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Systematics of the Ground State Rotational Band in Even–Even Nuclei with $158 \leqslant A \leqslant 182$ and $A \geqslant 230$[*]

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1. INTRODUCTION

A great amount of work dealing with the systematization of the properties of the excited states of even–even nuclei has been done up to the present. In the introduction to a recent paper, C. A. Mallmann[1] (hereafter referred to as Paper I) quotes a large list of references on this subject and, for the sake of avoiding repetition, it will not be given again here.

As a result of the work already done, one knows that, according to the structure of the lower part of their level schemes, nuclei may be crudely classified into four types:

(*a*) Those in which the spacing between the energy levels obeys a more or less harmonic (vibrational) pattern, i.e., the ratio between the energies of the levels belonging to the triplet of the second excited state and the energy of the first excited one lies between 2 and 3.

(*b*) Those in which the ratio between the energy of the second excited state and the energy of the first excited one takes values between 1 and 1.8 (Ref. 1). In these nuclei, a proton or neutron shell is closed.

(*c*) Those which have a spacing of the level energies nearly analogous to that of a spinning top (rotational level scheme).

(*d*) Those having level schemes which display characteristics that represent a transition between types (*a*) and (*c*).

In Paper I, Mallmann has studied the properties of the excited states of all nuclei with $40 \leqslant A$ belonging to types (*a*), (*b*) and (*d*); particularly the regions in which nuclides of type (*d*) are found have been clearly pointed out.

In this paper we will study the systematics of some properties of the level scheme of type (*c*) nuclides with $A \geqslant 40$. Their existence in regions of the nuclear chart where there are many particles outside of closed shells was explicitly pointed out by Bohr and Mottelson in a series of papers,[2–6] and by Hill and Wheeler.[7] These authors established special forms of the unified model formulation to give a coherent explanation of many empirical evidences (as enhanced transition probabilities, large quadrupole moments, extremely low-lying energy levels not attributable to particle excitations), referred to in detail in these publications. We will consider that outside the boundaries of the "rotational regions" are all those nuclei which, according to the criteria to be given in a later section, have a level scheme which diverges more than 1.5% from the "pure" rotational one. With the experimental data available to date this means, referring to values of A, N, and Z, that

$$158 \leqslant A \leqslant 182, \qquad A \geqslant 230,$$
$$64 \leqslant Z \leqslant 74,\ 94 \leqslant N \leqslant 108 \qquad Z \geqslant 90,\ N \geqslant 140,$$

where the Z- and N-conditions must hold simultaneously. When needed for reasons of completeness, nuclei of type (*d*) will also be taken into account.

As mentioned in Paper I, there exist other regions on the nuclear chart with $A \geqslant 40$ where the appearance of rotational-type levels could be expected, but up till now no experimental evidence for this has been found.

Litherland *et al.*[8] have shown on an experimental basis the existence of rotations in the vicinity of $A = 25$ and, more recently, also Rakavy[9] has pointed out the same for $18 \leqslant A \leqslant 26$. Also Failleros *et al.*[10] have mentioned collective effects near $A = 16$.

For the collection of experimental data we have made use of the usual sources in this field.[11–14]

2. THEORETICAL PREDICTIONS

Many of the regular features concerned with nuclear structure properties are accounted for by the shell model.[15]

As is often mentioned, particular success was achieved by the scheme of single-particle energy states which is obtained with two of the principal implications of the model, namely, the independent movement of each nucleon in an isotropic potential well averaged over the whole nucleus and strong spin-orbit coupling.

But in regions where N and Z values are simul-

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[*] This paper is an expanded version of a communication on "Energy systematics of the excited states of the ground state rotational band in even–even nuclei with $90 \leqslant N \leqslant 114$ and $56 \leqslant Z \leqslant 74$", by W. Lubomirsky and one of us (W. Scheuer) presented before the 30th meeting of the Argentine Association of Physics (1957).

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taneously away from closed shells, large quadrupole moments were found, which pointed to an anisotropic distribution of the proton charge and, hence, to deformed nuclei.[16] This deformation was attributed to effects of collective motions of the components of the nucleus[2–7]; this collective character gave, at the same time, an explanation for the very enhanced E2 transition probabilities which had been observed in the deformed nuclei.

Somewhat later,[17–20] the ordering of the single-particle states in the deformed nuclei was also given as a function of a parameter characterizing the deformation. This was done considering both the intrinsic motion of single nucleons in the deformed nucleus described by wave functions expanded in terms of the eigenvectors of an isotropic harmonic oscillator[18] and in terms of an anisotropic one[20]; both treatments are equivalent. The deviations of the particle level scheme so obtained, with respect to the one which results in the shell model, are very strong for the deformations corresponding to the large quadrupole moments observed (compare, for example, Fig. 5 of Ref. 18 and Fig. 1 of Ref. 20). The agreement between the proposed ordering and the spins and parities measured in the region is good.

The main features and consequences of the unified model are well known today through the original papers and some very useful review articles.[21–23] We only wish to state very briefly the predictions which can be—and many times have been—compared with the experimental data collected in the following sections. Our statements are based on the review of Alder *et al.*[22]:

(*a*) For even–even nuclei with axially symmetric (spheroidal) deformation (to which all of the following refers), the energy levels are given by the expression:

$$E = E_0 + \frac{\hbar^2}{2\mathcal{J}} I(I+1), \tag{2.1}$$

where I is the total angular momentum, $\mathcal{J}$ is the effective moment of inertia of the nucleus about an axis perpendicular to the symmetry axis, and E_0 is a constant term for each rotational band. This means that it corresponds to the energy of the particle or vibrational excitation onto which the rotations develop. The ground state band is then characterized by $E_0 = 0$.

(*b*) The total angular momentum I may be split in its components along a spatially-fixed axis, M, and along the nuclear symmetry axis, K; all three are constants of the motion in the deformed potential (however, the angular momentum of the individual nucleons, which is a good quantum number in the shell model, does not have such a property in this case). It is readily shown that

(1) K being coincident with the axis of symmetry, it corresponds to an angular momentum which did not originate in collective rotations;

(2) for the ground state $K = 0$;

(3) with the ground state ($K = 0$, $E = 0$) can only be associated rotational states with total angular momentum

$$I = 0, 2, 4, 6, 8, \ldots \text{ and even parity } (K = 0) \tag{2.2}$$

(4) E2 γ transitions go on between them, without crossovers.

(*c*) From (2.1) and (2.2) it follows that the ground state has zero energy, zero spin and even parity. This is in agreement with the striking consequence of the shell model formulation that all e–e nuclei should possess these characteristics. Even if direct experimental confirmations of these predictions are rather scarce, they have been generally accepted as rules, since a great amount of empirical data can be arranged in a coherent manner only if they actually hold.[15] We will designate the ground state by $^{1}0+$, which expresses the fact that it is the first state with 0 spin of even parity; a similar notation will be used for the other states.

The prediction of a first excited state of 2+ character also agrees with the general statement that, with only four known exceptions, all e–e nuclides possess first excited states of such character.

(*d*) A maximum for the energy of the $^{1}2+$ state ($K = 0$), if it is of rotational nature, is given by

$$E(^{1}2+) \approx \frac{13\hbar^2}{\mathcal{J}_{\text{rig}}}, \tag{2.3}$$

where

$$\mathcal{J}_{\text{rig}} = \tfrac{2}{5} AMR_0^2 (1 + 0.31\beta + \ldots) \tag{2.4}$$

is the moment of inertia for a rigid rotation of a spheroidal configuration and the deformation parameter β may be obtained from the expression of the electric quadrupole moment for a nucleus with spheroidal deformation assumed to be uniformly charged,

$$Q = \frac{3}{(5\pi)^{\frac{1}{2}}} ZR_0^2 \beta (1 + 0.16\beta + \ldots), \tag{2.5}$$

where A is the mass number, Z the nuclear charge, and $R_0 = \text{const} \times A^{\frac{1}{3}}$ cm is nuclear radius.

Equation (2.3) clearly shows that if the deformations in the heavy region and in the $158 \leqslant A \leqslant 182$ region are more or less the same, the energy of the $^{1}2+$ state in the former region should be in absolute value about one-half of that in the latter one.

(*e*) The relationship of Q and, hence, of the deformation to the reduced nuclear transition probability for E2 transitions is given by:

$$\begin{aligned} B(\text{E2};\, ^{1}0+ \to {}^{1}2+) &= \frac{5}{16\pi} \cdot e^2 Q \\ &= \frac{9}{20\pi} e^2 R_0^4 \{0.4 Z^2 \beta^2 (1 + 0.16\beta)^2\}. \end{aligned} \tag{2.6}$$

In the shell model the transition probability has to be accounted for entirely by the action of a single proton. As its Q is much smaller than that of the strongly deformed nuclei, it readily follows that these must have $B(\text{E2})$ values much larger than those predicted by the single-particle equation

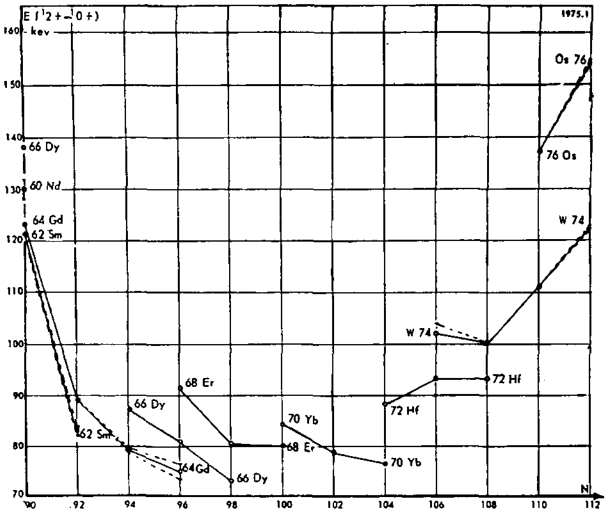

Figure 1. Energy of the first excited state as a function of N

$$B(\mathrm{E2};\, {}^{1}0+ \rightarrow {}^{1}2+)_{\mathrm{s.p.}} = \frac{9}{20\pi} e^2 R_0{}^4. \qquad (2.7)$$

A useful parameter is the factor of enhancement F, given by

$$F = \frac{B(\mathrm{E2};\, {}^{1}0+ \rightarrow {}^{1}2+)}{B(\mathrm{E2};\, {}^{1}0+ \rightarrow {}^{1}2+)_{\mathrm{s.p.}}} \qquad (2.8)$$

(*f*) For a K different from zero, the total angular momentum I of the nucleus may take values $I = K$, $K-1$, $K-2$, $K-3, \ldots$ and the parity of the corresponding rotational levels must be the same as that of the intrinsic level onto which they develop ($K \neq 0$).

3. EXPERIMENTAL DATA RELATED TO THE GROUND STATE ROTATIONAL BAND

Description of Chart I of Paper I

All of the most significant data connected with the first two states of the fundamental rotational band are given in Chart I of Paper I, which is, essentially, a 3-dimensional representation of the energy surface of the first excited state. The discussion of the collected experimental data will be preceded by a description of this chart. At first glance it will be realized that they resemble those of Paper I.

As ordinate and abscissa, we have chosen Z and N respectively. The isobaric lines are indicated by two short heavy segments, placed on each side of the central part of the Chart. The configurations which, according to the shell model, correspond to closed shells are entered along the coordinate axes. Along each isobaric line the following data are indicated when known:

(*a*) At the upper end of the segment which indicates the line in question, the upward reduced nuclear transition probability B(E2) as given by the (single particle) equation (2.7) is entered, in units of $e^2 \times 10^{-51}$ cm^4. The value $R = 1.2 \times 10^{-13} \times A^{\frac{1}{3}}$ cm has been adopted.

(*b*) For the odd–odd nuclei the half-life is given at the intersection of the corresponding coordinates; when an isomeric state *which feeds some level of neighboring even–even nuclei* exists, its half-life is also given. (These data are quoted as an aid for the planning of experimental work; no attempt has been made to ensure their extreme precision. The same is true for the little arrows which indicate the modes of decay.)

(*c*) For the even–even nuclei the following properties are given:

(1) At the crossing point of the coordinates, the energy of the first excited state, in Mev, is given. Taking this point as center,

(2) In the lower left quadrant a full triangle is introduced if, from a measurement of the magnetic moment, the zero spin value of the ground state has actually been determined; if it is only possible to ascertain that the spin value is ~0, a question mark is added.

(3) In the upper left quadrant the character of the first excited state is given, according to the following key:

—without parentheses if the level has been reached from the ground state by Coulomb excitation or if it has been characterized by an angular correlation which only involves other levels whose character may be given without parentheses;

—without parentheses and underlined if the Coulomb excitation starts from a $\underline{0+}$ (i.e.: ▲) ground state or if the angular correlation involves levels with underlined character;

—with parentheses if there exists some kind of indirect evidence, for the quoted character.

(4) In the upper right quadrant the value of F (equation (2.8)) is given.

(5) In the lower right quadrant the half-life and/or the natural abundance is given.

(*d*) The question marks near both ends of the line indicate the placing of those nuclei unknown at present which are nearest the (not represented) stability line.

In Figs. 1 and 2 we have represented, as usual,[24–27] the energy of the first excited state separately as a function of N and Z respectively. Errors, which are appreciable, have been indicated. From these figures, points of constant energy have been obtained which have been translated on Chart I, where they are found as little open circles on the coordinate lines. Wherever space allowed it, the corresponding energy value, in Mev, is noted beside each circle. Finally these points have been linked with full lines. On the normally fine reticulate of the chart, those coordinates corresponding to neutron or proton numbers for which major irregularities appear in the tracing of the so obtained isoergic lines, are distinguished by their heavily broken character. In the regions of Chart I considered in this paper the following lines have been represented when possible:

$0.100 \leqslant E(^{1}2+ — ^{1}0+) \leqslant 0.300$: each 0.025,
$0.045 \leqslant E(^{1}2+ — ^{1}0+) \leqslant 0.095$: each 0.005,

and the lines 0.0445, 0.04407, 0.0435, 0.0430, all energies being expressed in Mev.

The spacing of the lines is such that they certainly do not overlap within the error margin of their tracing.

Another kind of 3-dimensional representation of the energy surface has been given in reference 22.

Ground State: $^{1}0+$

In the region $158 \leqslant A \leqslant 182$, 31 even–even nuclei are known; in the region $A \geqslant 230$ the known ones are 14. Adding this to the equivalent figures obtained in Paper I, a total of 303 known even–even nuclei with $A \geqslant 40$ results.

In the rotational regions here considered, there is no nucleus in which a ~0 value has been measured for the ground state spin and only in three cases has it been determined to be effectively 0. Therefore the percentages of the known even–even nuclei with $A \geqslant 40$ for which experimental data for the ground state spin are available remain practically those of Paper I; in 13% it has been determined to be 0 and in 6% to be ~0.

First Excited State: $^{1}2+$

Character and Excitation Energy

As previously stated, the theoretical prediction for the character of this state is 2+ as is the case for all even–even nuclei, independent of the type of level scheme. In Section 2 it has also been mentioned that the $E(^{1}2+{-}{-}^{1}0+)$ excitation energy should be particularly low in the rotational regions, and lowest in the $A \geqslant 230$ region.

In Table 1, making use also of the results obtained in Paper I, some illustrative data connected with the $^{1}2+$ state have been summarized.

We will now examine Chart I for both of the rotational regions under consideration.

<u>$158 \leqslant A \leqslant 182$.</u>—This region is placed in the central part of the rectangle formed by the coordinates corresponding to proton and neutron numbers which close a shell at $Z = 50$, $Z = 82$; $N = 82$ and $N = 126$.

As seen in Paper I, in the neighborhood of these coordinates the isoergic lines should be evidently parallel to said coordinates. If as the model implies, no shell closing effects do exist in the region, it should be possible to make a rough approximation of the general tendency of the isoergic lines by concentric ellipses whose centers coincide with the center of the rectangle and whose major and minor axes are along $Z = 66$ and $N = 104$, respectively. The lowest value for $E(^{1}2+{-}^{1}0+)$ should be reached at the center of the ellipses.

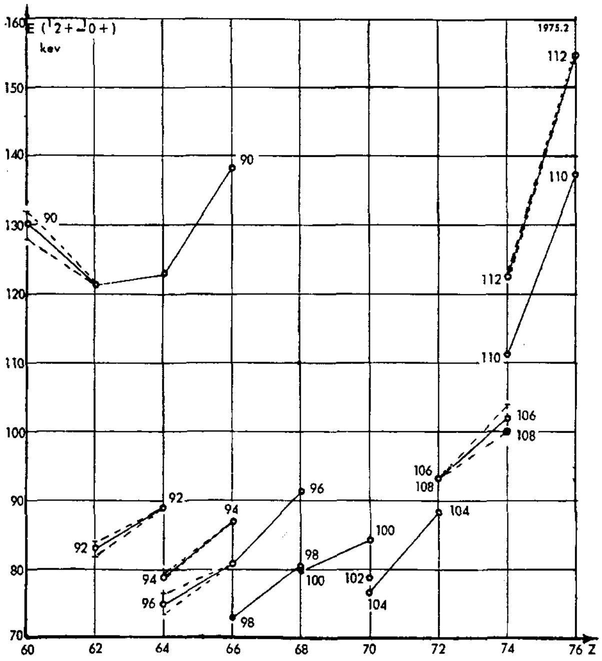

Figure 2. Energy of the first excited state as a function of Z

From the scarce data available it can be concluded that the general tendency expected is more or less accomplished, but $Z = 66$ does not seem to be the major axis mentioned. This is also seen in Figs. 1 and 2. In Fig. 1 the lowest Z-constant line should be $Z = 66$; in Fig. 2 each constant-N line should have its minimum at $Z = 66$. However, both these figures and the chart point to $Z = 62$. From the figures, the minor axis seems actually to be near $N = 104$.

In Chart I Er^{168} stands out as a localized irregularity much more clearly than in Figs. 1 and 2. The energy of its $^{1}2+$ state would fit into the general trend only if it were nearly 5 kev lower than the 79.9 ± 0.1 kev measured accurately.[27] (Note the 76.6 ± 0.1 kev value corresponding to the $^{1}2+$ state of Yb^{174}.) If the general tendency of the isoergic lines should be the one given by the 80 kev line between $N = 98$ and $N = 100$, the accurately measured $E(^{1}2+{-}{-}^{1}0+)$ value of Yb^{172} (Ref. 27) must be increased by 5 kev and, simultaneously, the

Table 1. Some Data Regarding the $^{1}2+$State

Region	*e–e nuclei with measured first excited state*		*e–e nuclei in which the state has character*			$E(^{1}2+{-}{-}^{1}0+)$ *(Mev)*		*F*	
	Number	*% of known e–e nuclei*	$\underline{2+}$	2+	(2+)	*Min.*	*Max.*	*Min.*	*Max.*
$158 \leqslant A \leqslant 182$	16	52	2	9	5	0.073	0.100	146	365
$A \geqslant 230$	18	53	—	2	16	0.042	0.059	142	257
$A \geqslant 40$	161	53	31	80	50	0.042	3.9	3.5	365

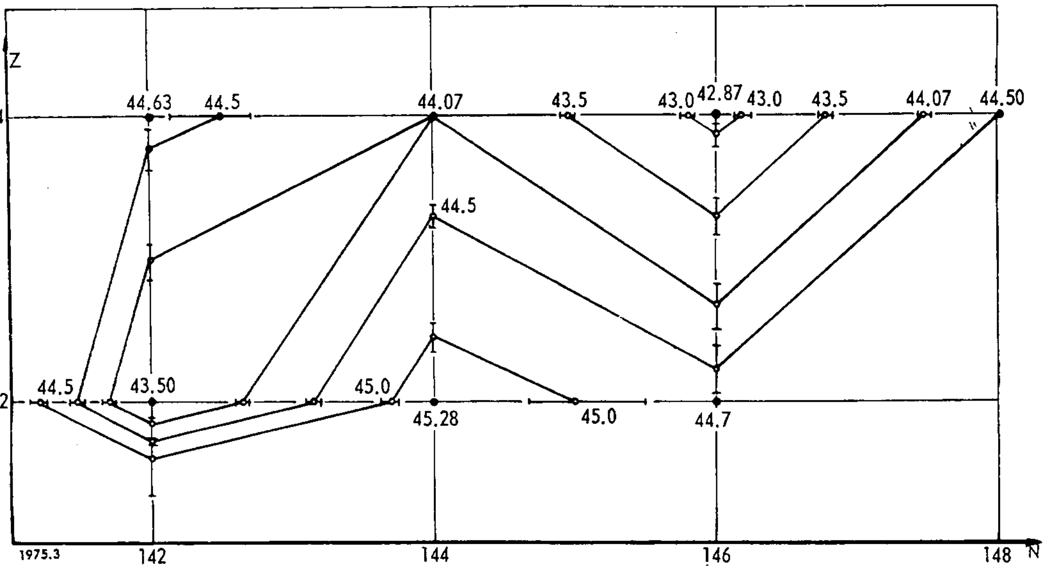

Figure 3

corresponding accurately measured value of Yb^{174} (Refs. 27, 28) by nearly the same amount. A lowering of the 80.25 ± 0.05 kev first excited state energy of Er^{166} (Refs. 27, 29, 30) alone would not eliminate the irregularity. Finally, it may be stated that the $E(^{1}2+ - {}^{1}0+)$ of Er^{168} has been determined from the decay of Tm^{168}, which is long-lived and is considered to be a class A nucleus; in the same experiment were accurately determined many other activities which are normally attributed to Tm isotopes.

In addition, the energy assigned by Brown *et al.*[31] to the $^{1}2+$ state in W^{180} seems to be about 5 kev too high. A further measurement would be desirable.

$A \geqslant 230$.—Nuclei in this region have $N > 126$ and $Z > 82$. With the experimental data available at present, certainly nothing based on the general tendency of the isoergic lines of Chart II of Paper I may be ascertained with respect to the closure of the next major shells in N and Z; the same conclusion would be arrived at if the auxiliary graphs not reproduced here were considered. Yet it should be stated that neither is there evidence against a closure of the proton shell at $Z = 126$. The general tendency of the few lines in relation to the coordinates $Z = 82$ and $N = 126$ is almost correct.

Hollander[26] first pointed out the crossing of lines which appears on the auxiliary graphs in the regions where $Z = 92$ or 94 and $N = 142$, 144, 146 or 148 simultaneously. In Chart I this is translated into a well-noticeable irregular region, which is reproduced in detail in Fig. 3. A fine structure analysis is possible due to the fact that several nuclei of the region have their first excited state energy determined with an error of about 0.1%. To visualize clearly the well on the level surface in which U^{234} lies, we have chosen as isoergic line 0.04407 Mev instead of 0.04400 Mev (the fact that in the neighborhood of U^{234} the errors of some of the interpolation points overlap is immaterial). A similar case, for Hg^{200}, appears in Paper I.

A reassignment of the mass number of the nuclides of this zone could perhaps avoid the above-mentioned irregularities. However, there are very strong links between the nuclides on account of the many measurements of β^-, β^+, ϵ and α decays; any change practically affects the rest of the table up to its upper end. All nuclides of the actually abnormal zone belong to class A. Regarding the determination of which in fact are the anomalous nuclei in this region, at least two assumptions may be made:

(*a*) The tendency of the lines between $N = 146$, 148 is the "correct" tendency, and the 43.50 kev value for U^{234} is "correct" also. Then the irregularity is centered on the $N = 144$ line, where the energies would be too high; the effect would be around 3.5%. However, $E(^{1}2+ - {}^{1}0+)$ of Pu^{246} then becomes too high also.

(*b*) The "correct" tendency is the tendency between $N = 144$ and 148; hence the only abnormal energy is the energy belonging to U^{234}. The well would represent a 6–7% irregularity.

It should be mentioned that Ghiorso *et al.*,[32] based on an ~3% anomaly in the α-decay energies, have indicated the existence of an abnormality at $N = 152$, of a nature essentially different from that attributed to the closing of shells. Rassey[20] and also Perlman *et al.*[33] point out that in the level scheme diagram for deformed nuclei, there appears a major gap at $N = 152$, which could be some indication towards a subshell (see Fig. 1 of Ref. 20). Particularly, Rassey mentions that the fact that there is no increase in $E(^{1}2+ - {}^{1}0+)$ as $N = 152$ is approached is no evidence against the closing of a subshell at this value, since the $^{1}2+$ states are of rotational character; according to Rassey this could be the only difference between the $N = 152$ subshell and those which do not lie in deformed nuclei regions.

Another gap exists at $Z = 66$, and it seems plausible to expect for this value of protons some effect indicative of increased stability. But, according to the preceding paragraph, the possible increase in $E(^{1}2+ - ^{1}0+)$ for nuclei with $Z = 66$ previously mentioned should not be correlated with this gap. Effectively, the gap exists for $0.20 \leqslant \xi \leqslant 0.30$; ϵ is equal to β only to the first order but is always greater than β; whereas, according to Table 2, Dy nuclei seem to have $\beta > 0.33$ (even if their $\bar{F}$'s lie on the exponential of Fig. 5).

Reduced Transition Probabilities

In Coulomb excitation experiments,‡ the reduced upward nuclear transition probability $B(\mathrm{E2}; ^{1}0+ \rightarrow ^{1}2+)$ for an excitation of given multipolarity can be obtained by measuring the yield of the unconverted deexcitation γ ray, of the corresponding conversion electron or of the inelastically scattered bombarding particle. For the E2 transitions here considered we have

$$B(\mathrm{E2}; ^{1}0+ \rightarrow ^{1}2+) = \frac{\sigma . \alpha}{E_{\mathrm{p}} - \beta . E(^{1}2+ - ^{1}0+)} \cdot \frac{1}{f_{\mathrm{E2}}\{E(^{1}2+ - ^{1}0+), E_{\mathrm{p}}, \gamma\}} \tag{3.1}$$

where σ is the measured excitation cross section, E_{p}, the energy of the bombarding particle and α, γ and β account for the charges and masses of the bombarding particle and of the target nucleus; the function $f_{\mathrm{E2}}(E(^{1}2+ - ^{1}0+), E_{\mathrm{p}}, \gamma)$ decreases strongly as the excitation energy increases, if the bombarding energy is constant. ϵ, always less than one, is given by:

$$\epsilon = \begin{cases} 1/(1+\alpha_{\mathrm{t}}) & \text{for } B(\mathrm{E2})_{\gamma};\ (\gamma\text{-detection}) \\ \alpha_{i}/(1+\alpha_{\mathrm{t}}) & \text{for } B(\mathrm{E2})_{e^-};\ (e^-\text{-detection}) \\ 1 & \text{for } B(\mathrm{E2})_{\mathrm{p}'};\ (\mathrm{p}'\text{-detection}), \end{cases} \tag{3.2}$$

where α_{t} is the total conversion coefficient and α_{i} (i = K, L, M . . .) are the partial conversion coefficients.

The same transition probability may be obtained from measurements of the half-life of the $^{1}2+$ state, through

$$B(\mathrm{E2}; ^{1}0+ \rightarrow ^{1}2+) = \frac{4.1 \times 10^{-13}}{1+\alpha_{\mathrm{t}}} \times \frac{0.693}{\tau(\mathrm{E2}; ^{1}2+ - ^{1}0+)} \times \frac{1}{\{E(^{1}2+ - ^{1}0+)\}^{5}} \tag{3.3}$$

where τ is the half-life.

If the energies are in kev, $B(\mathrm{E2}; ^{1}0+ - ^{1}2+)$ results in units of $e^{2} \times 10^{-48}$ cm^{4}, in both expressions.

Table 2 gives data connected with F, i.e. the ratio between the transition probability obtained in the experimental manner just described and the one predicted by the single particle equation (2.7). A table of similar nature was given by Alder *et al.*[22] for all those nuclei which have been Coulomb excited; they also showed that, where the comparison is possible, $B(\mathrm{E2})_{\mathrm{T}}$ agrees within the experimental error with the values derived from Coulomb excitation experiments.

The first three columns represent respectively the nucleus under consideration, the present best known value of $E(^{1}2+ - ^{1}0+)$ and the conversion coefficients used for the calculation of ϵ.

Since there exists less interest in knowing the exact value of $B(\mathrm{E2})$ of a given nucleus than the general trend of $B(\mathrm{E2})$ as a function of different parameters, a uniform criterion for selecting the conversion coefficients to be used in their evaluation should be used. The one employed by Alder *et al.* seems to be most reliable: K-shell values taken from tables[34] which consider the effect of finite nuclear size; L-shell values calculated from these K-shell values with K/L ratios obtained from tables[35] which do not take that effect into account; and finally

$$\alpha_{\mathrm{t}} = \alpha_{\mathrm{K}} + 1.3\alpha_{\mathrm{L}}.$$

All values of $B(\mathrm{E2})$ depend indirectly on the excitation energy through the values of the conversion coefficients, which are extremely sensitive to it. Since the review article of Alder *et al.* was written, the first excited state energy has been determined more accurately in several nuclei and the corresponding changes in $(1+\alpha_{\mathrm{t}})$ may amount to a maximum of 10%. This will produce a maximum change in $B(\mathrm{E2})_{\tau}$ and $B(\mathrm{E2})_{\gamma}$ of 10% also, in $B(\mathrm{E2})_{e^-}$ of 6% and in $B(\mathrm{E2})_{\mathrm{p}'}$ no change at all will be introduced for this reason. In γ detection the quoted error in the yield determinations is seldom as low as 10% and in e^- detection it is never less than 25%; but in many cases the half-lives are known with errors smaller than 10%. As a rule, all conversion coefficients have been recalculated with the energy quoted in column two, instead of taking them from the table of Alder *et al.*

In the regions of Z and

$$k = \frac{0.511}{E(^{1}2+ - ^{1}0+)}$$

here considered, a slight simplification can be introduced in the calculation of α_{t}.

From Fig. 1 of the table of Sliv *et al.* it could be supposed that for E2 transitions with $k < 0.5$, the percentage difference between the point- and finite-nucleus K-shell values will be 1%. Our Fig. 4 corroborates this. For the few cases in which higher $k \lesssim 1$ values have to be used, the difference is somewhat greater but $\alpha \ll 1$ and, since in the calculation of $B(\mathrm{E2})$ it is $(1+\alpha_{\mathrm{t}})$ that comes in, the influence of that difference on the value of $B(\mathrm{E2})$ still remains meaningless. Consequently,

$$\alpha_{\mathrm{t}} = \alpha_{\mathrm{K}}\ (\text{finite nucleus}) + 1.3\,\alpha_{\mathrm{L}}\ (\text{point nucleus})$$

is used with α_{L} obtained by log–log interpolation§ from the tables of Rose *et al.*[35]

In columns IV to VII of Table 2 the $B(\mathrm{E2})$ values obtained are entered according to the methods of

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‡ For a detailed analysis of the methods and results of the Coulomb excitation process, see the review article of Alder *et al.* (Ref. 22).

§ The only exceptions to this method are the coefficients for Th^{232} and U^{238}. For the former we have used the mean of the nearly equal values calculated by Davis *et al.*[36] and McGowan *et al.*[37] and for the latter we have used the mean of the also nearly equal values calculated by Davis *et al.*[36] and Newton[38]; all these values have been obtained from privately circulated tables of Rose *et al.*

Table 2. Data Regarding the *F* Values[a]

Nucleus	$E(^{1}2+ - {}^{1}0+)$ (*Mev*)	αt	αL	*γ detection*		*e⁻ detection*	*p′ detection*		*γ measurement*		*B*(E2)	$\overline{F}$	*β*	*Q* (10^{-24} *cm*²)
${}_{60}\mathrm{Nd}^{146}$	0.455	0.01		0.25 ± 0.07	H2[b]						0.25 ± 0.07	11 ± 3	0.09	1.6
${}_{60}\mathrm{Nd}^{148}$	0.300	0.05		0.71 ± 0.21	H2						0.71 ± 0.14	31 ± 6	0.15	2.7
				0.71 ± 0.20	S1									
${}_{60}\mathrm{Nd}^{150}$	0.130	0.88	0.24	2.48 ± 0.75	H2	2.5 ± 2.5					2.19 ± 0.25	93 ± 11	0.25	4.9
				2.16 ± 0.27	S1									
${}_{62}\mathrm{Sm}^{148}$	0.562	< 0.001		0.50 ± 0.15	H2						0.50 ± 0.15	13 ± 4	0.12	2.3
${}_{62}\mathrm{Sm}^{150}$	0.337	0.04		0.98 ± 0.29	H2						0.98 ± 0.29	41 ± 13	0.16	3.2
${}_{62}\mathrm{Sm}^{152}$	0.1213	1.17	0.38	3.30 ± 0.99	H2	3.22 ± 0.80	$2.66^{(x)}$	E1	$3.62^{(x)}$	S5	3.42 ± 0.19	142 ± 9	0.30	6.2
							3.20 ± 0.36	S2	3.54 ± 0.25	S3				
${}_{62}\mathrm{Sm}^{154}$	0.083	4.82	2.24	4.67 ± 1.40	H2	7.53 ± 1.88	$3.60^{(x)}$	E1			3.77 ± 0.42	154 ± 17	0.32	6.5
							3.45 ± 0.45	S2						
${}_{64}\mathrm{Gd}^{154}$	0.1232	1.16	0.41	$5.00 \pm 1.50^{(x)}$	H2	3.9 ± 1.9	3.38 ± 0.50	R1	3.9 ± 0.3	S3	3.80 ± 0.25	155 ± 10	0.31	6.5
				4.27 ± 1.25	H3									
${}_{64}\mathrm{Gd}^{156}$	0.08897	3.91	1.85	$9.33 \pm 2.80^{(x)}$	H2	7.4 ± 1.8	$5.0^{(x)}$	E1	5.4 ± 0.3	N1	4.83 ± 0.19	194 ± 8	0.35	7.4
				4.32 ± 1.30	H3		4.50 ± 0.25	R1						
${}_{64}\mathrm{Gd}^{158}$	0.0791	6.21	3.24	$12.6 \pm 3.8^{(x)}$	H2	7.6 ± 1.9	$\sim 4^{(x)}$	E1			5.37 ± 0.24	212 ± 10	0.36	7.8
				4.54 ± 1.36	H3		5.36 ± 0.25	R1						
${}_{64}\mathrm{Gd}^{160}$	0.075	7.64	4.17	$15.5 \pm 4.7^{(x)}$	H2	7.1 ± 1.8	$\sim 4^{(x)}$	E1			5.69 ± 0.24	219 ± 9	0.37	8.0
				4.32 ± 1.30	H3		5.71 ± 0.25	R1						
${}_{66}\mathrm{Dy}^{160}$	0.08700	4.64							5.6 ± 0.6	M2	5.6 ± 0.6	217 ± 23	0.35	7.9
${}_{66}\mathrm{Dy}^{162}$	0.0808	6.26	3.46	12.0 ± 3.6	H4	6.7 ± 1.7	$\sim 6^{(x)}$	E1			7.7 ± 1.5	293 ± 57	0.41	9.4
${}_{66}\mathrm{Dy}^{164}$	0.0730	8.05	4.52	9.90 ± 3.00	H4	9.6 ± 2.4	$\sim 6^{(x)}$	E1			9.72 ± 1.87	365 ± 70	0.46	10.6
${}_{68}\mathrm{Er}^{164}$	0.0913	4.22							6.1 ± 2.1	B1	6.1 ± 2.1	229 ± 78	0.35	8.3
${}_{68}\mathrm{Er}^{166}$	0.08025	7.10							5.92 ± 0.18	G1	5.74 ± 0.18	212 ± 7	0.34	8.0
									6.3 ± 0.8	M1				
${}_{70}\mathrm{Yb}^{170}$	0.08425	6.48							5.66 ± 0.18	G1	5.63 ± 0.16	202 ± 6	0.32	7.9
									5.6 ± 0.4	B2				
${}_{72}\mathrm{Hf}^{176}$	0.08835	5.94		6.10 ± 1.80	H2				5.60 ± 0.40	M3	5.62 ± 0.40	193 ± 13	0.31	7.9
${}_{72}\mathrm{Hf}^{178}$	0.09319	4.76		7.38 ± 2.20	H2						7.38 ± 2.20	249 ± 74	0.35	9.1
${}_{72}\mathrm{Hf}^{180}$	0.09328	4.76		6.56 ± 1.97	H2				5.0 ± 0.4	S3	5.08 ± 0.39	168 ± 13	0.29	7.5
${}_{74}\mathrm{W}^{182}$	0.10009	4.01	2.43	4.50 ± 0.54	M4	5.6 ± 1.4			4.41 ± 0.35	S3	4.48 ± 0.30	146 ± 10	0.26	7.0
${}_{74}\mathrm{W}^{184}$	0.11113	2.64	1.50	4.46 ± 0.44	M4	4.1 ± 1.0			5.1 ± 1.7	S7	4.44 ± 0.39	143 ± 13	0.26	7.0
${}_{74}\mathrm{W}^{186}$	0.12252	1.84	0.97	3.74 ± 0.36	M4	4.1 ± 1.0					3.78 ± 0.34	120 ± 11	0.23	6.4
${}_{76}\mathrm{Os}^{186}$	0.13721	1.32							3.1 ± ?	S3	3.1 ± ?	99 ± ?	0.21	5.8
${}_{76}\mathrm{Os}^{188}$	0.15495	0.85		2.91 ± 0.31	M4				2.6 ± 0.6	S3	2.89 ± 0.27	90 ± 8	0.20	5.6
				3.53 ± 1.06	B3									
${}_{76}\mathrm{Os}^{190}$	0.187	0.42		2.52 ± 0.26	M4				$2.5^{(x)}$	S4	2.51 ± 0.25	78 ± 9	0.18	5.2
				2.47 ± 0.75	B3									
${}_{76}\mathrm{Os}^{192}$	0.20575	0.31		2.05 ± 0.21	M4						2.05 ± 0.19	62 ± 6	0.16	4.7
				2.10 ± 0.63	B3									
${}_{78}\mathrm{Pt}^{194}$	0.32907	0.08		1.77 ± 0.27	S6						1.77 ± 0.27	53 ± 9	0.15	4.3
${}_{78}\mathrm{Pt}^{196}$	0.3542	0.06		1.29 ± 0.19	S6						1.29 ± 0.19	38 ± 6	0.13	3.7
${}_{78}\mathrm{Pt}^{198}$	0.403	0.04		1.35 ± 0.21	S6						1.35 ± 0.21	39 ± 6	0.13	3.8

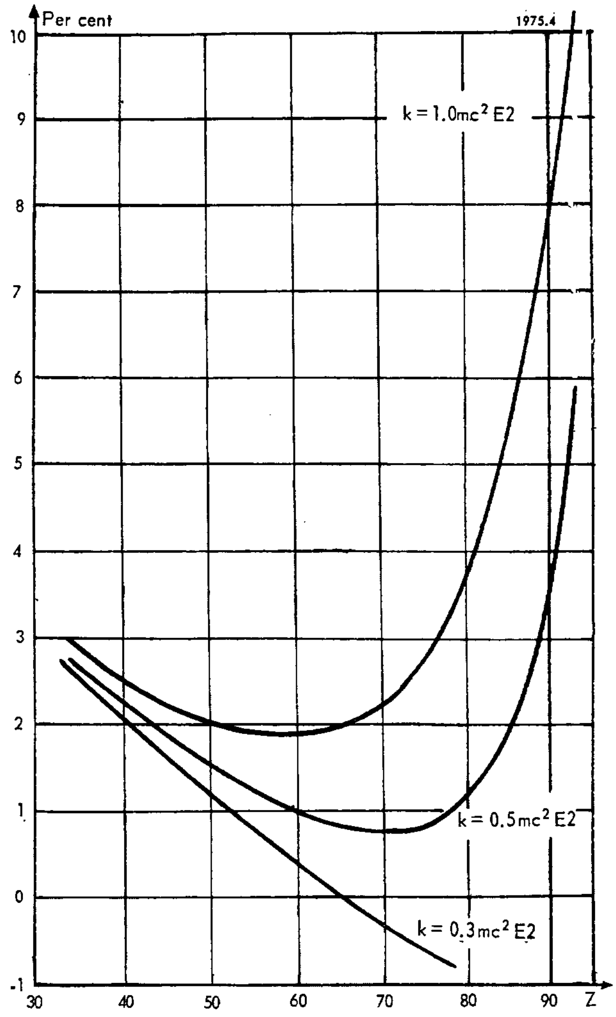

Figure 4. Percentage difference between the point- and finite-nuclear K-shell conversion coefficient values as a function of Z for different values of k

determination. References are quoted beside each value except for the e^- determinations which are all due to reference H1. Alder *et al.*[22] have pointed out the great discrepancy of the γ-ray yield measurements of Mark, McClelland, Goodman and Paulissen[39–44] as compared to those of other experimenters; data derived from these measurements are not reported in Table 2. The values given by Heydenburg *et al.* in Phys. Rev., *100*, 150 (1955), and later revised by them, are also excluded.

In connection with the errors quoted in these columns the following should be stated. No possible error in $(1+\alpha_t)$ and α_L obtained with the energy values in column II has been taken into account. The indirect dependence of the B(E2) on the energy through ϵ has already been considered. It remains to discuss the direct dependence of B(E2) deduced from Coulomb excitation experiments through Eq. (3.1). We have calculated an upper limit of 2% for the modifications in B(E2) due to the changes in B(E2) caused by the difference between the column II values and those used by the different experimenters. It has also been established that the error in the

[a] The values of $\bar{F}$ quoted in this table and in Fig. 5 for Er^{160}, Ra^{226}, Th^{232}, U^{238} and the Gd isotopes differ from those given in Chart I, because new data were available to us after Chart I went to press.

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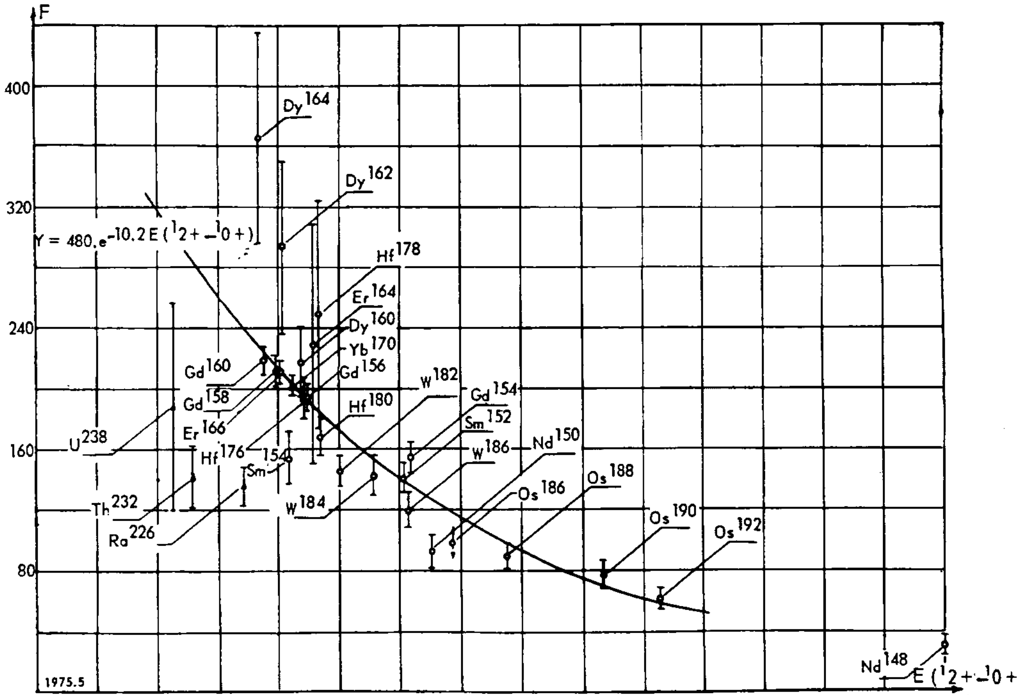

Figure 5. The F values as a function of the excitation energy E ($^12+—^10+$)

column II values introduces in all cases an error smaller than 2% in B(E2). Both effects are meaningless when compared with the yield determination error and are, therefore, not considered.

The $B(\mathrm{E2})_\tau$ values depend directly on $E(^12+—^10+)$ through its fifth power. Fortunately, in all cases in which we will take into account the τ measurements, the energy is accurately known and no error greater than 1.5% is introduced in $B(\mathrm{E2})_\tau$ on account of the uncertainty in energy. This error has also been disregarded.

As a conclusion, the error quoted in B(E2) arises from the errors reported in τ and in the yield determinations.

In column VIII the weighted average of the values of columns IV to VII is given. Before averaging we set the following rules:

(*a*) Those values in which the error is not known are excluded, except when for a given nucleus this value is the only existing one.

(*b*) Also excluded are those values which, within the errors, do not coincide with any other of the same nucleus. It comes out that only in Th^{232} and U^{238} do there appear such values. In each of them three measurements exist but no one pair coincides: we were then forced to average all three values.‖ Values of Huus *et al.* for some of the Gd isotopes are just on the limit of being excluded.

For Gd there exist ϵB(E2) values of Heydenburg and Temmer (privately circulated) and those published in reference H3. The latter were adopted. Excluded values are marked with an "x" in Table 2.

In column IX the F values calculated with $\overline{B(\mathrm{E2})}$ and $B(\mathrm{E2})_{\mathrm{s.p.}}$ are entered. These values have been represented as a function of the excitation energy in Fig. 5. In this figure a fact which is known to be general in the whole table is clearly seen; there are practically no even–even nuclei with $E(^12+—^10+)$ between 200 and 300 kev. All the nuclei with $E(^12+—^10+) \geqslant 300$ kev appear on Chart II, as seen from the rotational regions, on the other side of the heavy lines which indicate the center of the rotational–vibrational transition region.

In the region below 210 kev the points reported are rather scattered. In particular, the points corresponding to nuclei which belong to the region $158 \leqslant A \leqslant 182$ are very compressed on the energy scale and many have so large an error that it is completely impossible to give a law for their dependence regarding the excitation energy. Only as a guide for judging the general tendency we have drawn the curve $y = 480 \exp\{-10.2\, E(^12+—^10+)\}$, where the energy is expressed in Mev. Note the position of the points corresponding to nuclei of the heavy region.

The dispersion of the points does not diminish if, instead of F, $F_Z{}^2$ is represented as a function of $E(^12+—^10+)$.

As seen in Section 2, the values of B(E2) deformation parameters and quadrupole moments are mutually dependent. Values for the last two are given in columns X and XI respectively. Representation of these values as a function of N or A has been already given.[22, 45–47] In these papers the great discrepancy between the deformation derived from excitation

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‖ The values quoted for these nuclei in Table 2 and Chart I are different; this is because of a private communication of Heydenburg and Temmer, which modified our criterion for the application of rule (*b*) when Chart I was in press.

Table 3. Ratios between the Energies of the Higher States and the First State

Nucleus	$E(^{1}2+{-}^{1}0+)$ (*kev*)	$\frac{E(^{1}4+{-}^{1}0+)}{E(^{1}2+{-}^{1}0+)}$	$\frac{E(^{1}6+{-}^{1}0+)}{E(^{1}2+{-}^{1}0+)}$	$\frac{E(^{1}8+{-}^{1}0+)}{E(^{1}2+{-}^{1}0+)}$
${}_{62}\mathrm{Sm}^{152}$	121.3	3.00 – 0		
${}_{64}\mathrm{Gd}^{154}$	122.9	3.02 – 0	5.83 – 1 ?	
${}_{64}\mathrm{Gd}^{156}$	88.97	(3.24 – 0)	6.57 – 1 ??	
${}_{64}\mathrm{Gd}^{158}$	79.1	(3.30 – 2)		
${}_{66}\mathrm{Dy}^{160}$	87.00	3.27 – 1	6.69 – 1 ??	
${}_{66}\mathrm{Dy}^{162}$	80.8	(3.29 – 0)	6.80 – 2 ?	
${}_{66}\mathrm{Dy}^{164}$	73.0	3.40 – 15 ??	6.41 – 31 ??	
${}_{68}\mathrm{Er}^{166}$	80.5	3.29 – 0	(6.80 – 8)	
${}_{68}\mathrm{Er}^{168}$	79.9	(3.31 – 0)		
${}_{70}\mathrm{Yb}^{170}$	84.25	(3.30 – 0)		
${}_{70}\mathrm{Yb}^{172}$	78.70	(3.31 – 0)		
${}_{70}\mathrm{Yb}^{174}$	76.6	(3.32 – 2)	6.87 – 3	
${}_{72}\mathrm{Hf}^{176}$	88.35	(3.29 – 0)	(6.77 – 13)	
${}_{72}\mathrm{Hf}^{178}$	93.19	(3.29 – 1)	(6.94 – 3)	(11.50 – 9)
${}_{72}\mathrm{Hf}^{180}$	93.28	(3.31 – 1^{x})	(6.88 – 2^{x})	(11.64 – 6^{x})
${}_{74}\mathrm{W}^{182}$	100.09	3.29 – 0		
${}_{74}\mathrm{W}^{184}$	111.13	(3.16 – 2^{x})		
${}_{76}\mathrm{Os}^{186}$	137.21	(3.18 – 6	(6.39 – 12)	
${}_{76}\mathrm{Os}^{190}$	187	(2.92 – 2)	(5.59 – 6)	(8.88 – 8)
${}_{76}\mathrm{Os}^{192}$	205.75	(2.8 – ?)		
${}_{88}\mathrm{Ra}^{224}$	84.47	(3.00 – 1)		
${}_{88}\mathrm{Ra}^{226}$	67.76	3.10 – 1	(6.14 – 2)	
${}_{90}\mathrm{Th}^{226}$	72.13	(3.14 – 1)		
${}_{90}\mathrm{Th}^{228}$	57.48	(3.24 – 0)		
${}_{90}\mathrm{Th}^{230}$	52.8	(3.30 – 5)		
${}_{90}\mathrm{Th}^{232}$	51	3.24 – 17 ??		
${}_{92}\mathrm{U}^{232}$	47.2	(3.31 – 2^{x})	(6.80 – 4^{x})	
${}_{90}\mathrm{Th}^{234}$	48	3.33 ??		
${}_{92}\mathrm{U}^{234}$	43.50	(3.29 – 2)	(6.81 – 3)	(11.47 – 9^{x})
${}_{92}\mathrm{U}^{236}$	45.28	(3.26 – 3)	(6.92 – 12)	
${}_{92}\mathrm{U}^{238}$	44.7	3.13 – 58 ??		
${}_{94}\mathrm{Pu}^{238}$	44.07	(3.31 – 0)	(6.92 – 2)	(11.68 – 23)
${}_{94}\mathrm{Pu}^{240}$	42.87	(3.31 – 0)	(6.81 – 0)	
${}_{96}\mathrm{Cm}^{242}$	42.12	(3.30 – 1)	(6.86 – 12)	
${}_{96}\mathrm{Cm}^{246}$	42.9	(3.31 – 9)		
${}_{96}\mathrm{Cm}^{248}$	43.4	(3.30 – 6^{x})		
${}_{96}\mathrm{Cf}^{250}$	42	3.24 – 29 ??		

energy and half-life measurements is clearly pointed out and discussed. Alternatively, due to that direct dependence, it would be useless to represent deformations and moments as a function of $E(^{1}2+{-}^{1}0+)$.

We shall look briefly at the general agreement of the $B(E2)$ values given in Table 2 for a given nucleus; we shall consider only those values which were used in the averaging. At first glance it can be seen that—with the exception of Th^{232} and U^{238}—the agreement is good. It is particularly satisfactory that there is agreement between pairs of values (Sm^{152}, Gd^{154}, Er^{166}, Yb^{170}, W^{182}) with a relatively small error (7% to 15%); Gd^{156} is an exception to this.

The general situation is shown in the scheme given below. 3–6, placed at the intersection of column e⁻ and row p′ means that in 3 cases out of 6 in which it is possible to compare for a given nucleus e⁻ values and p′ values, these agree within their experimental errors.

Table 4

(158 ⩽ *A* ⩽ 182) + (*A* ⩽ 230) *Number of cases with known*					
$\frac{E(^{1}4+{-}^{1}0+)}{E(^{1}2+{-}^{1}0+)}$		$\frac{E(^{1}6+{-}^{1}0+)}{E(^{1}2+{-}^{1}0+)}$		$\frac{E(^{1}8+{-}^{1}0+)}{E(^{1}2+{-}^{1}0+)}$	
R_4	3	R_6	—	R_8	—
(R_4)	18	(R_6)	11	(R_8)	4
R_4?	—	R_6?	1	R_8 ?	—
R_4??	5	R_6??	2	R_8 ??	—
Total	26	Total	14	Total	4

	γ	e⁻	p′	τ
γ	5–11			
e⁻	12–12	—		
p′	6–6	3–6	—	
τ	8–8	4–4	2–3	2–2

Adding all the *x*–*y* pairs, one gets Σ = 42–52, i.e. disagreement in 20% of the comparable measurements. 13% is due to the quoted γ measurements on Th^{232} and U^{238}. 6% is due to the e⁻ values for Sm^{154}, Gd^{156} and Gd^{158}, all of which (as already mentioned for the Gd isotopes) also present great discrepancy with the γ and τ values for the same nuclei; the exclusion of the e⁻ values for the Gd isotopes in the averaging would introduce a lowering of the corresponding $\bar{F}$ values by less than 1% and that of the e⁻ value for Sm^{154} a lowering of nearly 5%. The remaining 1% is due to the said discrepancy between the well measured p′ and τ values for Gd^{156}.

Looking at the τ-row, one sees that the good agreement between τ and Coulomb excitation values stated by Alder *et al.*[22] still holds in the rotational regions.

Comparing the values (approximate or stated without error) of reference E1 in column VI of Table 2 with the other p′-measurements, there seems to be good agreement within the values obtained by this method.

$^14+$, $^16+$ and $^18+$ States

Excitation Energies and Characters

Theoretical predictions for these states are expressed by Eqs. (2.1) and (2.2).

From Eq. (2.1) are immediately obtained the ratios between the energies of the higher states and the first one:

$$\begin{aligned} R_4 &= \frac{E(^14+ - ^10+)}{E(^12+ - ^10+)} = 3.33, \\ R_6 &= \frac{E(^16+ - ^10+)}{E(^12+ - ^10+)} = 7, \qquad (3.4) \\ R_8 &= \frac{E(^18+ - ^10+)}{E(^12+ - ^10+)} = 12. \end{aligned}$$

We have quoted in Table 3 all the calculable energy ratios for the nuclei of the rotational and near rotational regions. In some cases, the ratio has been calculated making use of data obtained by different authors. The value which, separated from it by a hyphen, is placed beside a given R represents the error in its last figure or figures. Those errors estimated by us are marked with an x. The R values are quoted according to the following key:

—without parentheses if both states whose energy ratio is given by R are characterized without parentheses (whether or not this character is underlined);

—with parentheses if at least one of the intervening states has character stated with parentheses;

—with question mark if a transition has been measured which, if placed in cascade upon a member of the rotational band, gives rise to a level with a reasonable energy ratio, but no coincidence measurements involving that transition have been performed nor do conversion coefficient data exist to ascertain its E2 character;

—with double question mark: (i) if similar conditions to those just mentioned do exist but the γ ray which would de-excite the plausible rotational level also fits well between two higher known states of the scheme, or if the relative error in the energy ratio is greater than 4%; (ii) if due to some other particular situation the R value is uncertain by more than 4%.

This key, together with a knowledge of the certainty with which the character of the first excited state is known, immediately determines the certainty in the knowledge of the character of the higher states.

In all but one (Dy^{164}) of the cases belonging to the last two classes established for R, both levels involved in the ratios have been considered to be of rotational character by the experimenters who quote them.

Looking at column 3 of Table 3, it can be seen that R_4 increases steadily for the first nuclei; but once the value 3.30 is reached, for all the subsequent nuclei R_4 does not diverge from the theoretical value of 3.33 by more than 1.5%, within its experimental error; this situation is maintained until W^{184}.

In the region with $A \geqslant 224$ the first R_4 value which does not depart more than 1.5% from 3.33 belongs to Th^{232}; thence, until the end of the table, the 1.5% limit holds.

The boundaries set in the Introduction for the rotational regions arise from arbitrarily considering that all those nuclei with a R_4 value which does not differ by more than 1.5% from the theoretical value are of rotational character. The corresponding departure for both R_6 and R_8 is then, with the present data, 4.5%.

It should be pointed out that for Er^{168} an eventual decrease of 5 kev in $E(^12+ - ^10+)$ without change in energy of the transition $^14+ \rightarrow ^12+$ would raise its R_4 value from 3.31 to 3.47 (see Section 3). No R data exist for W^{180}.

In Table 4 some statistical data from both regions taken together are collected.

Description of Chart II

The significant data concerned with the ^{14}, ^{16}, and ^{18} states are given in Chart II of Paper I. In many respects it is similar to Chart I; the main differences are:

(*a*) Heavy unbroken lines are introduced to mark the center of the vibrational-rotational transition regions[1]; the most deformable, the "softest" nucleus[48, 49] of a transition zone is placed where two of these lines meet. The data related to the even–even nuclei which were used in Paper I to define the position of the lines are shown inside a circle. The nuclei farthest from the rotational regions for which the R values are quoted in Table 3 are just on these lines.

(*b*) For the e–e nuclei of the rotational and nearly-rotational regions R_4, R_6 and R_8 are entered in successive quadrants, when known.

(*c*) The e–e nuclei properties entered in all other regions are described in Paper I.

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