Angular Structure of $\pi\mathcal{N}$ Veneziano Models.

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In this letter we investigate whether a model for $\pi\mathcal{N}$ charge exchange scattering with Veneziano (1) terms can reproduce the observed angular distribution as a function of energy. It is shown that, though one is able to build up a model which fits the magnitude of forward and backward peaks as well as the widths of the most important low-lying resonances, no reasonable Veneziano model can account for the measured angular structure even relaxing some of these requirements. In particular, it is proved that the background terms ((s, u) terms near the forward direction) are not responsible for the filling-up of the dips predicted by the Regge terms, when an imaginary part is allowed on baryon trajectories. This is in contradiction with a recent paper (2).

Since the work of Igi (3), several papers on $\pi\mathcal{N}$ Veneziano models appeared (4). All of them faced the difficulty of fitting simultaneously forward and backward peaks and the widths of well-known resonances. We shall attempt to construct the simplest possible model with these properties, starting from a set of reasonable and fairly modest conditions which the invariant $A^{(-)}$ and $B^{(-)}$ amplitudes must satisfy. Our hope is that a model which reproduces the gross features of $\pi\mathcal{N}$ charge exchange scattering has the best chances to give an adequate description at all angles, but the negative conclusions which we reach are quite general and apply, *e.g.*, to the model of ref. (2).

Thus, we require that every Veneziano block included has

a) leading Regge asymptotic behaviour in both trajectories involved,

b) series of poles beginning at the position of the resonances to be fitted, *i.e.* $\mathcal{N}_\alpha(939)$, $\mathcal{N}_\gamma(1518)$, $\Delta_\delta(1236)$ and $\rho(765)$ (through forward peak).

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(1) G. Veneziano: *Nuovo Cimento*, **57** A, 190 (1968).
(2) A. W. Hendry, S. T. Jones and H. W. Wyld jr.: *Nucl. Phys.*, **15** B, 389 (1970).
(3) K. Igi: *Phys. Lett.*, **28** B, 330 (1968).
(4) S. K. Bose and K. C. Gupta: *Phys. Rev.*, **184**, 1572 (1969); M. A. Virasoro: *Phys. Rev.*, **184**, 1621 (1969); R. F. Amann: *Lett. Nuovo Cimento*, **2**, 87 (1969); E. Berger and G. C. Fox: *Phys. Rev.*, **188**, 2120 (1969); S. Y. Chu and B. R. Desai: *Phys. Rev.*, **188**, 2215 (1969); J. Namyslowski and M. Swiecki: *Nuovo Cimento*, **64** A, 555 (1969); S. Fenster and K. Wali: Argonne Nat. Lab. preprint (August 1969); G. Joshi and A. Pagnamenta: Rutgers Univ. preprint.

Condition b) says that there will be no satellite terms starting at $\frac{5}{2}$ or higher. Together with a), it implies that every block must contribute to high- and low-energy scattering, thus avoiding trivial fits. In this way, and assuming the $\mathcal{N}_\alpha$ and $\mathcal{N}_\gamma$ trajectories to be degenerate, one is led to the amplitudes

$$(1)\qquad \begin{cases} \mathscr{A}^{(-)} = a_1 C^-_{\Delta\rho}(\frac{3}{2}, 1) + a_2 C^-_{\mathcal{N}\rho}(\frac{3}{2}, 1) + a_3 C^-_{\mathcal{N}\Delta}(\frac{3}{2}, \frac{3}{2}) , \\ \mathscr{B}^{(-)} = b_1 B_{\mathcal{N}\mathcal{N}}(\frac{1}{2}, \frac{1}{2}) + b_2 C_{\mathcal{N}\mathcal{N}}(\frac{3}{2}, \frac{3}{2}) + b_3 C_{\Delta\Delta}(\frac{3}{2}, \frac{3}{2}) + b_4 B^+_{\mathcal{N}\rho}(\frac{1}{2}, 1) + b_5 C^+_{\mathcal{N}\Delta}(\frac{3}{2}, \frac{3}{2}), \end{cases}$$

with the usual notation.

However, it is straightforward to prove that (1) cannot give simultaneously the correct magnitude of the $\Delta(1236)$ width and of the high-energy backward peak (given by Δ-leading terms), having assumed absence of Δ parity partner (*). This inconsistency, common to previous models, is due to the smallness of asymptotic numbers compared to the large resonance residues.

Relaxing condition a) to admit terms with leading asymptotic behaviour in only one trajectory we cannot still solve the problem, since, as can be seen, none of them contributes either to the leading backward behaviour or to the $\Delta(1236)$ residue.

One is therefore forced to abandon restriction b) to introduce satellite terms starting at $\frac{5}{2}$ for Δ (**) (and at 2 or 3 for ρ), thus enlarging somehow the low-energy interval to be fitted, and requiring at the same time a vanishing parent trajectory residue of both amplitudes at this point. Within this frame, some terms are useless in the sense that they do not contribute either to the $\Delta(1236)$ residue or to the backward peak, whereas the contributions of others add trivially to those of the model eq. (1). We finally end up with

$$(2)\qquad \begin{cases} A^{(-)} = \mathscr{A}^{(-)} + a_4 D^-_{\Delta\rho}(\frac{5}{2}, 2) , \\ B^{(-)} = \mathscr{B}^{(-)} + b_6 D_{\Delta\Delta}(\frac{5}{2}, \frac{5}{2}) + b_7 D^+_{\Delta\rho}(\frac{5}{2}, 3) , \end{cases}$$

where $D_{ij}(m, n) = \Gamma(m - \alpha_i)\Gamma(n - \alpha_j)/\Gamma(m + n - 2 - \alpha_i - \alpha_j)$.

The coefficients were fixed by imposing the restrictions of correct nucleon pole residue, elastic widths of $\Delta(1236)$ and $\mathcal{N}_\gamma(1518)$ without parity partners, forward and backward high-energy peaks of $\mathrm{d}\sigma/\mathrm{d}\Omega$ and finally no $J = \frac{5}{2}$ pole on the Δ trajectory. The coefficient b_2 was set equal to zero since it contributes only to the $\mathcal{N}_\gamma$ residue (it actually does not contribute to the leading Δ backward behaviour (***)). Since there is one condition lacking, an ambiguity appears in the coefficients which in fact can be estimated in terms of the magnitude of the backward peak and turns out to be pretty small. The trajectories used in the fit are $\alpha(x) = \alpha_0 + \alpha' x$ with $\alpha_{0_\rho} = 0.5$, $\alpha_{0_\mathcal{N}} = -0.38$, $\alpha_{0_\Delta} = -0.02$ and $\alpha' = 1$.

(*) Details for these and further assertions will be given in a forthcoming paper.

(**) The reduction of the 6-point function for scalar particles may suggest to include Veneziano satellites starting at most at $\frac{5}{2}$ for Δ, $\frac{3}{2}$ for $\mathcal{N}$ and 1 for ρ trajectory (D. WONG: private communication). This hypothesis had to be violated for the ρ trajectory in our model.

(***) Note that b_1 does not contribute either but affects the residues of two low-energy poles.

The resulting coefficients a_i (1/GeV) and b_i (1/(GeV)^2) are

$$
(3)\qquad \left\{
\begin{array}{ll}
a_1 = 25.2 \pm \frac{1}{5}E_1 \mp \frac{1}{5}E_2\,, & b_1 = -134.1 \mp E_1\,,\\[2mm]
a_2 = 149.1 \mp \frac{1}{5}E_1 \pm \frac{1}{5}E_2\,, & b_3 = 210.4 \pm E_1\,,\\[2mm]
a_3 = 62.3 \mp \frac{1}{5}E_1 \pm \frac{1}{5}E_2\,, & b_4 = 316.6 \pm \frac{1}{2}E_1\,,\\[2mm]
a_4 = -37.1 \pm \frac{2}{5}E_1 \mp \frac{2}{5}E_2\,, & b_5 = -205.7 \mp E_1\,,\\[2mm]
 & b_6 = -17.4 \pm \frac{2}{15}E_1 \pm \frac{8}{15}E_2\,,\\[2mm]
 & b_7 = -22.9 \mp \frac{2}{15}E_1 \mp \frac{8}{15}E_2\,,
\end{array}
\right.
$$

where E_1 and E_2 are related by

$$E_1^2 + E_2^2 = 128\pi s \left.\frac{\mathrm{d}\sigma_{\mathrm{CEx}}}{\mathrm{d}\Omega}\right|_{u=0} \approx 32$$

and their relative sign is undetermined. It is appearent that there are too strong cancellations among the coefficients in order to reproduce the forward and backward peaks.

At this point, in order to give a numerical meaning to the $(s\text{-}u)$ terms one must assign an imaginary part α_I to the s-channel trajectory. However also the $(s\text{-}t)$ terms are affected substantially by such an assignment, *e.g.* $B^+_{\mathcal{N}\rho}(\frac{1}{2}, 1)$ can be expanded near the forward direction to give

$$
(4)\qquad B^+_{\mathcal{N}\rho}\left(\frac{1}{2}, 1\right) = \frac{\pi}{\Gamma[\alpha_\rho(t)]}(2M\alpha'\nu)^{\alpha_\rho(t)-1}\left\{\frac{1-\exp[-i\pi\alpha_\rho(t)]}{\sin\pi\alpha_\rho(t)} + \right.
$$
$$
+\frac{1-\alpha_\rho(t)}{2M\alpha'\nu}\left[\alpha_{0_{\mathcal{N}}} + \alpha'(M^2+\mu^2) + \frac{\alpha_{0\rho}-1}{2}\right]\frac{1+\exp[-i\pi\alpha_\rho(t)]}{\sin\pi\alpha_\rho(t)} -
$$
$$
\left. -2i\exp[-2\pi\alpha_I]\exp[2\pi i(\alpha_{0_{\mathcal{N}}} + \alpha' s)] + i\,\frac{1-\alpha_\rho(t)}{2M\alpha'\nu}\,\alpha_I\,\frac{\exp[-i\pi\alpha_\rho(t)]}{\sin\pi\alpha_\rho(t)}\right\}
$$

up to second order in Stirling's formula, having used $\nu = (s-u)/4M$. The first term corresponds to the parent trajectory and is the one we used to fit the asymptotic peak. The second one is the first daughter term which contributes to the dip at $\alpha_\rho(t) = 0$ but with a too fast decrease with energy. The experimental behaviour at this dip corresponds in fact to the parent trajectory and this is consistent with the standard Regge-pole fits which use a sense-choosing mechanism for the ρ coupling ([5]). The third

([5]) W. RARITA, R. RIDDEL jr., C. CHIU and R. PHILLIPS: *Phys. Rev.*, **165**, 1615 (1968); V. BARGER and R. PHILLIPS: *Phys. Rev.* (in press).

term is a non-Regge part which vanishes at $\alpha_\rho = 0$ and decays exponentially with s. Finally, the fourth is a spurious term which appears because of the violation of crossing due to the introduction of an imaginary part only in s-channel trajectories. It does not vanish at the dip position and with the assumption $\alpha_I \sim s$ asymptotically it behaves with energy like the parent Regge term, breaking, in this way, the Gell-Mann mechanism usually attributed to Veneziano models. Furthermore, the corresponding terms of $A^{(-)}$ do vanish at $\alpha_\rho = 0$ giving rise to a « mechanism » opposite to the sense-choosing implied by experimental fits (*).

Returning to the model eq. (2), with $\alpha_I = 0.1\,s$, which is a reasonable choice to fit the total widths of low-energy resonances, it turns out that the (s-u) terms, and the daughter, non-Regge and spurious part of the (s-t) terms may be larger than the parent trajectory contribution even at fairly high energy (*e.g.*, $p_{lab} = 18$ GeV). This is of course due to the very severe cancellations in our model which hold true only for the parent trajectory. This invalidates completely the predictive power of this model.

This failure is not inherent to our model. If one gives up the fit of the backward peak aiming only at reproducing the angular distribution in the near-to-forward region, one may obtain a rather impressive agreement in a fairly wide energy interval as the one got by HENDRY *et al.* [2]. However, the filling of the dip at $\alpha_\rho = 0$ in not due to the background (s-u) terms, as they argue, but to the spurious part of (s-t) terms which can easily be evaluated in their model to give the correct magnitude for the differential cross-section at the dip position. Moreover the amount of spurions term that comes into their model in the forward direction is even larger than the parent Regge-pole contribution to $A'^{(-)}$ in such a way that this model would predict a high-energy total cross-section $\sigma_{\pi^-} - \sigma_{\pi^+}$ about 40% of the experimental value.

Concluding, one may remark that these problems are a common feature of any naive Veneziano model with complex trajectories. Actually only the spurious terms could explain the filling-up of the dip, but this is of course prevented by a proper simultaneous fit to the forward peak and the asymptotic total cross-section. These troubles do not disappear when unitarizing the Veneziano model with Lovelace's K-matrix method, as discussed in ref. [6].

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(*) Note that this spurious term has no signature factor and therefore cannot be treated on the same footing with the parent Regge pole.

(6) C. LOVELACE: *Nucl. Phys.*, **12** B, 253 (1969).