EXPERIMENTAL AND THEORETICAL PARTICLE PARAMETERS FOR L ELECTRONS. DIRECTIONAL CORRELATIONS IN Hg^{199}

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Tabulations of particle parameters for electron-gamma directional correlations exist until now only for K electrons in the so-called point nucleus approximation. For L electrons no tabulations exist. In many cases, however, when the transition energy is too low to permit conversion in the K shell the need is obvious for L shell particle parameters. Also, the study of penetration effects is very often complicated by multipole admixtures, which brings out a need for more observables than K electrons.

Recently, however, we have - in collaboration with Sliv, Band and Listengarten - calculated the particle parameters for K, L_I and L_{II} electrons. These parameters [1] are based on matrix elements [2] corrected for screening and finite nuclear size, using the surface current model to incorporate the dynamical effects. With those theoretical values we have now a new tool, L electron correlations, which will be of value for future work in nuclear spectroscopy.

As a first application we have measured the 50 keV ($L_I + L_{II}$) - 158 keV γ correlation in Hg^{199}. The isomeric decay of Hg^{199} has been investigated by several authors [3-6].

The L electrons of the 50 keV transition were analyzed in a magnetic lens spectrometer, which is described elsewhere [7, 8]. The electrons were selected at a resolution of 0.7%. The automatically movable gamma detector consists of a 3" × 3" NaI(Tl) Harshaw integral line assembly. The coincidence unit is a Cosmic Rad. Lab. model 801.

The L-electron spectrum of the 50 keV transition is superimposed on a sum of the three continuous β-branches of the Au^{199} decay. Thus there is a β - 158 γ contribution to the 50 L - 158 γ coincidence rate which has to be subtracted. This background was found to be isotropic. While the L_{III} line was completely resolved in all cases, the L_I and L_{II} lines were not perfectly resolved from each other due to source extension.

Three different measurements were made with different L_I to L_{II} intensity ratios in the electron channel. The ratio was obtained by fitting the line shape of a pure line to the measured spectrum using the known L subshell ratios.

The result is, after correcting for finite solid angles of the detectors:

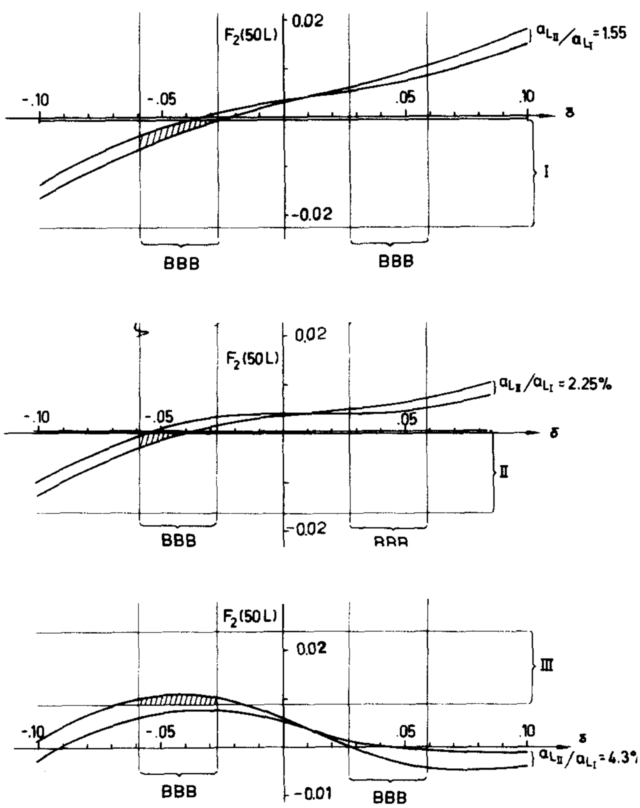

Fig. 1. F_2(50 L) plotted as a function of δ for different weights $a_{L_{II}}/a_{L_I}$ corresponding to our different runs. BBB refers to the δ of ref. 2. a, b, c corresponds to our different runs and the experimental result of F_2(50 L) is indicated by horizontal band in each case.

Table 1

Measurement	$a_{L_{II}}/a_{L_I}$	A_2
I	1.55 ± 0.25%	0.0062 ± 0.0060
II	2.25 ± 0.25%	0.0043 ± 0.0045
III	4.3 ± 0.4%	-0.0089 ± 0.0040

The correlation expression in this case is $W(\theta) = 1 + G_2A_2P_2(\cos\theta) + G_4A_4P_4(\cos\theta)$.

The A_4 term is $< 10^{-4}$ and can safely be neglected. Our measurement thus gives $A_2^{\text{measured}} = G_2 \cdot F_2(158\gamma) \cdot F_2(50\ \mathrm{L})$ where $F_2(50\ \mathrm{L})$ contains the information of interest. We thus obtain $F_2(50\ \mathrm{L}) = A_2^{\text{measured}}/G_2 \cdot F_2\ (158\gamma)$. As the 158 keV γ transition is of pure E2 character [4] we use the value of $F_2(158\gamma) = -0.535$ from Ferentz and Rosenzweig [9]. The factor G_2 contains contributions from the static quadrupole interaction and from the attenuation due to scattering of the electrons in the source. The effect of these two contributions is, however, very small (< 10%) and has been omitted since it will have a negligible effect on our results. The theoretical value $F_2(50\ \mathrm{L})$ is explicitly:

$$F_2(50\ \mathrm{L}) = (a_{\mathrm{L_I}} \cdot F_2(50\ \mathrm{L_I}) + a_{\mathrm{L_{II}}}F_2(50\ \mathrm{L_{II}}))/(a_{\mathrm{L_I}}+a_{\mathrm{L_{II}}}) ,$$

where

$$F_2(\mathrm{L}_i) = \frac{b_2(\mathrm{M1})_{\mathrm{L}_i} \cdot F_2(1,1) + 2p_{\mathrm{L}_i} \cdot b_2(\mathrm{M1+E2})_{\mathrm{L}_i} \cdot F_2(1,2) + p^2_{\mathrm{L}_i} \cdot b_2(\mathrm{E2})_{\mathrm{L}_i} \cdot F_2(2,2)}{1 + p^2_{\mathrm{L}_i}}$$

$$p_{\mathrm{L}_i} = \delta \cdot \surd(\alpha_2{}^{\mathrm{L}_i}/\beta_1{}^{\mathrm{L}_i}) , \qquad \delta = \langle \mathrm{E2}\rangle/\langle \mathrm{M1}\rangle$$

and a_{L_i} are the appropriate weighting factors.

For b_2 we use the recently calculated values [1] which for this case (Z = 80, k = 0.974) are given in table 2.

Table 2

	$\mathrm{L_I}$	$\mathrm{L_{II}}$
b_2(M1)	0.0046	0.246
b_2(M1 + E2)	-0.360	0.634
b_2(E2)	1.223	1.360

Conversion coefficients are taken from Sliv [10]. To analyse our result we have made a parametric representation of $F_2(50\ \mathrm{L})$ as a function of δ, where the parameter is the intensity ratio $\mathrm{L_{II}}/\mathrm{L_I}$. This is seen in fig. 1. The horizontal bands correspond to our different runs (I, II and III). Where these areas overlap the intensity ratio parameter limits, the δ-solution should be found. It is clear that all solutions are consistent. The result is

$$-0.068 < \delta < -0.038 .$$

For comparison the result of ref. 4 which is denoted BBB, is shown in fig. 1.

The 50 keV ($p\frac{3}{2} \rightarrow f\frac{5}{2}$) M1 transition in Hg^{199} is l-forbidden according to the shell model. The most recent contribution to the understanding of the non-zero rate of these kinds of transition is due to Sorensen [11] who has shown how to calculate the rate of these transitions in terms of the general pairing plus quadrupole force model. Moreover one should be able to predict the value of δ from this theory. It is therefore of great interest to determine experimentally the mixing amplitude for these transitions. L correlations can play here a very important role in the determination of mixing amplitudes.

Moreover, one of the most accurate methods to determine δ, namely the measurement of L subshell ratios has recently been shown to give inconsistent result [12]. This shows the need for more observables in order to resolve this inconsistency.

Concerning Hg^{199} the $\mathrm{L_{II}}$ correlation is very sensitive to δ. Due to experimental difficulties we are, however, not able to give results of this at present, but experiments are going on and results will be published later. It is of course obvious that for a general use of L correlations especially at higher energies one should aim at higher resolution in the electron channel. Experiments are going on in order to make use of high resolution spectrometers for this type of measurements.

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CONFIGURATION MIXING IN THE CONTINUUM AND NUCLEAR REACTIONS. - I

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In order to simplify the actual numerical calculations, the application of the particle-hole model has so far been mostly limited to bound configurational states. This has, for the description of positive energy nuclear states, obvious shortcomings associated with the introduction of a potential well having a totally wrong behaviour outside the nucleus. Several methods have already been proposed for the extension of the shell model to the continuum [1]. The method proposed here for the treatment of unbound configurations seems to be the most straightforward extension of the diagonalization method used for discrete configurations. It is free from the difficulties associated with antisymmetrization, boundary conditions at the nuclear surface or non-Hermitian secular problems encountered by other formulations. A similar method was applied to atomic problems in the case of a single channel by Fano [2]. In a subsequent paper Fano and Prats [3] suggested a formal generalization. We wish to show here the practical feasibility of the method by applying it to the scattering of a particle by a square well potential.

As usual, we split the nuclear Hamiltonian into an independent particle part H_0 and a residual interaction V. The problem of determining nuclear reaction amplitudes will be essentially reduced to the diagonalization of the Hamiltonian, using as a basis the Slater determinants, which are eigenfunctions of H_0. The basic single particle wave functions in the discrete spectrum are defined as usual. In the continuum, we choose the *standing wave* solutions orthonormalized to a δ-function of the energies. The behaviour of these functions for large r is:

$$\varphi(r) \sim \sqrt{\frac{2}{\pi k}}\,\frac{1}{r}\,Y_{lm}(\hat{r})\,\sin(kr+\delta)\;. \qquad (1)$$

Such a basis avoids the introduction of complex numbers in the numerical calculation, as far as possible.

A particularly simple and complete treatment of nuclear reactions can be given when the unperturbed configurations taken into account are only those in which one single nucleon at most is in a continuum state. Any wave function of the system may then be expanded as:

$$\Psi = \sum_{\alpha} a_{\alpha}\,|\alpha\rangle + \sum_{\beta}\int \mathrm{d}\epsilon\; a_{\beta}(\epsilon)\,|\epsilon,\beta\rangle\;, \qquad (2)$$

where $|\alpha\rangle$ denotes the Slater determinants with all $A+1$ nucleons in discrete states, and $|\epsilon,\beta\rangle$ the determinants with one nucleon of energy ϵ in the continuum. The Schrödinger equation may then be written

$$(H_{A+1}-E)a_{\alpha} + \sum_{\beta'}\int \mathrm{d}\epsilon'\,(\alpha\,|\,V|\,\epsilon'\beta')\,a_{\beta'}(\epsilon') = 0\;, \qquad (3a)$$