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ON THE MANY-BODY FOUNDATION OF THE NUCLEAR FIELD THEORY

D. R. BÈS[†]
Comisión Nacional de Energía Atómica, Buenos Aires, Argentina[††]

R. A. BROGLIA
Niels Bohr Institute, University of Copenhagen, Denmark

and

G. G. DUSSEL[†], R. J. LIOTTA and R. P. J. PERAZZO[†]
Comisión Nacional de Energía Atómica, Buenos Aires, Argentina[††]

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Abstract: The equivalence between the description of the many-body finite nuclear system in terms of Feynman diagrams involving only the fermion degrees of freedom and of Feynman diagrams involving fermion and phonon degrees of freedom is proved for intermediate states in the case of a general two-body residual interaction

1. Introduction

We discuss the problem of fermions moving in a set of single-particle levels, and interacting through a residual two-body force. This problem may be solved, either by performing a shell-model diagonalization [see, for instance, ref. [1])] or, in perturbation theory, using a Feynman diagrammatic expansion [see, for instance, refs [2, 3])].

Conceptual and practical simplifications are achieved by describing this physical situation in terms of fermion and collective variables. These two degrees of freedom are coupled through the particle-vibration interaction, which has been usually assumed to be linear in the phonon coordinate and quadratic in the fermion creation and annihilation operators This framework has been extensively used in nuclear physics, ever since suggested in ref [4]). However, for a long time, only the vertices of figs 6c and d were taken into account An important development, within this framework, consisted in relating the value of the collective parameters to the microscopic structure [5]), and also, to consider two particles [6]) or one superconducting quasiparticle [7]) interacting with the phonons.

[†] Fellow of the Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina

[††] Mailing address: Departamento de Física Nuclear, CNEA, Av. Libertador 8250, Buenos Aires (S 29), Argentina

The particle-phonon basis is overcomplete and violates the Pauli principle In 1967, Mottelson [8]) introduced the vertices of figs 6a and b, thus including all the possible orientations of the lines entering a given vertex Therefore, all the time permutations of the vertices were considered in a Feynman diagram corresponding to the particle-vibration interaction. In this way, the Pauli principle between the odd particles and the phonons was taken into account, at least to first order in the interaction strength Detailed applications of this formalism were performed in the Pb region [9, 10]) The particle-pairing phonon vertices (fig 13) were introduced in ref. [10])

However, there still remained questions concerning the order to which the Pauli principle and the overcompleteness of the basis were taken into account, the extent of the equivalence between fermion and field treatment of the residual interaction, the adequacy of the linear particle-phonon interaction, etc. An answer to these questions was given in ref [11]) It was shown there that the four-point vertices (fig 1) have to be included, in addition to the previous particle-phonon vertices (fig. 6 or 13). The rules for using them in a diagrammatic expansion were also provided These rules were verified in simple but non-trivial models [11, 12]). In all cases, the conventional shell-model results were reproduced to the order in which the perturbative field expansion was carried out

In the present paper we prove the equivalence between the Feynman diagrammatic expansion involving only fermions and a similar expansion for the nuclear field treatment of the residual interaction, for processes connecting intermediate states The contributions of field diagrams involving phonons, appear as a partial summation of Feynman diagrams, which is similar (but not equal) to the partial summation implied in the RPA (for instance, four-point vertices are to be kept here).

However, in Feynman diagrams the initial and final states are fermion states which satisfy the Pauli principle, while phonons are used in the basic states of the nuclear field theory The generalization of the proof of the equivalence between the two treatments to process connecting initial and final states is not considered in the present paper This problem has been dealt with only in an heuristic way †.

In this paper we also generalize the rules given in ref [11]) for the particle-vibration vertices, for the case of an arbitrary interaction

In sect 2, we revisit the formalism for the two-fermion propagator In sect 3, we construct a particle-hole field Hamiltonian using the collective degrees of freedom that arise from the partial summation of all the diagrams in which a pair of fermion lines successively interact with each other In sect 4, we study the interaction with an external one-particle field Q, within the field formalism. Applications to particular cases are discussed in sect 5 The particle-particle (hole-hole) case is briefly considered in the appendix

† For schematic models, it has been shown that the spurious components which are present in the initial or final states are eliminated through the field treatment giving rise to states which have zero matrix elements for any processes connecting them with the physical states [12])

2. The two-fermion Green function

The fermion Hamiltonian includes a single-particle term, $H_{s.p.}$, and a two-body residual interaction, $H_{t.b.}$,

$$H = H_{s.p.} + H_{t.b.}, \tag{1}$$

$$H_{s.p.} = \sum_j \varepsilon_j a_j^+ a_j, \tag{2a}$$

$$H_{t.b.} = \tfrac{1}{4} \sum_{j_i} \langle j_1\, j_2 | V | j_3\, j_4 \rangle a_{j_1}^+ a_{j_2}^+ a_{j_4} a_{j_3}. \tag{2b}$$

The matrix elements in (2b) are

$$\langle j_1\, j_2 | V | j_3\, j_4 \rangle = \int \mathrm{d}^3 r_1\, \mathrm{d}^3 r_2\, \varphi_{j_1}^*(1) \varphi_{j_2}^*(2) V(1,2) [\varphi_{j_3}(1)\varphi_{j_4}(2) - \varphi_{j_3}(2)\varphi_{j_4}(1)]. \tag{3}$$

In what follows, we will label with k (i) the states lying above (below) the Fermi energy All the possibilities for the two-body matrix elements (3) are represented in fig. 1

The quantities ε_j in (2a) are the single-particle energies. We assume that all the Hartree-Fock contributions have been included already in the ε_j. Correspondingly, all the diagrams, including processes such as those of fig 2, should be disregarded. The one-fermion free propagator can be written as

$$G_{\mathrm{HF}}(j, t_2 - t_1) = (1 - n_j) \mathrm{e}^{-i\varepsilon_j(t_2 - t_1)} \theta(t_2 - t_1) - n_j\, \mathrm{e}^{-i\varepsilon_j(t_2 - t_1)} \theta(t_1 - t_2), \tag{4a}$$

$$\theta(t) = \begin{cases} 1 & \text{if } t \geqq 0 \\ 0 & \text{if } t < 0, \end{cases} \tag{4b}$$

where $n_j (= 1, 0)$ is the occupation number of the state j.

The interaction (2b) can be treated using a Feynman perturbative expansion. In a general Feynman diagram, the fermions freely propagate [eq. (4)] between any two of the interaction vertices shown in fig. 1 A diagram may contain sections which are joined withthe remaining parts through four "external" fermion lines, j_1, j_2, j_3 and j_4, interacting at the instants t_1, t_2, t_3 and t_4, respectively The contributions of these sections to the total propagator is given by the two-fermion Green function, which is defined as follows·

$$G(j_1 t_1, j_2 t_2, j_3 t_3, j_4 t_4) = \langle \psi | T\{a_{j_1}^+(t_1) a_{j_2}(t_2) a_{j_3}^+(t_3) a_{j_4}(t_4)\} | \psi \rangle. \tag{5}$$

In eq. (5), $|\psi\rangle$ stands for the real ground state of the system; $a_j^+(t)$ is a fermion creation operator in the Heisenberg picture,

$$a_j^+(t) = \mathrm{e}^{iHt} a_j^+ \mathrm{e}^{-iHt},$$

and $T\{\ \ .\}$ is the time-ordering operator. The propagator (5) can be expanded in powers of the interaction. The zero-order term is

$$G^{(0)} = \delta_{j_3 j_4} \delta_{j_1 j_2} G_{\mathrm{HF}}(j_4, t_4 - t_3) G_{\mathrm{HF}}(j_2, t_2 - t_1) - \delta_{j_1 j_4} \delta_{j_3 j_2} G_{\mathrm{HF}}(j_4, t_4 - t_1) G_{\mathrm{HF}}(j_2, t_2 - t_3), \tag{6}$$

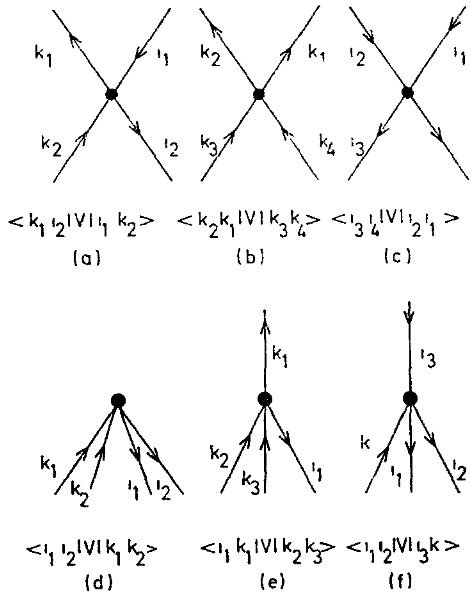

Fig. 1 The four-point vertices of the residual two-body interaction

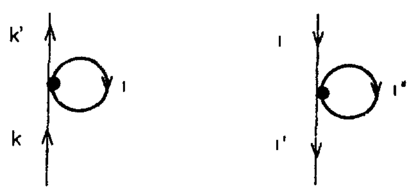

Fig 2. Hartree-Fock corrections to the single-fermion propagator

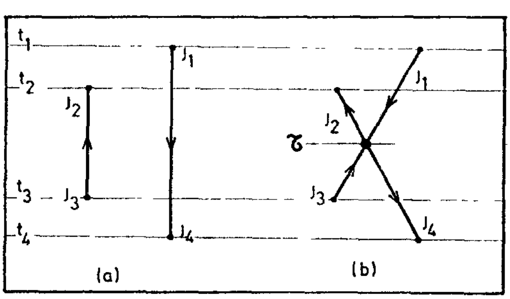

Fig 3 Zero- and first-order contributions to the propagator (5) for the time ordering $t_1 > t_2 > t_3 > t_4$.

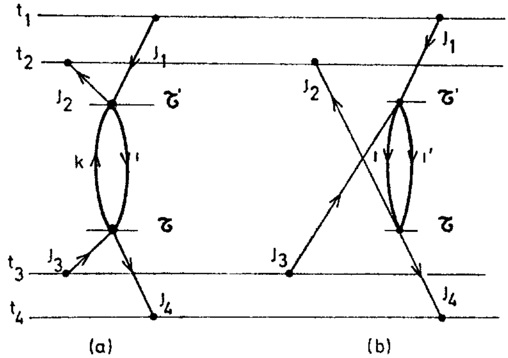

Fig 4 The second-order contributions to the propagator (5) for the time ordering $t_1 > t_2 > t_3 > t_4$. Only contributions of the type 4a are included in (8)

and describes the propagation of two fermion lines without interaction. We shall discuss first the particle-hole case, i.e. t_1, t_2, t_3 and t_4 are such as those of fig 3

The first-order contribution to (5), pictured in fig. (3b), is

$$G^{(1)} = -i\langle j_2\, j_4|V|j_1\, j_3\rangle \int_{-\infty}^{+\infty} \mathrm{d}\tau\, G_{\mathrm{HF}}(j_2, t_2-\tau) G_{\mathrm{HF}}(j_1, \tau-t_1) G_{\mathrm{HF}}(j_3, \tau-t_3) \times G_{\mathrm{HF}}(j_4, t_4-\tau). \quad (7)$$

Since in (7) the intermediate time τ is integrated from $-\infty$ to $+\infty$ the four-point vertices that are involved are those of figs. 1d ($\tau > t_1 > t_2 > t_3 > t_4$), 1f ($t_1 > \tau > t_2 > t_3 > t_4$), 1a ($t_1 > t_2 > \tau > t_3 > t_4$), etc.

Of all the second-order contributions to (5), we will consider those processes in which the two vertices have a particle-hole pair in common (fig. 4a), i e.

$$\mathscr{G}^{(2)}_{\mathrm{ph}} = -(-i)^2 \sum_{ik} \langle j_2\, i|V|j_1\, k\rangle\langle k j_4|V|i j_3\rangle \int_{-\infty}^{+\infty} \mathrm{d}\tau \int_{-\infty}^{+\infty} \mathrm{d}\tau'\, G_{\mathrm{HF}}(j_2, t_2-\tau')$$
$$\times G_{\mathrm{HF}}(j_1, \tau'-t_1) G_{\mathrm{HF}}(k, \tau'-\tau) G_{\mathrm{HF}}(i, \tau-\tau') G_{\mathrm{HF}}(j_3, \tau-t_3) G_{\mathrm{HF}}(j_4, t_4-\tau). \quad (8)$$

The second-order contributions to (5) which are included in (8), involve the propagation of a particle-hole pair (ki) from τ to τ'. There are also higher-order contributions in which the particle and the hole propagate between τ and τ' interacting an arbitrary number of times through all irreducible vertex parts †. These contributions are taken into account if the product of the two fermion free propagators $-G_{\mathrm{HF}}(k, \tau'-\tau) G_{\mathrm{HF}}(i, \tau-\tau')$ is replaced by the full Green function (5) with

† An irreducible vertex part is defined [13]) as a part of a diagram which has a four-point vertex at time t where the line j_3 enters and the line j_4 leaves the graph, and a four-point vertex at time zero, where the line j_1 enters and the line j_2 leaves; it is irreducible if it is not possible to separate it into two parts by breaking only two fermion lines at the same time.

$t_1 = t_2 = \tau$ and $t_3 = t_4 = \tau'$

$$\sum_{\nu \geq 2} \mathscr{G}_{\text{ph}}^{(\nu)} = (-i)^2 \sum_{jj'j''j'''} \langle j_2 j|V|j_1 j'\rangle\langle j''j_4|V|j'''j_3\rangle \int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} d\tau' G_{\text{HF}}(j_2, t_2-\tau')$$
$$\times G_{\text{HF}}(j_1, \tau'-t_1) G_{\text{ph}}(j'j, j''j''', \tau'-\tau) G_{\text{HF}}(j_3, \tau-t_3) G_{\text{HF}}(j_4, t_4-\tau), \quad (9)$$

where

$$G_{\text{ph}}(j'j, j''j''', \tau'-\tau) = G(j\tau', j'\tau', j''\tau, j'''\tau). \quad (10)$$

The Green function (10) has been extensively discussed in the literature [see e.g. refs [13-15]]. We expand (10) in terms of a complete set of eigenstates $|\psi_n\rangle$ of H, having excitation energies ω_n:

$$G_{\text{ph}}(j'j, j''j''', \tau'-\tau) = \sum_n \langle\psi|a_j^+ a_{j'}|\psi_n\rangle\langle\psi_n|a_{j''}^+ a_{j'''}|\psi\rangle e^{-i\omega_n(\tau'-\tau)}\theta(\tau'-\tau)$$
$$+\langle\psi|a_{j''}^+ a_{j'''}|\psi_n\rangle\langle\psi_n|a_j^+ a_{j'}|\psi\rangle e^{i\omega_n(\tau'-\tau)}\theta(\tau-\tau'). \quad (11)$$

By replacing (11) into (9), we obtain

$$\sum_{\nu \geq 2} \mathscr{G}_{\text{ph}}^{(\nu)} = (-i)^2 \int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} d\tau' G_{\text{HF}}(j_2, t_2-\tau') G_{\text{HF}}(j_1, \tau'-t_1)$$
$$\times \sum_n \{\Lambda(j_2 j_1, n)\Lambda^*(j_3 j_4; n) e^{-i\omega_n(\tau'-\tau)}\theta(\tau'-\tau) + \Lambda(j_4 j_3, n)\Lambda^*(j_1 j_2, n)$$
$$\times e^{i\omega_n(\tau'-\tau)}\theta(\tau-\tau')\} G_{\text{HF}}(j_3, \tau-t_3) G_{\text{HF}}(j_4, t_4-\tau), \quad (12)$$

where

$$\Lambda(j_a j_b n) = \sum_{jj'} \langle j_a j|V|j_b j'\rangle\langle\psi\, a_j^+ a_{j'}|\psi_n\rangle. \quad (13)$$

In (10) and (13) the restriction is made that there should be one particle and one hole present in each pair (jj') and $(j''j''')$.

3. Equivalence between the fermion and the nuclear field treatment of the residual interaction

In this section we discuss how the partial summation in (9) corresponds to replace the original Hamiltonian (1) by another Hamiltonian in which extra collective (particle-hole) degrees of freedom are included †.

The factor within curly brackets in (12) has the same time-reversal properties as the propagator of a free phonon [16]) with energy ω_n The summation over n accounts for all the diagrams in which a particle and a hole line mutually interact ν number of times ($\nu \geq 2$), through all possible irreducible vertex parts. The propagation of each mode n is represented by a wavy line. The question whether a wavy line corresponds to the propagation of a true phonon or of a particle-hole pair is irrelevant, since there is no way to distinguish between them. to investigate this difference, another fermion line has to interact with either the particle or the hole line. However,

† The existence of a holomorphic mapping between the Feynman-Goldstone and the nuclear field theory diagrammatic expansions was shown in ref [19]) In this case, however, the proof was carried out only for the energy, and in the framework of the single-particle propagator formalism.

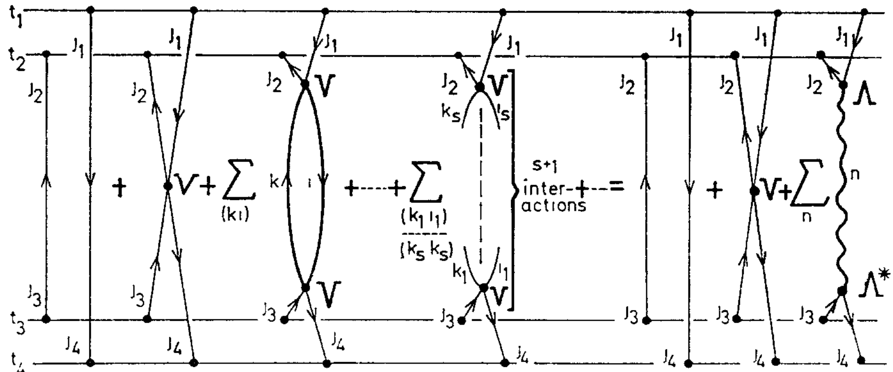

Fig. 5. Correspondence between the Feynman diagrammatic expansion of the propagator (5) in the pure fermion treatment (left) with the Feynman diagrams associated with the same propagator in the nuclear field treatment (right)

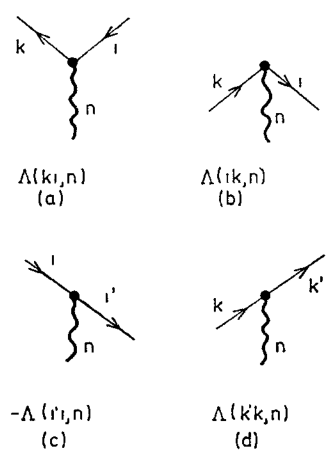

Fig 6. The particle-vibration coupling vertices for the particle-hole bosons

we have assumed that either the particle-hole pair or the phonon propagates without interacting with any other line of the diagram between their creation and annihilation.

The crossing of a wavy line with another line of the diagram (either a fermion line or another wavy line) corresponds to an even number of crossings between fermion lines Therefore, it does not introduce any extra minus sign, as the crossing between two fermion lines does. This is a further evidence of the identical behavior of the propagators of a boson and a particle-hole pair.

The factors $\Lambda^*(j_a j_b, n)$ $(\Lambda(j_a j_b; n))$ represent the amplitude for the creation (annihilation) of a phonon n and a single fermion transition from the state j_b to the state j_a (fig. 5). They should thus be considered as the strength of the particle-phonon

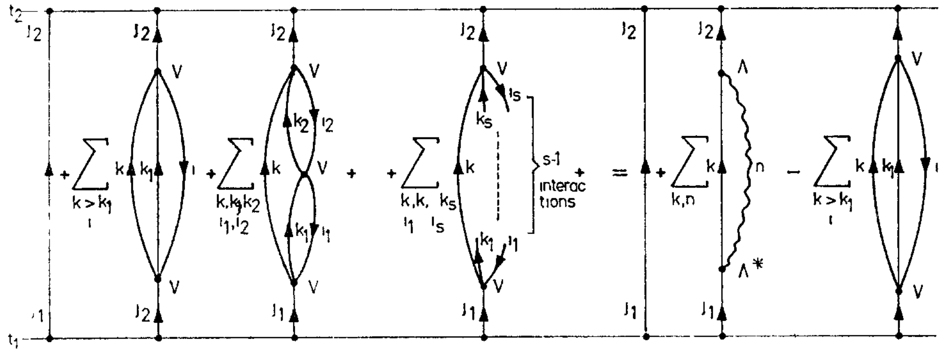

Fig 7 Correspondence between the Feynman diagrammatic expansion of the one-body propagator in the pure fermion treatment (left) with the Feynman diagrams associated with the same propagator in the nuclear field treatment (right).

interaction However, these vertices do not exhaust the effects of the two-body residual interaction In fact, the four-point vertices still give a contribution through $G^{(1)}$ We thus justify the replacement of the fermion Hamiltonian (1) by the nuclear field Hamiltonian introduced in ref [11]), namely

$$
\begin{aligned}
H_{\mathrm{f}} &= H_{\mathrm{s\,p}} + H_{\mathrm{t\,b}} + H_{\mathrm{b}} + H_{\mathrm{p\,v}}\,, \\
H_{\mathrm{s\,p}} &= \sum_{j} \varepsilon_j a_j^+ a_j\,, \\
H_{\mathrm{t\,b}} &= \tfrac{1}{4} \sum_{j} \langle j_1 j_2 | V | j_3 j_4 \rangle a_{j_1}^+ a_{j_2}^+ a_{j_4} a_{j_3}\,, \\
H_{\mathrm{b}} &= \sum_{n} \omega_n \Gamma_n^+ \Gamma_n\,, \\
H_{\mathrm{p\,v}} &= \sum_{n} \sum_{j_1 j_2} \{ \Lambda^*(j_1 j_2 n) \Gamma_n^+ a_{j_2}^+ a_{j_1} + \Lambda(j_1 j_2 n) \Gamma_n a_{j_1}^+ a_{j_2} \}\,,
\end{aligned}
\tag{14}
$$

where Γ_n^+ is the phonon creation operator.

In (14) the two-body interaction $H_{\mathrm{t\,b}}$ does not contain Hartree-Fock contributions These are incorporated in the single-particle term Hamiltonian $H_{\mathrm{s\,p}}$ Both the vertices of $H_{\mathrm{t\,b}}$ and the particle-vibration interaction $H_{\mathrm{p\,v}}$ (figs. 1 and 6) have to be included, and all time permutations of these vertices have to be taken into account Diagrams involving the simultaneous creation of a particle-hole pair of lines, any number of succesive interactions between them, and the simultaneous annihilation of the pair (bubbles), should be disregarded Because of this last rule, only a diagrammatic treatment of (14) appears to be feasible

Note that in eqs. (14) the collective degrees of freedom are independent of the single-particle ones, namely

$$[\Gamma_n^+, a_j^+] = [\Gamma_n, a_j^+] = 0.$$

The state $|0\rangle$ is the vacuum, both for fermions and bosons,

$$\Gamma_n|0\rangle = a_i^+|0\rangle = a_k|0\rangle = 0.$$

There are four possible particle-phonon interaction vertices (13), according to whether the fermion states j_a, j_b correspond to particles or holes (fig. 6),

$$\Lambda(ki, n) = \langle 0|a_i^+ a_k H_{\mathrm{p\,v}}\, \Gamma_n^+|0\rangle = \sum_{k'i'} \langle ki'|V|ik'\rangle\lambda(k'i', n) + \langle kk'|V|ii'\rangle\mu(k'i', n),$$

$$\Lambda(ik, n) = \langle 0|H_{\mathrm{p\,v}}\, \Gamma_n^+ a_k^+ a_i|0\rangle = \sum_{k'i'} \langle ii'|V|kk'\rangle\lambda(k'i', n) + \langle ik'|V|ki'\rangle\mu(k'i', n),$$

$$\Lambda(k_1 k_2, n) = \langle 0|a_{k_1} H_{\mathrm{p\,v}}\, a_{k_2}^+ \Gamma_n^+|0\rangle = \sum_{k'i'} \langle k_1 i'|V|k_2 k'\rangle\lambda(k'i', n) + \langle k_1 k'|V|k_2 i'\rangle\mu(k'i', n),$$

$$\Lambda(i_1 i_2, n) = -\langle 0|a_{i_2}^+ H_{\mathrm{p\,v}}\, a_{i_1} \Gamma_n^+|0\rangle = \sum_{k'i'} \langle i_1 i'|V|i_2 k'\rangle\lambda(k'i', n) + \langle i_1 k'|V|i_2 i'\rangle \times \mu(k'i', n), \tag{15}$$

where the amplitudes λ and μ are defined as

$$\lambda(ki, n) = \langle\psi|a_i^+ a_k|\psi_n\rangle; \qquad \mu(ki, n) = \langle\psi|a_k^+ a_i|\psi_n\rangle. \tag{16}$$

The equivalence between the Hamiltonian (1) and (14) is thus established for sections which are joined with the remaining part of the diagram through four external fermion lines.

This equivalence may be similarly established for sections which are linked with the remaining part by only two external lines However, an additional rule must be introduced in this case †.

The zero-order contribution to the corresponding one-body Green function represents the propagation of a free fermion (fig 7) There are no first-order terms The second-order contribution contains two equivalent fermion lines. We select the higher contributions in which the same pair of particle-hole lines successively interact through all irreducible vertex parts. All these contributions can be replaced by a diagram corresponding to the free propagation plus a diagram containing a phonon, following the same procedure as before However, this procedure implies an independent summation over the intermediate single-particle states for each of the two equivalent fermion lines in the second-order contribution. Thus, the second-order term is taken into account twice and, therefore, it must be subtracted once; whenever there is a fermion line and a boson line which appear and disappear at the same vertices, we must include another diagram in which the phonon line is replaced by a particle-hole pair, and which is evaluated with an additional minus sign. In this diagram the summation over the intermediate two-particle states should be ordered, as is usual in the Feynman diagrammatic treatment [17]) The two-particle lines and the hole line do not interact between the two vertices.

In ref. [11]) the frequencies ω_n and the fermion-boson interacting strengths $\Lambda(j_a j_b, n)$ are obtained using the RPA. Consequently, only the irreducible vertex parts consisting of the four-point RPA vertices (figs. 1a and d) are included

† The difficulties connected with the treatment of the single-particle states with the particle-vibration coupling were pointed out to us by M Baranger

in the construction of the phonons This requirement corresponds to a particular solution of the two-body Green function (10) (possibly the only practical solution) but the Hamiltonian (14) is, in principle, more general

All the second-order processes in which the two vertices have a pair of particles (holes) in common (fig 4b) are not considered in eq (8). Correspondingly, the higher-order terms in which the intermediate pair of particles (holes) interact an arbitrary number of times, are left outside the partial summation (9) Hence all these diagrams should be explicitly considered when the particle-hole field (14) is used A straight-forward generalization of (14) consists in including also the particle-particle (hole-hole) field Hamiltonian, in order to replace the corresponding set of diagrams (see the appendix).

4. Interaction with an external field

The interaction of the system with an external field is proportional to the one-body operator

$$Q = \sum_{jj'} \langle j|Q|j'\rangle a_j^+ a_{j'} \tag{17}$$

Any fermion diagram that describes the effect of (17) contains two fermion lines (jj') having a common vertex $\langle j|Q|j'\rangle$ at an instant t_0 These two lines are included within a section of the total diagram, such that this section joins the remaining part of the diagram through the two "external" fermion lines $(j_1 j_2)$ at the instants t_1 and t_2 $(t_1 \neq t_2)$ respectively (see fig 8) The contribution of this section to the time evolution of the system is

$$G_Q = \sum_{jj'} \langle j|Q|j'\rangle G(j_1 t_1, j_2 t_2, j t_0, j' t_0) \tag{18}$$

In addition to the case of free propagation within the section, let us consider those cases in which the two lines $(j_1 j_2)$ have a common interaction vertex at the time τ, which is also common to the "internal" particle-hole pair, $(j_3 j_4)$ We also assume that (jj') corresponds to a particle-hole pair Applying the same arguments as in

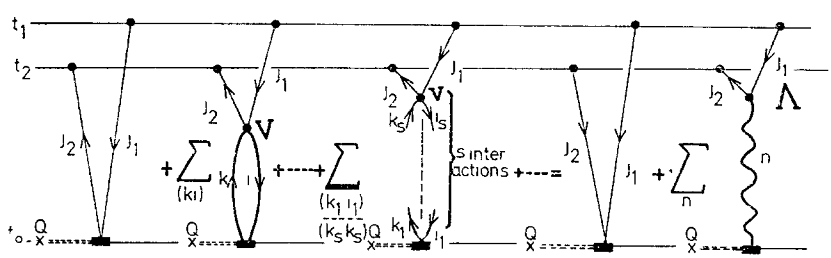

Fig 8 Correspondence between the Feynman diagrammatic expansion describing the effect of an external single operator Q in the pure fermion treatment (left) with the Feynman diagrams associated with the same process in the nuclear field treatment (right)

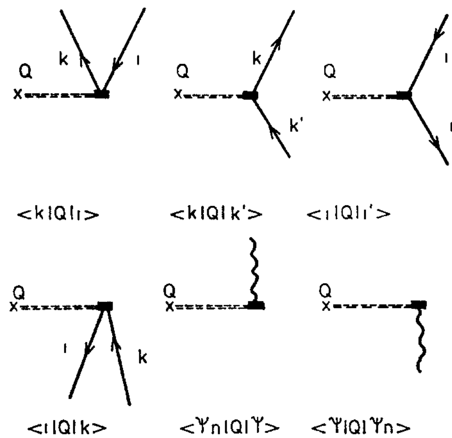

Fig 9 The vertices of the single-particle operator Q within the field treatment

sect. 2, eq. (18) can be written in terms of the usual particle-hole Green function (10) which depends on a single time difference $\tau - t_0$:

$$G_Q = -\langle j_2|Q|j_1\rangle G_{\mathrm{HF}}(j_2\,,\, t_2-t_0)G_{\mathrm{HF}}(j_1\,,\, t_0-t_1)-(-i)$$

$$\times \sum_{j_3 j_4}\sum_{jj'}\left\{\langle j|Q|j'\rangle\langle j_2\, j_4|V|j_1\, j_3\rangle \int_{-\infty}^{+\infty} \mathrm{d}\tau\, G_{\mathrm{HF}}(j_2\,,\, t_2-\tau)G_{\mathrm{HF}}(j_1\,,\, \tau-t_1)\right.$$

$$\left. \times\, G_{\mathrm{ph}}(j_3\, j_4\,,\, jj',\, \tau-t_0)\right\}. \qquad (19)$$

In order to obtain the field representation of the operator Q, we use in (19) the expansion (11):

$$G_Q = -\langle j_2|Q|j_1\rangle G_{\mathrm{HF}}(j_2\,,\, t_2-t_0)G_{\mathrm{HF}}(j_1\,,\, t_0-t_1)-(-i)\int_{-\infty}^{+\infty}\mathrm{d}\tau$$

$$\times\, G_{\mathrm{HF}}(j_2\,,\, t_2-\tau)G_{\mathrm{HF}}(j_1\,,\, \tau-t_1)\sum_n \{\Lambda(j_2\, j_1\,,\, n)\langle\psi_n|Q|\psi\rangle \mathrm{e}^{-i\omega_n(\tau-t_0)}\theta(\tau-t_0)$$

$$+\Lambda^*(j_1\, j_2\, n)\langle\psi|Q|\psi_n\rangle \mathrm{e}^{i\omega_n(\tau-t_0)}\theta(t_0-\tau)\}, \qquad (20)$$

where

$$\langle\psi_n|Q|\psi\rangle = \sum_{jj'}\langle j|Q|j'\rangle\langle\psi_n|a_j^+ a_{j'}|\psi\rangle = \sum_{ki}\{\langle k|Q|i\rangle\lambda^*(ki,\, n)+\langle i|Q|k\rangle\mu^*(ki,\, n)\},$$

$$\langle\psi|Q|\psi_n\rangle = \sum_{jj'}\langle j|Q|j'\rangle\langle\psi|a_j^+ a_{j'}|\psi_n\rangle = \sum_{ki}\{\langle k|Q|i\rangle\mu(ki,\, n)+\langle i|Q|k\rangle\lambda(ki,\, n)\}. \qquad (21)$$

The first term in (20) represents the amplitude for the transition between the single-particle states $(j_1 j_2)$ The second term accounts for a whole set of diagrams in which an initial particle and an initial hole state successively interact through all possible irreducible vertex parts, until the final external lines are simultaneously created. This set of diagrams is replaced by a single one in the field language (fig. 8). Thecoefficients (21) represent the amplitude with which the phonon n is created or annihilated by the operator Q

In the field formalism, the operator Q contains both a fermion term Q_{f} plus a boson term Q_{b},

$$Q = Q_{\mathrm{f}} + Q_{\mathrm{b}}, \tag{22a}$$

$$Q_{\mathrm{f}} = \sum_{jj'} \langle j|Q|j'\rangle a_j^+ a_{j'}, \tag{22b}$$

$$Q_{\mathrm{b}} = \sum_n \langle \psi_n|Q|\psi\rangle \Gamma_n^+ + \langle \psi|Q|\psi_n\rangle \Gamma_n \tag{22c}$$

Thus, in the field formalism, a one-body operator has the vertices shown in fig. 9.

5. Evaluation of the phonon frequencies and the fermion-boson vertices within the RPA

The arguments of the preceding sections are completely valid for the most general Green function (10). If the *exact* propagator is known, all the diagrams of fig. 10 are "forbidden" diagrams

However, in practice, the propagator (10) can only be evaluated performing various approximations. The most important is to consider the four-point vertices figs. 1a and d as the only irreducible vertex parts in all the intermediate processes between τ and τ'. This "ladder" approximation leads [5]) to the RPA eigenvalue equations

$$\begin{gathered}(\varepsilon_k - \varepsilon_i)\lambda(ki, n) + \sum_{k_1 i_1} \{\langle ki_1|V|ik_1\rangle \lambda(k_1 i_1, n) + \langle kk_1|V|ii_1\rangle \mu(k_1 i_1, n)\} = \omega_n \lambda(ki, n), \\ -(\varepsilon_k - \varepsilon_i)\mu(ki, n) - \sum_{k_1 i_1} \{\langle ki_1|V|ik_1\rangle \mu(k_1 i_1, n) + \langle kk_1|V|ii_1\rangle \lambda(k_1 i_1, n)\} = \omega_n \mu(ki, n),\end{gathered} \tag{23}$$

where the amplitudes $\lambda(ki, n)$ and $\mu(ki, n)$ [eq. (16)] fulfill the orthonormalization condition

$$\sum_{ki} \{\lambda(ki, n)\lambda^*(ki, n') - \mu(ki, n)\mu^*(ki, n')\} = \delta_{nn'}. \tag{24}$$

In this case, diagrams such as 10d and e must be explicitly taken into account, since they are not included in the definition of the phonons; the diagrams 10a, b and c remain "forbidden"

Another possible further approximation to (10) is to exclude the vertex 1d from the construction of the phonon (TDA) In such a case, 10b becomes a legitimate diagram, which is neither accounted for in the propagator of the boson nor in the particle-boson vertices These vertices are readily obtained from (15) by setting $\mu(ki, n) = 0$

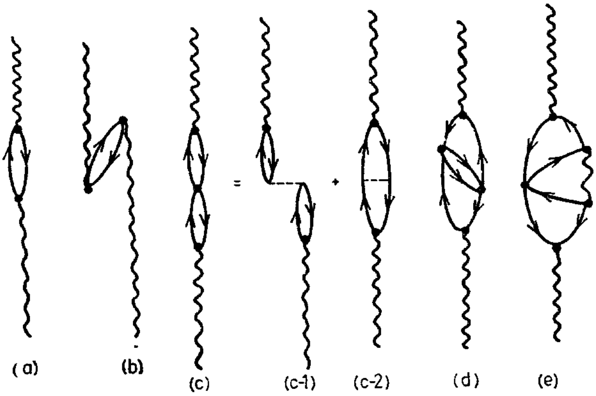

Fig 10 Examples of diagrams that are forbidden if the exact propagator (10) is used

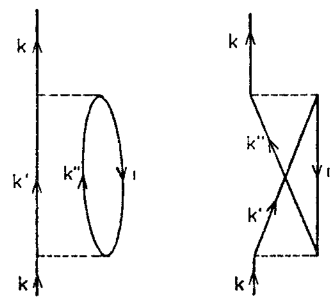

Fig 11. The second-order processes in the propagation of a fermion line

An alternative approximation is to consider only the direct part of the four-point vertices 1a and d which appear in the construction of the boson. This procedure may be useful when the residual interaction becomes separable within this approximation (such as for a multipole force). In this case, the matrix elements of the interaction have the form

$$\langle j_1 j_2 | V | j_3 j_4 \rangle = -V_{j_1 j_3} V_{j_2 j_4} g. \tag{25}$$

The eigenvalue equation (23) is replaced by the dispersion relation

$$1 = \sum_{ki} g |V_{ki}|^2 2(\varepsilon_k - \varepsilon_i)/[(\varepsilon_k - \varepsilon_i)^2 - \omega^2], \tag{26}$$

and the amplitudes $\lambda(ki, n)$, $\mu(ki, n)$ have the value

$$\lambda(ki, n) = \frac{V_{ki}}{(\varepsilon_k - \varepsilon_i) - \omega_n} \left\{ \sum_{ki} \frac{|V_{ki}|^2 4(\varepsilon_k - \varepsilon_i)\omega_n}{[(\varepsilon_k - \varepsilon_i)^2 - \omega_n^2]^2} \right\}^{-\frac{1}{2}}, \tag{27}$$

$$\mu(ki, n) = \frac{V_{ki}}{(\varepsilon_k - \varepsilon_i) + \omega_n} \left\{ \sum_{ki} \frac{|V_{ki}|^2 4(\varepsilon_k - \varepsilon_i)\omega_n}{[(\varepsilon_k - \varepsilon_i)^2 - \omega_n^2]^2} \right\}^{-\frac{1}{2}}. \tag{28}$$

Introducing (25), (27) and (28) in (15), we obtain for the particle-vibration vertices

$$\Lambda(jj', n) = -g V_{jj'} \Lambda_n,$$

with

$$\Lambda_n = \sum_{ki} V_{ki}[\lambda(ki, n)+\mu(ki, n)] = \frac{1}{2g}\left[\omega_n \sum_{ki} \frac{|V_{ki}|^2(\varepsilon_k-\varepsilon_i)}{[(\varepsilon_k-\varepsilon_i)^2-\omega_n^2]^2}\right]^{-\frac{1}{2}}.$$

In this case, one has to distinguish between the direct and exchange vertices in any diagram and disregard those diagrams presenting successive direct interactions between a particle and a hole line In the example of fig. 10c, only diagram 10c-1 should be disregarded, while the diagram 10c-2 should explicitly be taken into account. In this way, one gains a greater simplification in the construction of the phonon, at the expense of including more diagrams in the perturbative expansion An advantage of this procedure appears when we are interested in separating diagrams contributing to different powers of Ω^{-1}, Ω being related to the degeneracy of the single-particle states [12]) According to eq. (26), $g^{\frac{1}{2}} V_{ki}$ is of the order $\Omega^{-\frac{1}{2}}$ with respect to the (zero-order) particle-hole energy $\varepsilon_k-\varepsilon_i$ or to the frequencies ω_n Therefore, Λ_n is of order unity, the particle-phonon vertices of order $\Omega^{-\frac{1}{2}}$, and the four-point vertices of order Ω^{-1}. Since each closed loop of the fermion lines introduces a contribution of order Ω, the diagram 10c-1 is of order $\Lambda^2(ki, n)V_{ki}^2\Omega^2 = O(\Omega^0)$ while the diagram 10c-2 is of order $\Lambda^2(ki, n)V_{ki}^2\Omega = O(\Omega^{-1})$.

Another advantage of including only the direct terms of a separable interaction in the contribution of the phonons is that we do not need to subtract any longer the second-order graph which appears when there is a fermion and a phonon line in parallel (fig. 7) This is due to the fact that the two lines k' and k'' in fig. 11 are no longer equivalent and, thus, both k' and k'' are summed independently over all single-particle states

6. Conclusion

There is a holomorphic mapping between the Feynman diagrams corresponding to the pure fermion Hamiltonian [1]) and to thenuclear field Hamiltonian (14) of ref. [11]) The subset of fermion diagrams which differ only within a particular section and such that this section represents the propagation of a particle-hole pair is replaced by three diagrams within the field treatment The two diagrams corresponding to the zero- and first-order propagators remain as in the fermion treatment and a single diagram containing a phonon substitutes the remaining elements of the subset

The fermion Hamiltonian (1) and the nuclear field Hamiltonian (14) yield identical values for the two-particle propagators, provided that the phonon frequencies and the strength of the particle-phonon vertices are adequately calculated The RPA is the most convenient (although not the most general) way of obtaining these quantities Similar considerations are made for the vertices corresponding to any external one-body operator.

The nuclear field Hamiltonian treats the fermion and phonon degrees of freedom on an equal footing The eigenfunctions of the zero-order Hamiltonian in eqs. (14) are the elementary modes of excitation of the nuclear system. The product of these eigenfunctions thus provides a basis for dealing with the many-body nuclear problem, which displays the basic features of the nuclear correlations The nuclear field theory diagrammatic expansion of the residual interactions is a systematic method for correcting the different short-comings of this basis. violations of the Pauli principle, overcompleteness, effects of the two-body interaction which are not included in the zero-order wave functions, etc.

Discussions with M. Baranger, A. Bohr, P F. Bortignon, H. Feshbach, A. Kerman, B. Mottelson, J. Negele, T. Marumori and H. Sofía have greatly stimulated this work.

Note added in proof: After this paper was submitted for publication, the work of ref [18]), which discusses some of the subjects dealt with in this paper came to our attention.

Appendix

THE PARTICLE-PARTICLE AND THE HOLE-HOLE PHONONS

The particle-hole case is discussed in the previous sections. The same arguments can be carried over for the situation in which the succesive interactions take place between two-particle(or two-hole) lines.

We start by assuming a different time ordering than that of fig 3, namely with $t_2 > t_1$ (fig 12)

Out of all the second-order contributions, we have now to consider only the processes in which the two vertices have a pair of particles (or holes) in common (fig. 12c) By replacing the intermediate free propagator by the full particle-particle Green function, we can write the equivalent of eq (9)

$$\sum_{\nu \geq 2} \mathscr{G}^{(\nu)}_{\mathrm{pp}} = (-i)^2 \sum_{\substack{j<j' \\ j''<j'''}} \langle j_4 j_2 | V | j j' \rangle \langle j'' j''' | V | j_1 j_3 \rangle \int_{-\infty}^{+\infty} \mathrm{d}\tau \int_{-\infty}^{+\infty} \mathrm{d}\tau' G_{\mathrm{HF}}(j_4, t_4 - \tau')$$

$$\times G_{\mathrm{HF}}(j_2, t_2 - \tau') G_{\mathrm{pp}}(jj', j''j''', \tau' - \tau) G_{\mathrm{HF}}(j_3, \tau - t_3) G_{\mathrm{HF}}(j_1, \tau - t_1), \qquad \text{(A 9)}$$

where

$$G_{\mathrm{pp}}(jj', j''j''', \tau - \tau') = G(j''\tau, j\tau', j'''\tau, j'\tau') \qquad \text{(A.10)}$$

is the particle-particle Green function [14, 17]). The restriction upon the pairs (jj') and $(j''j''')$ is that they are either two particles or two holes The expansion (11) now reads

$$G_{\mathrm{pp}}(jj', j''j''', \tau - \tau') = \sum_n \langle \psi | a_j a_{j'} | \psi_n \rangle \langle \psi_n | a^+_{j''} a^+_{j'''} | \psi \rangle \mathrm{e}^{-i\omega_n(\tau' - \tau)} \theta(\tau' - \tau)$$

$$+ \langle \psi | a^+_{j''} a^+_{j'''} | \psi_n \rangle \langle \psi_n | a_j a_{j'} | \psi \rangle \mathrm{e}^{i\omega_n(\tau' - \tau)} \theta(\tau' - \tau) \qquad \text{(A.11)}$$

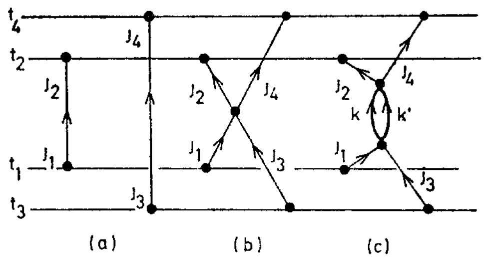

Fig 12 Zero-, first- and second-order contributions to the propagator (5) corresponding to the time ordering $t_4 > t_2 > t_1 > t_3$

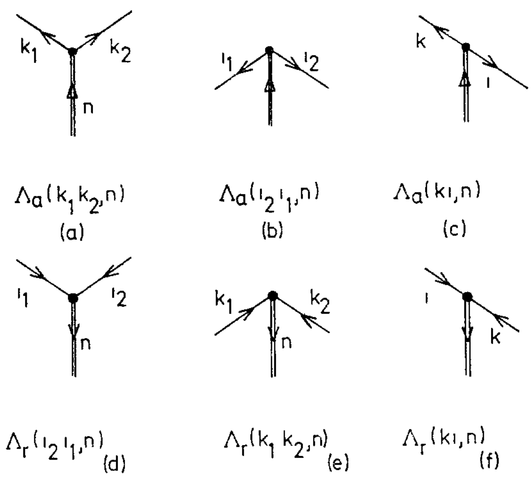

Fig. 13 The particle-vibration coupling vertices for the addition and removal pairing bosons

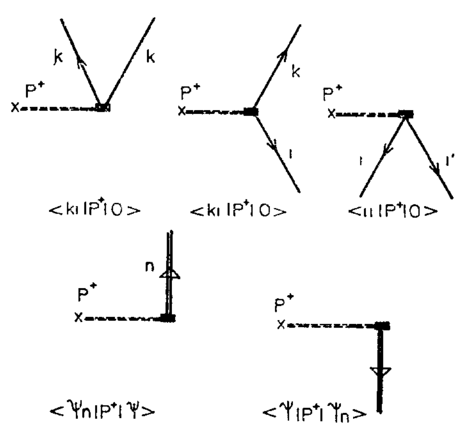

Fig 14 The vertices of the two-body creation operator P^+ within the field treatment

The field representation of (A.9) is obtained using (A.11):

$$\sum_{\nu \geq 2} \mathscr{G}^{(\nu)}_{\mathrm{pp}} = -\int_{-\infty}^{+\infty} \mathrm{d}\tau \int_{-\infty}^{+\infty} \mathrm{d}\tau' G_{\mathrm{HF}}(j_4, t_4-\tau) G_{\mathrm{HF}}(j_2, t_2-\tau)$$

$$\times \sum_n \{\Lambda_{\mathrm{a}}^*(j_1 j_3, n)\Lambda_{\mathrm{a}}(j_4 j_2, n)\mathrm{e}^{-i\omega_n(\tau-\tau')}\theta(\tau'-\tau)$$

$$+\Lambda_{\mathrm{r}}^*(j_4 j_2, n)\Lambda_{\mathrm{r}}(j_1 j_3, n)\mathrm{e}^{i\omega_n(\tau-\tau')}\theta(\tau-\tau')\} G_{\mathrm{HF}}(j_1, \tau'-t_1) G_{\mathrm{HF}}(j_3, \tau'-t_3). \quad (\mathrm{A.12})$$

In this case the particle-vibration vertices correspond to the addition and to the removal phonons:

$$\Lambda_{\mathrm{a}}(j_a j_b, n) = \sum_{j>j'} \langle j_a j_b | V | jj' \rangle \langle \psi | a_{j'} a_j | \psi_n \rangle, \quad (\mathrm{A.13a})$$

$$\Lambda_{\mathrm{r}}(j_a j_b, n) = \sum_{j>j'} \langle jj' | V | j_a j_b \rangle \langle \psi | a_j^+ a_{j'}^+ | \psi_n \rangle. \quad (\mathrm{A.13b})$$

The set of diagrams in which two particle(hole) lines interact ν number of times ($\nu \geq 2$) is replaced by a single diagram in which an addition-type (removal-type) boson is present. The field Hamiltonian now reads

$$H_{\mathrm{f}} = H_{\mathrm{s\,p}} + H_{\mathrm{t\,b}} + H_{\mathrm{b}} + H_{\mathrm{p\,v}}, \quad (\mathrm{A.14a})$$

$$H_{\mathrm{b}} = \sum_n \{\omega_{\mathrm{a},n} \Gamma_{\mathrm{a},n}^+ \Gamma_{\mathrm{a},n} + \omega_{\mathrm{r},n} \Gamma_{\mathrm{r},n}^+ \Gamma_{\mathrm{r},n}\}, \quad (\mathrm{A\ 14b})$$

$$H_{\mathrm{p\,v}} = \sum_n \sum_{j_1>j_2} \{(\Lambda_{\mathrm{a}}^*(j_1 j_2, n)\Gamma_{\mathrm{a},n}^+ + \Lambda_{\mathrm{r}}(j_1 j_2, n)\Gamma_{\mathrm{r},n}) a_{j_2} a_{j_1}$$

$$+(\Lambda_{\mathrm{a}}(j_1 j_2, n)\Gamma_{\mathrm{a},n} + \Lambda_{\mathrm{r}}^*(j_1 j_2, n)\Gamma_{\mathrm{r},n}^+) a_{j_1}^+ a_{j_2}^+\}. \quad (\mathrm{A.14c})$$

The particle-vibration vertices are pictured in fig. 13. An operator creating two particles has the form (fig. 14)

$$P^+ = \sum_{j>j'} \langle jj' | P^+ | 0 \rangle a_j^+ a_{j'}^+ + \sum_n \{\langle \psi_n | P^+ | \psi \rangle \Gamma_{\mathrm{a},n}^+ + \langle \psi | P^+ | \psi_n \rangle \Gamma_{\mathrm{r},n}\}, \quad (\mathrm{A.22})$$

where

$$\langle \psi_n | P^+ | \psi \rangle = \sum_{j>j'} \langle jj' | P | 0 \rangle \langle \psi_n | a_j^+ a_{j'}^+ | \psi \rangle, \quad (\mathrm{A.21a})$$

$$\langle \psi | P^+ | \psi_n \rangle = \sum_{j>j'} \langle jj' | P | 0 \rangle \langle \psi | a_j^+ a_{j'}^+ | \psi_n \rangle. \quad (\mathrm{A.21b})$$

Again, the most practical way of obtaining the frequencies ω and the vertices (A 13) and (A.21) is through the RPA

In the case of pairing phonons, one does not subtract an extra diagram when there are a fermion and a boson line which appear (and disappear) at the same vertices as for particle-hole phonons.

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