($|(1\mathrm{ap}0)(1\mathrm{rn}0)\rangle$), we obtain the processes of fig. 2. The corresponding reaction amplitudes (relative to the g.s.) are also given in the fig. 2. The g.s. transition matrix element is $T(1, \mathrm{a}, \mathrm{p}, 0)\, T(1, \mathrm{r}, \mathrm{n}, 0)$.

For the sake of simplicity of the final states we focus our present study to the reaction ^{208}Po(^{14}C, ^{14}O)^{208}Pb but other nuclei (and eventually targets easier to use) can be similarly considered. The pair exchange amplitudes can be estimated by means of the expressions appearing in fig. 2 for each final state. The values of the two-nucleon transfer amplitudes (1) are obtained from the experimental cross sections of (t p) and (p t) reactions reported in refs. [5,6] (we assume that the ratios between the T-matrix elements are the same for protons as for neutrons). The resulting squared transition amplitudes for populating states in ^{208}Pb via the direct pair exchange reaction (relative to the g.s.) are given in the fourth column of table 1.

An improved calculation of the reaction process must take into account, for instance, the Q-effects and angular momentum mismatches in the entrance and exit channels, the influence of the S = 1 component of the transferred

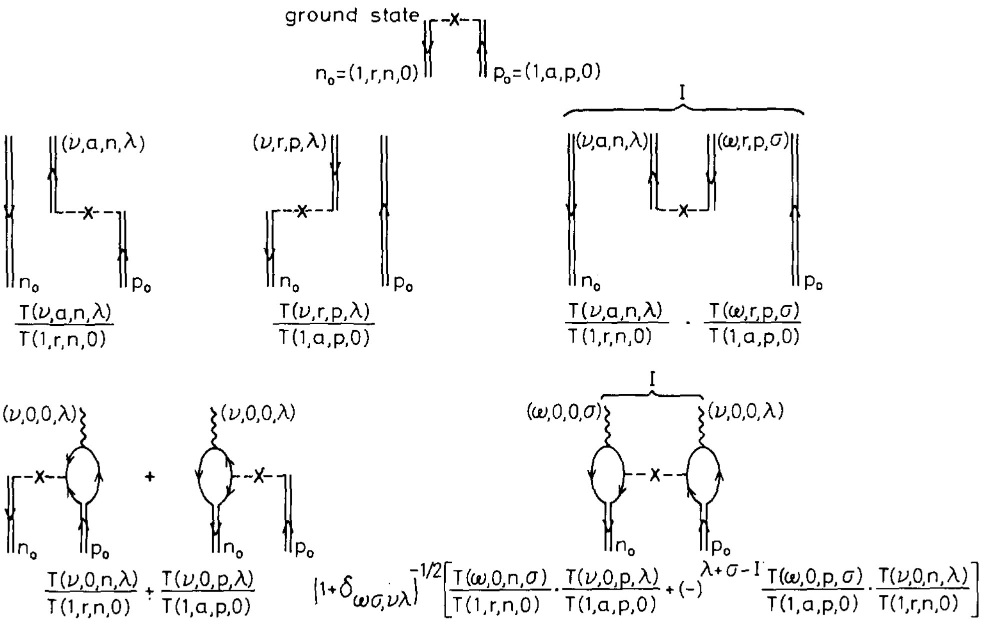

Fig. 2. Pair exchange processes.

pair, etc. Such a calculation has been made [9] under the following simplifying assumptions: (a) the reaction proceeds via a one-step process (DWBA); (b) the responsible interaction is an effective zero-range force between the centre of mass of the transferred diproton and dineutron; (c) the relative coordinate in the initial and final channels is the same as the relative coordinate $\boldsymbol{R}$ between the two cores ^{206}Pb and ^{12}C (no recoil approximation); (d) the transfer only takes place along the line joining the two cores (Buttle–Goldfarb type of approximation); (e) the overlaps between initial and final intrinsic motion in the dinucleons are calculated using the same size parameter in both the lighter and heavier nuclei; (f) the motion of the diproton and dineutron in the ejectile and projectile, respectively, is obtained from a pure $(p_{1/2})^2$ configuration. With these assumptions the nuclear matrix element becomes [9]

$$\langle {}^{14}\mathrm{O}; \mathrm{B}, JM|V|\mathrm{A}, \mathrm{g.s.}; {}^{14}\mathrm{C}\rangle = -(4\pi^{3/2})^{-1} V_0 Y^*_{JM}(\Omega_R)\, \mathrm{i}^{J_\mathrm{n}+J_\mathrm{p}} \langle J_\mathrm{n} J_\mathrm{p} 00|J0\rangle \hat{J}^{-1}$$

$$\times \sum_{\substack{n_\mathrm{p}S_\mathrm{p}I_\mathrm{p}\\ n_\mathrm{n}S_\mathrm{n}I_\mathrm{n}}} \int \mathrm{d}r\, \Psi^*(1-n_\mathrm{p}-S_\mathrm{p}, S_\mathrm{p}; |r-R|)\, \Psi^*(N_\mathrm{n}, I_\mathrm{n}; r)\, \Psi(N_\mathrm{p}, I_\mathrm{p}; r)\, \Psi(1-n_\mathrm{n}-S_\mathrm{n}, S_\mathrm{n}; |r-R|)$$

$$\times \prod_{\tau=\mathrm{n,p}} \mathrm{i}^{I_\tau-S_\tau-J_\tau}(-)^{N_\tau+L_\tau} \hat{I}_\tau \hat{L}^2_\tau \hat{S}^3_\tau \langle I_\tau S_\tau 00|J_\tau 0\rangle \begin{Bmatrix} 1 & 1 & S_\tau \\ 1/2 & 1/2 & 1/2 \end{Bmatrix} \begin{Bmatrix} S_\tau & S_\tau & S_\tau \\ I_\tau & J_\tau & L_\tau \end{Bmatrix}$$

$$\times (1+\delta_{S_\tau 0})^{-1/2} \sum_{j_{1\tau}\geqslant j_{2\tau}} \chi(\nu_\tau, \alpha_\tau, \tau, J_\tau; j_{1\tau}, j_{2\tau})\, \hat{j}_{1\tau} \hat{j}_{2\tau} \begin{Bmatrix} j_{1\tau} & l_{1\tau} & 1/2 \\ j_{2\tau} & l_{2\tau} & 1/2 \\ J_\tau & L_\tau & S_\tau \end{Bmatrix} \langle n_\tau S_\tau, N_\tau I_\tau, L_\tau | n_{1\tau} l_{1\tau}, n_{2\tau} l_{2\tau}, L_\tau\rangle, \quad (4)$$

where V_0 is the strength of the zero-range force and the standard notation for the racahlogy and h.o. transformation brackets has been used. The wave functions in the integrand represent the radial motion of the centre of mass of the dinucleon in a Saxon–Woods potential with parameters that have been adjusted in order to yield empirical separation energies. The structure coefficients χ [eqs. (2) and (4)] in the heavier systems have been calculated using amplitudes X [eq. (2)] given by

$$X(\nu, \alpha, \tau, \lambda; j, j') = \frac{\Lambda(\nu, \alpha, \tau, \lambda)\langle j \| Y_\lambda \| j' \rangle (-)^{n_j + n_{j'}}}{[\epsilon_j + \epsilon_{j'} - \omega(\nu, \alpha, \tau, \lambda)](1 + \delta_{jj'})^{1/2}}, \tag{5}$$

where the Λ's are normalization constants. All the single-particle states belonging to the shell above and below have been considered in (5). Empirical energies are used for these states and for the phonon frequencies ω.

Results of a calculation as the one just described are given in the fifth column of table 1.

An estimate of absolute intensities can be made by performing a similar DWBA calculation for the reaction $^{40}Ca(^{14}C, ^{14}O)^{40}Ar$ at 51 MeV, already observed [1]. On the assumption that the normalization constant obtained at this energy and mass region can be taken to the Pb region at 250 MeV, we predict a peak differential cross section for the ground-state transition of about 0.8 μb/sr.

An analysis of the relative intensities of table 1 shows the largest cross section for the neutron pairing vibrational state at 4.86 MeV. The cross sections presented by the other pairing two-phonon (and the 10.16 MeV four-phonon) states are predicted one order of magnitude smaller: the presence of the $S = 1$ pair channels impairs the rough estimates of column four (where only the two-particle $S = 0$ transfers are considered). Each S two-particle transfer channel carries comparable strength giving rise to important enhancements or cancellations. However, the population of all states belonging to the pairing phonon family is considerably larger than the population of particle–hole phonon states. In particular the population of two particle–hole phonon states would be vanishingly small, reflecting the fact that the processes represented in figs. 1c and 1d are of next higher order than those corresponding to figs. 1a and 1b from the point of view of nuclear field theory [thus, the factors $\hat{k}$ or $\hat{h}$ appearing in the corresponding denominators in (2)].

Table 1
Predicted cross sections for the reaction $^{208}Po(^{14}C, ^{14}O)^{208}Pb$.

Pair exchange residual state			$d\sigma/d\sigma_{g.s.}$	
E_x [a)] (MeV)	J^π	phonon description	eq. (4) [b)]	DWBA [c)]
0.0	0^+	(1, 0, 0, 0)	1.0	1.0
2.61	3^-	(1, 0, 0, 3)	0.66	0.31
3.20	5^-	(1, 0, 0, 5)	1.5	1.3
4.86	0^+	(1, a, n, 0)(1, r, n, 0)	0.44	2.3
5.20*	6^+	$(1, 0, 0, 3)^2$	0.01	2×10^{-4}
5.30*	0^+	(1, a, p, 0)(1, r, p, 0)	2.3	0.15
5.55 + 5.80	2^+	(1, a, n, 2)(1, r, n, 0)	1.1	0.15
5.81	8^+	(1, 0, 0, 3)(1, 0, 0, 5)	0.05	6×10^{-3}
6.37*	2^+	(1, a, p, 2)(1, a, p, 0)	1.8	0.13
6.40*	10^+	$(1, 0, 0, 5)^2$	0.04	5×10^{-3}
10.16*	0^+	(1, a, n, 0)(1, r, n, 0)(1, a, p, 0)(1, r, p, 0)	1.0	0.22

a) Asterisks denote estimated values of the excitation energies.
b) Calculated using formulae (4). Positive coherence is assumed between the two ratios contributing to particle–hole states.
c) DWBA calculations for incident $E(^{14}C) = 250$ MeV. The optical model parameters are taken from ref. [7] and the same set used for entrance and exit channels. Results obtained with other choice (ref. [8]) differ by only few percents.

Finally, it should be noted that ^{208}Pb is not the only doubly magic nucleus that may be studied in the pair exchange reactions. Similar considerations can be made for ^{48}Ca, using the ^{48}Ti (^{14}C, ^{14}O) reaction and for ^{56}Ni and ^{146}Gd if one performs the reactions (^{16}O, ^{16}C) on ^{56}Fe and ^{146}Sm, respectively. It is expected that these cases will be more favoured from the point of view of the reaction mechanism than the Pb one, because the differences between the grazing angular momentum for entrance and exit channels are smaller.

Obviously other residual nuclei can also be populated with the advantage of using easier targets. However, the drawbacks of non-doubly closed shell final nuclei are (i) the splitting of single states due to the presence of a larger background of non-collective excitations and (ii) the fact that the parentage between target and residual nucleus may prevent the population of interesting collective states.

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