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THE RPA OF RENORMALIZED FERMIONS

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Abstract: The RPA modes are constructed using data associated with different kinds of renormalized one-particle Green functions as input. A general normalization condition is derived and the formalism applied to the ^{40}Ca nucleus.

1. Introduction

The p-h (or p-p) configurations entering into an RPA calculation for a closed shell system, are built with data that can be extracted from its odd neighbours. Single-particle energies are usually defined performing an average of the excitation energies of the odd systems with the spectroscopic factors for one-body transfer as weighting factors [1]):

$$\varepsilon_i = \sum_n |\langle 0|c_i|n\rangle|^2 (E_n - E_0) + \sum_N (E_0 - E_N)|\langle N|c_i|0\rangle|^2 . \qquad (1)$$

In (1) i labels the states of an arbitrary independent particle basis, $|0\rangle$ is the exact ground state of the doubly closed shell system of A particles and $|n\rangle$ ($|N\rangle$) is a complete set of eigenstates of the $A+1$ ($A-1$) system having energies E_n (E_N).

From a diagrammatic point of view the set ε_i defined in this fashion is determined by all the ladder-type series of diagrams (fig. 1A) in which the repeated irreducible diagram is one in which the bare fermion line is connected only once [1]) to any set of interacting fermion lines (figs. 1B, C). One relevant diagram of this type is that associated with Hartree-Fock contributions (fig. 1B).

The purpose of the present paper is to propose an alternative choice of the s.p. space circumventing the step implied in eq. (1). This amounts to the use of fermion propagators renormalized by higher-order processes than those of the HF type. We are particularly interested in exploring the consequences of relaxing the conditions of one insertion diagrams (figs. 1B, C) and allowing processes as the one shown in fig. 1D that are responsible for the fragmentation [1,2]) of the s.p. strength in the odd neighbour systems.

One possibility along this direction is the use of the experimental data for 1p and 1h systems for an empirical definition of the one-fermion propagator. This is possible

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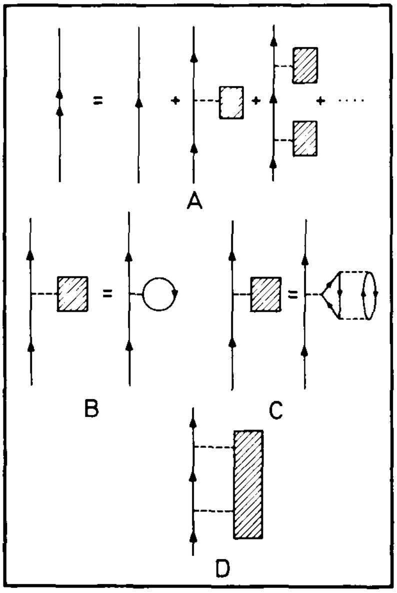

Fig. 1. (A) Ladder-type series of "one-insertion diagrams" defining a dressed one-body Green function (double arrowed line). (B–C) Examples of "one-insertion diagrams" (B Hartree–Fock contributions, C higher order) that are contained in the s.p. energies ε_i defined with eq. (1). (D) "Two insertion diagram", that is excluded in the series A. This kind of process when used to define a renormalized one-body Green function causes fragmentation of the one-body transfer strength.

since its poles are associated to the experimental energies E_n and E_N and its residues are linked to the spectroscopic amplitudes for one-body transfer.

Another alternative is the use of one-fermion propagators obtained theoretically by renormalizing bare lines with some given process [2]). Both procedures amount to use a more complicated elementary mode of excitation than HF particles or holes.

The most striking consequence of using such a one-body Green function to build RPA modes, is the appearance of non-vanishing amplitudes that populate a one-boson state by creating a particle and a hole in states above the Fermi surface. This feature invalidates the usual RPA normalization conditions which require a proper generalization.

In sect. 2 we formulate the theoretical framework to obtain the RPA modes with one-body propagators that are not of the HF type paying special attention to the normalization conditions of the eigenvectors. In sect. 3 we consider a schematic Lipkin model in which the fermion lines are renormalized by the emission and subsequent absorption of an RPA boson. In addition we analyze the octupole modes in ^{40}Ca in which the s.p. strength presents a large fragmentation. A comparison with other RPA theories using renormalized propagators is discussed together with the conclusions in sect. 4.

2. Theory

2.1. THE RPA EQUATIONS

We start by considering the hamiltonian[†]

$$H = H_{\text{s.p.}} + V = \sum_{1} \varepsilon_1 c_1^+ c_1 + \tfrac{1}{2} \sum_{1234} V_{12,34} c_1^+ c_4^+ c_3 c_2 \,. \tag{2}$$

In (2) the arabic numbers label the s.p. basis that makes $H_{\text{s.p.}}$ diagonal.

We use for the one-body Green function:

$$G(12; t) = \langle 0 | T\{c_1(t) c_2^+(0)\} | 0 \rangle \,, \tag{3}$$

where $T\{\ \}$ is the time ordering operator, $|0\rangle$ is the exact g.s. of the A-particle systems and the creation (annihilation) operator $c^+(t)$ $(c(t))$ is written in the Heisenberg picture. The spectral representation of (3) is:

$$G(12; k) = \sum_n \frac{\langle 0|c_1|n\rangle\langle n|c_2^+|0\rangle}{k-(E_n-E_0)} + \sum_N \frac{\langle 0|c_2^+|N\rangle\langle N|c_1|0\rangle}{k-(E_0-E_N)} \,. \tag{4}$$

The positive (negative) energy poles of G are the excitation energies of the eigenstates $|n\rangle$ $(|N\rangle)$ of the $A+1$ $(A-1)$ system and its residues are directly related to the spectroscopic factors for one-body transfer induced on the closed core target of A nucleons. We will refer to the $|n\rangle$ and $|N\rangle$ states as "particle" and "hole" states respectively. All the parameters specifying $G(12, \text{k})$ are either the output of a renormalized one-body Green function [2]) calculation or can be obtained from experiment. We shall thus consider that

$$\begin{aligned} X_n(12) &= \langle 0|c_1|n\rangle\langle n|c_2^+|0\rangle \,, \\ X_N(12) &= \langle 0|c_2^+|N\rangle\langle N|c_1|0\rangle \,, \end{aligned} \tag{5}$$

are known quantities and use them as input data to our RPA problem.

For the sake of concreteness we restrict our arguments to a "particle"-"hole" two-fermion RPA Green function. The extension to the "particle"-"particle" case offers no additional conceptual difficulty. Our problem is therefore to find the poles ω_p and residues of the Green function

$$S(12, 34; k) = \sum_p \frac{\langle 0|c_2^+ c_1|p\rangle\langle p|c_3^+ c_4|0\rangle}{k-\omega_p} - \sum \frac{\langle 0|c_3^+ c_4|p\rangle\langle p|c_2^+ c_2|0\rangle}{k+\omega_p} \tag{6}$$

evaluated by summing a ladder series where the fermion lines correspond to renormalized one-fermion propagators. We thus write the usual equation for S in the ladder approximation

$$S(12, 34; k) = S_0(12, 34; k) + \sum_{5,6,7,8} S_0(56, 34; k) V_{56,78} S(12, 78; k) \,. \tag{7}$$

[†] We use the notation $V_{12,34} = \langle 14|V|23\rangle$ for the non-antisymmetrized matrix elements. If $m(i)$ labels particle (hole) states then for a separable interaction $V_{m'i',mi} = \langle m'i|V|i'm\rangle = -xQ_{mi}Q_{m'i'}$. Thus the role of initial or final p-h states in a diagram representing the two-body interaction is clearly displayed.

In (7) S_0 represents the Fourier transform of the free propagator for a pair of "particle"-"hole" lines, namely:

$$S_0(12,34;t) = -G(13;t)G(24;-t)\,. \tag{8}$$

If the residues of $S(k)$ at ω_p are evaluated with the aid of (7) a set of homogeneous linear equations is obtained that determine the amplitudes $\langle 0|a_3^+a_4|p\rangle$ and $\langle p|a_2^+a_1|0\rangle$ within an arbitrary normalization constant:

$$\begin{aligned}\langle p|c_3^+c_4|0\rangle &= \sum_{5678} V_{56,78}\left\{\sum_{nN}\frac{X_n(53)X_N(46)}{\omega_p - E_{nN}} - \frac{X_n(46)X_N(53)}{\omega_p+E_{nN}}\right\}\langle p|c_7^+c_8|0\rangle\,,\\ \langle 0|c_3^+c_4|p\rangle &= -\sum_{5678} V_{56,78}\left\{\sum_{nN}\frac{X_n(53)X_N(46)}{\omega_p + E_{nN}} - \frac{X_n(46)X_N(53)}{\omega_p-E_{nN}}\right\}\langle 0|c_7^+c_8|p\rangle\,.\end{aligned} \tag{9}$$

In (9) we have made use of the notation $E_{nN} = E_n - E_N$. In order to have a non-trivial solution for $\langle p|c_3^+c_4|0\rangle$ and $\langle 0|c_3^+c_4|p\rangle$, ω_p must be a root of the determinant. This determinantal equation can easily be seen to lead to the standard RPA non-hermitian eigenvalue problem whenever the residues (5) tend to their (Hartree-Fock) limiting value. This can be checked if we choose, for instance[†], $3 > k_F$ and $4 < k_F$,

$$X_n(53) = \delta_{5,3}\,, \qquad X_N(46) = \delta_{4,6}\,,$$

$$X_n(46) = 0\,, \qquad X_N(53) = 0\,,$$

$$E_{nN} \to \varepsilon_{34} \text{ (p-h energy)}\,.$$

2.2. NORMALIZATION CONDITIONS

One normalization condition can be obtained from eq. (7) by calculating the residues of S at the poles E_{nN}. If we multiply both sides of (7) by $(k - E_{nN})$ and let k tend to E_{nN} we obtain:

$$\begin{aligned}X_n(13)X_N(42) &= \sum_{5678} X_n(53)X_N(46)V_{56,78}\\ &\quad\times\left\{\sum_p \frac{\langle 0|c_2^+c_1|p\rangle\langle p|c_7^+c_8|0\rangle}{\omega_p - E_{nN}} + \frac{\langle 0|c_7^+c_8|p\rangle\langle p|c_2^+c_1|0\rangle}{\omega_p + E_{nN}}\right\},\\ X_N(13)X_n(42) &= \sum_{5678} X_N(53)X_n(46)V_{56,78}\\ &\quad\times\left\{\sum_p \frac{\langle 0|c_2^+c_1|p\rangle\langle p|c_7^+c_8|0\rangle}{\omega_p + E_{nN}} - \frac{\langle 0|c_7^+c_8|p\rangle\langle p|c_2^+c_1|0\rangle}{E_{nN} - \omega_p}\right\}.\end{aligned} \tag{10}$$

[†] The two equations (9) are redundant if the indices 3, 4 have no restriction, both equations are required if we instead restrict for instance $3 \geq 4$ assuming some ordering of the s.p. states.

By summing over (nN) and subtracting both equations we obtain, making use of (9),

$$\sum_{nN} \{X_n(13)X_N(42) - X_N(13)X_n(42)\}$$
$$= \sum_{p} \langle 0|c_3^+ c_1|p\rangle\langle p|c_3^+ c_4|0\rangle - \langle 0|c_3^+ c_1|p\rangle\langle p|c_2^+ c_1|0\rangle \,. \tag{11}$$

The normalization condition (11) requires the knowledge of all the poles ω_p of (6). This feature makes eq. (11) of little practical use since the number of roots ω_p may be very large. We thus need another condition in which the summation does not take place over the dressed states $|p\rangle$ as in (11) but over the bare labels (12) and (34).

In order to do so, we first note that S and S_0 may be regarded as (k-dependent) matrices whose rows and columns are labeled by the (bare) s.p. indices (12) and (34) respectively. With this notation we can write

$$\boldsymbol{S}(k) = (\boldsymbol{S}_0^{-1}(k) - \boldsymbol{V})^{-1} = \boldsymbol{R}^{-1}(k)\,. \tag{7$'$}$$

The matrix $\boldsymbol{S}(k)$ becomes singular whenever k is equal to each root ω_p of the determinantal equation previously mentioned, or alternatively, whenever an eigenvalue $E_p(k)$ of the matrix $\boldsymbol{R}(k)$ vanishes. We may write, in the neighbourhood of each eigenfrequency ω_p:

$$E_p(k) \cong (k - \omega_p)\left(\frac{\partial E_p(k)}{\partial k}\right)_{k=\omega_p} = \mu_p(k - \omega_p)\,. \tag{12}$$

If $\boldsymbol{Z}(k)$ is the (non-singular) matrix that brings $\boldsymbol{R}(k)$ to its diagonal form $\boldsymbol{E}(k)$, we write for $\boldsymbol{S}(k)$

$$\boldsymbol{S}_{12,34}(k) = (\boldsymbol{R}^{-1}(k))_{12,34} = \sum_p \boldsymbol{Z}_{12,p} \frac{\mu_p^{-1}}{k - \omega_p} (\boldsymbol{Z}^{-1})_{p,34}\,. \tag{13}$$

Finally, (13) and (6) are consistent with each other it we set[†]

$$\begin{aligned} \langle 0|c_2^+ c_1|p\rangle &= \boldsymbol{Z}_{12,p}\mu_p^{-1/2}\,, \\ \langle p|c_3^+ c_4|0\rangle &= \mu_p^{-1/2}(\boldsymbol{Z}^{-1})_{p,34}\,. \end{aligned} \tag{14}$$

The weighting factors μ_p can be worked out explicitly from the definition in (12) namely

$$\frac{\partial E_p(k)}{\partial k} = \sum_{1234} \frac{\partial(\boldsymbol{Z}^{-1})_{p,12}}{\partial k} \boldsymbol{R}_{12,34}\boldsymbol{Z}_{34,p}(k)$$
$$+ \sum_{12,34} (\boldsymbol{Z}^{-1})_{p,12} \frac{\partial \boldsymbol{R}_{12,34}}{\partial k} \boldsymbol{Z}_{34,p} + \sum_{12,34} (\boldsymbol{Z}^{-1})_{p,12}\boldsymbol{R}_{12,34} \frac{\partial \boldsymbol{Z}_{34,p}}{\partial k}\,. \tag{15}$$

[†] In comparing (13) and (6) we have restricted our analysis to the positive energy roots ω_p (i.e. advanced part of S). This comparison is nevertheless general enough since on the other hand we have made no assumption on the relative ordering of the indices 1, 2 and 3, 4.

By properly inserting the unit matrix $\boldsymbol{Z}\boldsymbol{Z}^{-1}$ and evaluating the derivative at the point $k=\omega_p$ in which $E_p=0$, we obtain

$$\mu_p=\sum_{1234}(\boldsymbol{Z}^{-1}(\omega_p))_{p,12}\frac{\partial \boldsymbol{R}_{12,34}}{\partial k}\boldsymbol{Z}_{34,p}(\omega_p)\,. \tag{16}$$

We now replace (14) in (16) and get

$$\sum_{1234}\langle p|c_2^+c_1|0\rangle\left[\frac{\partial \boldsymbol{R}_{12,34}}{\partial k}\right]_{k=\omega_p}\langle 0|c_3^+c_4|p\rangle=1\,, \tag{17}$$

that is the required normalization condition for the set of residues of S that are associated with each of the poles ω_p. Moreover, the unitary property of the matrix $\boldsymbol{Z}$ implies another consistency condition, namely

$$\sum_p \boldsymbol{Z}_{12,p}(\boldsymbol{Z}^{-1})_{p,34}=\delta_{12,34}=\sum_p \mu_p\langle 0|c_2^+c_1|p\rangle\langle p|c_3^+c_4|0\rangle\,, \tag{18}$$

that is a system of linear homogeneous equations in the unknowns μ_p.

In general eq. (17) may be rather difficult to use since one has to evaluate the inverse of a k-dependent matrix $S_0(12,34;k)$. If we however consider that only two major shells are active in the determination of the one-body propagator (4), there are no two s.p. levels within that space having the same angular momentum and parity. As a consequence, the residues $X_n(13)$ and $X_N(42)$ that enter in the definition of S_0 fulfill

$$X_n(13)=X_n(11)\delta_{1,3}\,,$$
$$X_N(42)=X_N(22)\delta_{2,4}\,,$$

and thus S_0 is a diagonal matrix. Under these assumption we obtain:

$$\left.\frac{\partial R_{12,34}}{\partial k}\right|_{k=\omega_p}=\delta_{12,34}\left(\sum_{nN}\frac{A_{nN}(12)}{\omega_p-E_{nN}}-\frac{A_{nN}(21)}{\omega_p+E_{nN}}\right)^{-2}\left(\sum_{nN}\frac{A_{nN}(12)}{(\omega_p-E_{nN})^2}-\frac{A_{nN}(21)}{(\omega_p+E_{nN})^2}\right), \tag{19}$$

with

$$A_{nN}(12)=X_n(11)X_N(22)\,.$$

Eq. (19) can readily be seen to reduce to the standard RPA metric tensor when the one-body Green functions G involved in eq. (8) are pure HF propagators.

We want to stress that the normalization condition (17) was derived with *no particular assumptions made* upon S_0 and V. In fact, a relevant particular case of (17) is when the propagator S is obtained from eq. (7) by replacing the two-body interaction V by an energy-dependent kernel [2,3]) and taking the propagator S_0 to be built up by HF lines. In such circumstances eq. (18) is also simple to evaluate and can be seen to correspond to eq. (27) of ref. [3]). It also reduces, for the case of one-fermion renormalized propagators [2]), to the Ward identity.

3. Applications and examples

3.1. SEPARABLE INTERACTIONS

In this subsection we derive for future use, the particular expression of the formalism developed in sect. 2 for the case of separable interactions. We thus assume:

$$V_{12,34} = -VQ_{12}Q^*_{34} . \tag{20}$$

With this simplification the determinantal equation can easily be seen to reduce to a dispersion equation

$$-\frac{1}{V} = \sum_{nN} |Q_{nN}|^2 \frac{E_{nN}}{k^2 - E^2_{nN}} . \tag{21}$$

The corresponding matrix element Q_{nN} of the one-body operator involved in eq. (21), between dressed one fermion states $|n\rangle$ and $|N\rangle$, is:

$$Q_{nN} = \sum_{12} Q_{12}\langle 0|c_1|n\rangle\langle 0|c^+_2|N\rangle . \tag{22}$$

The residues of the dressed propagator S can be obtained following an argument similar to that of subsect. 2.2. We first note that whenever the residual interaction fulfills (20) the ladder expansion turns into a geometrical series. This can be summed giving:

$$S(12, 34; k) = S_0(12, 34; k) - \frac{(\sum_{\alpha\beta} S_0(12, \alpha\beta; k)Q^*_{\alpha\beta})(\sum_{\mu\nu} S_0(\mu\nu, 34; k)Q_{\mu\nu})}{V^{-1} - \sum_{\substack{\gamma\delta \\ \rho\sigma}} S_0(\gamma\delta, \rho\sigma; k)Q_{\gamma\delta}Q^*_{\rho\sigma}} . \tag{23}$$

Eq. (23), by the way, provides an alternative derivation of eq. (21) since the singularities of S must correspond to the zeros of the denominator appearing in the l.h.s. By comparing (23) with the spectral representation of S we obtain the (properly normalized) residues of S, namely:

$$\langle 0|c^+_2 c_1|p\rangle = \left\{ \sum_{\substack{\alpha\beta \\ \rho\sigma}} Q_{\alpha\beta}Q^*_{\rho\sigma} \left[\frac{\partial S_0(\alpha\beta, \rho\sigma; k)}{\partial k} \right]_{k=\omega_p} \right\}^{-1/2} \sum_{\mu\nu} S_0(12, \mu\nu; \omega_p)Q^*_{\mu\nu} . \tag{24}$$

3.2. A SCHEMATIC MODEL

In this subsection we discuss an application of the present formalism to a schematic situation in which the fermion building blocks contain, up to infinite order, the process of the emission and subsequent absorption of an RPA boson [2]). With this purpose we consider a model consisting of two equally degenerate levels with pair degeneracy Ω, separated by an energy gap ε and with a residual monopole interaction acting among the particles. These are assumed to fill completely the lower level. Within this scheme there is only one collective RPA boson with a frequency:

$$\omega_{\mathrm{RPA}}=\varepsilon(1-4V\Omega/\varepsilon)^{1/2}\equiv\varepsilon(1-x)^{1/2}\,. \tag{25}$$

The boson is coupled to the bare (HF) fermion lines through the vertex functions:

$$\Lambda=2V\Omega(\varepsilon/2\Omega\omega_{\mathrm{RPA}})^{1/2}\,. \tag{26}$$

The dimensionless parameter x is restricted to be less than 1 and represents the ratio of the strength of the residual interaction V to the critical value V_c for which the RPA equation becomes unstable. The energies of the normalized fermion lines are [2]):

$$\begin{aligned} E_{n_1}&=-E_{N_1}=\tfrac{1}{2}\varepsilon+\Lambda^2\left(\frac{1}{\varepsilon+\omega}-\frac{1}{\omega}\right)+\mathrm{O}(\Lambda^4)\,,\\ E_{n_2}&=-E_{N_2}=\tfrac{1}{2}\varepsilon+\omega+\Lambda^2\left(\frac{1}{\varepsilon+\omega}+\frac{1}{\omega}\right)+\mathrm{O}(\Lambda^4)\,. \end{aligned} \tag{27}$$

The positive (negative) energies (27) correspond to states of the odd system $A+1$ $(A-1)$ that have a structure of predominantly 1p(1h) or 1p plus one boson (1h plus one boson). The poles of the "p-h" propagator S_0 are:

$$\begin{aligned} E_{n_1N_1}&\approx\varepsilon+2\Lambda^2\left(\frac{1}{\varepsilon+\omega}-\frac{1}{\omega}\right)+\mathrm{O}(\Lambda^4)\,,\\ E_{n_2N_2}&\approx\varepsilon+2\omega+2\Lambda^2\left(\frac{1}{\varepsilon+\omega}+\frac{1}{\omega}\right)+\mathrm{O}(\Lambda^4)\,,\\ E_{n_1N_2}&=E_{n_2N_1}\approx\varepsilon+\omega+\frac{2\Lambda^2}{\varepsilon+\omega}+\mathrm{O}(\Lambda^4)\,. \end{aligned} \tag{28}$$

We can thus see that the corresponding roots of eq. (21) are one p-h pair like (root closer to $E_{n_1N_1}$) or similar to one p-h pair plus one or two bosons (roots closer to $E_{n_2N_2}$ or $E_{n_1N_2}$, respectively).

The residues of the renormalized propagator that are the second input of our particular RPA problem are:

$$\begin{aligned} \langle n_1|c_1^+|0\rangle&=\langle N_1|c_{\bar{1}}|0\rangle=1-\tfrac{1}{2}\Lambda^2\left[\frac{1}{\omega^2}+\frac{1}{(\varepsilon+\omega)^2}\right]+\mathrm{O}(\Lambda^4)\,,\\ \langle n_2|c_1^+|0\rangle&=\langle N_2|c_{\bar{1}}|0\rangle=\frac{\Lambda}{\omega}+\mathrm{O}(\Lambda^3)\,,\\ \langle n_1|c_{\bar{1}}^+|0\rangle&=-\langle N_1|c_1|0\rangle=-\frac{\Lambda^2(\varepsilon+2\omega)}{\varepsilon\omega(\varepsilon+\omega)}+\mathrm{O}(\Lambda^4)\,,\\ \langle n_2|c_{\bar{1}}^+|0\rangle&=-\langle N_2|c_1|0\rangle=\frac{\Lambda}{\varepsilon+\omega}+\mathrm{O}(\Lambda^3)\,. \end{aligned} \tag{29}$$

Where 1 ($\bar{1}$) labels the states above (below) the Fermi surface.

We can now proceed to obtain in lowest order the root of (21) that lies closer to ω_{RPA} (or ε). We set $\omega = \omega_{RPA} + \alpha/\Omega$, and we neglect terms of higher order than x^3/Ω. We obtain

$$\omega = \omega_{RPA} - \frac{\varepsilon x^2}{8\Omega}(1+3x) . \tag{30}$$

As expected, the change in the RPA frequency ω_{RPA} to this order [4]) corresponds to the one introduced by the diagrams of fig. 2.

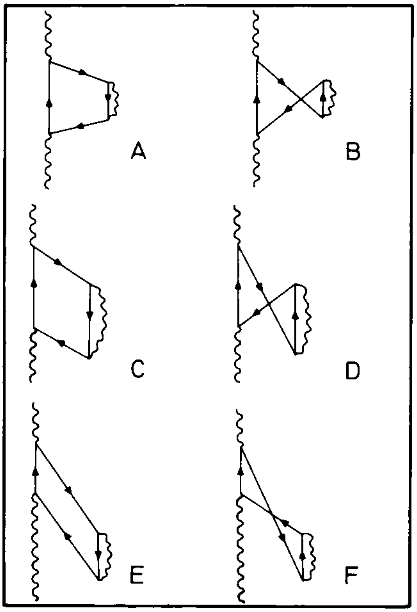

Fig. 2. All the diagrams contributing up to the order x^3/Ω to the energy shift $\delta\omega = \omega - \omega_{RPA}$ [see ref. [4])]. The remaining time permutations contribute to a higher power of x. The diagrams A and B appear only once, while all the remaining ones (C, D, E, F) must be counted twice thus accounting for the time permutation $t \to -t$.

We can now turn to the evaluation of the residues of S with the use of eq. (24). After a straightforward but lengthy calculation we obtain:

$$\begin{aligned}
\langle n_0 | c_{\bar{1}}^{+} c_{\bar{1}} | 0 \rangle &= \frac{\Lambda}{\varepsilon - \omega_{RPA}} \left(1 - \frac{5x^2}{32\Omega} \right) + O(x^3/\Omega) , \\
\langle n_0 | c_{\bar{1}}^{+} c_{1} | 0 \rangle &= \frac{\Lambda}{\varepsilon + \omega_{RPA}} + O(x^3/\Omega) , \\
\langle n_0 | c_{1}^{+} c_{1} | 0 \rangle &= \frac{3\Lambda x}{8\varepsilon\Omega} + O(x^3/\Omega) , \\
\langle n_0 | c_{\bar{1}}^{+} c_{\bar{1}} | 0 \rangle &= -\frac{3\Lambda x}{8\varepsilon\Omega} + O(x^3/\Omega) .
\end{aligned} \tag{31}$$

The "scattering residues" $\langle n_0|c_1^+c_1|0\rangle$ and $\langle n_0|c_{\bar{1}}^+c_{\bar{1}}|0\rangle$ have been calculated to the lowest non-vanishing order. The corresponding diagrammatic representation of the renormalized residues (31) is displayed in fig. 3.

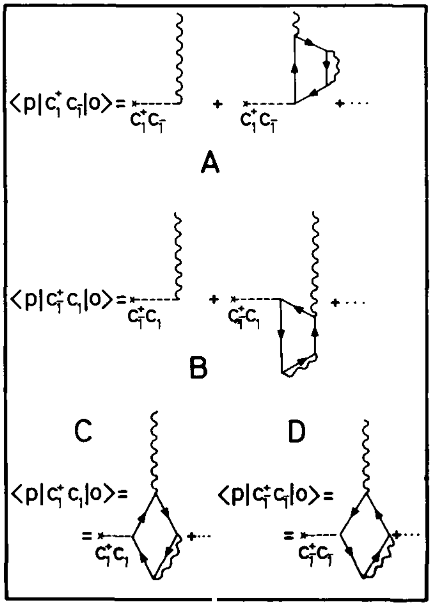

Fig. 3. Diagrams corresponding to the lowest order in the calculation of the amplitudes to populate the collective state $|p_0\rangle$ by acting with a pair of operators $c_\sigma^+c_{\sigma'}$. The zero order of the amplitudes to $c_1^+c_{\bar{1}}$ $(c_{\bar{1}}^+c_1)$ correspond to the ordinary RPA forward (backward) amplitudes (figs. A, B). Within the renormalized RPA framework appear $(1/\Omega)$ amplitudes corresponding to $c_1^+c_1$ $(c_{\bar{1}}^+c_{\bar{1}})$ i.e. the annihilation and subsequent creation of a fermion on the same side of the Fermi level (figs. C, D).

The normalization condition (17) can also be verified to be fulfilled up to the required order. The evaluation of $\partial R/\partial k$ is lengthy since it requires the calculation of the inverse of a k and Ω dependent four by four matrix. This complication is chiefly due to the fact that the model that we consider assumes the existence of levels of the same parity and angular momentum at both sides of the Fermi surface. Neglecting terms of higher order than x^2/Ω eq. (17) becomes

$$2\Omega\left\{\frac{16\Omega}{x^2}[\langle n_0|c_1^+c_1|0\rangle^2-\langle n_0|c_{\bar{1}}^+c_{\bar{1}}|0\rangle^2]+\left(1+\frac{5x^2}{16\Omega}\right)\langle n_0|c_1^+c_{\bar{1}}|0\rangle^2 \right.$$
$$\left. -\langle n_0|c_{\bar{1}}^+c_1|0\rangle^2-3\langle n_0|c_1^+c_{\bar{1}}|0\rangle[\langle n_0|c_1^+c_1|0\rangle+\langle n_0|c_{\bar{1}}^+c_{\bar{1}}|0\rangle]\right\}=1\,. \qquad (32)$$

The replacement of (31) in (32) shows that the normalization condition is properly satisfied to the required order.

3.3. A REALISTIC SITUATION

In this section we apply the formalism explained in sect. 2 to the study of the 3^- strength in ^{40}Ca. In this example we make no assumption about the structure of the fermionic degrees of freedom borrowing instead from experiment all the data concerning the poles and the residues of the one-body Green function.

The available experimental information [5]) of the neighbouring odd systems show that the one-body strength is spread among many levels. The present treatment is particularly suited to deal with this situation since it is able to include in the building blocks the particle-vibration coupling that is responsible for such fragmentation [6,7]).

The experimental one-body spectroscopic factors must fulfill the sum rule:

$$1 = \sum_n |\langle 0|c_j|n\rangle|^2 + \sum_N |\langle 0|c_j^+|N\rangle|^2 . \tag{33}$$

In table 1 we include all states of a given j^π that exhaust more than 60% of the sum rule. In addition (33) is imposed and the corresponding experimental values are normalized to add up to unity. The error bars are assumed to be large enough (~25%) to compatibilize different DWBA analyses of the same data.

We want to stress that in the summation (33) the states of the $A+1$ *and* $A-1$ system are simultaneously involved. The ^{39,41}Ca have states of unnatural parity with measurable spectroscopic factors [i.e. states of negative (positive) parity in the ^{39}Ca (^{41}Ca)]. The population of such states is achieved through processes as the ones displayed in fig. 6 of ref. [2]).

To study the distribution of the 3^- strength we assume that a residual separable, octupole-octupole interaction is active. The derivations of subsect. 3.1 are thus applicable to this case. The eigenenergies are obtained by solving the dispersion equation (21) in which the energies E_{nN} are taken from the experimental data summarized in table 1. The matrix elements Q_{nN} involve the residues [eq. (22)] associated to the spectroscopic factors for one-body transfer that are also given in table 1. The matrix element Q_{12} between Hartree-Fock levels involve radial wave functions that are assumed to be those of a three dimensional harmonic oscillator. The residues of G that are normalized according to eq. (24), are linked directly to the B(E3) value. If we denote by $|p_0\rangle$ the collective state we have:

$$\langle p_0|Q^{(3)}|0\rangle = \sum_{12} \langle p_0|c_1 c_2^+|0\rangle Q_{12}^{(3)} ,$$

with

$$Q_{12}^{(3)} = \begin{cases} e(1+e_{\text{pol}})\langle 1|r^3 Y_{3\mu}|2\rangle & (1, 2, \text{proton states}) , \\ e e_{\text{pol}}\langle 1|r^3 Y_{3\mu}|2\rangle & (1, 2, \text{neutron states}) . \end{cases}$$

We compare the results of an ordinary RPA with those obtained with renormalized fermions. With this purpose we define the s.p. energies with the experimental data of table 1 together with the Baranger [1]) prescription [eq. (1)]. The

TABLE 1

Experimental data of ^{40}Ca obtained from ref. [5])

l_j	s.p. energies [2])		Experimental data			
	neutrons	protons	neutrons		protons	
	E (MeV)	E (MeV)	E (MeV)	S^2 [b])	E (MeV)	S^2 [b])
$d_{5/2}$	−6.28	−7.207	−9.50	0.010	−9.75	0.016
			−9.28	0.017	−9.10	0.016
			−9.19	0.021	−8.90	0.016
			−9.07	0.032	−8.55	0.037
			−8.70	0.042	−8.43	0.052
			−8.50	0.035	−8.17	0.062
			−8.36	0.053	−7.78	0.016
			−8.19	0.027	−7.43	0.052
			−7.97	0.058	−7.20	0.016
			−7.70	0.017	−6.96	0.031
			−7.88	0.017	−6.77	0.016
			−7.21	0.037	−6.50	0.016
			−6.92	0.016	−6.34	0.300
			−6.45	0.042	−5.61	0.145
			−6.16	0.210	−5.27	0.205
			−5.49	0.085		
			−5.32	0.032		
			−5.13	0.210		
			−4.46	0.017		
			11.01	0.010		
			12.57	0.010		
$s_{1/2}$	−2.527	−3.303	−4.02	0.050	−4.10	0.080
			−2.47	0.950	−2.52	0.920
$d_{3/2}$	0.00	0.00 [c])	0.00	1.000	0.00	0.930
					9.320	0.070
$f_{7/2}$	6.97	5.989	−2.79	0.030	−2.820	0.060
			7.270	0.970	7.246	0.940
$p_{3/2}$	9.430	8.059	9.22	0.720	−5.83	0.012
			9.74	0.240	−3.020	0.011
			11.89	0.040	8.960	0.890
					9.650	0.087
$p_{1/2}$	9.032		−8.800	0.080		
			−7.520	0.040		
			10.890	0.100		
			11.220	0.610		
			12.040	0.170		

[a]) Single-particle energies were obtained with eq. (1).

[b]) The spectroscopic amplitudes shown in the table have been normalized, i.e. they satisfy eq. (33).

[c]) The proton energies were shifted as to make the energy of the $d_{3/2}$ level equal for protons and neutrons.

above-mentioned states of unnatural parity have a significant contribution to the s.p. energies since their small residues are multiplied by a large energy difference.

The use of the dressed fermion states gives rise to 198 "particle"-"hole" configurations instead of only 11 in an ordinary RPA calculation. The value of the coupling constant in both cases was adjusted to give the experimental excitation energy of the first 3^- state in ^{40}Ca. The values thus obtained are:

$$\kappa_{\mathrm{RPA}} = 0.0021\ \mathrm{MeV} \cdot \mathrm{fm}^{-6}, \qquad \kappa_{\mathrm{RRPA}} = 0.0025\ \mathrm{MeV} \cdot \mathrm{fm}^{-6}$$

for the ordinary and for the renormalized RPA versions, respectively. The circumstance that a 20% stronger residual interaction is needed when dressed (experimental) fermion data is used is a consequence of a large particle vibration coupling that reduces the value of the residues of the states that are predominantly of pure s.p. nature. As can be seen from eqs. (21) and (22), the appearance of such residues amount to a change in the value of the coupling constant that has to be increased to compensate for smaller values of the matrix element of $Q^{(3)}$.

The effect of the fragmentation of s.p. states can also be traced in the B(E3) values through its dependence on the polarization charge (fig. 4). For small values of the polarization charge e_{pol} the ordinary and RPA values yield essentially the same result. In this limit only the proton levels are active and these show a smaller fragmentation than the neutron states. The influence of neutron levels is more important as e_{pol} becomes larger.

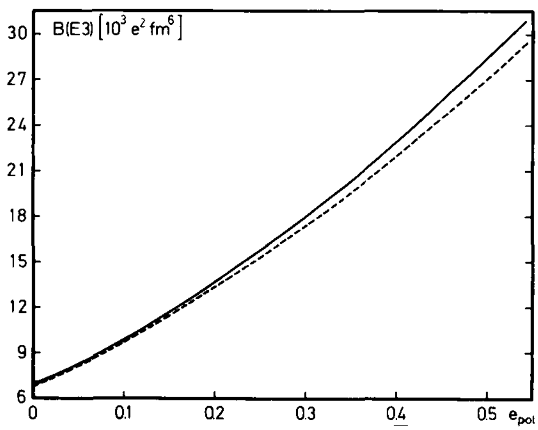

Fig. 4. The B(E3) value as a function of e_{pol}. The dashed (full) line correspond to the ordinary (renormalized) RPA results.

The most striking difference of both results is the response function for the E3 excitation. A standard RPA calculation shows that all the E3 strength that is missing from the lowest (collective) state is concentrated in rather few states. The situation is the opposite (see fig. 5) if renormalized fermion building blocks are used. A much larger density of states is found, each collecting a portion of the E3 strength that is missing from the collective root and thus showing a much broader distribution.

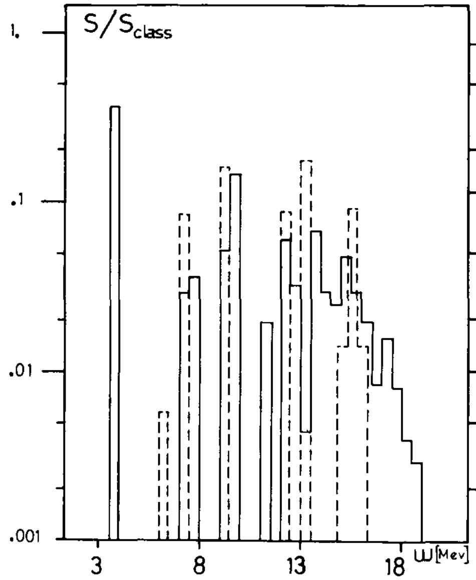

Fig. 5. Distribution of the B(E3) strength given as the fraction of the classical sum rule plotted as a function of the excitation energy ω. The dashed (full) line corresponds to the ordnary (renormalized) RPA results. The large concentration at the collective root (~3.73 MeV) is the same for both calculations.

4. Conclusions

Since the early times of the RPA in nuclear physics [8]) the method has been continuously improved to describe the collective p-h and p-p states in closed shell systems. The experimental information available in the neighbouring odd nuclei has always been used in different ways. The first approaches only borrowed experimental s.p. energies with the implicit assumption that these contained all the Hartree-Fock contributions. A more profound use of the experimental evidence is very much tied, in later calculations, to the method used to improve beyond the simplest possible ladder approximation implied in the RPA equations. Generally speaking there are two extreme philosophies that are equally applicable to including higher-order corrections. One consists in the renormalization of the residual interaction and one-body operators but keeping at all stages bare fermion lines in all diagrams. The other is to use bare interactions and renormalize instead the propagators, defining in this fashion new, more elaborate modes of excitation of the many-body system. Clearly any procedure in between the two has to be worked out with extreme care since it is bound to face severe problems of counting twice similar effects. Along the first approach fall the comprehensive calculations that link the RPA treatment to the Migdal theory of Fermi liquids [9]). The experimental (renormalized) propagators are mainly used, within these frameworks, to parametrize effective two-body interactions as rigorously as possible. The second picture is instead used, for instance, within the nuclear field theory [10]) (NFT) that is specially tailored to study the coupling of s.p. and collective degrees of freedom of a many-body system. These two approaches hinge, respectively, on two concepts that, as expected can be related to each other. The ruling idea within the philosophy of introducing higher-order corrections in the two-body interaction is that of a quasi-particle. Basically it is assumed that a fermion, added to a many-body correlated core, keeps its own identity with some of its physical parameters (e.g. mass, charge, etc) corrected by the interactions with all the A particles of the core. The inclusion of higher-order processes in the propagator relies instead in the idea of elementary modes of excitation (of collective and s.p. nature) defined by the (infinite) summation of families of diagrams. In the present paper we follow this picture using as an elementary building block a generalization of the fermionic modes of excitation of the system.

Both approaches depart from each other when the idea of a quasiparticle is no longer fully applicable. This situation arises when non-local effects due to the dynamical coupling of fermionic and collective modes are relevant. These processes are typically represented by diagrams such as fig. 1D. On the other hand when these effects are negligible the concept of a quasiparticle and of a fermionic mode of excitation are, for all practical purposes, the same. The signature of large coupling between collective and s.p. states (that is a highly non-local effect) is a large fragmentation of the one-body transfer strength as found in the odd neighbours of ^{40}Ca. In this paper we show that it is nevertheless possible to build a (collective) RPA mode using as input data a strongly renormalized one-fermion propagator that can only marginally be interpreted as a quasiparticle. We have shown that this procedure can be carried in two possible ways. One is a "first principle" fashion: first a fermionic propagator is constructed renormalizing the HF one with some given set of processes, next the new collective "elementary" mode is constructed through the RPA procedure as presented here. The other possible way circumvents the first step borrowing from experiment the necessary data to fully specify the fermionic ("single particle") degrees of freedom. We have focused our attention on the p-h situation. The p-p case offers no additional conceptual difficulties. The advantage of the method that is presented here is that no assumption of any kind has to be made upon the strength of the coupling of collective and s.p. modes or, which is the same, the degree of fragmentation of the s.p. strength. This, by the way, can be achieved since we proceed via the solution of Dyson equations and therefore, infinite summations are implied. A price has to be paid to be able to work with building blocks more complicated than a pure quasiparticle, in the Migdal sense. This is the use of normalization properties for the residues of the Green function that are not obvious. These stem from generalized Ward identities and are interpreted within this framework in terms of sum rules of various kinds[†].

[†] Within the framwork of renormalized interactions the Ward identities appear instead linked to consistency conditions of effective interactions and one-body operators [9]).

We have applied our method to study the lowest 3^- state in ^{40}Ca with a residual separable octupole-octupole p-h interaction. Clearly the method is not limited to this rather schematic situation and can easily be worked out with more sophisticated two-body forces. The aim here is to detect which are the most prominent physical differences introduced by the use of fragmented one-body propagators. The collective features are not drastically altered thus showing the consistency of the expansion in inverse powers of an effective degeneracy that is implied in the NFT procedure. However the modifications found are not at all negligible since they amount to a ~20% change in the strength of the residual interaction. In addition some remarkable differences with an ordinary RPA approach are to be noted. The first is linked to the appearance of "scattering residues" that imply that there is a non-vanishing amplitude to populate the collective vibration by annihilating and subsequently creating a fermion on the same side of the Fermi surface. These non-vanishing residues, that correct the amplitude for populating the collective mode, show that a new kind of correlation has been introduced in the ground state of A particles. The influence of these residues in the normalization condition of the boson components is clearly displayed in the schematic model in eq. (32). These correlations are however not effective in the Ca calculations since the shell-model orbitals above and below the Fermi level are all of the same parity and can thus not couple to $J^{\pi} = 3^-$. This situation changes for instance in the Pb region in which spin-orbit intruders make possible this coupling. The second feature is the distribution of the E3 strength that appears to be spread over a very large number of excited states. This can be understood in terms of the appearance of a large number of fragmented "particle"-"hole" configurations with proper spin and parity and should be expected whenever one finds a large fragmentation of the one-body strength in the neighbouring odd nuclei. The next difference with a standard RPA procedure is related to the appearance of unnatural-parity states in the odd neighbours. The inclusion of these in the Baranger prescription cause a significant change in the energy gap between particle and hole states and gives rise to positive-parity "particle"-"hole" states at an energy somewhat lower than $2\hbar\omega_0$. These states however do not display strong collective features.

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