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ON THE (d, ^{6}Li) REACTION

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Abstract: General features of the (d, ^{6}Li) reaction are discussed in connections with some residual forces. An enhancement of ground state transitions among superconducting nuclei is found. Some examples in the 1p shell are also worked out and coherence is found between the $T = 0$ and $T = 1$ channels of the residual force.

1. Introduction

The purpose of the present paper is to study some features of the α-particle transfer and in particular of the (d, ^{6}Li) reactions. The method is an extension of the usual procedure applied in the case of two-particle transfer reactions [1]) which involves an expansion in terms of fractional parentage coefficients and a separation of the c.m. and relative coordinates of the transferred group of particles.

In sect. 2 the general formalism is developed. Although the (d, ^{6}Li) reaction is specifically treated, the methods can be extended to different reactions. A modified form factor is defined that describes the motion of the c.m. of the transferred α-cluster and the whole problem can be treated with ordinary DWBA codes for one-particle transfer reactions.

In sect. 3 some specific examples are worked out, the first is devoted to α-transfer reactions within the pairing scheme and the second to transitions among 1p shell nuclei. As it was previously pointed out [2]), in the first case an enhancement is found of ground state transitions among superconducting nuclei. Features of the form factor are discussed together with the contribution of the interior in this kind of reactions. In the case of the p-shell calculations a comparison is made between the results of the present paper and some of the available experimental evidence. In the same section some additional calculations are performed using model residual forces acting separately in the $T = 0$ and $T = 1$ channels. The purpose of this is to study coherence effects in the nuclear structure part of the cross section.

2. General formalism

The differential cross section for the (d, ^{6}Li) reaction is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{m_{\mathrm{d}}^* m_{\mathrm{Li}}^*}{(2\pi\hbar^2)^2} \frac{k_{\mathrm{Li}}}{k_{\mathrm{d}}} \frac{1}{(2J_{\mathrm{i}}+1)(2J_{\mathrm{d}}+1)} \sum_{\substack{M_{J_{\mathrm{i}}} M_{J_{\mathrm{f}}} \\ M_{\mathrm{d}} M_{\mathrm{Li}}}} |T_{\mathrm{d,Li}}|^2, \tag{1}$$

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where m^* denotes the reduced mass. The reaction amplitude is

$$T_{\mathrm{d,Li}} = \langle \phi_{\mathrm{Li}}^{(-)}(\boldsymbol{k}_{\mathrm{Li}}, \boldsymbol{R}_{\mathrm{Li}}) \psi_{\mathrm{Li}}(J_{\mathrm{Li}}\, T_{\mathrm{Li}}\, M_{\mathrm{Li}}\, 0) \psi_A(\Gamma_{\mathrm{f}}\, J_{\mathrm{f}}\, T_{\mathrm{f}}\, M_{J_{\mathrm{f}}}\, M_{T_{\mathrm{f}}}) | V(\rho) \times | \psi_{A+4}(J_{\mathrm{i}}\, T_{\mathrm{i}}\, M_{J_{\mathrm{i}}}\, M_{T_{\mathrm{i}}}) \psi_{\mathrm{d}}(J_{\mathrm{d}}\, T_{\mathrm{d}}\, M_{\mathrm{d}}\, 0) \phi_{\mathrm{d}}^{(+)}(\boldsymbol{k}_{\mathrm{d}}, \boldsymbol{R}_{\mathrm{d}}) \rangle. \tag{2}$$

As usual $\phi_{\mathrm{Li}}^{(-)}$ and $\phi_{\mathrm{d}}^{(+)}$ are the outgoing and incoming distorted waves of the ^{6}Li and the deuteron, ψ_{Li} and ψ_{d} are their internal wave functions and ψ_{A+4} and ψ_A are the wave functions of the target and residual nuclei respectively. The quantum number Γ_{f} labels the different states of the A nucleons with total angular momentum J_{f} and total isospin T_{f}. The target is assumed to be in its ground state. The pick-up potential $V(\rho)$ is a function only of the relative distance ρ between the deuteron and the transferred α-cluster c.m.

The general procedure involves two mutually related tasks, the first is to separate ψ_{A+4} in a product of two functions describing the motion of the A nucleons and of the four transferred particles, the second aims to the splitting of the intrinsic and center of mass degrees of freedom of the transferred group. When separating ψ_{A+4} into ψ_A and ψ_4 we have to face the problem of completely antisymmetrizing the four-body wave function. A way around this is to perform the expansion in terms of a chain of two-particle fractional parentage coefficients and to use the properly antisymmetrized wave function for the outgoing ^{6}Li. The wave function ψ_{A+4} is therefore expanded by separating first the nucleons 1 and 2 and next 3 and 4; each two-particle wave function is antisymmetric and both are vector coupled to total angular momentum J_4 and isospin T_4,

$$\begin{aligned}\psi_{A+4}(J_{\mathrm{i}}\, T_{\mathrm{i}}\, M_{J_{\mathrm{i}}}\, M_{T_{\mathrm{i}}}) = \sum & \langle \Gamma' J' T', j' j''' J_{34}\, T_{34} \{ | J_{\mathrm{i}}\, T_{\mathrm{i}} \rangle \langle \Gamma'' J'' T'', j j'' J_{12}\, T_{12} \{ | \Gamma' J' T' \rangle \\ & \times U(J'' J_{12}\, J_{\mathrm{i}}\, J_{34}, J' J_4) U(T'' T_{12}\, T_{\mathrm{i}}\, T_{34}, T' T_4) \\ & \times [\psi_A(\Gamma'' J'' T'') \{ \psi_2(j j'' J_{12}\, T_{12}) \psi_2(j' j''' J_{34}\, T_{34}) \}^{J_4 T_4}]^{J_{\mathrm{i}} T_{\mathrm{i}}}. \end{aligned} \tag{3}$$

The summation in (3) runs over all quantum numbers other than those of ψ_{A+4}. The U-coefficients are the unitary 6-j symbols. The four-particle wave function can now be split into two parts, one containing only intrinsic and the other the c.m. coordinates,

$$\begin{aligned} & \{ \psi_2(j j'' J_{12}\, T_{12}) \psi_2(j' j''' J_{34}\, T_{34}) \}^{J_4 T_4}_{M_{J_4} M_{T_4}} \\ & \quad = \sum \begin{pmatrix} l & l'' & L_{12} \\ \frac{1}{2} & \frac{1}{2} & S_{12} \\ j & j'' & J_{12} \end{pmatrix} \begin{pmatrix} l' & l''' & L_{34} \\ \frac{1}{2} & \frac{1}{2} & S_{34} \\ j' & j''' & J_{34} \end{pmatrix} \begin{pmatrix} L_{12} & L_{34} & L_4 \\ S_{12} & S_{34} & S_4 \\ J_{12} & J_{34} & J_4 \end{pmatrix} \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_4 \\ \Lambda_1 & \Lambda_2 & \Lambda_4 \\ L_{12} & L_{34} & L_4 \end{pmatrix} \\ & \quad \times (-)^{\lambda_4 + \Lambda_4 - L_4} U(\Lambda_4 \lambda_4 J_4 S_4, L_4 J_4') \langle \nu_1 \lambda_1, N_1 \Lambda_1; L_{12} | n l, n'' l''; L_{12} \rangle \\ & \quad \times \langle \nu_2 \lambda_2, N_2 \Lambda_2; L_{34} | n' l', n''' l'''; L_{34} \rangle \langle N_r \Lambda_r, N \Lambda; \Lambda_4 | N_1 \Lambda_1 N_2 \Lambda_2; \Lambda_4 \rangle \\ & \quad \times \{ [\phi_{N_r \Lambda_r}(\nu^{\frac{1}{2}} \boldsymbol{R}_r) \phi_{N\Lambda}(2\nu^{\frac{1}{2}} \boldsymbol{R})]^{\Lambda_4} [\{ \phi_{\nu_1 \lambda_1}(\sqrt{2}\nu^{\frac{1}{2}} \boldsymbol{r}_{12}) \phi_{\nu_2 \lambda_2}(\sqrt{2}\nu^{\frac{1}{2}} \boldsymbol{r}_{34}) \}^{\lambda_4} \{ \chi^{S_{12}} \chi^{S_{34}} \}^{S_4}]^{J'_4} \}^{J_4}_{M_{J_4}} \\ & \quad \times \{ \chi^{T_{12}} \chi^{T_{34}} \}^{T_4}_{M_{T_4}}. \end{aligned} \tag{4}$$

In eq. (4) the summation again runs over all quantum numbers other than the ones on the left-hand side. Use has been made of the unitary 9-j coefficients and of harmonic oscillator transformation brackets [3]). The functions $\phi_{nl}(\nu^{\frac{1}{2}}\boldsymbol{r})$ are harmonic oscillator wave functions. The oscillator parameter ν corresponds to the central potential produced by the A nucleons and in which the four transferred particles are assumed to move; r_{ij} and R_r measure the relative distance of the i- and j-nucleons and of the pairs (1, 2) and (3, 4) respectively, $\boldsymbol{R}$ is the coordinate of the c.m. of the four particles referred to the c.m. of the A-nucleons system. Finally $\chi^S\chi^T$ are the spin-charge wave functions.

The next problem is now to properly write the internal wave function of the outgoing ^{6}Li. With this purpose one can do the plausible assumption that the ground state of this nucleus is well described by an α-particle plus two nucleons moving in the 1p shell. Disregarding the small $L \neq 0$ contributions of the two 1p particles, one has

$$\psi_{\mathrm{Li}}(J_{\mathrm{Li}} = 1, T_{\mathrm{Li}} = 0, M_{\mathrm{Li}}, 0)$$
$$= \{\psi_\alpha((L = 0, S = 0)J = 0, T = 0)\psi_2((L = 0, S = 1)J = 1, T = 0)\}^{J_{\mathrm{Li}}=1,\, T_{\mathrm{Li}}=0}_{M_{\mathrm{Li}},\,0}. \tag{5}$$

When expressed in terms of relative coordinates, the α-particle wave function can be cast into the form

$$\psi_\alpha((L = 0, S = 0)J = 0, T = 0) = \phi^0_{00}(\eta^{\frac{1}{2}}r_{12}/\sqrt{2})\phi^0_{00}(\eta^{\frac{1}{2}}r_{34}/\sqrt{2})\phi^0_{00}(\eta^{\frac{1}{2}}R_r)\phi^0_{00}(\tfrac{2}{3}\eta^{\frac{1}{2}}\rho)$$
$$\times \sum_{(S_2+T_2=1)} (-)^{T_2}(2)^{-\frac{1}{2}}\chi((S_2\,S_2)S_\alpha = 0)\chi((T_2\,T_2)T_\alpha = 0), \tag{6}$$

where the ^{6}Li size parameter is denoted by η. The spin-charge part has both the singlet and triplet contributions. The wave function of the two remaining particles is

$$\psi_2((L = 0, S = 1)J = 1, T = 0)$$
$$= \sum_{n_{\mathrm{d}}N_{\mathrm{d}}\Lambda_{\mathrm{d}}J'_{\mathrm{d}}} \langle n_{\mathrm{d}}\Lambda_{\mathrm{d}}, N_{\mathrm{d}}\Lambda_{\mathrm{d}}; 0|01, 01; 0\rangle U(\Lambda_{\mathrm{d}}\Lambda_{\mathrm{d}}11, 0J'_{\mathrm{d}})$$
$$\times \{\phi_{N_{\mathrm{d}}\Lambda_{\mathrm{d}}}(\tfrac{2}{3}(2\eta)^{\frac{1}{2}}\boldsymbol{\rho})[\phi_{n_{\mathrm{d}}\Lambda_{\mathrm{d}}}((\tfrac{1}{2}\eta)^{\frac{1}{2}}\boldsymbol{r}_{\mathrm{d}})\chi(S'_{\mathrm{d}} = 1)]^{J'_{\mathrm{d}}}\}^{J=1}\chi(T_{\mathrm{d}} = 0). \tag{7}$$

One can now use eqs. (7), (6), (5) and (4) to calculate the overlap integral (2). The different factors thus appearing in the expression of the reaction amplitude can be grouped as

$$T_{\mathrm{d,Li}} = \delta_{T_{\mathrm{i}}T_{\mathrm{f}}}\delta_{M_{\mathrm{d}}M_{\mathrm{Li}}} \sum_{NLM} \langle J_{\mathrm{f}}LM_{J_{\mathrm{f}}}M_L|J_{\mathrm{i}}M_{J_{\mathrm{i}}}\rangle(2L+1)^{\frac{1}{2}}\mathcal{G}_{NL}(J_{\mathrm{i}}\,T_{\mathrm{i}}\,\Gamma_{\mathrm{f}}\,J_{\mathrm{f}})B^M_{NL}(\boldsymbol{k}_{\mathrm{Li}}, \boldsymbol{k}_{\mathrm{d}}). \tag{8}$$

The factor B^M_{NL} contains all the kinematic information while the nuclear structure

factor $\mathscr{G}_{NL}$ is

$$\mathscr{G}_{NL}(J_i\,T_i\,\Gamma_f J_f) = \frac{1}{\sqrt{2}}\binom{6}{4}^{\frac{1}{2}}\binom{4}{2}^{\frac{1}{2}}\frac{1}{\sqrt{2L+1}}\sum_{N_1\Lambda_1 N_2\Lambda_2 jj'j''j'''J_{12}J_{34}(S_2+T_2=1)}(-)^{L+\Lambda_2+J_{12}+1}$$
$$\times\frac{((2J_{12}+1)(2J_{34}+1))^{\frac{1}{2}}}{2S_2+1}U(J_{12}J_{34}\Lambda_1\Lambda_2, LS_2)\begin{pmatrix} l & l'' & \Lambda_1\\ \frac{1}{2} & \frac{1}{2} & S_2\\ j & j'' & J_{12}\end{pmatrix}\begin{pmatrix} l' & l''' & \Lambda_2\\ \frac{1}{2} & \frac{1}{2} & S_2\\ j' & j''' & J_{34}\end{pmatrix}$$
$$\times[v_1 0, N_1\Lambda_1;\Lambda_1|nl, n''l'';\Lambda_1][v_2 0, N_2\Lambda_2;\Lambda_2|n'l', n'''l''';\Lambda_2]$$
$$\times[N_r 0, NL; L|N_1\Lambda_1, N_2\Lambda_2; L]\mathscr{C}^{J_iT_i}_{\Gamma_fJ_f}(jj'j''j'''J_{12}J_{34}T_2L). \tag{9}$$

Throughout, the notation

$$[v0, N\Lambda;\Lambda|nl, n'l';\Lambda] = \langle v0, N\Lambda;\Lambda|nl, n'l';\Lambda\rangle\langle u_{00}(\eta^{\frac{1}{2}}r)|u_{v0}(\nu^{\frac{1}{2}}r)\rangle \tag{10}$$

is used, u_{nl} stands for the radial oscillator wave function which corresponds to each case.

The coefficients $\mathscr{C}$ play the rôle of a four-body spectroscopic factor

$$\mathscr{C}^{J_iT_i}_{\Gamma_fJ_f}(jj'j''j'''J_{12}J_{34}T_2L) = \binom{A+4}{2}^{\frac{1}{2}}\binom{A+2}{2}^{\frac{1}{2}}\sum_{\Gamma'J'T'}\frac{(-)^{T_i+T_2-T'}}{((2T_i+1)(2T_2+1))^{\frac{1}{2}}}$$
$$\times U(J_fJ_{12}J_iJ_{34}, J'L)\langle\Gamma_fJ_fT_i, jj''J_{12}T_2\{|\Gamma'J'T'\rangle\langle\Gamma'J'T', j'j'''J_{34}T_2\{|J_iT\rangle. \tag{11}$$

Additional statistical factors are included due to the antisymmetry of the total wave functions in the initial and final states. The kinematical factor reads

$$B^M_{NL}(\boldsymbol{k}_{Li}, \boldsymbol{k}_d) = (2L+1)^{-\frac{1}{2}}\langle\phi^{(-)}_{Li}(\boldsymbol{k}_{Li}, \boldsymbol{R}_{Li})f(\rho)|V(\rho)|\phi^M_{NL}(2\nu^{\frac{1}{2}}\boldsymbol{R})\phi^{(+)}_d(\boldsymbol{k}_d, \boldsymbol{R}_d)\rangle, \tag{12}$$

where $f(\rho)$ is

$$f(\rho) = \sum_{n_dN_d}[n_d 0, N_d 0; 0|01, 01; 0]\phi^0_{00}(\tfrac{2}{3}\eta^{\frac{1}{2}}\rho)\phi^0_{00}(\tfrac{2}{3}\sqrt{2}\eta^{\frac{1}{2}}\rho). \tag{13}$$

Little can be said about the influence of $f(\rho)$ since usually is lumped together with the pick-up potential $V(\rho)$ and both are jointly treated within very schematical models.

The role of the c.m. wave function ϕ^M_{NL} can be better understood if a modified form factor is defined as

$$\Phi_{LM}(\boldsymbol{R}) = \sum_N \mathscr{G}_{NL}(J_i\,T_i\,\Gamma_f J_f)\phi^M_{NL}(2\nu^{\frac{1}{2}}\boldsymbol{R}). \tag{14}$$

Since the $\mathscr{G}_{NL}$ factor is the probability of finding in the $(A+4)$ nucleus an α-particle as it comes out in the outgoing ^{6}Li, the function Φ_{LM} can be interpreted as the wave function of the picked up cluster.

By using formula (8) the differential cross section of eq. (1) can finally be written as the coherent sum over the c.m. radial quantum number N

$$\frac{d\sigma}{d\Omega} = \frac{m^*_d m^*_{Li}}{(2\pi\hbar^2)^2}\frac{k_{Li}}{k_d}\frac{1}{(2J_f+1)}\sum_{LM}(2L+1)|\sum_N \mathscr{G}_{NL}B^M_{NL}|^2. \tag{15}$$

With the definition (14) of a modified form factor it is possible to use ordinary single-nucleon transfer DWBA codes in order to compute the differential cross section of (15).

3. Specific examples

3.1. THE PAIRING SCHEME

Let us consider transitions among $J = 0$ states, that are supposed to be well described by a $T = 1$ pairing residual interaction acting among the nucleons. If only shell-seniority zero states are considered, the spectroscopic factors $\mathscr{C}$ are equal to

$$\mathscr{C}^{0T}_{\Gamma_f 0}(jjj'j'001) = \frac{1}{(\Omega_j \Omega_{j'}(2T+1))^{\frac{1}{2}}} \langle A+4,\, T||\{A^+(j')A^+(j)\}^{00}||A,\, \Gamma_f\, T\rangle,$$

$$\Omega_j = j+\tfrac{1}{2}, \quad (16)$$

where the three components of $A^+(j)$ are the pair creation operators with isospin 1

$$A^+_1(j) = \sum_m \tfrac{1}{2}(-)^{j-m} n^+_{jm} n^+_{j-m}, \qquad A^+_{-1}(j) = \sum_m \tfrac{1}{2}(-)^{j-m} p^+_{jm} p^+_{j-m},$$

$$A^+_0(j) = \sum_m \frac{1}{2\sqrt{2}}(-)^{j-m}(n^+_{jm} p^+_{j-m} + p^+_{jm} n^+_{j-m}), \quad (17)$$

the operators $p^+_{jm}(n^+_{jm})$ create a proton (neutron) with angular momentum j and projection m. The $\mathscr{G}_{NL}$ factor is then reduced to

$$\mathscr{G}_{NL}(0T\ \Gamma_f 0)$$
$$= -\delta_{L,0}\sqrt{45}(2T+1)^{-\frac{1}{2}} \sum_{JJ'N_1N_2} (\Omega_j \Omega_{j'})^{-\frac{1}{2}} \langle A+4,\, T||\{A^+(j)A^+(j')\}||A,\, \Gamma_f\ T\rangle$$
$$\times\ [\nu_1 0, N_1 0; 0|nl, nl; 0][\nu_2 0, N_2 0; 0|n'l', n'l'; 0][N_r 0, N0; 0|N_1 0, N_2 0; 0]. \quad (18)$$

The behaviour of the cross section as the strength of the residual pairing interaction changes can be easily investigated if a symmetric two-level model [2]) is used in order to calculate the matrix elements in eq. (18). In this particular case the spectroscopic factor for the four-body transfer can be taken outside the summation and the cross section will be proportional to the square of it. In fig. 1 are plotted the values of $\langle||\{A^+A^+\}||\rangle$ as a function of the ratio G/G_c, G_c being the value of the coupling constant for which the system undergoes a phase transition from the normal to the superconducting phase. The reactions that are considered are those populating states of the systems with $A = 4\Omega_j$ and $A = 4\Omega_j+4$ nucleons. The transitions going to excited states show a different kind of behaviour than those populating the ground states. The matrix elements associated with the second kind are bigger in the limit of $G/G_c \gg 1$ than for $G/G_c \ll 1$, these transitions connect states belonging to the seniority $v = 0$ rotational band of the pairing model. The first kind are those populating the so called "pairing vibration" mode of the residual nucleus and are slightly peaked in the transition region. These effects increase if the degeneracy of the levels that are involved becomes larger.

In the pairing rotational limit the $\mathscr{C}$-coefficients tend asymptotically to

$$\mathscr{C}^{0T}_{\Gamma_f 0}(jjjj001) = \frac{1}{2\Omega\sqrt{3}}\{(2\Omega+T+1-m)(2\Omega-T-m)(2\Omega+2+m-T)(2\Omega+T+m+3)\}^{\frac{1}{2}}, \quad (19)$$

$$A_{\text{residual}} = 4\Omega+2m.$$

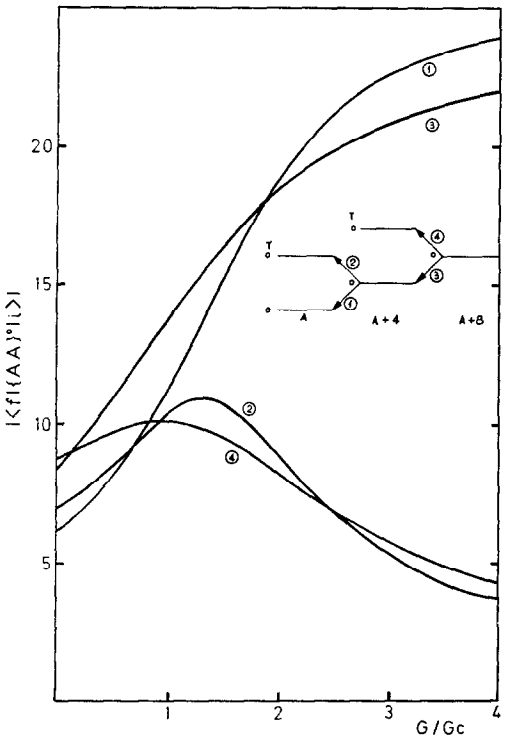

Fig. 1. The absolute value of the four-particle transfer matrix element as a function of the pairing coupling constant in a two symmetric level model with $\Omega_1 = \Omega_2 = 4$. The system with A nucleons is assumed to have one of the two valence levels completely filled. A level scheme clarifies the transitions that are considered. All states have $T = 0$.

TABLE 1

Structure factors for an α-transfer reaction in the ^{56}Ni region as calculated within the pairing scheme

N	$\mathscr{G}_{NL}$
0	−0.0018
1	0.0241
2	−0.1471
3	0.4974
4	−0.9593
5	1.0000
6	−0.4373

The pair degeneracies that were chosen for the calculation correspond to a rough model of the ^{56}Ni region, replacing the $j = \frac{3}{2}$ and $j = \frac{5}{2}$ levels by only one with effective pair degeneracy $\Omega = 4$. In this symmetric two-level case the summation of

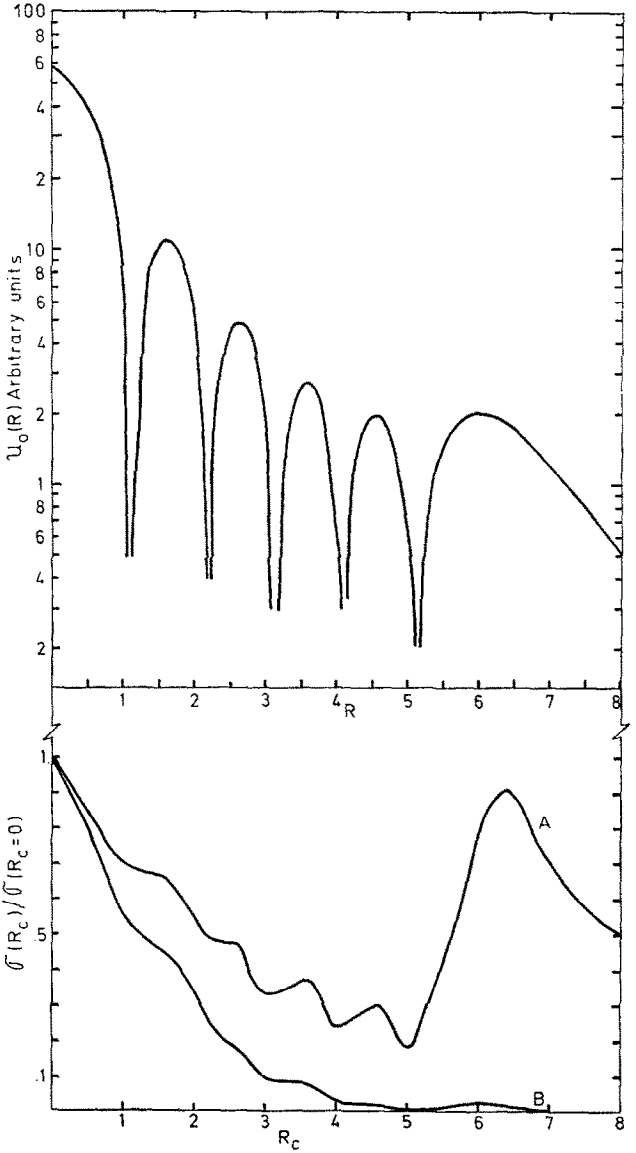

Fig. 2. Upper part: The modified form factor $u_0(R)$ as a function of R as calculated with the structure factors of table 1. Lower part: The ratios $\sigma(R_c)/\sigma(R_c = 0)$ are plotted as a function of the lower cut-off radius R_c. Curve A corresponds to volume absorption and curve B to surface absorption. The corresponding optical-model parameters are given in table 2.

the harmonic oscillator transformation brackets and overlap integrals can be performed independently. In table 1 the results of this summation are given, normalized so that the largest contribution has the value 1. The rest of the information referring to the actual number of valence pairs of the target nucleus, the strength of the residual force and the particular final state that is populated is contained in the reduced matrix elements of (18) that acts as an overall multiplicative factor to get the true $\mathscr{G}_{NL}$. The appearance of opposite signs show a coherent effect since the asymptotic sign of the ϕ^M_{NL} function of formula (14) is $(-)^{N+1}$. The modified form factor as calculated in formula (14) is plotted in the upper part of fig. 2 where it can be seen that the nuclear surface for the chosen binding energy is near 6 fm. Due to the fact that four particles are transferred the form factor displays many nodes although the involved oscillator shells have not such a large N. In order to investigate the contribution of the nuclear interior the total cross section σ has been calculated with different lower cut-off radii R_c. In the lower part of fig. 2 the ratios $\sigma(R_c)/\sigma(R_c = 0)$ have been plotted as a function of R_c for the two different sets of optical parameters of table 2, which

TABLE 2

Optical-model parameters corresponding to the lower curves of fig. 2 for a (d, ^{6}Li) reaction in the ^{56}Ni region

Curve	Particle	E	V_0	r_0	a_0	W_v	W_D	r_v, r_D	a_v, a_D
A	d	28	100	1.25	0.65	40		1.25	0.65
	^{6}Li	26.5	300	1.25	0.65	40		1.25	0.65
B	d	28	100	1.25	0.65		20	1.25	0.65
	^{6}Li	26.5	300	1.25	0.65		20	1.25	0.65

These are not assumed to correspond to any realistic situation and are only used to show the effect of volume and surface absorption in the participation of nuclear interior. Energies are in MeV and lengths in fm.

correspond to volume and surface absorptive potentials. The attenuation of the incoming and outgoing waves only through the surface allow a large contribution of the nuclear interior as shown by the steepness of curve B. In the case of the curve A interference effects allow the contribution of the nuclear surface.

3.2. THE p-SHELL

Let us consider as a second example α-transfer reactions among 1p shell nuclei. The wave function of the target and the residual nuclei are assumed to be well described by the configuration $(1s)^4(1p)^{A-4}$ and can be expanded in terms of L-S coupled wave functions as

$$|\Gamma JT\rangle = \sum_{f\rho S\kappa L} A^{\Gamma JT}_{[f]\rho TS\kappa L}|[f]\rho TS\kappa LJ\rangle. \tag{20}$$

In (20) the quantum numbers $[f]$, ρ, T, S, κ, L allow a complete classification of the basis. The formula for the $\mathscr{G}_{NL}$, if the L-S scheme is used, can be obtained from (9)

by dropping the 9-j symbols. The two-particle fractional parentage coefficients [4]) appearing in (11) will depend upon the two-body residual interaction that is used through the numbers A^{IJT} of formula (20).

Two reactions are studied, that populating states of ^{8}Be from the target ^{12}C and the one going from ^{16}O to ^{12}C.

In the first case the Cohen and Kurath [5]) POT effective interaction was used in order to get the wave functions of the target and residual nuclei. The corresponding structure factors are summarized in table 3. Only even values of J are listed since for an $(l)^n$ configuration the selection rules contained in the Moshinsky transformation brackets forces the $\mathscr{G}_{NL}$ to be zero for odd values of L.

TABLE 3

Structure factors for the reaction ^{12}C(d, ^{6}Li)^{8}Be

J_f	T_f	E_f	L	N	$\mathscr{G}_{NL}$
0	0	0.0	0	0	−0.106
			0	1	2.058
			0	2	−10.893
0	0	18.5 [a])	0	0	0.002
			0	1	−0.034
			0	2	0.182
2	0	2.90	2	0	−1.862
			2	1	11.780
2	0	14.62 [a])	2	0	−0.378
			2	1	2.392
2	0	17.85 [a])	2	0	0.303
			2	1	−1.920
4	0	11.4	4	0	−11.017
4	0	18.41 [a])	4	0	−0.569

[a]) Theory (Cohen and Kurath POT effective interaction).

The similar values of the size parameters in the ^{8}Be and the outgoing ^{6}Li cause the radial overlaps of (10) to be small unless the corresponding relative radial quantum numbers are zero. The main contribution are therefore due to the largest possible value of the center of mass quantum number N as appears in table 3. This fact disappears if higher shells than the 1p are considered as shown in table 1.

There is some recent experimental evidence [6]) for the reaction under consideration. An attempt of fitting theoretically the corresponding angular distributions was done and the results for the ground and first 2^+ states are shown in fig. 3. The relative strengths are well reproduced by the theory. The optical parameters for the outgoing ^{6}Li at the involved energy are yet unknown and a rather arbitrary choice had to be done on them, without any exhaustive search. The ^{6}Li set used in ref. [7]) for a deuteron laboratory energy of 19.5 MeV was chosen.

The direct reaction in which a pair with a given (J, T) is transferred is a specific tool to investigate the strengths of the corresponding components of the two-body

residual force. The (d, ^{6}Li) reactions is not so clearly related to any part of the two-body interaction. It is therefore interesting to try to establish whether constructive or destructive interference is going to occur among contributions of different physically important channels. In order to advance some steps in this study the calculations of the ^{16}O(d, ^{6}Li)^{12}C reaction have been performed with different kinds of residual

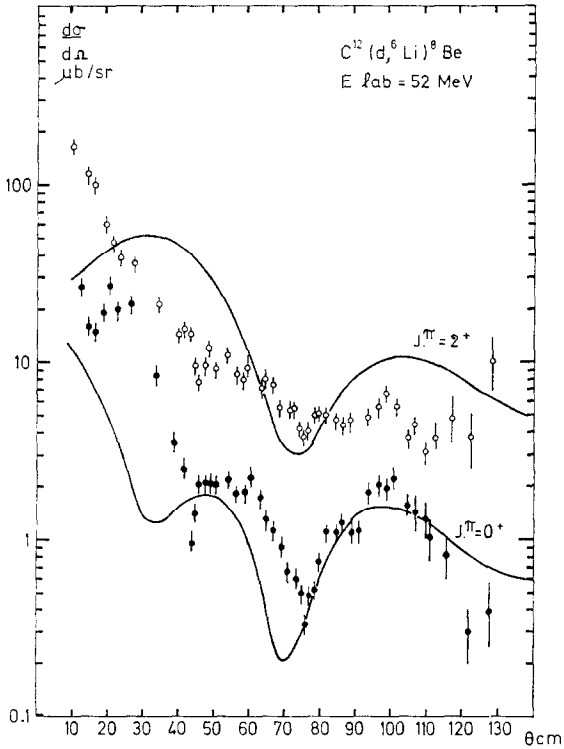

Fig. 3. Theoretical and experimental [6]) angular distributions for the ground and first 2^+ states of ^{8}Be. Optical parameters are obtained from ref. [8]) for the deuteron ($V = 71.8$ MeV, $r_V = 1.25$ fm, $a_V = 0.7$ fm, $W = 11.0$ MeV, $r_W = 1.25$ fm, $a_W = 0.7$ fm, $W = 11.0$ MeV, $r_W = 1.25$ fm, $a_W = 0.7$ fm, $r_c = 1.3$ fm) and from ref. [7]) for the ^{6}Li ($V = 40.4$ MeV, $r_V = 1.09$ fm, $a_V = 0.54$ fm, $W = 6.8$ MeV, $r_W = 1.05$ fm, $a_W = 0.84$ fm, $r_c = 1.3$ fm).

forces. First the wave functions corresponding to the POT effective interaction were used.

Next diagonalizations were done on the assumption that only the $(J, T) = (0, 1)$ and $(1, 0)$ channels of this force were separately active. The resulting sets of wave functions were used to get the $\mathscr{G}_{NL}$ factors. Finally the same calculations was done for the pairing (H_p) and "deuteron-pairing" (H_d) model Hamiltonians. These are defined as

$$\langle p^2 TSLJ|H_p|p^2 TS'L'J\rangle = -\tfrac{3}{2}G_p \delta_{T,1}\delta_{S,0}\delta_{L,L'}\delta_{L,0}, \tag{21}$$

$$\langle p^2 TSLJ|H_d|p^2 TS'L'J\rangle = -\tfrac{3}{2}G_d \delta_{T,0}\delta_{S,1}\delta_{L,L'}\delta_{L,0}. \tag{22}$$

In each column of table 4 the $\mathscr{G}_{N0}$ factors corresponding to the ground state transition are given for the indicated seven residual interactions. In all calculations the splitting of the $p_{\frac{3}{2}}$ and $p_{\frac{1}{2}}$ shells was taken to be 1.29 MeV. The coupling constants appearing in (21) and (22) were chosen $G_p = G_d = \frac{8}{3}$ MeV.

TABLE 4

Structure factors for the ground state transition in the ^{16}O(d, ^{6}Li)^{12}C reaction using POT and model residual interactions

					$\mathscr{G}_{NL}$						
					POT			Model residual interactions			
J_f	T_f	E_f	L	N	$(JT) = (01)$	$(JT) = (10)$	Complete	H_p	H_d	$H_p + H_d$	$H_p - H_d$
0	0	0.0	0	0	−0.442	−0.498	−0.526	−0.437	−0.501	−0.575	−0.361
				1	3.295	3.712	3.926	3.257	3.734	4.288	2.696
				2	−6.727	−7.579	−8.015	−6.649	−7.623	−8.754	−5.504

4. Conclusions

The values listed in table 4 indicate that the contributions of each separate (J, T) channel of the residual force do not interfere destructively. The combined effect of both components is always larger than the one due to each one separately. This fact depends however upon the relative signs of the terms of the residual force, for instance, if one of the model Hamiltonians is taken repulsive the coherence is destroyed, as follows from the last column of table 4.

The similar values for the structure factors using the model and realistic interactions suggest that the most important contributions to the $\mathscr{G}_{NL}$ are due to the $(J, T) = (0, 1)$ and $(1, 0)$ components of the residual force. Bearing in mind this together with the fact that both channels have shown to be attractive one can also conclude that a behaviour such as the one found in subsect. 3.1 will not be cancelled out by any destructive interference due to different components of a more realistic two-body force.

The strengths G_p and G_d that were used are close to one-half the values that would have been necessary to reproduce the corresponding matrix elements of POT. In spite of this fact the structure factors obtained with the model and realistic interactions have values that are very close to each other. This suggest that the $\mathscr{G}_{N0}$ tend to a limiting asymptotic value for large values of the coupling constants, being this related to the maximum possible deformations that the wave functions can acquire in both spin and isospin spaces.

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