PARTITION FUNCTION FOR N = 2 SUPERSTRING ON THE TORUS

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ABSTRACT

A general 1-loop partition function for the 4 dimensional superstring is explicitly constructed as a tensor product of N=2 superconformal theories. Its vanishing is shown by using a generalized Riemann identity.

ICSU No.: 11.17.+y

A predictive theory of the real world will be provided by string theory only if the problem of the large degeneracy of vacua is solved. As is well known, despite the almost uniqueness of 10 dimensional superstring theory, when compactification is called for, many conformal field theories can be used for the internal compactified space.

An interesting compactification scheme, based on N=2 superconformal field theory, has recently been proposed by D. Gepner [1]. The massless spectra of these models agree exactly with those of manifolds with vanishing first Chern class, obtained by the compactification of the field theory limit of the heterotic string [2]. This equivalence suggests that deeper understanding of such models might help to classify the possible classical ground states of string theory.

Gepner's models are constructed by considering the tensor product of r copies of N=2 superconformal minimal models [3], such that the internal central charge, given by

$$c = \sum_{i=1}^{r} 3 \, \frac{(m_i - 2)}{m_i} \qquad (1)$$

equals 9. The space-time degrees of freedom are provided in the light cone gauge by two free bosons X^{μ} and two free fermions Ψ^{μ} for each chiral sector of the four dimensional superstring. They contribute with an extra 3 to the central charge leading to a c=12 anomaly-free theory. Of all the possible combinations built in this way, only those states which are related by the action of space-time supersymmetry and which have odd integer total U(1) charge must be considered.

Supersymmetry projection should guarantee the vanishing of the partition function of the theory. A generalized Riemann-theta identity was found in [4,5] to show this vanishing for the 1^9 model, i.e. 9 copies of c=1 theories for the heterotic string.

In this article, an explicit construction of the partition function for the general case of r internal theories satisfying (1) is presented. By adapting the Riemann identity derived in [4] we will prove explicitly the vanishing of the corresponding partition functions. We will follow Gepner's general prescriptions summarized above but will make an alternative choice for the characters of the minimal models which seems more convenient for our purposes.

The primary fields in the Neveu-Schwarz (NS) sector of each of the internal theories are given in the Coulomb gas formalism [6] by

$$N_{jk} = :e^{i(1/2-j)\alpha_+\phi + i(1/2-k)\alpha_+\bar{\phi}}: \tag{2}$$

with $j,k \in \mathbb{Z}+1/2$; $0<j,k,j+k\leq m-1$ and $\alpha_+^2=1/m$. They have conformal dimension

$$h_{jk} = \frac{4jk-1}{4m}$$

and U(1) charge

$$Q_{jk} = \frac{j-k}{m}$$

The characters of the unitary representations generated by these fields are [7], in the A+ sector

$$\chi_{jk}^{A+(m)}(\tau,z) = \frac{\vartheta[{}^0_0](\tau,z)}{\eta^3(\tau)} \; x^{h_{jk}-c/24} \, y^{Q_{jk}} \Gamma_{jk}^{(m)}(\tau,z) \tag{3}$$

where

$$\Gamma^{(m)}_{jk} = \prod_{n=1}^{\infty} \frac{(1-x^{mn+j+k-m})(1-x^{mn-j-k})(1-x^{mn})^2}{(1+x^{mn-j}y)(1+x^{mn+j-m}y^{-1})(1+x^{mn-k}y^{-1})(1+x^{mn+k-m}y)}$$

and $x = e^{2\pi i\tau}$, $y = e^{2\pi iz}$.

The characters in the other sectors can be easily obtained from (3) by shifting z by 1/2, $\tau/2$ and $\tau/2+1/2$. In fact, using that

$$\chi^{A+(m)}_{j,k}(\tau, z+n\tau/2) = x^{-n^2C/24}\, y^{-nC/6}\, \chi^{A+(m)}_{j-n/2,k+n/2}(\tau, z) \qquad (4)$$

we have

$$\begin{aligned}
\chi^{A-}_{jk}(\tau,0) &= \chi^{A+}_{jk}(\tau,1/2) \\
\chi^{P+}_{j-1/2,k+1/2}(\tau,0) &= \chi^{A+}_{jk}(\tau,\tau/2) \\
\chi^{P-}_{j-1/2,k+1/2}(\tau,0) &= \chi^{A+}_{jk}(\tau,1/2+\tau/2)
\end{aligned} \qquad (5)$$

Notice that a shift in $z=\tau/2$ leads to the integer indices $j'=j-1/2$, $k'=k+1/2$ as corresponds to a Ramond state.[1]

Characters for the c=12 theories can now be constructed by considering the products of the r internal theories for different values of j and k, times the space-time part (with the constraint given by (1)). Thus,

1Notice that the characters here are built from the primary states by considering all possible descendants, while in [1] they are grouped into sets having descendants differing by even integer U(1) charges.

$$\chi^{A+(m)}_{jk}(\tau,z) = \frac{\vartheta[{0 \atop 0}](\tau,z)}{\eta(\tau)} \prod_{i=1}^{r} \chi^{A+(m_i)}_{j_i k_i}(\tau,z) \tag{6}$$

The fact that all component characters are evaluated at the same z-argument guarantees that they all lie in the same sector. By shifting z by the different values considered in (5) we obtain the total character for the four sectors i.e. NS^+, NS^-, R^+, R^-.

Recall also that, in order to ensure that space-time has the same boundary conditions as the internal sectors, the ϑ function for the space-time character is the same as the one that appears in the internal part. It corresponds to a linear combination of the characters of the representations of the SO(2) algebra.

As already mentioned, only those characters related by supersymmetry must be kept. The Susy operator for the internal theory can be written as the direct product of the operators [6,8,9]:

$$S_i = R^+_{01} = \sigma^+_i \ :e^{1/2\ \alpha_+\phi_i - 1/2\ \alpha_+\bar{\phi}_i}: \tag{7}$$

The action of S on a NS state yields, from OPE, the Ramond state

$$S\, N_{jk} = \sigma^+(z)\ :e^{i(1-j)\alpha_+\phi\ -\ ik\alpha_+\bar{\phi}}: \ = R^+_{j-1/2,k+1/2}$$

The character for the field $S^n N_{jk}$ can then be written as

$$S^n\chi^{A+}_{jk}(\tau,z) = x^{n^2c/24}\ \ y^{nc/6}\ \chi^{A+}_{jk}(\tau,z+n\tau/2) = \chi_{j-n/2,k+n/2}(\tau,z)$$

Therefore the full Susy operator acting on (6) reads

$$S^n\chi^{(m)}_{jk}(\tau,z) = x^{\frac{1}{24}n^2c_T}\, y^{\frac{1}{6}nc_T}\, \vartheta[{0 \atop 0}](\tau,z+n\tau/2) \prod_{i=1}^{r} \chi^{(m_i)}_{j_i k_i}(\tau,z+n\tau/2)$$

where $c_T = 3 + \sum_{i=1}^{r} c_i$.

To construct the supersymmetrized character, all n's must be summed over, including a minus sign for the odd-n Ramond states. For the sake of simplicity we will first consider the case in which all the internal theories have the same central charge c=3(m-2)/m. (In this case, the only allowed theories have c= 1, 3/2, 9/5, 9/4). The most general case will be discussed at the end.

We propose

$$\chi^{SUSY}_{jk}(\tau,z) = \sum_{n=0}^{2m-1} \sum_{s=0}^{2m-1} (-1)^{n+s}\, x^{\frac{1}{24}n^2c_T}\, \chi^{A+}_{jk}(\tau,z+n\tau/2+s/2) \qquad (8)$$

Notice that due to the symmetries

$$\chi_{jk}(\tau,z) = \chi_{j-m,k+m}(\tau,z) \propto S^{2m}\chi_{jk}(\tau,z)$$

$$\vartheta[{0 \atop 0}](\tau,z) = \vartheta[{m \atop 0}](\tau,z) \propto \vartheta[{0 \atop 0}](\tau,z+m\tau)$$

the sum ranges from 0 to 2m-1. Moreover, the sum over s ensures that only those combinations of characters with odd integer total U(1) charge are selected. This can be seen by rewriting (8) as

$$\chi^{SUSY}_{jk}(\tau,0) = \sum_{s,n=0}^{2m-1} (-1)^{n+s}\, x^{\frac{1}{24}n^2c_T}\, \frac{1}{2}\left\{e^{2\pi i\sum_i(Q_i)s/2}\left[\chi^{A+}_{jk}(\tau,n\tau/2)+\chi^{A-}_{jk}(\tau,n\tau/2)\right]+\right.$$

$$+\ e^{2\pi i\sum_i (Q_i+1)s/2}\left[\chi^{A+}_{jk}(\tau,n\tau/2)-\chi^{A-}_{jk}(\tau,n\tau/2)\right]\Big\}$$

The sum over s selects either $\chi^{A+}+\chi^{A-}$ when the internal charge is odd and spacetime charge is even, or $\chi^{A+}-\chi^{A-}$ when they are even and odd respectively.

It is now simple to put together the left and right sectors in order to obtain the full modular invariant and supersymmetric partition function with $c_T=12$:

$$Z(\tau) = \sum_{lq\bar{l}\bar{q}} N_{lq\bar{l}\bar{q}}\ \chi^{SUSY}_{lq}(\tau)\ \ \chi^{SUSY}_{\bar{l}\bar{q}}(\bar{\tau}) \qquad (10)$$

where we have redefined the indices such that $l=j+k$ and $q=j-k$ with $l=1,\dots,m-1$ and $q=1-l,3-l,\dots,l-1$(by steps of 2) and $N_{lq\bar{l}\bar{q}}=\prod_{i=1}^{r} N_{l_i q_i \bar{l}_i \bar{q}_i}$, with $N_{l_i q_i \bar{l}_i \bar{q}_i}$ the modular coefficients [10] which ensure the modular invariance of (10). With our starting expression (8) for the total character the coupling of the spacetime part satisfies $n=\bar{n}$ mod.2. In particular we choose

$$N_{lq\bar{l}\bar{q}} = \delta(l-\bar{l}).\delta(q-\bar{q}) = \frac{1}{4m^2}\sum_{\alpha,\beta=0}^{2m-1} e^{i\pi\alpha(q-\bar{q})/m}\ e^{i\pi\beta(l-\bar{l})/m} \qquad (11)$$

The partition function (10) can be rewritten as

$$Z(\tau,\bar{\tau}) = \prod_{i=1}^{r}\left\{\frac{1}{4m^2}\sum_{\alpha_i,\beta_i=0}^{2m-1}\right\} Z_L(\tau,\alpha_i,\beta_i)\ \bar{Z}_R(\bar{\tau},\bar{\alpha}_i,\bar{\beta}_i)$$

with

$$Z_L(\tau,\alpha,\beta) = \sum_{n,s=0}^{2m-1} (-1)^{n+s}\, x^{n^2/2}\, \vartheta[{}^0_0](\tau, n\tau/2+s/2) \quad \times$$

$$\prod_{i=1}^{r} \sum_{l_i q_i} e^{i\pi(\alpha_i q_i+\beta_i l_i)/m}\, \chi_{l_i q_i}(\tau, s/2+n\tau/2) \qquad (12)$$

and a similar expression for $\bar{Z}_R$ (Recall that here NS (R) states are coupled with $\overline{\mathrm{NS}}$ ($\bar{\mathrm{R}}$) states). Its modular invariance can be checked explicitly by using the modular transformation properties given in [8,10].

This partition function corresponds to the 4-dimensional superstring; the heterotic string may be obtained from it by modifying $\bar{Z}_R$ as indicated in [1].

We will show that Z_L vanishes identically. For this purpose we will express Z_L as proportional to a product of theta functions, such that a generalized Riemann identity can be used.

We start by observing that (12) can be cast in the following form:

$$Z_L = \sum_{s=0}^{2m-1} \sum_{n=0}^{1} (-1)^{n+s}\, x^{n^2/2}\, \vartheta[{}^0_0](\tau, s/2+n\tau/2) \quad \times$$

$$\prod_{i=1}^{r} \sum_{l_i q_i} \chi_{l_i q_i}(\tau, s/2+n\tau/2+\alpha_i)\, e^{i\pi\beta_i l_i/m} \Bigg|_{2\Sigma\alpha_i \in m\mathbb{Z}} \qquad (13)$$

This can be shown by dividing the sum over n's into a sum over even integers n=2p and odd integers n=2p+1, where p=0,...,m−1. The sum over p can then be performed since, due to the periodicity $\chi_{l,q-2m}=$

$\chi_{l,q}$,

$$\sum_{p=0}^{m-1} \sum_{q_i} e^{2i\pi\alpha_i q_i/m} \; \chi_{l_i,q_i-2p}(\tau,z) = \sum_{p=0}^{m-1} e^{4i\pi p\alpha_i/m} \sum_{q_i} e^{2i\pi\alpha_i q_i} \; \chi_{l_i,q_i}(\tau,z) =$$

$$= \sum_{q_i} \chi_{l_i,q_i}(\tau,z+\alpha_i)\Big|_{2\Sigma\alpha_i\in\, m\mathbb{Z}} \tag{14}$$

In the last equality the sum over p leads to the condition $2\Sigma\alpha_i \in m\mathbb{Z}$. Besides, the α_i can be written inside the argument as may be seen from (3) or, more easily, from

$$\chi_{q,l}(\tau,z) = f_l(\tau)\frac{\vartheta\begin{bmatrix}0\\0\end{bmatrix}(\tau,z)}{\vartheta\begin{bmatrix}(-q-l+m)/2m\\0\end{bmatrix}(m\tau,z)\;\;\vartheta\begin{bmatrix}(-q+l-m)/2m\\0\end{bmatrix}(m\tau,z)} \tag{15}$$

with

$$f_l(\tau) = \vartheta\begin{bmatrix}-\frac{l}{m}+\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(m\tau,0)\;\prod_{n=1}^{\infty}(1-x^{mn})^3\;\frac{e^{i\pi(\frac{2l-m}{2m})}}{\eta^3(\tau)}\quad x^{-\frac{l}{4m}+\frac{m-1}{8}}$$

a function independent of q and z.

The aim now is to evaluate explicitly the sum over the q_i's and to show that it decouples from the sum over the l_i's.

By using the periodicities $\chi_{q,l} = \chi_{q+m,m-l} = \chi_{q+2m,l}$ the q's and l's can be reorganized by grouping the values $l=1,m-1$; $l=2,m-2$;... such that

$l=1,\dots,(m-1)/2 \qquad q = -2k+m-l \qquad$ for m odd,

$l=1,\ldots,m/2 \qquad q=-2k+m-l-1 \qquad \text{for } m \text{ even}$

with k ranging from 0 to m-1, independently of l.

We will consider here only the odd m case, since a similar reasoning holds for even m.[2]

The sum over q in (13) leads to

$$\sum_{k=0}^{m-1} \frac{1}{\vartheta\begin{bmatrix} k/m \\ 0 \end{bmatrix}(m\tau,z)\ \vartheta\begin{bmatrix} (k+1)/m \\ 0 \end{bmatrix}(m\tau,z)} = \frac{\sum_{k=0}^{m-1}\prod_{a=0}^{m-3}\vartheta\begin{bmatrix} (k+b_a)/m \\ 0 \end{bmatrix}(m\tau,z)}{\prod_{k=0}^{m-1}\vartheta\begin{bmatrix} k/m \\ 0 \end{bmatrix}(m\tau,z)}$$

where we have extracted the common denominator and have used the invariance of ϑ's under shifts of the characteristics by integers. The b_a are integers, all distinct mod m and $b_a \neq 0, l$.

Notice that $\prod_{k=0}^{m-1}\vartheta\begin{bmatrix} k/m \\ 0 \end{bmatrix}(m\tau,z)$ has the same zeros as the $\vartheta\begin{bmatrix} 0 \\ 0 \end{bmatrix}(\tau,z)$ appearing in the numerator of eq.(15). The two expressions must therefore be proportional because $\vartheta(\tau,z)$ are well defined functions on the Jacobi variety $J=T/n\tau+m$. In fact it is possible to check that the proportionality factor is

$$g(\tau) = x^{\sum_{k=0}^{m-1}(\frac{k}{2m}) - \sum_{k=0}^{(m-1)/2}(k^2-\frac{1}{2})} \prod_{n=1}^{\infty}\left[\frac{(1-x^{mn})^m}{(1-x^n)}\right] \tag{17}$$

2 For even m, we are double counting the term $l=m/2$, so we must include a factor 1/2 for $f_{m/2}(\tau)$.

The sum in the numerator of (16) can now be rewritten as

$$\prod_{a,b=0}^{m-3}\left(\frac{1}{m}\sum_{p_{ab}=0}^{m-1}\right)\prod_{a=0}^{m-3}\left[\sum_{k_a=0}^{m-1}\left(\vartheta\begin{bmatrix}(k_a+b_a)/m\\ 0\end{bmatrix}(m\tau, z+\sum_{b=0}^{m-3}(p_{ab}-p_{ba}))\right)e^{-2i\pi b_a\Sigma(p_{ab}-p_{ba})/m}\right]$$

$$=\prod_{a,b=0}^{m-3}\left(\frac{1}{m}\sum_{p_{ab}=0}^{m-1}\right)e^{-2i\pi\sum_a b_a\sum_b(p_{ab}-p_{ba})/m}\prod_{a=0}^{m-3}\vartheta\begin{bmatrix}0\\ 0\end{bmatrix}\left(\frac{\tau}{m},\frac{z}{m}+\sum_{b=0}^{m-3}(p_{ab}-p_{ba})/m\right)$$

(18)

where we have used a different k_a for each ϑ in order to be able to interchange the sum and the product, and then identified them by introducing delta functions as sums over p_{ab}. The sum over k_a was performed in the second member.

By collecting the results from (15), (16), (17) and (18) and inserting them in (12), it is possible to see that Z_L is a linear combination of factors of the form:

$$\tilde{Z}_L(\tau,\alpha_i^a) = \sum_{s=0}^{2m-1}\sum_{n=0}^{1}(-1)^{n+s}\vartheta\begin{bmatrix}n/2\\ s/2\end{bmatrix}(\tau,0)\prod_{i=1}^{r}\prod_{a=0}^{m-3}\vartheta\begin{bmatrix}n/2\\ s/2m\end{bmatrix}\left(\frac{\tau}{m},\frac{\alpha_i^a}{m}\right)e^{-2i\pi n\sum\alpha_i c_i/6} \tag{19}$$

where we defined $\alpha_i^a = \alpha_i + \sum_b p_{ab}-p_{ba}$.

A generalized Riemann identity can now be borrowed from [4]:

$$\sum_{s=0}^{2m-1}(-1)^s\,\vartheta^{(4m)}\begin{bmatrix}0\\ s/2m\end{bmatrix}(\frac{\tau}{m},X) = \sum_{s=0}^{2m-1}(-1)^s\,\vartheta^{(4m)}\begin{bmatrix}1/2\\ s/2m\end{bmatrix}(\frac{\tau}{m},TX) \tag{20}$$

where $(TX)_i = X_i-\frac{1}{2m}\sum_j X_j$.

As $4m=r(m-2)+m$, the following choices can be made for X_i:

$X_{(i-1)(m-2)+a+1} = \frac{\alpha_i^a}{m}$, for $i=1,\dots,r$ and $a=0,\dots,m-3$; and

$X_{r(m-2)+1},\ldots,X_{4m}=0$, such that $\sum_{i=1}^{4m} X_i = (m-2)\sum_{i=1}^{r}\frac{\alpha_i}{m}$. By including the constraint $2\sum\alpha \in m\mathbb{Z}$, (20) reads

$$\sum_{s=0}^{2m-1}\sum_{n=0}^{1}(-1)^{s+n}\,\vartheta^{(4m)}\begin{bmatrix}n/2\\ s/2m\end{bmatrix}(\frac{\tau}{m},X)\;e^{-2\pi inc\Sigma\alpha_i/6}=0 \tag{21}$$

By expanding the $\vartheta^{(4m)}$ as an infinite sum over $n_1,\ldots,n_{4m}$ and identifying

$$n_{(i-1)(m-2)+1}=\ldots=n_{i(m-2)}\equiv n_i$$
$$n_{r(m-2)+1}=\ldots=n_{4m}\equiv n_0$$

we obtain

$$\sum_{s=0}^{2m-1}\sum_{n=0}^{1}(-1)^{n+s}\,\vartheta\begin{bmatrix}n/2\\ s/2\end{bmatrix}(\tau,0)\prod_{i=1}^{r}\prod_{a=1}^{m-2}\vartheta\begin{bmatrix}n/2\\ s/2m\end{bmatrix}(\frac{\tau}{m},\frac{\alpha_i^a}{m})\;e^{-2\pi inc\Sigma\alpha_i/6}=0 \tag{22}$$

which coincides exactly with our previous expression (19) for $\tilde{Z}_L(\tau,\alpha_i^a)$.

For completeness let us briefly indicate which changes must be made when r internal theories with different central charge c_i are considered (the possible combinations such that $\Sigma c_i=9$ were listed in [11])

To construct the supersymmetrized character (8), summation over the n's must be extended from 0 to 2m-1, where m is now the least common multiple of the m_i. The left-right coupling condition (11) to be imposed is now

$$\delta(l_i-\bar{l}_i).\delta(q_i-\bar{q}_i) = \sum_{\alpha_i,\beta_i=0}^{2m-1} e^{i\pi\alpha_i(q_i-\bar{q}_i)/m_i}\, e^{i\pi\beta_i(l_i-\bar{l}_i)/m_i} .$$

The constraint the α_i's have to satisfy after summing over p in (13) is now $\sum c_i\alpha_i/3 \in \mathbb{Z}$. Finally, the equality $4m=m+\sum m\frac{(m_i-2)}{m_i}$ suggests the following identifications in the Riemann ϑ identity (20): in the sum over 4m indices, m n_i's are used to reconstruct the spacetime $\vartheta(\tau,z)$, $n_{3m+1}=\ldots=n_{4m}=n_0$. To obtain each of the (m_i-2) $\vartheta(\frac{\tau}{m},\frac{z}{m})$'s it is necessary to identify $\frac{m}{m_i}$ n_i's. So the Riemann identity (20) also ensures the vanishing of $\tilde{Z}_{L^-}$ when considering different internal theories, which in turn implies the vanishing of the full partition function for the 4-dimensional superstring.

Notice that, although we made a specific choice for the modular coefficient (11), more general ones could have been used. In fact, recall that the $\delta(l-\bar{l})$ factor was not used in the derivation and could have been replaced by more general functions $L_{l\bar{l}}$ which preserve the modular invariance of $Z(\tau)$ (see, for example, [10]).

As a final remark, we see that the essential steps in our proof which lead to the Riemann identity (20) can be used to show the vanishing of the general partition function for the superstring in 6 and 8 spacetime dimensions.

ACKNOWLEDGEMENTS

We are specially thankful to T. Jayaraman for his active collaboration in part of this work. Two of us (G.A. and C.N.) acknowledge SISSA for hospitality during the initial steps of this research.

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