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Application of the Yrast Concept to the Pairing Degree of Freedom

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Abstract

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The Yrast approximation is applied to the $T=1$ pairing collective Hamiltonian. A formalism is thus obtained which may incorporate anharmonic effects in a simple way. The experimentally observed anharmonicities in the ^{56}Ni region are satisfactorily reproduced.

1. Introduction

This symposium is devoted to high spin nuclear states and thus it may appear not to be the place to present a contribution dealing only with $J^\pi=0^+$. However, we treat the high values of the isospin and of the number of particles in a similar way as other contributors to this symposium study the problem associated with high angular momentum.

Using the transformation properties of the collective variables α_μ (i.e., those of a quadrupole tensor)

$$\alpha'_\nu = \sum_\mu D^2_{\mu,\nu}\,\alpha_\mu \tag{1}$$

A. Bohr [1] in 1952, constructed a collective hamiltonian depending on the three components of the angular momentum operator and on two intrinsic deformation variables β and γ.

$$H = T_{\rm rot} + T_{\rm vib} + V$$

$$T_{\rm rot} = \frac{1}{8B\beta^2}\left\{\frac{I_x^2}{\sin^2(\gamma-2\pi/3)} + \frac{I_y^2}{\sin^2(\gamma+2\pi/3)} + \frac{I_z^2}{\sin^2\gamma}\right\}$$

$$T_{\rm vib} = \frac{1}{2B}\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} - \frac{1}{2B\beta^2}\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma};$$

$$V = V(\beta^2, \beta^3\cos 3\gamma) \tag{2}$$

In our case the relevant properties of the collective variables Λ_μ is that they transform like the two-body transfer operator, i.e., like a vector in isospace and like an operator creating two particles under rotations in gauge space.

$$\Lambda'_\nu = e^{2i\phi}\sum_\mu D^1_{\mu\nu}(\theta)\,\Lambda_\mu \tag{3}$$

Using (as in (2)) these transformation properties [2], the Pauli quantization principle and symmetry considerations, we arrive to the collective hamiltonian

$$H = T_{\rm rot} + T_{\rm vib} + V$$

$$T_{\rm rot} = \frac{1}{2B\Lambda^2}\left\{\frac{(T_x+M)^2}{\cos^2 2\Gamma} - 2MT_x\frac{(1-\sin 2\Gamma)}{\cos^2 2\Gamma} + \frac{T_y^2}{\cos^2\Gamma} + \frac{T_z^2}{\sin^2\Gamma}\right\}$$

$$T_{\rm vib} = \frac{1}{2B}\frac{1}{\Lambda^5}\frac{\partial}{\partial\Lambda}\Lambda^5\frac{\partial}{\partial\Lambda} - \frac{1}{2B\Lambda^2}\frac{1}{\sin 4\Gamma}\frac{\partial}{\partial\Gamma}\sin 4\Gamma\frac{\partial}{\partial\Gamma}$$

$$V = V(\Lambda^2, \Lambda^4\cos^2 2\Gamma) \tag{4}$$

The deformation variables Λ and Γ play the same role as the variables β and γ in (2). The variable Λ corresponds approximately to the gap parameter in the BCS solution and Γ measures the difference between the neutron and proton gaps in the intrinsic system.

The eigenfunctions of (4) carry the total isospin T and the number of pairs of particles M as good quantum numbers. Thus, the solutions of (4) yield energy eigenvalues and reduced matrix elements for the two-body operator, between $J^\pi=0^+$ states characterized by the pair of quantum numbers (M, T). In the region $40 \lesssim A \lesssim 70$ about 40 states and a similar number of transitions within the collective band have been identified.

It is easy to find [3, 4] the solutions of [4] for some limiting cases, namely (*a*) $V=\frac{1}{2}c\Lambda^2$ (harmonic vibrations; $\omega=(C/B)^{\frac{1}{2}}$, $b=(CB)^{-1/4}$), (*b*) $V=-c\delta(\Lambda-\Lambda_{\rm eq})\delta(\Gamma-\Gamma_{\rm eq})$; $\Gamma_{\rm eq}=0$, $\pi/4$ (rigid rotations with axial symmetry).

These experimental results however point out to the existence of a phase transition between normal and superconducting systems. A treatment of (4) similar to those of Baranger and Kumar [5], Dussel and Bes [6] or Gneuss and Greiner [7] for the quadrupole case becomes impractical for values of M and T larger than 2. On the contrary, the application of the yrast concept [8] seems suitable here. The central idea is that there exists certain states in which centrifugal forces become so important as to produce stable deformations. For these states, the phase transition could be trivially described.

We must minimize $T_{\rm rot}$ with respect to Λ, Γ, T_x, T_y and T_z, for fixed values of M and T. If (M, T) are sufficiently large, the energy has a minimum for

$$T_x = -M,\ T_y = T_z, \text{ and } \Gamma_{\rm eq} = \pi/4 \tag{5}$$

and

$$T = M = |T_x|,\ T_y = T_z = 0 \tag{6}$$

The values of $\Lambda_{\rm eq}$ depend on the potential V. For the harmonic potential

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Table I. *Reactions populating the different degrees of freedom within the yrast aproximation*

Here Y indicates a transition with $\Delta Y = \pm 1$; Γ means $\Delta n_\Gamma = \pm 1$; etc., all the other quantum numbers remaining constant

Experiment	$A < A_0$	$A > A_0$
(t,p)	Γ	Y
(h,p), (t,n)	Γ, θ	Y
(h,n)	$Y, \Gamma, \theta, \Lambda$	Y, Γ
(p,t)	Y, Γ	$Y, \Gamma, \theta, \Lambda$
(p,h), (n,t)	Y	Γ, θ
(n,h)	Y	Γ

$$\Lambda_{eq} = (M/B\omega)^{\frac{1}{2}} \tag{7}$$

The states satisfying (5), (6) and (7) define the yrast band. Each member of it is characterized by the quantum number $Y(=M=T)$. The band is only semirigid because of the stretching effect (7). They have $T=Y$ and all the isospin directed along the intrinsic x-axis. The two-body transfer matrix elements are of order

$$O(\Lambda_{eq} M^{\frac{1}{2}}) \cong O(M) \tag{8}$$

between two consecutive yrast states.

The yrast states are physically interpreted as being the single closed shell nuclei. For instance, the ground states of the Ni isotopes constitute an yrast band.

The equation for the Γ-motion is reduced to that of a two dimensional harmonic oscillator in the variable $\eta \cong \frac{1}{2}\cos 2\Gamma$ and

$$\Lambda = (T_x + M)/2 = \pm n_\Gamma;\ \pm(n_\Gamma - 2);\ \ldots\ 0 \text{ or } 1 \tag{9}$$

are the eigenvalues of the angular momentum in two dimensions. If $n_\Gamma = \Lambda = 1$ we describe the ground state of nuclei with two protons more than the yrast states (Zn isotopes) if $n_\Gamma = -\Lambda = 1$, those with two protons less (Fe isotopes). The matrix elements $\langle n_\Gamma = 1 \rangle$ are of order $(2Y)^{\frac{1}{2}}$.

We also obtain excitations of the yrast states by allowing motion in the Λ-direction (excited states with the same M, T as the ground state) or by switching rotational quanta from the x to the $y-z$ directions (producing antianalogue states). The order of magnitude of the two-body transfer matrix elements corresponding to $\Delta n_\Delta = \pm 1$ or $\Delta n_\theta = \pm 1$ is also $(2Y)^{\frac{1}{2}}$.

2. Conclusions

1. These elementary excitations may be combined to form more complex states. The classification according to 5 quantum numbers Y, n_θ, n_Δ, n_Γ, and Λ can be shown to be complete (apart from trivial $2T+1$ degeneracy in the laboratory frame).

2. The yrast approximation is checked by comparing with the known exact solution for the harmonic potential which are reproduced to leading order in Y. We obtain in addition a physical interpretation of the different orders of magnitude of the matrix elements.

3. All the (t, p) transitions reported by Casten et al. [9] correspond to $\langle n_\Gamma = \pm 1 \rangle$ transitions. The ratio between (h,p) transfer processes reported by Hansen and Nathan [10] correspond to the ratio $d\sigma(\Delta n_\theta = 1)/d\sigma(\Delta n_\Gamma = 1)$. It is clear then that the potential energy surface is still at best half explored for $A < 56$. For instance, we do not have information about the strong yrast transitions or about the Δn_Δ reactions. Table I shows the relevant reactions for $A < 56$ and for $A > 56$.

Table II. *Values of D'/C and K satisfying (11) and (12) for the values of D/C given in the first column*

D/C	D'/C	K
5.0	−2.6	0.083
1.0	−0.56	0.11
0.30	−0.19	0.14
0.10	−0.083	0.17
0.00	−0.022	0.23

4. Symmetry considerations limit the form of the anharmonic terms [2]. In the potential energy surface the simplest anharmonic terms are of the form Λ^4, $\Lambda^4 \cos^2 2\Gamma$. In the kinetic energy, the term $\cos 2\Gamma(T_y^2 - T_z^2)$ mixes the values of T_x.

The inclusion of these terms is made as follows: First we minimize the potential

$$V_{\text{eff}}(\Lambda) = \frac{Y^2}{2B\Lambda^2} + \tfrac{1}{2} C\Lambda^2 + D\Lambda^4/b^2 \tag{10}$$

with respect to Λ. From the corresponding cubic equation we obtain (Λ_{eq}/b) as a function of Y and D/C.

Secondly we use the experimental results on the ratios between (t, p) cross sections

$$R = \frac{\sigma(Y,0000 \leftrightarrow Y,001 \pm 1)/\sigma(Y,0000 \leftrightarrow Y,001 \pm 1)_H}{\sigma(4,0000 \leftrightarrow 4,001 \pm 1)/\sigma(4,0000 \leftrightarrow 4,001 \pm 1)_H} \tag{11}$$

$$= \Lambda_{eq}^4 b_\Gamma^2 / (\Lambda_{eq}^4 b_\Gamma^2)_{Y=4}$$

with

$$b_\Gamma^2 = \frac{1}{Y}\left[1 + 8\frac{D'}{C}\left(\frac{\Lambda_{eq}}{b}\right)^6 \frac{1}{Y^2}\right]^{-\frac{1}{2}}$$

in order to determine D'/C for a given D/C. The subindex H denotes the cross sections corresponding to the harmonic approximation.

Similarly, the experimental ratio between the transitions $\Delta n_\theta = 1$ and $\Delta n_\Gamma = 1$ determines the strength of the anharmonic kinetic energy term $\omega K 2\gamma_1 (T_y^2 - T_z^2)$ if one assumes D/C (and D'/C) known.

Treating this term in perturbation theory,

$$S = \frac{\sigma(Y,0000 \to Y,1000)/\sigma(Y,0000 \to Y,001 - 1)}{\sigma(Y,0000 \to Y,1000)_H/\sigma(Y,0000 \to Y,001 - 1)_H} \tag{12}$$

where

$$\frac{\sigma(Y,0000 \to Y,1000)}{\sigma(Y,0000 \to Y,001-1)} = \frac{1}{b_\Gamma^2 Y} \times \left[\frac{1 - b_\Gamma^2\left(KY + 16\left(\frac{\Lambda_{eq}}{b}\right)^2 \frac{D'}{C}\left[-2 + 6Y^2/\left(\frac{\Lambda_{eq}}{b}\right)^4\right]^{-1}\right)}{1+K}\right]^2 \tag{13}$$

The corresponding values of D/C, D'/C and K are given in Table II. We have verified that the fitting to the experimental data is almost equally good in the range $0.1 \leqslant D/C \leqslant 1.0$. The

Table III. *Relative intensities of* $\Delta n_\Gamma = \pm 1$, (t,p) *reactions* (*eq.* (*11*))

The states are labelled by $(Y, n_\theta\, n_\Delta\, n_\Gamma \Lambda, T)$, Where $T = Y - \Lambda$

Target state	Residual state	R(exp)[9]	R(th.)
^{46}Ca(4,0011,3)	^{48}Ca(4,0000,4)	1.00	1.00
^{48}Ti(3,0011,2)	^{50}Ti(3,0000,3)	0.90	0.81
^{50}Cr(2,0011,1)	^{52}Cr(2,0000,2)	0.63	0.68
^{48}Ca(4,0000,4)	^{50}Ca(4,001–1,5)	1.00	1.00
^{50}Ti(3,0000,3)	^{52}Ti(3,001–1,3)	0.83	0.81
^{52}Cr(2,0000,2)	^{54}Cr(2,001–1,3)	0.56	0.68
^{54}Fe(1,0000,1)	^{66}Fe(1,001–1,2)	0.50	0.58

Table IV. *Relative intensities of* $\Delta n_\theta = 1$ *and* $\Delta n_\Gamma = 1$ *transitions populated in* (h,p) *reactions* (*eq.* (*12*))

The states are labelled as in Table III

Target state	Residual states	S(exp)[10]	S(th.)
^{48}Ca(4,0000,4)	^{50}Sc(4,001–1,5)(4,1000,4)	0.27	0.13
^{50}Ti(3,0000,3)	^{52}V(3,001–1,4)(3,1000,3)	0.13	0.23
^{52}Cr(2,0000,2)	^{54}Mn(2,001–1,3)(2,1000,2)	0.33	0.37
^{54}Fe(1,0000,1)	^{56}Co(1,001–1,2)(1,1000,1)	0.67	0.65

final selection of $D/C = 0.3$ used in tables 3–5 was made by adjusting the yrast transitions.

$$U = \frac{\sigma(Y,00\,n_\Gamma 1 \leftrightarrow Y+1,00\,n_\Gamma 1)/\sigma(Y,00\,n_\Gamma 1 \leftrightarrow Y+1,00\,n_\Gamma \Lambda)_{\mathrm{H}}}{\sigma(3,0011 \leftrightarrow 4,0011)/\sigma(3,0011 \leftrightarrow 4,0011)_{\mathrm{H}}} \tag{14}$$

where

$$\frac{\sigma(Y,00\,n_\Gamma 1 \leftrightarrow Y+1,00\,n_\Gamma \Lambda)}{\sigma(Y,00\,n_\Gamma 1 \leftrightarrow Y+1,00\,n_\Gamma \Lambda)_{\mathrm{H}}} = \left(\frac{I_{\mathrm{eq}}}{b}\right)^2 \frac{1}{Y} \cdot \left[1 + \frac{Kb_\Gamma^2}{4}(2+3n_\Gamma - \Lambda) - 8b_\Gamma^2 \left(\frac{I_{\mathrm{eq}}}{b}\right)^2 \frac{D'}{C}\left[-2 + 6Y^2/\left(\frac{I_{\mathrm{eq}}}{b}\right)^4\right]\right]^2 \tag{15}$$

The best fit to the data shown in Tables III–V is made with the values $D/C = 0.30$, $D'/C = -0.19$ and $K = 0.14$. We note however that the yrast expansion in powers of $1/Y$ may require substantial corrections in the present region of application. We have not attempted yet a detailed microscopical calculation of the constants D/C, D'/C and K, although it is worthwhile to observe that the empirical ratio $D'/D = -0.60$ is close to the value -0.50 corresponding to the pure pairing force.

Table V. *Relative intensities of yrast transitions* (*eq.* (*14*))

The experimental values are Q-corrected. The values corresponding to (p, t) and (t, p) reactions are normalized to 1, for the corresponding transitions $Y = 3 \leftrightarrow Y = 4$

Target state	Residual state	U(exp).	U(th.)
^{60}Ni(2,0000,2)	^{58}Ni(1,0000,1)	1.48[11]	1.39
^{62}Ni(3,0000,3)	^{60}Ni(2,0000,2)	1.31[11]	1.03
^{64}Ni(4,0000,4)	^{62}Ni(3,0000,3)	1.04[11]	0.91
^{64}Zn(3,0011,2)	^{62}Zn(2,0011,1)	1.12[11]	1.15
^{66}Zn(4,0011,3)	^{64}Zn(3,0011,2)	1.00[11]	1.00
^{68}Zn(5,0011,4)	^{66}Zn(4,0011,3)	0.81[11]	0.95
^{70}Zn(6,0011,5)	^{68}Zn(5,0011,4)	0.60[11]	0.74
^{64}Zn(3,0011,2)	^{66}Zn(4,0011,3)	1.00[12]	1.00
^{66}Zn(4,0011,3)	^{68}Zn(5,0011,4)	0.81[12]	0.95
^{68}Zn(5,0011,4)	^{70}Zn(6,0011,5)	0.66[12]	0.74
^{56}Fe(1,001–1,2)	^{58}Fe(2,001–1,3)	1.18[9]	1.84
^{58}Fe(2,001–1,3)	^{60}Fe(3,001–1,4)	1.41[9]	1.27

5. So far the applications of the collective hamiltonian (4) have been restricted to sufficiently heavy nuclei with similar number of protons and neutrons, i.e., to the surrounding of $A = 56$. Residual effects of the $T = 1$ pairing force may be obtained whenever we are near to a single-closed shell. For instance the region of 82 neutrons seem also suitable for doing (h,n) and (h,p) experiments.

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