Z. Phys. A - Atoms and Nuclei 306, 259-263 (1982)

Zeitschrift für Physik A Atoms and Nuclei

The Effect of Statistical Fluctuations on the Measurement of the Total Energy and Multiplicity of γ Rays Following Deep-Inelastic Reactions

A.J. Pacheco* and L.G. Moretto
Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California, USA

Received February 2, 1982

We discuss the effect of statistical fluctuations on the first and second moments of both the intrinsic rotational energy and the sum of spin magnitudes in deep-inelastic fragments, as extracted from measurements of the total γ-ray energy and the γ-ray multiplicity, respectively. The calculations were done in the framework of a model that considers the thermal excitation of rotational modes in the intermediate dinuclear complex, accounting exactly for the correlations between the angular momenta generated in both nuclei.

1. Introduction

The simple relation between the spin and the energy is a well known property of the rotational states in nuclei. From an experimental standpoint, this fact provides the rationale for the use of total γ-ray energy detectors ("sum spectrometers") to investigate high-spin states of evaporation residues following compound-nucleus and deep-inelastic reactions [1-3]. It has been shown that by gating on different regions of the sum-energy spectrum one can select subsets of the total population having different values of the average spin [2], and in this regard the technique has proven very useful in the study of rotational structures [3].

Whereas sum spectrometers can be used effectively as spin-selecting devices, the extraction of more quantitative information based on the measured total energy is hindered by several uncertainties of experimental and theoretical nature. In the case of deep-inelastic reactions a fundamental source of uncertainties is associated with fluctuations, statistical or otherwise, introduced by the reaction mechanism. The most significant problem in this regard is the distribution in both magnitude and orientation of the spins of the emitting population [4-10].

The purpose of the present paper is to evaluate the contribution of spin fluctuations to the distributions of sum energy and γ-ray multiplicities. Our analysis will be restricted to the case of binary reactions between heavy ions in which the detection of one of the two fragments determines a reaction plane with the orbital angular momentum perpendicular to it. We shall base our calculation on a statistical model [9, 11]. The effects of the spin fluctuations on the first and second moments of the total-energy distribution will be calculated as a function of the temperature, total angular momentum, and mass asymmetry of the system assuming that the collective modes of the dinuclear complex are in thermal equilibrium with the internal degrees of freedom.

2. Theory and Calculations

The equilibrium statistical model utilized in this calculation has been described in detail in [9] and [11]; thus only a brief sketch will be given here.

* Permanent address: Comisión Nacional de Energía Atómica, Buenos Aires, Argentina

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics and by Nuclear Sciences of the Basic Energy Sciences Program of the U.S. Department of Energy under Contract W-7405-ENG-48.

0340-2193/82/0306/0259/$01.00

The intermediate dinuclear system formed during the reaction is described by means of two touching rigid spheres with all the associated degrees of freedom. Removing the motion of the center of mass and taking into account the conservation of the total angular momentum $\mathbf{I}$, the most general configuration of the system is defined by six normal modes (one tilting, one twisting, two wriggling, and two bending). The angular momenta associated with these modes are given by:

$$\xi_{W_{x(z)}} = W_H(\mathbf{s}_H - \mathbf{r}_H)_{x(z)} + W_L(\mathbf{s}_L - \mathbf{r}_L)_{x(z)},$$
$$\xi_{B_{x(z)}} = B_H(\mathbf{s}_H - \mathbf{r}_H)_{x(z)} + B_L(\mathbf{s}_L - \mathbf{r}_L)_{x(z)},$$
$$\xi_{TW} = (\mathbf{s}_H - \mathbf{r}_H)_y = -(\mathbf{s}_L - \mathbf{r}_L)_y,$$
$$\xi_{TI} = \mathbf{I})_y, \tag{1}$$

where $\mathbf{s}_H(\mathbf{s}_L)$ is the spin of the heavy (light) fragment, $\mathbf{r}_H(\mathbf{r}_L)$ is the spin of the heavy (light) fragment arising from rigid rotation, and the cartesian directions (x, y, z) are chosen so that the y axis coincides with the line between centers of the complex. All the variables ξ_j can independently take any value from $-\infty$ to ∞ (from $-|\mathbf{I}|$ to $|\mathbf{I}|$ in the case of ξ_{TI}) without violating the conservation of the total angular momentum in the system. In terms of these normal coordinates the total rotational energy of the system reads

$$E = \frac{|\mathbf{I}|^2}{2\mathscr{I}_\perp} + \frac{1}{2} \sum_{\text{(six modes)}} \lambda_i \xi_i^2. \tag{2}$$

The coefficients W_H, W_L, B_H, and B_L as well as the stiffness coefficients λ_i may be expressed as a function of two variables, the total mass and the mass asymmetry of the complex [11]. At symmetry, these coefficients take the following values:

$$B_H = -B_L = W_H = W_L = \frac{1}{\sqrt{2}},$$
$$\lambda_W = \tfrac{7}{5}\mathscr{I}_{\text{sym}}^{-1}, \quad \lambda_B = \mathscr{I}_{\text{sym}}^{-1}, \quad \lambda_{TW} = 2\mathscr{I}_{\text{sym}}^{-1},$$
$$\lambda_{TI} = \tfrac{5}{14}\mathscr{I}_{\text{sym}}^{-1}. \tag{3}$$

Here, $\mathscr{I}_{\text{sym}}$ is the moment of inertia of a sphere having half the mass of the complex.

If one considers a system with temperature $T=0$, the average intrinsic rotational energy is given by the simple expression

$$\langle E_H + E_L \rangle = \frac{|\mathbf{r}_H|^2}{2\mathscr{I}_H} + \frac{|\mathbf{r}_L|^2}{2\mathscr{I}_L} = \frac{\mathscr{I}_\parallel}{2\mathscr{I}_\perp^2}|\mathbf{I}|^2, \tag{4}$$

where $\mathscr{I}_H$ and $\mathscr{I}_L$ are the moments of inertia of the two spheres, $\mathscr{I}_\parallel = \mathscr{I}_H + \mathscr{I}_L$, $\mathscr{I}_\perp = \mathscr{I}_\parallel + \mu d^2$, and d is the distance between centers. If $T \neq 0$, the following integral must be evaluated:

$$\langle E_H + E_L \rangle = \int [E_H(\{\xi\}) + E_L(\{\xi\})]\, P(\{\xi\})\, d^6\xi, \tag{5}$$

where $P(\{\xi\})$ is the normalized probability distribution function

$$P(\{\xi\}) = \frac{\sqrt{\lambda_W^2 \lambda_B^2 \lambda_{TI} \lambda_{TW}}}{(2\pi T)^3 \operatorname{erf}\left\{|\mathbf{I}| \sqrt{\frac{\lambda_{TI}}{2T}}\right\}} \exp\left[-\frac{\sum_i \lambda_i \xi_i^2}{2T}\right]. \tag{6}$$

Note that by working with the set of normal variables $\{\xi\}$ one automatically accounts for the conservation of the vector $\mathbf{I}$, and therefore all the correlations between the spins induced in both nuclei are properly included.

It is convenient to express the analytical result of the integral (5) in units of the intrinsic rotational energy at $T=0$ (4),

$$\left(\frac{\mathscr{I}_\parallel |\mathbf{I}|^2}{2\mathscr{I}_\perp^2}\right)^{-1} \langle E_H + E_L \rangle = 1 + \alpha Z^2. \tag{7}$$

The variable Z depends on the total mass, temperature, and total angular momentum of the system:

$$Z = \frac{\sqrt{\mathscr{I}_{\text{sym}} T}}{|\mathbf{I}|}. \tag{8}$$

The constant α is a function of the mass asymmetry only

$$\alpha = \frac{1}{\mathscr{I}_{\text{sym}}} \left(\frac{2a_W}{\lambda_W} + \frac{2a_B}{\lambda_B} + \frac{a_{TI} F}{\lambda_{TI}} + \frac{a_{TW}}{\lambda_{TW}}\right), \tag{9}$$

where

$$a_W = \frac{\mathscr{I}_\perp^2}{\mathscr{I}_\parallel}\left(\frac{B_L^2}{\mathscr{I}_H} + \frac{B_H^2}{\mathscr{I}_L}\right), \quad a_B = \frac{\mathscr{I}_\perp^2}{\mathscr{I}_\parallel}\left(\frac{W_L^2}{\mathscr{I}_H} + \frac{W_H^2}{\mathscr{I}_L}\right),$$
$$a_{TI} = \left(\frac{\mathscr{I}_\perp}{\mathscr{I}_\parallel}\right)^2 - 1, \quad a_{TW} = \frac{\mathscr{I}_\perp^2}{\mathscr{I}_\parallel}\left(\frac{1}{\mathscr{I}_L} + \frac{1}{\mathscr{I}_H}\right),$$
$$F = 1 - \frac{2}{\sqrt{\pi}}\, t \frac{\exp(-t)}{\operatorname{erf}(t)}, \quad \text{with } t = \frac{1}{Z}\sqrt{\frac{\mathscr{I}_{\text{sym}} \lambda_{TI}}{2}}. \tag{10}$$

With the aid of (3) and (10), and recalling that at symmetry $\mathscr{I}_\parallel = 2\mathscr{I}_{\text{sym}}$ and $\mathscr{I}_\perp = 7\mathscr{I}_{\text{sym}}$, one obtains the simplified expression

$$\left(\frac{\mathscr{I}_\parallel |\mathbf{I}|^2}{2\mathscr{I}_\perp^2}\right)^{-1} \langle E_H + E_L \rangle = 1 + 140 Z^2, \tag{11}$$

valid for mass-symmetric reactions and small values of Z $(F \to 1)$. The exact expression is plotted as a function of Z in Fig. 1 (dark solid line). As an example, let us consider the system ^{165}Ho$+^{165}$Ho at 8.5 MeV/u. We estimate a total angular momentum $|\mathbf{I}| = 350\hbar$ and assume a temperature $T = 2.5$ MeV,

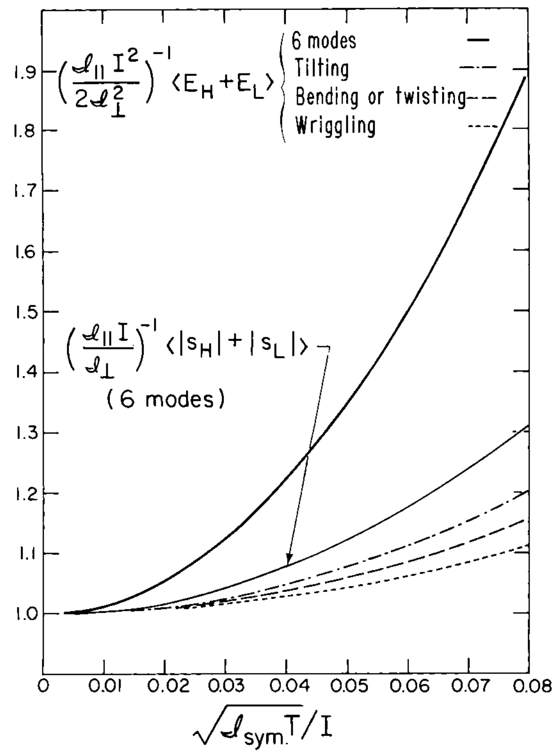

Fig. 1. Reduced average intrinsic rotational energy (dark solid curve) and average sum of the spin magnitudes (light solid curve) as a function of the parameter Z, for a mass-symmetric reaction. The three lowest curves illustrate the contribution to the rotational energy from the individual modes

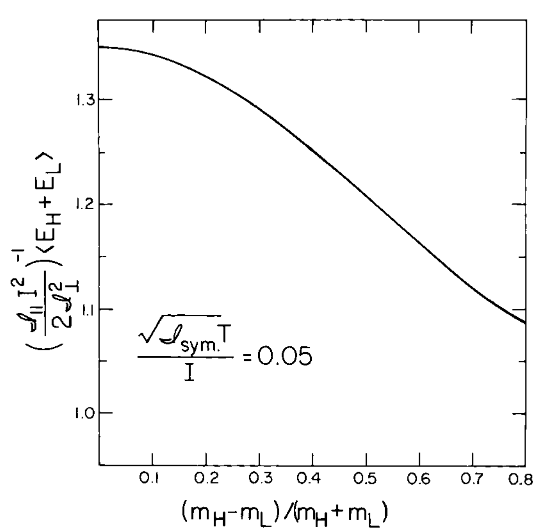

Fig. 2. Reduced average intrinsic rotational energy as a function of mass asymmetry for a fixed value of the parameter Z. If the thermal fluctuations were not included a constant value equal to 1 would be obtained

therefore, $Z \simeq 0.038$. From the figure we see that the statistical fluctuations introduce a 20% increase of the average internal rotational energy, over the rigid rotation prediction. Figure 1 also illustrates the individual contributions from the normal modes (dash-dot, long-dash, and short-dash curves). The dependence of the average reduced rotational energy on mass asymmetry, at constant Z, is shown in Fig. 2. As larger asymmetries are considered, the effect of the fluctuations decreases due to the progressive suppression of some of the modes.

So far we have examined the relation between the average internal rotational energy and the zero-temperature limit. Let us now turn to the connection between the rotational energy and the sum of the spin magnitudes. The question is relevant to any experiment in which, for example, the total γ-ray energy and the γ-ray multiplicity are measured in coincidence with deep-inelastic fragments. The rotational energy (extracted from the sum energy) and the sum of the spin magnitudes (extracted from the multiplicity) are not related by (7) as might at first appear. This is so because the average spins are also affected by the fluctuations. As shown in [11] the following relation holds:

$$\left(\frac{\mathscr{I}_\parallel}{\mathscr{I}_\perp}|\mathbf{I}|\right)^{-1}\langle|\mathbf{s}_H|+|\mathbf{s}_L|\rangle = 1+\frac{\mathscr{I}_{\mathrm{sym}}^{-1}}{2}\left[\frac{a_{TI}}{\lambda_{TI}}+\frac{a_W}{\lambda_W}+\frac{a_B}{\lambda_B}+\frac{a_{TW}}{\lambda_{TW}}\right]Z^2, \tag{12}$$

which is also plotted in Fig. 1 (light solid curve). Unlike the exact expression for the rotational energy, (12) is valid only for $Z \ll \frac{\mathscr{I}_L}{\mathscr{I}_\perp}\sqrt{\lambda \mathscr{I}_{\mathrm{sym}}}$ although this is not a severe restriction in general. Let us again consider the previous example of a mass-symmetric system with $Z=0.038$. If we estimate the average rotational energy from the average sum of spins using the expression $\frac{\langle|\mathbf{s}_H|+|\mathbf{s}_L|\rangle^2}{2\mathscr{I}_\parallel}$, the "true" value (as given by (7)) would be underpredicted by approximately 5%.

Finally, it is also interesting to consider the variances of the distributions of both rotational energies and sum of spin magnitudes. The reduced variance of the intrinsic rotational energy can be evaluated exactly.

$$\sigma^2_{(E_H+E_L)} = \left(\frac{\mathscr{I}_\parallel |\mathbf{I}|^2}{2\mathscr{I}_\perp^2}\right)^2 (\beta_2 Z^2 + \beta_4 Z^4). \tag{13}$$

In this equation

$$\mathscr{I}_{\mathrm{sym}}\beta_2 = \frac{a_{WTi}}{\lambda_W}+\frac{a_{BTi}}{\lambda_B},$$

$$\mathscr{I}_{\mathrm{sym}}^2\beta_4 = \frac{4a_W^2}{\lambda_W^2}+\frac{4a_B^2}{\lambda_B^2}+\left[\frac{2a_{TI}^2}{\lambda_{TI}^2}F-\frac{a_{WTi}^2}{\lambda_W\lambda_{Ti}}-\frac{a_{BTi}^2}{\lambda_B\lambda_{Ti}}\right]F+\frac{2a_{TW}}{\lambda_{TW}^2}+\frac{2a_{BW}}{\lambda_W\lambda_B} \tag{14}$$

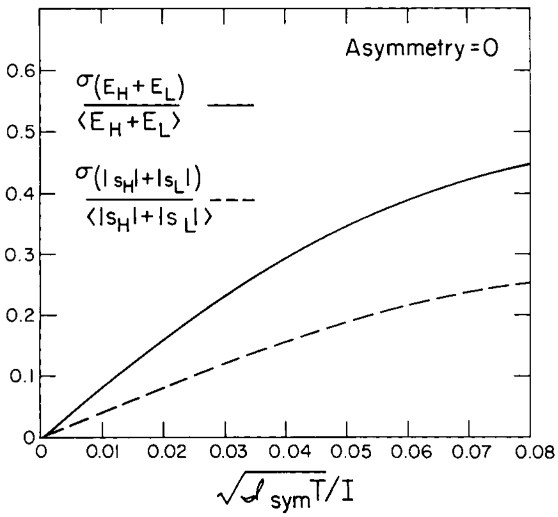

Fig. 3. Width-to-average ratios as a function of the parameter Z for a mass-symmetric reaction

where

$$a_{BW}=\frac{2\mathscr{I}_\perp^2}{\mathscr{I}_\parallel}\left(\frac{W_L B_L}{\mathscr{I}_H}+\frac{W_H B_H}{\mathscr{I}_L}\right),$$

$$a_{BTi}=\frac{2\mathscr{I}_\perp}{\mathscr{I}_\parallel}(W_H-W_L),$$

$$a_{WTi}=\frac{2\mathscr{I}_\perp}{\mathscr{I}_\parallel}(B_H-B_L). \tag{15}$$

Figure 3 (solid curve) shows the ratio $\sigma_{(E_H+E_L)}/\langle|E_H|+|E_L|\rangle$ as a function of Z for a mass-symmetric system. For comparison, the dashed line in the same figure represents the ratio $\sigma_{(|\mathbf{s}_H|+|\mathbf{s}_L|)}/\langle|\mathbf{s}_H|+|\mathbf{s}_L|\rangle$. The variance associated with the sum of spin magnitudes, $\sigma_{(|\mathbf{s}_H|+|\mathbf{s}_L|)}$ has been calculated using the expression derived in [11]:

$$\left(\frac{\mathscr{I}_\parallel}{\mathscr{I}_\perp}\right)^{-2}\sigma^2_{(|\mathbf{s}_H|+|\mathbf{s}_L|)}=\frac{\mathscr{I}_\perp^2}{\mathscr{I}_\parallel^2}\cdot\left[\frac{1-2B_L B_H}{\lambda_W}+\frac{1-2W_L W_H}{\lambda_B}\right]\mathscr{I}_{\mathrm{sym}}^{-1}Z^2, \tag{16}$$

which again is valid for $Z\ll\frac{\mathscr{I}_L}{\mathscr{I}_\perp}\sqrt{\lambda\mathscr{I}_{\mathrm{sym}}}$. It is remarkable that γ-ray multiplicity measurements in several systems have yielded anomalously large widths of the distribution, which cannot be explained from a triangular distribution of the incoming l-waves [12, 13]. For example, the relative width predicted by the statistical calculation for the ^{86}Kr $+{}^{144}$Sm using the values $I_{\mathrm{MAX}}\simeq 190\hbar$ and a temperature corresponding to an average total excitation energy of ~ 100 MeV, is $\frac{\sigma_{(|\mathbf{s}_H|+|\mathbf{s}_L|)}}{\langle|\mathbf{s}_H|+|\mathbf{s}_L|\rangle}=0.22$. When added in quadrature to the variance arising from the triangular l-wave distribution ($\simeq 0.35$) one obtains a total ratio of 0.41, which compares reasonably with an experimental value of approximately 0.5 [12].

3. Summary and Conclusions

The existence of fluctuations in the spins imparted to the nuclei in a deep-inelastic reaction represents a substantial contribution to the observed average values of physical quantities having an angular-momentum dependence. We have focused our attention on the intrinsic rotational energy and on the sum of spin magnitudes, which, in principle, may be experimentally obtained from measurements of the total γ-ray energy and γ-ray multiplicity. The calculations have been performed in the framework of an equilibrium statistical model that considers the thermal excitation of angular-momentum-bearing modes of the dinuclear complex. Under these assumptions, it has been shown that the average rotational energy may be, typically, 20% or 30% higher than the rigid-rotation prediction for a system with zero temperature. The magnitude of the effect decreases with increasing mass asymmetry as a consequence of the increasing stiffness of some of the rotational modes. A similar, although not so strong, effect is predicted for the average sum of the magnitudes of the spins. Second moments of the distributions have also been calculated, and the results indicate that the thermal excitation of the rotational modes of the complex may contribute significantly to the observed widths. Finally, it is important to notice that in the calculations presented here, the *correlations between the spins in both fragments have been fully taken into account*, in such a way that the conservation of the total angular momentum is guaranteed.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics and by Nuclear Sciences of the Basic Energy Sciences Program of the U.S. Department of Energy under Contract W-7405-ENG-48.

References

1. Tjøm, P.O., Stephens, F.S., Diamond, R.M., Boer, J. de, Meyerhof, W.E.: Phys. Rev. Lett. **33**, 593 (1974)
2. Körner, H.J., Hillis, D.L., Roulet, C.P., Aguer, P., Ellegaard, C., Fossan, D.B., Habs, D., Neiman, M., Stephens, F.S., Diamond, R.M.: Phys. Rev. Lett. **43**, 490 (1979)
3. Folkmann, F., Garrett, J.D., Hagemann, G.B., Harakeh, M.N., Herskind, B., Hillis, D.L., Ogaza, S., Emling, H., Grosse, E., Schwalm, D., Simon, R.S., Tjøm, P.O.: Nucl. Phys. A **361**, 242 (1981)

4. McDonald, R.J., Pacheco, A.J., Wozniak, G.J., Bolotin, H.H., Moretto, L.G., Schück, C., Shih, S., Diamond, R.M., Stephens, F.S.: Nucl. Phys. A **373**, 54 (1982)
5. Broglia, R.A., Pollarolo, G., Dasso, C.H., Dossing, T.: Phys. Rev. Lett. **43**, 1649 (1979)
6. Ayik, S., Wolschin, G., Nörenberg, W.: Z. Phys. A - Atoms and Nuclei **286**, 271 (1978)
7. Wolschin, G.: Nucl. Phys. A **316**, 146 (1979)
8. Vandenbosch, R.: Phys. Rev. C **20**, 171 (1979)
9. Moretto, L.G., Schmitt, R.P.: Phys. Rev. C **21**, 204 (1980)
10. Randrup, J.: Lawrence Berkeley Laboratory Report No. LBL-12676 (1981) and Nucl. Phys. (in press)
11. Schmitt, R.P., Pacheco, A.J.: Nucl. Phys. A **379**, 313 (1982)
12. Christensen, P.R., Folkmann, F., Hansen, Ole, Nathan, O., Trautner, N., Videbæk, F., Werf, S.Y. Van der, Britt, H.C., Chestnut, R.P., Freiesleben, H., Pühlhofer, F.: Nucl. Phys. A **349**, 217 (1980)
13. Dayras, R.A., Stokstad, R.G., Hensley, D.C., Halbert, M.L., Sarantites, Westerberg, L., Barker, J.H.: Phys. Rev. C **22**, 1485 (1980)

A.J. Pacheco
L.G. Moretto
Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720
USA