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NUCLEAR STRUCTURE STUDIES IN THE LEAD REGION WITH (d, t) REACTIONS

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Abstract: Energy distributions of tritons from (d, t) reactions in the isotopes of Pb, Bi and Tl were measured with ≈ 100 keV resolution. Evidence is given on the location of the missing $h_{\frac{9}{2}}$ state in Pb^{207}. As theoretically expected, the spectrum from Bi^{209}(d, t) is quantitatively similar to that from Pb^{208}(d, t) except that all levels are split into two or more components. The energies of the states excited by Pb^{207}(d, t) agree well with those predicted from the True and Ford calculation. The cross sections for exciting these agree reasonably well with calculations from the True and Ford wave functions; but several changes in the latter are suggested, including a reduction of the $(p_{\frac{1}{2}})^{-2}$ configuration in the ground state from 73 % to about 55 %. The long sought 0^+ state apparently occurs at 1.19 MeV. The spectra from Pb^{206}(d, t) and from Tl^{205}(d, t) show marked similarities to each other and a general similarity with the spectrum from Pb^{208}(d, t); it is shown that this is to be expected. In general, the shell model of nuclear structure with inverse stripping reaction theory gives a satisfactory quantitative explanation of all the data.

1. The Experiment

Neutron pick-up reactions have been used by Harvey [1]) and by Cohen and Mosko [2]) for nuclear structure studies in the region near the doubly closed shell Pb^{208}; in both cases, however, the resolution was rather poor (≈ 400 keV) and measurements were made at only a single angle, so that practically no quantitative conclusions could be reached. In this paper, studies of this type are reported using (d, t) reactions and employing much better resolution (≈ 100 keV). Data are reported at several angles and relatively detailed comparisons with theoretical predictions are made.

The experimental method has been described previously [3]). Thin targets were bombarded by 15 MeV deuterons from the University of Pittsburgh 119 cm cyclotron; reaction products are passed through a 60° wedge magnet spectrograph and into the emulsions of photographic plates located on the spectrograph focal plane. After the bombardment, the plates are developed, and the tracks at various positions in the photographic emulsions are counted under a microscope to obtain an energy spectrum. Only that region of the

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spectrum where the magnetic rigidity for tritons is greater than that for any other reaction product was studied. An example of typical data is shown in fig. 1. Data were obtained at scattering angles of 45°, 60° and 90° for the Pb^{206}, Pb^{207} and Pb^{208} targets; and at 90° for the Bi and Tl targets. The lead isotope targets were not isotopically pure (isotopic analyses are shown in table 1)

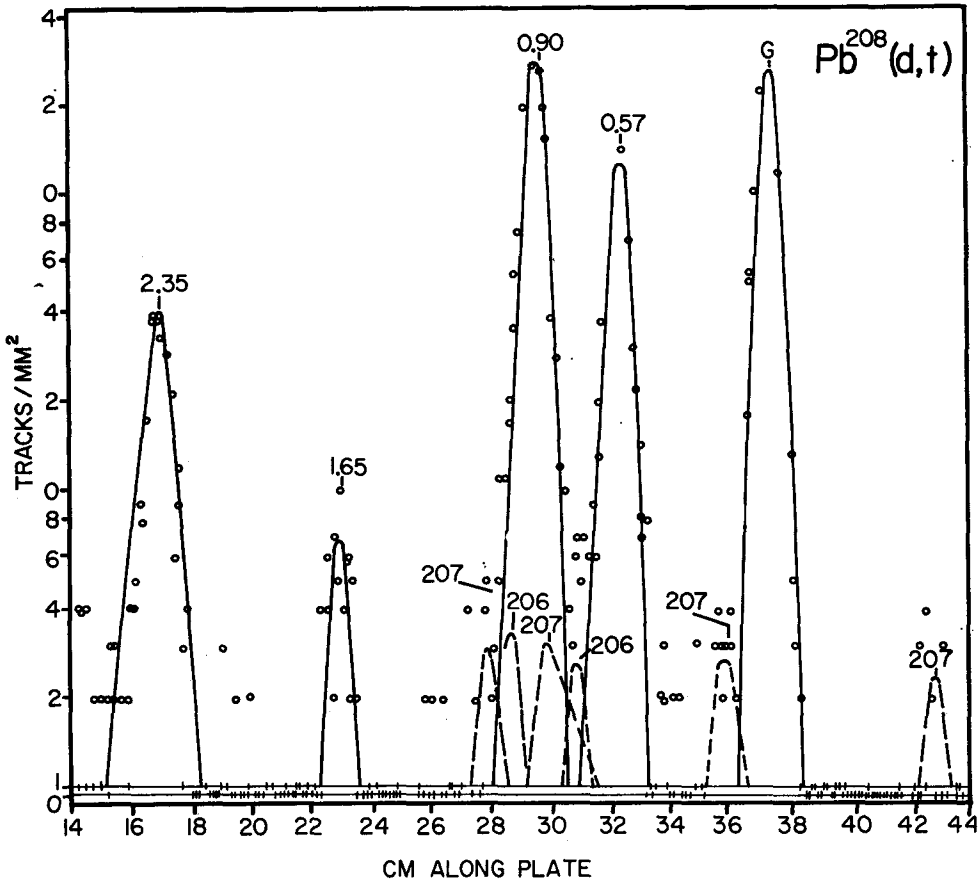

Fig. 1. Energy distribution of tritons from $Pb^{208}(d, t)Pb^{207}$. Angle of observation is 45°. Background at left side is from elastic deuterons, it is considerably lower at larger angles. Different symbols used for ordinates equal 0 and 1 are only to facilitate clarity in the figure.

TABLE 1

Composition of lead isotope targets

Designation	Composition (%)		
	Pb^{206}	Pb^{207}	Pb^{208}
Pb^{206}	80.4	11.7	7.9
Pb^{207}	3.5	71.5	25.0
Pb^{208}	1.2	2.8	96.0

so that careful corrections were necessary. In addition, the targets were thicker (11 mg/cm^2) than is desirable for optimum energy resolution [4]). The bismuth target was 7 mg/cm^2 thick and was evaporated on 0.9 mg/cm^2 gold backing. The thallium target was TlO powder on a mylar backing [5]). There are no triton peaks from oxygen or carbon in the region studied.

2. Results and Conclusions

Surveys of the triton energy spectra are shown in figs. 2 and 3; the 90° data for the lead isotopes is qualitatively very similar to the 45° and 60° data but the resolution is considerably poorer [4]).

$Pb^{208}(d, t)Pb^{207}$

The most easily understood results are those for the $Pb^{208}(d, t)Pb^{207}$ reaction, as that reaction excites the well known single "hole" states in the shell which closes at 126 neutrons. The spins are known from other work, so that the shell model assignments may be made unambiguously as shown in figs. 2 and 3. The only difficulty is the absence of the $h_{\frac{9}{2}}$ hole state; it will now be shown that it very probably lies within 40 keV of the $f_{\frac{7}{2}}$ state. The differential cross section for a (d, t) reaction may be expressed as

$$\frac{d\sigma}{d\Omega} \propto f(l_n, Q, \theta) S c^2, \tag{1}$$

where S is the coefficient of fractional parentage, c is the overlap integral of the initial state minus a neutron and the final state, and f is a factor obtained from nuclear reaction theory. In Butler stripping theory, f is a function of the angular momentum l_n of the neutron in the nucleus, the energy Q, released in the reaction, the angle θ at which the triton is emitted, and the nuclear radius. This will be assumed to be the case here. The difference in nuclear size among the lead isotopes will be neglected. S may be simply calculated [6]) and for the $Pb^{208}(d, t)Pb^{207}$ reaction, $c = 1$ for all single hole states. Thus, the observed data may be used to determine f. The results at each angle are shown in fig. 4. In that figure, a linear dependence on a semi-log plot is assumed for interpolation or extrapolation purposes. Actually, fig. 4 is presented for use in later calculations where the interpolations and extrapolations are relatively small; however, it may be used to obtain at least a rough estimate of the expected intensity of the $h_{\frac{9}{2}}$ hole state as a function of its Q-value. From this it may be concluded that it would have been observed if its Q-value were greater than -4.3 MeV, unless it is almost coincident with another peak. Since that Q-value seems much too small (all other states in the shell have Q-values between -1.1 and -3.4 MeV), the latter situation seems more probable. It cannot be below the $i_{\frac{13}{2}}$ state, for if it were, the $i_{\frac{13}{2}}$ state would not be isomeric. It cannot be very

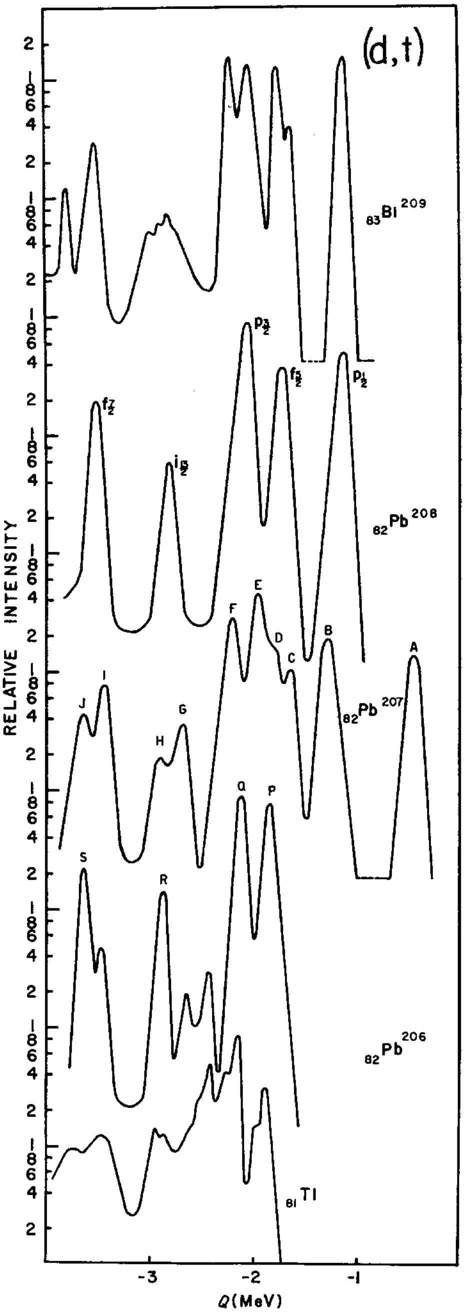

Fig. 2. Survey of energy spectra of tritons observed at 60° from (d, t) reactions in Pb isotopes, Bi and Tl.

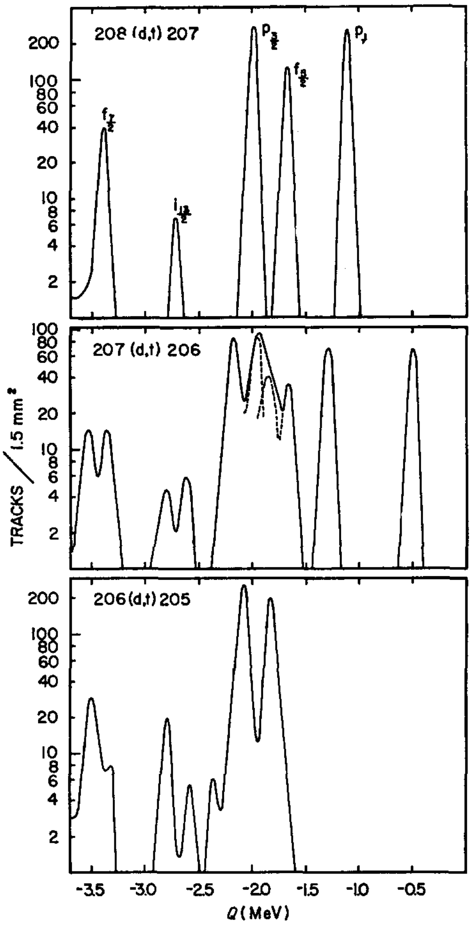

Fig. 3. Energy spectra of tritons observed at 45° from (d, t) reactions in the Pb isotopes.

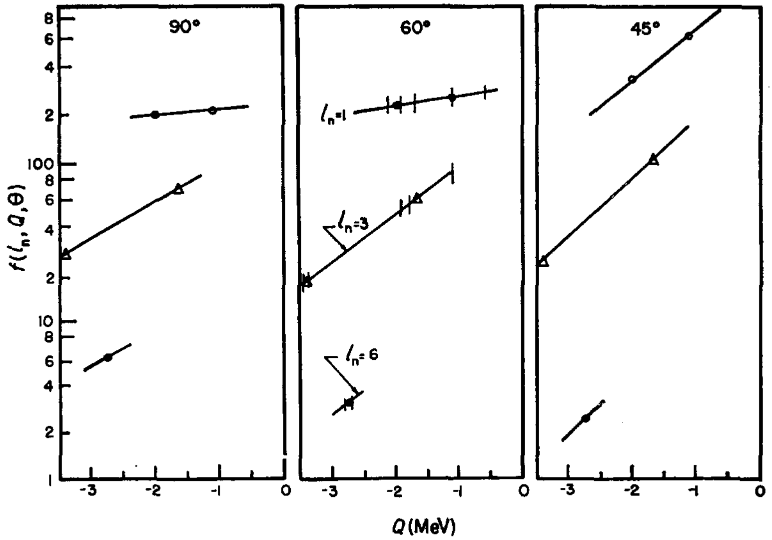

Fig. 4. $f(l_n, Q, \theta)$ versus Q for $\theta = 45°$, 60° and 90°. Pointers are determinations from Pb^{208}(d, t) Pb^{207}. Vertical lines in 60° data show values of Q for which these curves are used in calculation of intensities from Pb^{207}(d, t)Pb^{206} reactions.

close to the $i_{\frac{13}{2}}$ or else the Bi^{207} electron capture would decay almost exclusively to it rather than to the $i_{\frac{13}{2}}$. Thus the only alternative is to assume that the $h_{\frac{9}{2}}$ lies very close to the $f_{\frac{7}{2}}$.

The ratio of peak height to peak area is smaller for the $f_{\frac{7}{2}}$ state than for any of the other peaks in the spectrum, but it is only smaller than the average by about 6 %. While this is perhaps within the error of the determination, it lends at least some additional support to the conclusion, and indicates that the separation of the two levels must be less than 40 keV.

$Bi^{209}(d, t)Bi^{208}$

The next simplest reaction is $Bi^{209}(d, t)Bi^{208}$ as this again should excite single hole states in the **126** neutron shell. The spectrum should thus be identical with that from $Pb^{208}(d, t)$ except that for every level excited in the latter reaction there should be a group of levels corresponding to the various couplings of the extra $h_{\frac{9}{2}}$ proton in Bismuth to the single neutron hole. For example, the ground state which is $\frac{1}{2}^-$ in the Pb^{208} reaction, must be split into two states of spin 5^+ and 4^+ in the Bi reactions.

From fig. 2 both the correspondence between the two cases in the gross structure and the splitting in the Bi fine structure are clearly seen. The excitation energies of the states and the cross sections are listed in table 2. The correspondence in both energies (averaged in the case of Bi) and cross sections is seen to be quite close for all cases except for the $f_{\frac{7}{2}}$ state where the Bi cross section is rather too small and the average energy is shifted. The simplest explanation would be that the splitting of this level is very large and some of the

TABLE 2

Levels excited by (d, t) reactions in Pb^{208} and Bi^{209}

Excitation energy		$d\sigma/d\Omega$ (mb/sr at 60°)		Designation in Pb^{208}	Peak height/area ratio in Bi
Bi^{209}	Pb^{208}	Bi^{209}	Pb^{208}		
0†	0	3.0	3.0	$p_{\frac{1}{2}}$	0.92
0.49 0.61	0.57	0.48 1.5 } 2.0	2.3	$f_{\frac{5}{2}}$	1.5 1.5
0.87† 1.05†	0.90	2.7 2.3 } 5.0	5.7	$p_{\frac{3}{2}}$	0.85 1.2
1.65†	1.64	0.28	0.28	$ir_{\frac{13}{2}}$	0.27
2.34† 2.60	2.35	0.43 0.13 } 0.56	1.0	$f_{\frac{7}{2}}$	1.0 1.4

† More than one level as indicated by last column

components are not seen. This is difficult to reconcile with the facts that (1) there are no other sufficiently excited levels within 600 keV, and (2) the $f_{\frac{5}{2}}$ level is not strongly split and indeed none of the other levels is split by nearly as much as 600 keV. One interesting possibility is that a component of this level in Pb^{207} is $h_{\frac{9}{2}}$ as indicated by previous arguments and there is an extra strong coupling between the $h_{\frac{9}{2}}$ proton and the $h_{\frac{9}{2}}$ hole in the neutron shell. Such an interaction would be the analogue in isobaric spin space of the well known pairing interaction in ordinary spin space.

Also listed in table 2 are the ratios of peak height to areas under peaks for the various peaks in the Bi spectrum. A ratio of 1.5 is expected for a single isolated peak, so that the peaks for which this ratio is appreciably less than 1.5 are a combination of two or more unresolved peaks; the energies of these are marked by an asterisk in the table. In the case of the $i_{\frac{13}{2}}$, there must be at least 6 peaks if they are all of equal intensity, and since the spectrum indicates that the latter is not the case, there are probably 10 or more peaks.

$Pb^{207}(d, t)Pb^{206}$

The results for the reaction $Pb^{207}(d, t)Pb^{206}$ are much more complex to analyze since there are strong couplings between two holes in the same major shell. However, nuclear structure calculations for Pb^{206} have been made by True and Ford [7]) and by Kearsley [8]), so that detailed comparisons with theory are possible. Since the True and Ford paper contains complete wave functions, comparisons will be made with it, but the results of the two papers are very similar.

Since the ground state of Pb^{207} is pure $(p_{\frac{1}{2}})^{-1}$, the states of Pb^{206} excited by the $Pb^{207}(d, t)$ reaction are those with components in their wave functions of the form $(p_{\frac{1}{2}})^{-1}(x)^{-1}$ where x is $p_{\frac{1}{2}}$, $f_{\frac{5}{2}}$, etc. The states of this type from ref. [7]) are listed in table 3 together with their energies, spins, parities and the pertinent

TABLE 3

Energy levels excited by (d, t) reactions in Pb^{207}

Symbol	Excitation energy			Predicted I^{π}	True-Ford configuration
	This work	Alburger	Predicted		
A	0	0	0	0^+	0.73 $(p_{\frac{1}{2}})^2$
B	0.81	0.803	0.72	2^+	0.57 $p_{\frac{1}{2}} f_{\frac{5}{2}} + 0.30$ $p_{\frac{1}{2}} p_{\frac{3}{2}}$
C	1.19		1.36	0^+	0.18 $(p_{\frac{1}{2}})^2$
D	1.34	1.341	1.40	3^+	$p_{\frac{1}{2}} f_{\frac{5}{2}}$
E	1.49		1.39	2^+	0.37 $p_{\frac{1}{2}} f_{\frac{5}{2}} + 0.60$ $p_{\frac{1}{2}} p_{\frac{3}{2}}$
F	1.72		1.74	1^+	0.99 $p_{\frac{1}{2}} p_{\frac{3}{2}}$
G	2.19	2.200	2.17	7^-	0.90 $p_{\frac{1}{2}} i_{\frac{13}{2}}$
H	2.38	2.385	2.35	6^-	0.97 $p_{\frac{1}{2}} i_{\frac{13}{2}}$
I	2.95		3.01	4^+	0.90 $p_{\frac{1}{2}} f_{\frac{7}{2}}$
J	3.11		3.08	3^+	$p_{\frac{1}{2}} f_{\frac{7}{2}}$

part of their wave functions. The experimental energies from this work are also listed in table 3, and the agreement between theory and experiment is very good. In a few cases, levels known from other work 9) are listed in table 3, and it is seen that the agreement between the experiments is quite satisfactory.

One important conclusion from table 3 is the identification of a 0^+ state at 1.19 MeV. From ref. 7) alone, the 0^+ state could be the 1.19, 1.34 or 1.49 MeV state. However, the 1.34 MeV state is known to be 3^+ from ref. 9), and the cross section results below would preclude the identification of the 1.49 MeV state as the 0^+.

The theoretically expected cross sections for exciting the various states in Pb^{206} may be calculated from eq. (1) by taking $f(l_n, Q, \theta)$ from fig. 4, c^2 from ref. 1) (or table 3), and calculated values[†] of S. The values of f needed from fig. 4 are shown by the vertical cross lines in the 60° portion of that figure, and it is clear that interpolations and extrapolations from the data used in obtaining that figure are very small in all cases. This is most fortunate, as any large extrapolations would be very unreliable.

The results of the calculations are shown in table 4 where they are compared with the experimental values. The agreement is, in general, reasonably good. The most striking difficulty is in the $i_{\frac{13}{2}}$ level at forward angles. However, its intensity is very low at such angles, and the intensity is changing quite rapidly with angle. It is even possible that the level is excited by some other process to a sufficient extent to compete with the neutron pick-up reaction.

TABLE 4

Cross sections for excitation of various levels in $Pb^{207}(d, t)Pb^{206}$ reactions. All values are relative to the ground state transition from $Pb^{208}(d, t)Pb^{207}$ at that angle.

Level symbol	45°		60°		90°		meas./calc. ratio		
	meas.	calc.	meas.	calc.	meas.	calc.	45°	60°	90°
A	0.33	0.53	0.32	0.43	0.30	0.38	0.62	0.75	0.79
B	0.34	0.52	0.43	0.65	0.57	0.71	0.66	0.66	0.80
C	0.16	0.05	0.22	0.087	0.36	0.090	3.2	2.6	4.0
D	0.19	0.24	0.22	0.39	0.51	0.57	0.79	0.56	0.89
E	0.45	0.45	0.86	0.81	0.80	0.83	1.00	1.06	0.96
F	0.41	0.34	0.60	0.68	0.65	0.68	1.20	0.88	0.95
G	0.029	0.012	0.077	0.044	0.14	0.09	2.4	1.75	1.5
H	0.024	0.011	0.042	0.037	0.12	0.09	2.3	1.13	1.3
I	0.067	0.073	0.18	0.16	0.27	0.30	0.92	1.12	0.90
J	0.067	0.058	0.10	0.13	0.24	0.25	1.15	0.77	0.96

All other discrepancies between calculated and observed cross sections in table 4 may be explained as errors in the True and Ford wave function 7); to remove these discrepancies, the amplitudes of the respective terms should be

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[†] The authors are greatly indebted to J. B. French for his calculations of the S values for transitions between single hole and two hole states.

multiplied by the ratio of experimental to calculated cross sections. The most clear cut case of this type is the relative distribution of the $(p_{\frac{1}{2}})^{-2}$ state between the two 0^+ states (A and C): while the True and Ford wave function gives A to be 73 % $(p_{\frac{1}{2}})^{-2}$ and C to be 18 % $(p_{\frac{1}{2}})^{-2}$, the experiments indicate that each is about 55 % $(p_{\frac{1}{2}})^{-2}$. There is additional evidence from (d, p) reactions that A is definitely less than 73 % $(p_{\frac{1}{2}})^{-2}$ and 55 % is in reasonable agreement with those results. This will be discussed in the following paper [10]). Table 4 also indicates that the amplitude of $(p_{\frac{1}{2}}f_{\frac{5}{2}})_3$ in D should be decreased slightly, and that the amplitudes of one or both parts of the wave function for B should be decreased.

Pb^{206}(d, t)Pb^{205}

The energy levels of Pb^{205} excited by the (d, t) reaction on Pb^{206} are listed in table 5. The relative cross sections at 45° and 60° are also tabulated. According to either the True and Ford wave function or its modifications discussed above, a large part of the time the ground state of Pb^{206} is the same as the ground state of Pb^{208} except that the $p_{\frac{1}{2}}$ shell is empty instead of full. Since there can be no interaction between either completely empty or completely full shells and other states, the spectrum from Pb^{206}(d, t) should be largely identical with that from Pb^{208}(d, t) except that the $p_{\frac{1}{2}}$ level should be missing. Qualitatively, such a similarity is apparent from figs. 2 and 3 in that the four large peaks from Pb^{206}(d, t) seem to correspond to the $f_{\frac{5}{2}}$, $p_{\frac{3}{2}}$, $i_{\frac{13}{2}}$ and $f_{\frac{7}{2}}$ peaks from Pb^{208}(d, t).

TABLE 5

Energy levels excited by (d, t) reactions in Pb^{206}

Excitation energy (MeV)	Relative intensity		Symbol
	45°	60°	
0	1.0	1.0	P
0.24	1.3	1.1	Q
0.54	0.031	0.038	
0.77	0.028	0.026	
0.99	0.10	0.19	R
1.53	0.039	0.060	
1.72	0.15	0.28	S

TABLE 6

Cross section ratios between corresponding levels from Pb^{206}(d, t) and Pb^{208}(d, t). Capital letters refer to states from Pb^{206} reaction (cf. table 5), and spectroscopic notation refers to states from Pb^{208}(d, t).

Angle of observaiton	$P/f_{\frac{5}{2}}$	$Q/p_{\frac{3}{2}}$	$R/i_{\frac{13}{2}}$	$S/f_{\frac{7}{2}}$
45°	1.5	0.9	2.7	0.9
60°	1.1	0.7	2.2	1.0
90°	1.1	0.6	2.5	1.1

A quantitative test of the correspondence is shown in table 6, where the cross section ratios of corresponding levels from the two reactions are listed. If the True and Ford wave function were correct, all ratios in table 6 would be at least 0.73 whereas if the altered wave function deduced above is accepted, they should be at least 0.55. The evidence is far from conclusive, but there is perhaps some indication here that the True and Ford value is too high.

Tl(d, t)

Natural thallium is 70 % Tl^{205} which has the same number of neutrons as Pb^{206}. There is some evidence [11]) that an odd, low angular momentum proton coupling to a single-particle neutron state does not cause very wide splittings. Thus it might be hoped that the extra $s_{\frac{1}{2}}$ proton hole in Tl^{205} will not cause the spectrum from Tl^{205} (d, t) to differ markedly from the spectrum from Pb^{206}(d, t). It is evident from fig. 2 that this is indeed the case. There is essentially a one-to-one correspondence between the peaks in the two spectra, and there is at least a rough similarity in the relative intensities of the various peaks.

3. Discussion

From the results and their analyses described above, it is clear that the combination of the shell model description of nuclear structure and deuteron stripping reaction theory can successfully explain all the major features of the energy spectra of tritions from (d, t) reactions in the region near Pb^{208}. The energies are very well explained, and the cross sections can be at least semi-quantitatively calculated. A few problems arise, but these can always be explained away without major changes in the theory. The True and Ford wave functions [7]) for the states of Pb^{206} seem to be reasonably good, but several changes in them are suggested by the experimental results. No investigation has yet been made of what interactions are necessary in the theory to give these changes.

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