CAB/1974/2

Structure of the ρ-Singularity in a Multicluster Model

A. García, L. Masperi and N. Parga

Centro Atómico Bariloche and Instituto de Física Balseiro (Comisión Nacional de Energía Atómica and Universidad Nacional de Cuyo), Bariloche, Argentina.-

Summary.- We construct a multicluster model for πN_{CEx} amplitude in the forward direction which, in contrast to conventional ρ-Regge models, reproduces within error bars the data for $\sigma^{tot}_{\pi^- p} - \sigma^{tot}_{\pi^+ p}$ and $d\sigma_{CEx}/dt\big|_{t=0}$ for $3 \leq p_{lab} \leq 60$ GeV/c and fairly agrees with preliminary Batavia results. The model for the imaginary part shows an oscillation around a traditional ρ-pole value due to the onset of one large rapidity gap intermediate states. We propose that this oscillation, which fits well into Serpukhov enhancement, is the counterpart of similar threshold effects in $\sigma^{tot}_{\pi N}$ and σ^{tot}_{NN} arising at much higher energies. Absorptive corrections between clusters are considered to add consistency to the model.

Septiembre 1974
Bariloche - Argentina

1.- Introduction.

A very remarkable feature of the highest energy experimental πN data is that the charge exchange amplitude still seems to be qualitatively described by a linear ρ-Regge pole. However a closer analysis of the data gives quantitative discrepancies which suggest a more complicated structure of the ρ-singularity even at $t=0$.

In the first place, regarding the difference of total cross-sections $\Delta\sigma = \sigma^{tot}_{\pi^- p} - \sigma^{tot}_{\pi^+ p}$ a fluctuation of the effective ρ-trajectory intercept around its traditional low-energy value $\alpha(0) = .55$ is apparently necessary. In fact the Brookhaven and Serpukhov data need an increased value around[1] .67 whereas the preliminary Batavia results[2] indicate a decrease towards the original .55. On the other hand, the $d\sigma_{CEx}/dt\big|_{t=0}$ behaviour which is compatible[1], [3] with $\alpha(0) = .55$ up to Serpukhov energies seems to decrease even more with the preliminary Batavia data[2]. Clearly all this information is not understandable in terms of a pure pole amplitude. This is also transparently seen in the decreasing energy behaviour of $R = \mathrm{Re}\, F^{(-)}/\mathrm{Im}\, F^{(-)}\big|_{t=0}$, obtained using the relation $32\,\pi\, d\sigma_{CEx}/dt\big|_{t=0} = (\Delta\sigma)^2\,(1 + R^2)$, which falls below one for Serpukhov and Batavia energies[2], [4].

The purpose of this work is to show that a model based on cluster production is capable of fitting all the available high-energy experimental data within their error bars, in constrast to conventional combinations of poles and cuts. This is possible because the imaginary part of the amplitude given by the cluster model can be thought of as that of an effective pole whose intercept oscillates around a mean value ~ .55, being compatible with .67 at Brookhaven-Serpukhov energy and decreasing below .50 beyond the range of the preliminary Batavia data. Eventually the model predicts an extremely slow increase, starting at formidably high energy, towards its mean value.

Consequentely the real part does not follow the imaginary part behaviour since the above mentioned steep decrease of the latter gives, through dispersion relations, a comparatively smaller value for the former in the high-energy experimental range, i.e. R not greater than one above 20 GeV/c.

The above mentioned oscillation in the imaginary part of the amplitude is caused by the threshold effect for production of one large rapidity gap intermediate states which turns out to be the relevant one in the available energy range.

Next Section is devoted to a discussion of the dispersion relation which will be used to calculate the real part of the amplitude from different inputs for the imaginary one and to a description of the handling of data. In Sect. 3 it is shown why conventional combinations of Regge poles and cuts fail to explain data. In Sect. 4 the multicluster model is proposed and results are presented. In Sect. 5 corrections due to rescattering between clusters are used to improve the model.

2.- The choice of the dispersion relation.

For the different models to be discussed later, the procedure is as follows. A fit to $\Delta\sigma$ is performed in order to fix the free parameters of the models. These are used as input for the imaginary part of the amplitude beyond the experimental region, to calculate its real part through a dispersion relation (d. r.). In this way $d\sigma_{CEx}/dt\big|_{t=0}$ and R can be evaluated. Latest Batavia data are not included in the fit due to the preliminar character of their errors. We have therefore chosen to compare them with the predictions of the models in the corresponding range.

The most commonly used d.r. are the unsubstracted [(*)]

$$\mathrm{Re}\,F^{(-)}(\omega_L) = 16\,\pi\,\frac{m\,\omega_L\,f^2}{\omega_L^2 - \omega_B^2} + \frac{2}{\pi}\,\omega_L \fint_{\mu}^{\infty} d\,\omega\,\frac{\mathrm{Im}\,F^{(-)}(\omega)}{\omega^2 - \omega_L^2} \quad , \tag{1}$$

where $\omega_B = -\mu^2/(2\,m)$, with m and μ the nucleon and pion mass respectively, and the twice-substracted one

$$\mathrm{Re}\,F^{(-)}(\omega_L) = \left(1 + \frac{\mu}{m}\right) \frac{8\,\pi\,m}{3\,\mu}\,(a_1 - a_3)\,\omega_L - \frac{16\,\pi\,f^2\;m\,\omega_L\,(\omega_L^2 - \mu^2)}{(\mu^2 - \omega_B^2)\,(\omega_L^2 - \omega_B^2)}$$

$$+ \frac{2}{\pi}\,(\omega_L^2 - \mu^2)\;\omega_L \fint_{\mu}^{\infty} \frac{d\,\omega}{\omega^2 - \mu^2}\,\frac{\mathrm{Im}\,F^{(-)}(\omega)}{\omega^2 - \omega_L^2} \quad . \tag{2}$$

As noted in Ref. (5), the latter is not appropriate to predict the high-energy real part because of the increasing influence of low-energy experimental data errors. For instance these authors obtain a vanishing real part for laboratory energy ω_L between 5 and 20 GeV and negative thereafter using a value of s-wave scattering length $(a_1 - a_3)$ derived from the same d. r.[(6)]. Though a relatively small real part in the above range could help us to solve our puzzle, this extreme result is in obvious contradiction with a rough ρ-pole dominance.To save this deficiency, sizeable Coulomb corrections were introduced and important variations in the right sense were obtained[(7)]. However, it is well known[(8)] that the subtracted d. r. is also very unstable against small variations of the low-energy parameters due to severe cancellations among its three terms[(**)], already above 1 GeV. Moreover in eq. (2) the high-energy model is strongly suppressed, the high-energy real part being mainly determined by the low-energy input, so that its use is obviously inadequate for our purposes.

On the other hand, the unsubtracted d. r. is suitable for studying the influence of the high-energy model since it picks up more balanced contribution from low, medium and asymptotic energy regions with no cancellations. E. g., variation of f^2 and inclusion of Coulomb corrected data[5] up to 300 MeV is seen not to influence the results in the region $\geq$ 5 GeV. In what follows we will therefore use the d.r. eq. (1) for calculating real parts.

3.- Conventional models.

As discussed in Sect. 1, we shall describe quantitatively the impossibility of reproducing within error bars the experimental data for the total cross-sections difference[10] and for the extrapolation[4],[11] to $t = 0$ of $d\sigma_{CEx}/dt$, with a simple ρ-model.

If one takes a ρ-intercept .673 which gives the best fit to $\Delta\sigma$ data in the Brookhaven-Serpukhov region, using eq.(1) an enormous $d\sigma_{CEx}/dt\big|_{t=0}$ value turns out above 5 GeV/c as it could be expected (Fig. 1).

If we now make a fit to $\Delta\sigma$ for $p_{lab} \gtrsim 3.6$ GeV/c, where smooth behaviour starts, we obtain instead $\alpha(0) = .603$. This fit still goes through the error bars at Serpukhov energies but predictions for $d\sigma_{CEx}/dt\big|_{t=0}$ and R are clearly above them already above 5 GeV/c.

In order to obtain a good fit to the differential cross-section up to 50 GeV/c a still lower value $\alpha(0) \approx .55$ is needed. However the prediction for $\Delta\sigma$ falls below error bars at Serpukhov energies and the values of R stay above them. Incidentally, this intercept agrees with preliminary Batavia data on total cross-sections difference but the prediction for $d\sigma_{CEx}/dt\big|_{t=0}$ in this region is well above the corresponding experimental data. This will be shown in Fig. 5. Table I shows the Regge parameters for the three possibilities discussed above.

The usual refinements to ρ-pole model do not improve the situation. In fact the inclusion of a second ρ'-pole is useful for a $\Delta\sigma$ fit provided the ρ-intercept is chosen high enough to fit the high-energy region where the ρ' contribution has already exhausted but in this way the corresponding $d\sigma/dt|_{t=0}$ turns out to be obviously bad. If one fixes the ρ-intercept at a low value compatible with $d\sigma/dt$ data, the two-pole fit to $\Delta\sigma$ rejects the ρ'.

The absorptive ρ-P cut correction, which gives a negative contribution $s^{\alpha(0) - 1}/\ln s$ does not improve significantly the fit to total cross-sections difference since its influence is to increase the effective intercept more in the low- than in the high-energy range, just opposite to the mechanism needed to understand the Brookhaven-Serpukhov enhancement. Therefore, though in a limited range of energy it is possible to obtain[12] an effective intercept greater for $\Delta\sigma$ than for $d\sigma_{CEx}/dt|_{t=0}$ due to absorption stronger in the imaginary than in the real part of the non-flip amplitude, agreement in a larger energy region needs the assumption that, e.g., all Serpukhov $\Delta\sigma$ data are systematically high[13]. Either, if one insists in a correction like[14] $- s^{\alpha_c(0) - 1}/(\ln s)^{\lambda}$, we obtain the outrageous value $\lambda \simeq 14$ for a fit in the range 10-60 GeV/c with $\alpha_c(0) = .46$; moroever it disagrees strongly with data below 10 GeV/c and the prediction for $d\sigma_{CEx}/dt|_{t=0}$ is bad in the Batavia range.It seems more reasonable to think that the absorbed one-pole model is not valid beyond a certain energy where diffractive dissociation effects become relevant[15].

This analysis shows that there are no conventional ρ-models capable of explaining πN_{CEx} amplitude problems[***], unless unknown systematic errors vitiate the above arguments. We choose to consider that the contradictions emerging from experimental data actually exist, even though we alleviate them by taking the largest error available data in the Serpukhov region.

4.- A multicluster model.

We now propose a model based on the speculation that the relatively slow decrease of $\Delta\sigma$ in the Brookhaven-Serpukhov region is a physical phenomenon of the same nature as the increase of pp total cross-section at NAL-ISR energies.

Following Pokorski and Van Hove[18] we take the view that uncorrelated cluster production is a general feature of high-energy hadronic reactions. If, moreover, the production of clusters of particles is assumed to follow a multiperipheral scheme, one ends up with a model of the type of that of Frazer, Snider and Tan[19] for the vacuum exchange. In this model, asymptotia is reached far beyond the presently accessible energy range and the rise of pp cross-section is explained through threshold effects[†]. The most important of them comes from the threshold for production of one high-mass cluster well separated in rapidity from the leading particle. This contribution adds to that of the bare pomeron which corresponds to final states with no rapidity gaps greater than a characteristic value $\Delta \simeq 2 - 3$. The output Regge singularities, which govern the true asymptotic regime, appear through the contributions of final states with arbitrary number of clusters. Since clusters are diffractively produced, this model admits an expansion in powers of the small triple pomeron coupling and, up to ISR energies, only a few terms are significant.

For $\pi^- p \to \pi^0 n$, we start extending these ideas by classifying every intermediate state according to the number of rapidity gaps greater than Δ. Whereas in the pp case the links between clusters are assumed to the true pomerons, in πN_{CEx} one of them necessarily is a true ρ-singularity and the other a true P as the most relevant exchanges (Fig. 2). Here, the no-gap contribution gives the bare ρ-pole while those with one or more gaps switch on as soon as the energy grows above the corresponding thresholds, to build the structure of the true ρ-singularity.

These ideas can also be interpreted in the frame of Harari's two components view of multihadron production amplitudes[21]: a multiperipheral M, and a diffractive one D. In this scheme the ρ-contribution to the elastic πN amplitude, through the optical theorem, arises from: i) terms contained in "M^2" corresponding to exchange of ρ and P' quantum numbers between neighbouring rungs, which prefer a uniform rapidity distribution and should therefore mainly go to the no-gap contribution, i.e. the bare ρ; ii)those terms contained in "D^2" with pomeron exchanges in different places in each chain, which go to the two or more gap contributions since the effect of each pomeron is likely to be noticeable only when the corresponding subenergy is large; iii) those interference terms "M D" giving rise to exchange of ρ quantum numbers, which should be assigned to the one or more gap contributions.

There is another important difference between pp and πN_{CEx} models. For pp a distinction is made among low- and high-mass clusters. The first ones, mainly dominated by resonances, should be dual to P'. In the high-mass ones the role of P' is taken over by the pomeron in its bare version[19] since by definition no rapidity gap is allowed in the subenergy corresponding to the cluster. In πN_{CEx} the low-mass clusters must obviously be assigned to the bare ρ while, since there is no counterpart for the pomeron, there are two alternatives for the high-mass ones: either a) they contribute negligibly, or b) continue to build the bare ρ-singularity. In both cases, Fig. 3 should be the dual equivalent to Fig. 2; nevertheless the energy behaviour of the model depends crucially on the alternative chosen.

According to the simplified phase-space calculation of DeTar[22]*, the contribution of the N-clusters diagram to $\Delta\sigma$ is of the form

$$(\Delta\sigma)_N \propto s^{\alpha_o - 1} \int_o dx_1 \int_\Delta dx_2 \int_o dx_3 \cdots \int_\Delta dx_{2N-2} \cdot \int_o dx_{2N-1} \;\delta(Y - \Sigma x_i), \tag{3}$$

where Y is the total rapidity and x_i are the sizes in rapidity of clusters or gaps. The rapidity of particles within clusters have been integrated over - the slope of all exchanges are neglected - giving the bare ρ-pole factor $s^{\alpha_o - 1}$. In alternative a) the integrations extend over all the available phase-space, while for alternative b) the upper limit for the size of a cluster is a small number L.

If we consider that internal charged or neutral clusters are equally probable, there would be 2^{N-1} terms like eq. (3) for the N-clusters contribution. On the other hand it is an experimental fact for pp that the internal clusters are mainly neutral[18]. Assuming that the same thing occurs for πN_{CEx} and keeping only neutral internal clusters - it is clear that the external or leading clusters may be charged or neutral with the same probability - we only have two terms for each fixed N. In this case we obtain after integration

$$\Delta\sigma = 2\,\beta\, s^{\alpha_o - 1} \Big\{ 1 + 2 \sum_{N=2}^{\infty} G^{2(N-1)} \frac{\big(Y - (N-1)\,\Delta\big)^{2(N-1)}}{\big(2\,(N-1)\big)!} \,\theta(Y-(N-1)\,\Delta) \Big\}, \tag{4.a}$$

for alternative a) or

$$\Delta\sigma = 2\,\beta\, s^{\alpha_o - 1} \Big\{ 1 + 2 \sum_{N=2}^{\infty} G^{2(N-1)} \frac{\big(Y - (N-1)\Delta\big)^{N-2}}{(N-2)\,!} L^N \,\theta(Y-(N-1)\,\Delta) \Big\}, \tag{4.b}$$

for alternative b), where β and α_o are the bare ρ-pole residue and intercept and G is the triple $\rho P \rho_o$ coupling constant which is presumably small due to its relation with the internal pomeron coupling. The step functions θ determine the threshold in rapidity at which every N-clusters diagram sets in.

An important consequence of the smallness of the pion mass compared to the nucleon one is that the first threshold effects will appear at relatively low energy provided the rapidity gap Δ is universal (this should be true for internal clusters but probably an approximation for near to leading ones). Thus, while in pp scattering they show up at NAL energies, the corresponding effects for πN should be noticeable at Serpukhov (††) or even below according to the above choice.

The results for the parameters of the models given by eq. (4.a,b) are summarized in Table II. The set identified as Solution (a_2) is the one capable of reproducing the available experimental data within their errors as will immediately be discussed.

For model eq. (4.b) one could in fact expect its inability to solve the difficulties of the ρ-singularity because the high threshold $\sim 2\Delta$ for the first correction term prevents the imaginary part to decrease rapidly enough after the Serpukhov region. This is confirmed by Solution (b) which is a most favourable (steepest) fit within errors in the 3.6 - 60 GeV/c region and gives a Δ value in agreements with the assumed universality(8). The real part obtained from this parametrization through d.r. is too large and consequently at 50 GeV/c,e.g., $d\sigma/dt\big|_{t=0}$ = .073 mb/GeV^2 and R = 1.31 clearly above error bars.

In contrast, model eq. (4.a) has better chances of giving good results because its lower threshold for corrections allows a decrease of $\Delta\sigma$ after an enhancement in the Brookhaven-Serpukhov region.However a simple best fit to $\Delta\sigma$, Solution (a_1), is not yet satisfactory: though it provides a better χ^2-value than the best Regge pole fit $\alpha(0) = .603$, it does not improve the predictions for $d\sigma_{CEx}/dt|_{t=0}$ above 5 GeV/c. The ratio R turns out to be still considerably greater than the expected values even though it shows a decrease with energy due to the mechanism already discussed.

At present it seems however more sensible not to look for a best fit to one single piece of information but rather to find out a model which can predict all the available experimental data within the quoted error bars. We have already seen that there are no conventional Regge models capable of doing this. Note that the inclusion of Denisov et al. (1973) data, which essentially reduces the errors of those quoted in the Compilation[10], emphasizes the discrepancy with a simple pole model. As said before we chose the largest errors in order to put in evidence the necessity of a more complicated structure of the ρ-singularity. In this spirit we see that Solution (a_2) for the model eq. (4a), which provides the steepest fit to $\Delta\sigma$ up to 60 GeV/c inside error bars, gives very good predictions for $d\sigma/dt|_{t=0}$ and R in the same range (Fig. 4). We point out that the extreme Solution (a_2) slightly misses from below four data point bars of the Denisov et al. (1973) set (not shown in the figure). Nevertheless, a pretty small change of the parameters of this solution allows to find a new one with increased values of both $\Delta\sigma$ and $d\sigma_{CEx}/dt|_{t=0}$ which passes through them all.

Most remarkable is the fact that Solution (a_2) is also the only one which predicts values in agreement with preliminary Batavia data (Fig. 5). The oscillation of the model around hundred GeV/c fits well into the data and explains the quick decrease of Batavia values for $\Delta\sigma$. This oscillation is caused by the combined effect of a strong dominance of the first correction term $\sim (\ln s)^2$ and the low α_o value which takes over immediately after

Serpukhov energies. The same effect allows, through d.r., to obtain values of $d\sigma/dt|_{t=0}$ close to the experimental Batavia data though complete agreement is impossible since there is a clear mismatch with the Serpukhov trend probably caused by uncertainties in the extrapolation to $t = 0$. We add finally that the only new value for R which can be evaluated at 100 GeV/c is also <1, though with a large error.

It is interesting to point out that the rise of the pomeron contribution to $\sigma^{tot}_{\pi^- p} + \sigma^{tot}_{\pi^+ p}$ due to diffractive dissociation corrections should manifest itself at an energy μ/m times lower than for pp; however this effects is quite surely masked by the decreasing contribution of P' to πN. It has moreover been noted that corrections to bare pomeron need at least one high-mass cluster so that the corresponding threshold in rapidity is twice as big as that for ρ; consequently, whereas for the latter the first threshold effect is already decreasing in the Batavia range, this is clearly not the case for the pomeron.

The asymptotic behaviour of model eq. (4.a) is not as simple as a Regge pole. In fact this is true with a renormalized intercept $\alpha(0) = \alpha_o + G$ if one neglects the threshold factors in the Y power expansion. But in general complex poles appear in addition and the oscillation in $\Delta\sigma$ could be a manifestation of them. In any case the validity of eq. (4.a) at ultrahigh energy is very dubious since the effects of considering the slopes of the Regge exchanges probably start to be relevant at ISR energies[19] changing the behaviour in powers of Y to powers on ℓn Y. This change would be favourable in our case since, while in the experimental range the expression of $\Delta\sigma$ in powers of Y is a good approximation, the relative decrease at higher energies will diminish the real part.

Fortunately the ultimate asymptotic behaviour is not too important at present. The G^2 value turns out to be small (‡) and therefore suitable for a perturbative expansion for all accessible energy. The only relevant correction to the bare pole in this region comes from the first term ~ $(\ell n\ s)^2$ in eq. (4.a); it amounts to ~ 50 % of the total cross-sections difference at the position of its maximum ($p_{lab} \simeq$ 30-50 GeV/c) and to 80 % at 1000 GeV/c where the second correction ~ $(\ell n\ s)^4$ reaches its maximum with a 10 % contribution[(#)]. This property determines that,beyond the required enhancement in the Brookhaven-Serpukhov region due to the first threshold effect, the behaviour of $\Delta\sigma$ corresponds to a slowly decreasing $\alpha(0)$ much smaller than its ultimate asymptotic value $\alpha_o + G$.It also follows that the convergence of the exact sum to this limit is extremely slow; in fact it turns out for our parametrization that only after $Y = 500$, where 170 terms must be included in the sum, $\alpha(0)$ starts to increase.

Note that it is likely that another threshold effect **is responsible**, together with diffractive dissociation, for the increase of pp total cross section: the $N\bar{N}$ production[(25)]. To be produced a nucleon pair needs at least a separation in rapidity from each leading particle Δ_p which may be estimated to be of the order of Δ . Therefore in the correction to the pomeron, both threshold effects of one large-mass diffractive dissociation and one-pair creation start at a total rapidity $Y \simeq 2\ \Delta$. In contrast, for πN_{CEx} the correction to the no-gap term due to one-pair creation is of the form

$$(\Delta\sigma)_{1p} = 2\ \beta\ s^{\alpha_o - 1}\ G_p^2\ (Y - 2\ \Delta_p)\ \Theta\ (Y - 2\ \Delta_p), \tag{5}$$

which must be added to eq. (4.a) and where G_p^2 is related to the coupling strength to produce one pair. It is clear that this correction will appear now much beyond the diffractive dissociation threshold Δ and will consequently give very slight improvement to the results already discussed.

To close this Section, we remark that there exists a model, Solution (a_2), which solves the ρ-contradictions within experimental errors. However it is clear that we have chosen an extreme solution favourable for our purposes. This is evident in several aspects: one in the sense that the fit to $\Delta\sigma$ is the steepest possible; another concerns the charged internal clusters which have been completely left out (the inclusion of them in any sizeable amount worsens the results but only slightly since they do not modify the first correction term); and finally in the smallness of the bare intercept α_o, an unesthetical but irrelevant feature since the α_o-behaviour is nowhere seen in the smooth $\Delta\sigma$ region.

These extreme characteristics may however be attenuated if the slopes of Reggeon exchanges are considered and/or absorptive corrections are taken into account. We shall only discuss the latter.

5.- Absorptive corrections.

We have up to now considered a model with t-channel iterations. Probably s-channel iterations are also necessary to obtain a correct solution for the amplitude[(21),(26)]. It must be clear that t-channel iterations are essential since rescattering corrections alone lead to an absorbed ρ-pole model which we have already seen cannot solve the troubles.

We shall see that absorptive corrections due to the s-channel iteration of pomeron exchange between clusters is able to decrease the effective intercept, in $\Delta\sigma$, beyond Serpukhov compared to that of model eq. (4.a). This feature is in the right trend to reproduce data even starting from a more realistic bare intercept α_o.

Our analysis is based on a similar treatment of absorptive corrections to diffractive dissociation contributions to total cross-sections by Ciafaloni and Marchesini[(27)].

We limit ourselves to corrections to two-cluster graphs, as shown in Fig. 6. Corrections to higher order graphs are likely to be unnecessary since, as we have seen, truncation of the model after the term $\sim (\ell n\, s)^2$ does not change things appreciably.

We consider all possible pomeron exchanges between pairs of intermediate particles in different clusters and assume that the distribution of particles can be approximated by a continuous one inside each cluster so that the exponentiation procedure can be used. In this way the correction factor to be inserted into the integral of eq. (3) is, neglecting correlations,

$$A = \prod_{i=1}^{n_L} \prod_{j=1}^{n_R} (1 - T_{ij}) \approx \exp \left\{ - \int d^3 v_L \int d^3 v_R \right.$$

$$\left. \cdot\ \eta\,(v_L)\ \ \eta\,(v_R)\ T\,(v_R - v_L) \right\} , \qquad (6)$$

where the impact parameter-rapidity differential is $d^3 v = \frac{d^2 b}{\pi} dy$ and n_L (n_R) is the number of particles in the left (right) cluster. The density of particles in a multiperipheral chain building the bare ρ-pole is

$$\eta\,(v) = \frac{g_1\, g_2}{\alpha_o'} \cdot \frac{\exp \{ - b^2/(4\alpha_o'\ y) \}}{y} , \qquad (7)$$

where g_1 (g_2) is the coupling in the lower (upper) link of the chain and α_o' is the slope of the output bare ρ-trajectory. The pomeron propagator between particles v_L and v_R is

$$T\,(v_R - v_L) = \frac{\gamma_P^2}{\pi \alpha_P'}\ \frac{\exp \{ - (\vec{b}_R - \vec{b}_L)^2 \ / \ \left[4\ \alpha_P'\ (y_R - y_L) \right] \}}{y_R - y_L} , \qquad (8)$$

where α_P' is the slope of the pomeron and γ_P its coupling to a particle.

Integrating out the impact parameter variables, we obtain

$$A = \exp \left\{- \frac{(g_1 g_2 \gamma_P)^2}{\pi} \int_0^{y_1} dy_L \int_{y_2}^{Y} dy_R \frac{1}{\alpha'_P (y_R - y_L) + \alpha'_o (y_R + y_L)} \right\}. \quad (9)$$

To calculate $\Delta\sigma$, two extreme alternatives are considered

1) $\alpha'_P \ll \alpha'_o$:

$$\Delta\sigma = 2 \beta s^{\alpha_o - 1} \left\{ 1 + 2 \cdot G^2 \int_0^{Y-\Delta} dx_1 \int_0^{Y-\Lambda-x_1} dx_3 \left(\frac{Y}{Y + x_1}\right)^{\varepsilon Y} \right.$$

$$\left. \cdot \left(\frac{Y - x_3 + x_1}{Y - x_3}\right)^{\varepsilon(Y-x_3)} \left(\frac{Y - x_3 + x_1}{Y + x_1} \right)^{\varepsilon x_1} \right\} ; \quad (10)$$

2) $\alpha'_P \simeq \alpha'_o$:

$$\Delta\sigma = 2 \beta s^{\alpha_o - 1} \left\{ 1 + 2 G^2 \int_0^{Y-\Delta} \frac{dx_1}{Y^{\frac{\varepsilon}{2} x_1}} \frac{Y^{\frac{\varepsilon}{2} x_1 + 1} - (\Lambda + x_1)^{\frac{\varepsilon}{2} x_1 + 1}}{\frac{\varepsilon}{2} x_1 + 1} \right\}. \quad (11)$$

For $0 < \varepsilon = (g_1 g_2 \gamma_P)^2 / (\pi \alpha'_o) \leq 1$ a negative correction is obtained which is more important the greater the energy is. This new parameter is related to the triple $P \rho_o \rho_o$ coupling squared which is not necessarily as small as G^2 since in the former the pomeron is elastically coupled whereas in the latter the internal inelastic coupling appears.

The more realistic approximation $\alpha'_P \ll \alpha'_o$ gives stronger absorptive corrections. For this case and the parameters of Solution (a_1) the resulting predictions for $\varepsilon = .8$ are shown in Figs.4 and 5. Regarding $\Delta\sigma$, this obsorbed model gives a very good compromise for all data including Batavia (This is true also if Denisov et al. (1973) data are considered).

For $d\sigma_{CEx}/dt|_{t=0}$ and R, on the other hand, predictions are not as good as Solution (a_2) but are quite close to them with the advantage of having a more reasonable value $\alpha_o = .304$.

6.- Conclusions.

We have seen that a multicluster model seems to be an adequate frame to get consistency among experimental data on $\Delta\sigma = \sigma^{tot}_{\pi^- p} - \sigma^{tot}_{\pi^+ p}$ and $d\sigma_{CEx}/dt|_{t=0}$ up to the highest available energy. In contrast to the simple ρ-pole model and its conventional modifications, our model for $\Delta\sigma$ can be interpreted in terms of an effective energy dependent oscillating intercept which enhances Im $F^{(-)}$ in the Brookhaven-Serpukhov region and depresses it immediately above it to give agreement also with Batavia data. (See Fig.7 where a schematic plot is given for the behaviour of the model referred to that of the average pole corresponding to $\alpha(0) \simeq .55$). In this way the real part for $p_{lab} \leq 10$ GeV/c, through d.r., gets two compensating corrections, a positive one coming from the enhanced region and a negative one due to the depressed high-energy tail, resulting in a simple .55 behaviour. As the energy increases the negative correction takes over giving a real part monotonically decreasing with respect to .55 for all reasonable energies. This structure determines that $d\sigma/dt|_{t=0}$ still continues to almost coincide with the .55 prediction up to the energies where the corrections to the real and imaginary parts cancel each other, i.e. ~ 50 GeV/c. From there on, it will start to fall compared to .55 in agreement with preliminary Batavia data.

This mechanism is all due to the threshold for production of one large rapidity gap and closely resembles that originating in complex poles approach (See G.F. Chew, Ref. (20)). The πN_{CEx} amplitude should therefore be a proper laboratory to test the oscillation which in pomeron exchanges should show up at much higher energy due to thresholds in rapidity twice as big as for ρ. Provided one assumes similar rapidity behaviour of the oscillation in both exchanges and taking into account mass differences it can be expected, on the one hand, that a small wiggle associated to ρ-exchange appears at ~ 50 GeV/c for πN (precisely the one we are seeing in $F^{(-)}$) and correspondingly at ~ 300 GeV/c for NN, whereas on the other hand a maximum of the total cross-section associated to pomeron exchange should be observable at ~ 10^3 and ~ 10^4 GeV/c for πN and NN respectively. Noteworthily, this last prediction agrees with the results of the fit of Ref. (28) to pp and $\bar{p}p$ data and with the general considerations about a universal half-length ~ 4 in rapidity for total cross-sections[29].

Absorptive corrections between clusters have been shown to improve the model in the sense that they allow to start with a more reasonable bare pole intercept (even though still fairly small) through a further depression of $\Delta\sigma$ at high-energy. Still another improvement may be obtained if the slopes of reggeons are taken into account in the multicluster chain. This changes the $(\ln s)^n$ asymptotic behaviour of the diffractive dissociation graphs to a $(\ln \ln s)^n$ beyond the present experimental range, emphasizing the depression of $\Delta\sigma$ shown in Fig. 7.

Hopefully, from a more balanced scheme of s- and t-channel iterations a solution will emerge valid in the "second asymptotic" region[26].

We finally remark that an extension of the model to $t \neq 0$ ought to be performed to check its consistency with the angular distribution of πN_{CEx} differential cross-section.

****

We thank heartily Luciano Bertocchi for many valuable suggestions and remarks. Useful discussions with Naren Bali and Carlos García are also acknowledged. We express our gratitude to Dr. U. Amaldi and Roberto Odorico for their sending us the preliminary Batavia data.

Table I : Parameters of simple Regge pole models.
Pole 1 is the best fit with $\Delta\sigma = 2\ \beta\ s^{\alpha(0)-1}$ for 10 GeV/c < P_{lab} < 60 GeV/c and Pole 2 for 3.6 GeV/c < P_{lab} < 60 GeV/c.
Pole 3 is a best fit to $d\sigma_{CEx}/dt\big|_{t=0}$ up to 48 GeV/c.
Batavia data are not included in the fits.

	$\alpha(0)$	$2\ \beta$ (mb)
Pole 1	.673	5.13
Pole 2	.603	6.64
Pole 3	.551	7.47

Table II :

Parameters of multicluster models.
Solution (a_1) is the best fit to $\Delta\sigma$ for 3.6 GeV/c < P_{lab} < 60 GeV/c with eq. (4.a).
Solution (a_2) is the steepest fit in the same region.
Solution (b) is the steepest fit to $\Delta\sigma$ with eq. (4.b) truncated after N = 3; redefined parameters $2\ \beta\ (1+2\ G^2\ L^2)$ and $\{ G^4\ L^3\ /(1+2\ G^2\ L^2)\}^{1/2}$ are quoted.
Batavia data are not included in the fits.

	α_o	$2\ \beta$ (mb)	G^2	Δ
Solution (a_1)	.304	11.39	.107	3.02
Solution (a_2)	.106	17.82	.222	3.38
Solution (b)	.459	9.24	.147	2.34

Figure captions

Fig. 1 - Fits and predictions with a single Regge pole

- - - $\alpha(0) = .673$, ——— $\alpha(0) = .603$, -..-.. $\alpha(0) = .551$.

Re $F^{(-)}$ is calculated with eq. (1). In a) Denisov et al. (1973) data[10] are not shown in order to present the most favourable situation for a pole model. In b) the experimental points are the most trustworthy data according to Refs. (4) and (11).

Fig. 2 - An N-clusters graph for πN_{CEx} .

Fig. 3 - An N-clusters contribution to the renormalized ρ-singularity propagator obtained dualizing the clusters in Fig. 2.

Fig. 4 - Fit and predictions with multicluster model.

——— Solution (a_1) and -·-·-·Solution (a_2) for eq.(4.a),

—— —Absorption corrected model eq. (10) with parameters of Solution (a_1).

Fig. 5 - Predictions for the Batavia range.

Extrapolation to $t=0$ of differential cross-sections if performed by hand for Batavia data.

Serpukhov data; Batavia preliminary results.

- - - - Solution (a_1); ——— Solution (a_2);

—— —Absorbed model;

-.-. Pole 1 ; -..-.. Pole 2 ; -...-... Pole 3

Fig. 6 - Absorptive corrections to a two-clusters graph.

Fig. 7 - Schematic behaviour of the multicluster model referred to average Regge pole with $\alpha(0) \simeq .55$.

Arbitrary units are used to show an effective intercept behaviour.

—— Im $\left(F^{(-)} - F^{(-)}_{\text{av. pole}}\right)$; - - - Re $\left(F^{(-)} - F^{(-)}_{\text{av. pole}}\right)$;

-.-. $d\sigma_{CEx}/dt\big|_{t=0} - d\sigma_{\text{av. pole}}/dt\big|_{t=0}$.

(*) $F^{(-)}$ stands for the usual difference of $\pi^- p$ and $\pi^+ p$ forward amplitudes with the normalization $\mathrm{Im}\, F^{(-)} = q\sqrt{s}\,\Delta\sigma$, where q and $\sqrt{s}$ are the c.m. momentum and total energy respectively.

(**) In fact, using Höhler values[9] for the coupling constant $f^2 = .076$ and $(a_1 - a_3) = .271\ \mu^{-1}$ we obtain for $\omega_L \geq 5$ GeV a real part much greater than the imaginary one.

(***) This assertion is based on an analysis of extrapolated experimental data to $t = 0$. The relative freedom of this extrapolation allowed Berger and Sivers[16] to introduce unitarity effects to bend the ρ-trajectory[4], [17], $\alpha(t) \simeq 55 + t$, near $t = 0$. However the curvature thus obtained was not enough to accommodate the data and the extrapolation failed to pass through the ρ mass at $\alpha = 1$.

(†) The relevance of threshold effects for the pp rise, in the frame of multicluster production,has been stressed by several authors[20].

(††) Incidentally, this agrees with the idea of Ref. (23) of extending the popular $(\ln s)^2$ parametrization to $\sigma^{tot}_{K^+p}$ with an energy threshold considerably lower than for pp.

(ε) We remark that the model was truncated after the term $\sim \ln s$ in order to limit to four the number of parameters redefining the residue as $\beta(1 + 2\, G^2\, L^2)$ and the square of the triple coupling as $\{G^4\, L^3 / (1 + 2\, G^2\, L^2)\}^{1/2}$ which are the values quoted in Table II. This truncation introduces no alteration of results in the available energy range.

(‡) In this context it is remarkable to notice the similarity of $2\, G^2$ with the equivalent parameter δ^2 of Ref. (24) where a ρ-bootstrap within the multiperipheral model is attempted without any reference to clusters. In this model the renormalization of the bare singularity turns out to be of the same order of magnitude as ours.

(#) This explains why results changed only slightly (~ .1%) when the sum eq. (4.a) is truncated after the first correction ~ $(\ell n\ s)^2$.

References

(1) V.N. Bolotov, M.I. Devishev, M.N.Devisheva, M.I.Grachev, V.V. Isakov, V.A. Kachanov, R.N. Krasnokutsky, V.M. Kutyin, Yu. D. Prokoshkin, E.A. Razuvaev, A.N. Sitin, V.N. Semenov, R.S. Shuvalov and L.M. Vasilyev: Phys. Lett. 38 B, 120 (1972).

(2) Data Sheets - A.N. Diddens (plenary speaker), XVIII High-Energy Conference, London (1974).

(3) G. Höhler and H.P. Jakob: Tables of pion-nucleon forward amplitudes, Univ. of Karlsruhe TKP 23/72 (1972).

(4) V.N. Bolotov, V.V. Isakov, D.B. Kakauridze, V.A. Kachanov, V.M. Kutyin, Yu. D. Prokoshkin, E.A. Razuvaev,V.A. Semenov, V.A. Senko and V.G. Rybakov: Nucl. Phys. B 73, 365 (1974).

(5) A.A. Carter and J.R. Carter: A tabulation of pion-nucleon forward amplitudes ut po 30 GeV/c, Rutherford Lab.Report.R.L.-73-024(1973).

(6) D.V. Bugg, A.A. Carter and J.R. Carter: Phys. Lett. 44 B, 278 (1973).

(7) D.V. Bugg and A.A. Carter: Phys.Lett. 48 B, 67 (1974).

(8) G. Höhler, H.P. Jakob and P. Kroll: Z.Physik 261, 401 (1973);
G. Höhler: Tehran and Smolenice Lecture Notes, Univ. of Karlsruhe TKP-17/73 (1973).

(9) G. Höhler, J. Baacke and F. Steiner:Z.Physik 214, 381 (1968).

(10) High-Energy Reactions Analysis Group: CERN/HERA 72-1 (1972);
S.P. Denisov, S.V. Donskov, Yu.P. Gorin, A.I.Petrukhin, Yu. D. Prokoshkin and D.A. Stoyanova: Nucl. Phys.B 65, 1 (1973).

(11) J. Dronkers and P. Kroll: Nucl.Phys. B 47, 291 (1972).

(12) R.W. Hanson: Phys.Rev.D 5, 1227 (1972);
S.T. Sukhorukov and R.A.Ter-Martirosyan:Nucl. Phys. B 54, 522 (1973).

(13) W. Drechsler: Ann. of Phys. 80, 157 (1973).

(14) V.D. Barger and R.J.N. Phillips:Nucl. Phys. B 40, 205 (1972).

(15) N. Bali and J.W. Dash: Argonne preprint ANL/HEP 7380 (1974).

(16) E.L. Berger and D. Sivers: Phys.Rev. D 7, 2032 (1973).

(17) V.D. Barger and R.J.N. Phillips: Nucl. Phys. B 33, 22 (1971).

(18) S. Pokorski and L. Van Hove: CERN preprint TH 1772 (1973).

(19) W.R. Frazer, D.R. Snider and C. -I Tan: Phys. Rev. D 8, 3180 (1973).

(20) G.F. Chew: Phys. Rev. D 7, 3525 (1973) and Phys. Lett. 44 B, 169 (1973);

M. Bishari and J. Koplik: Phys. Lett. B 44, 175 (1973);

T.L. Neff: Phys. Lett. 45 B, 349 (1973);

A. Capella and M.-S. Chen: Phys. Rev. D 8, 2097 (1973);

D. Amati, L. Caneschi and M. Ciafaloni: Nucl. Phys. B 62, 173 (1973);

J.W. Dash: Phys. Rev. D 9, 9 (1974);

J. Finkelstein: Phys.Rev. D 8, 4176 (1973);

H. Harari and E. Rabinovici: Weiszmann Inst. preprint WIS 73/41 Ph (1973);

R.F. Peierls and L.-L. Wang:Phys. Rev. D 9, 2495 (1974);

M. Święcki: Nucl. Phys. B 68, 12 (1974).

(21) H. Harari: Scottish Summer School Lecture Notes, Weiszmann Inst. preprint WIS 73/32 Ph (1973).

(22) C. DeTar: Phys. Rev. D 3, 128 (1971).

(23) L.D. Soloviev: Serpukhov preprint IHEP 73/59 (1973).

(24) D.R. Snider: Phys. Rev. D 5, 3162 (1972).

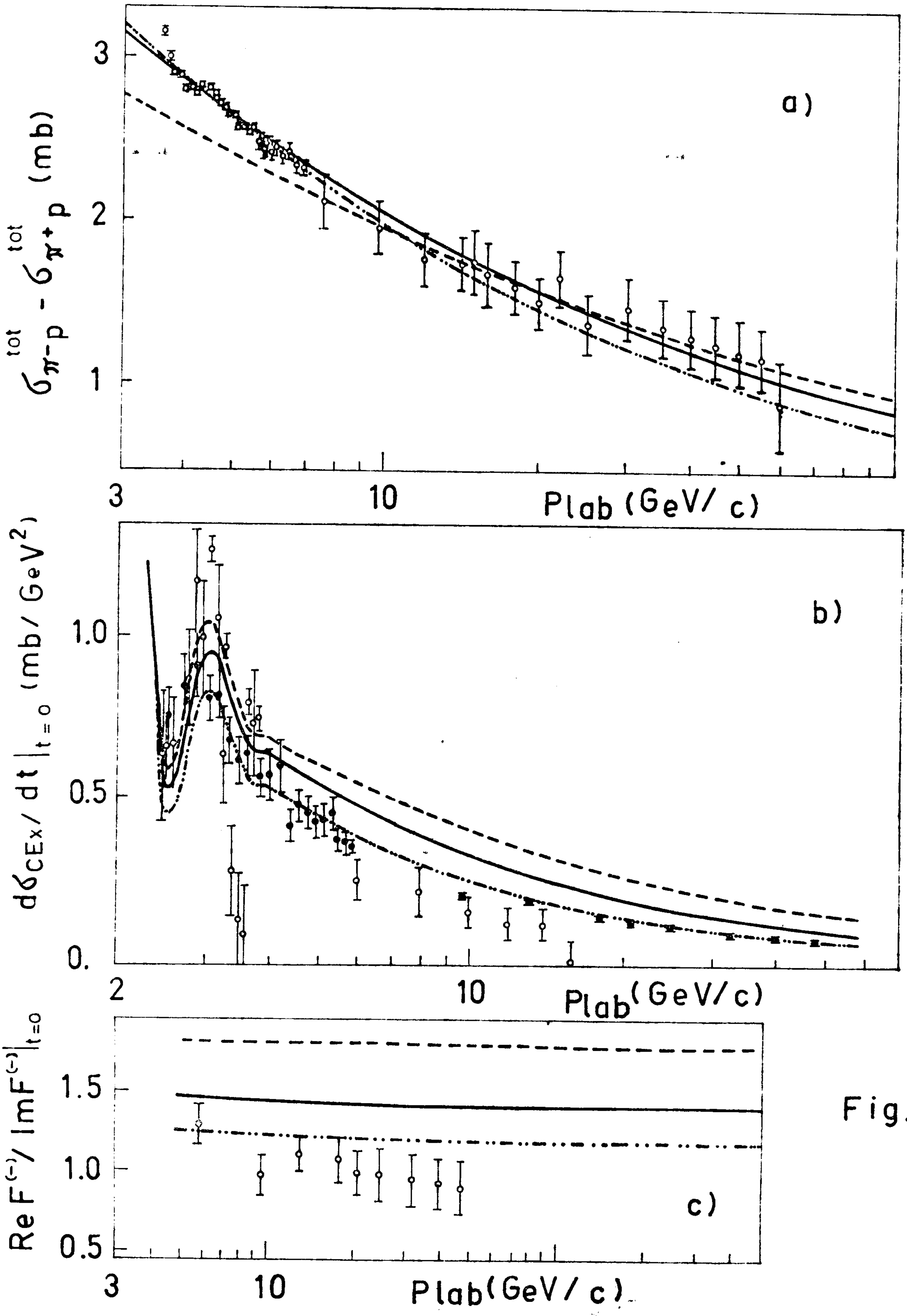
(25) T.K. Gaisser and C.-I Tan: Phys. Rev. D 8, 3881 (1973).

(26) D. Amati: CERN preprint TH 1782 (1973).

(27) M. Ciafaloni and G. Marchesini: Nucl. Phys. B 71, 493 (1974).

(28) C.-F. Chan, C.K. Chen and W. Rarita: Phys.Lett. B 47, 512 (1973).

(29) G.F. Chew and J. Koplik: Phys. Lett. B 48, 221 (1974).



$\sigma^{tot}_{\pi^- p} - \sigma^{tot}_{\pi^+ p}$ (mb)
3
2
1
a)
3
10
Plab (GeV/ c)
$d\sigma_{CEx}/dt|_{t=0}$ (mb/ GeV^2)
1.0
0.5
0.
b)
2
10
Plab (GeV/c)
$ReF^{(-)}/ImF^{(-)}|_{t=0}$
1.5
1.0
0.5
c)
3
10
Plab (GeV/ c)

Fig.1

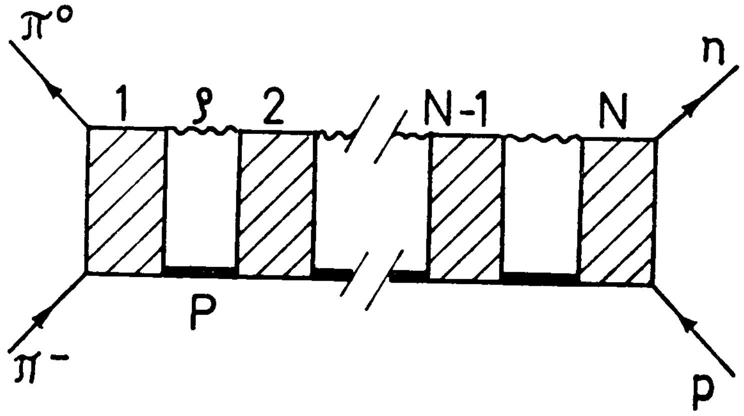


Fig. 2

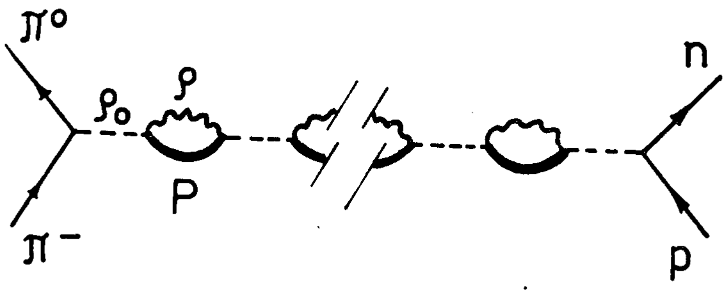
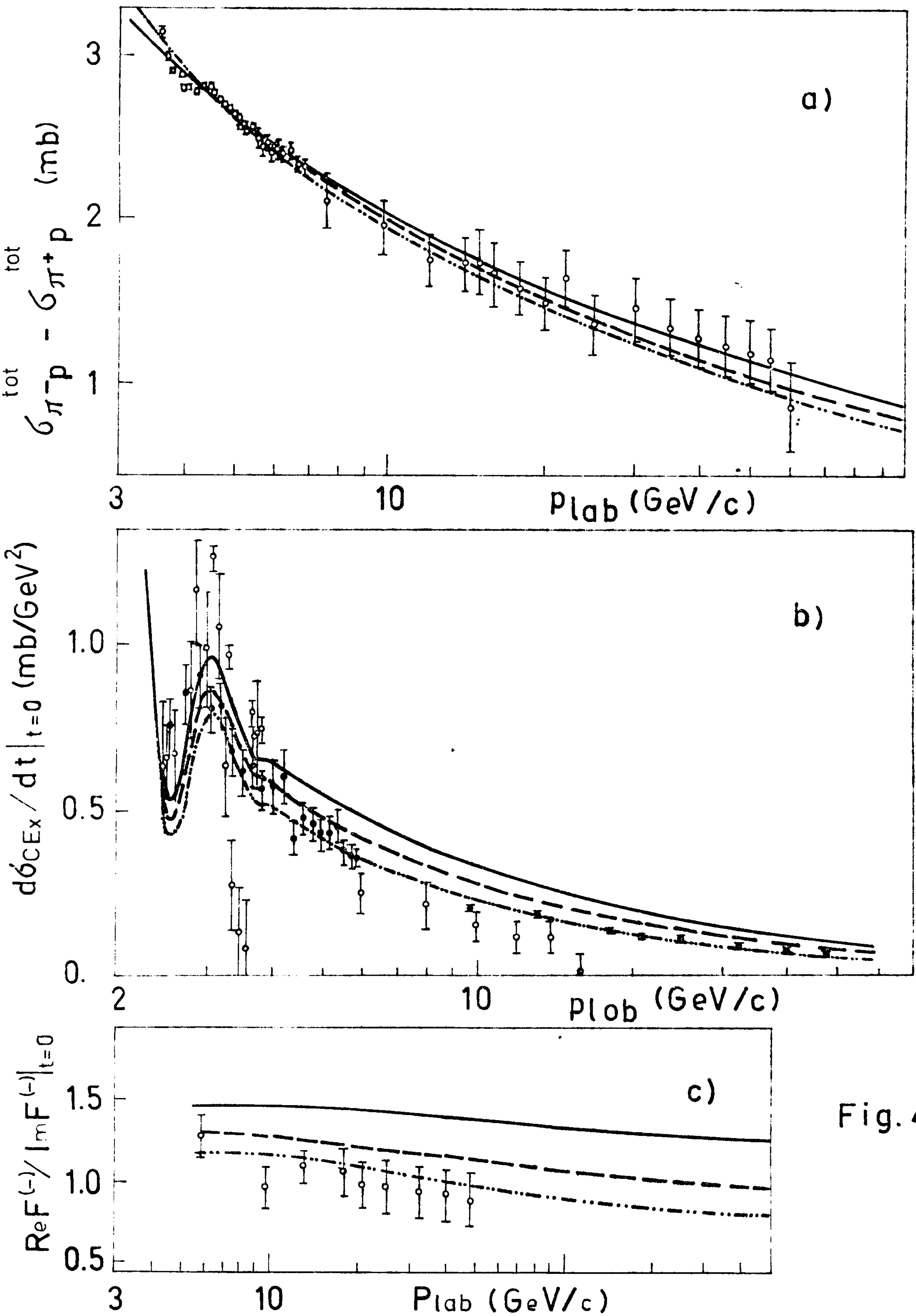


Fig.3



a)
b)
c)
$\sigma^{tot}_{\pi^- p} - \sigma^{tot}_{\pi^+ p}$ (mb)
p_{lab} (GeV/c)
$d\sigma_{CEx}/dt|_{t=0}$ (mb/GeV^2)
p_{lab} (GeV/c)
$ReF^{(-)}/ImF^{(-)}|_{t=0}$
P_{lab} (GeV/c)

Fig. 4

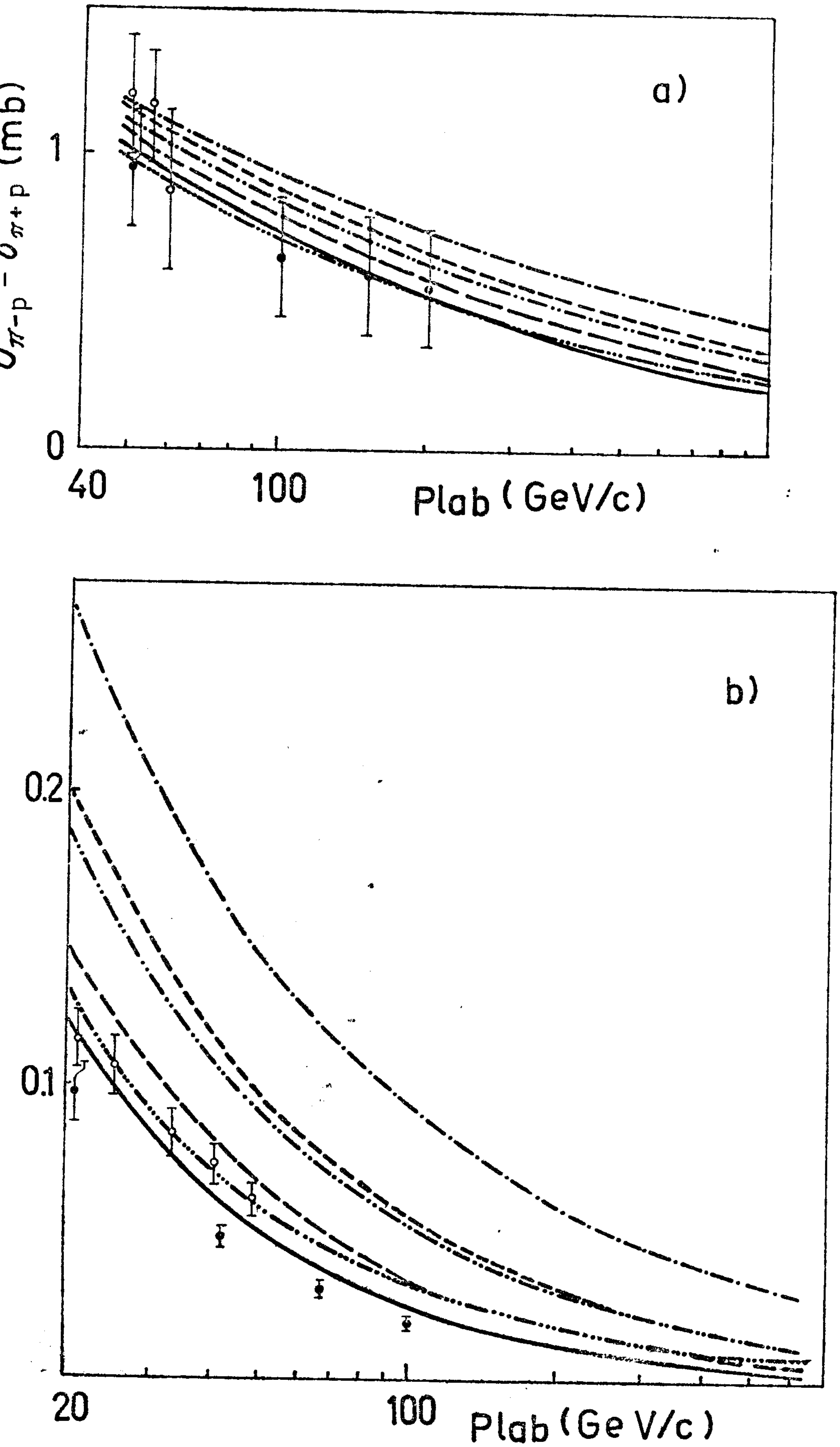

a)
$\sigma^{tot}_{\pi^- p} - \sigma^{tot}_{\pi^+ p}$ (mb)
1
0
40
100
Plab (GeV/c)
b)
$d\sigma_{CEX}/dt|_{t=0}$ (mb/GeV^2)
0.2
0.1
20
100
Plab (GeV/c)

Fig.5

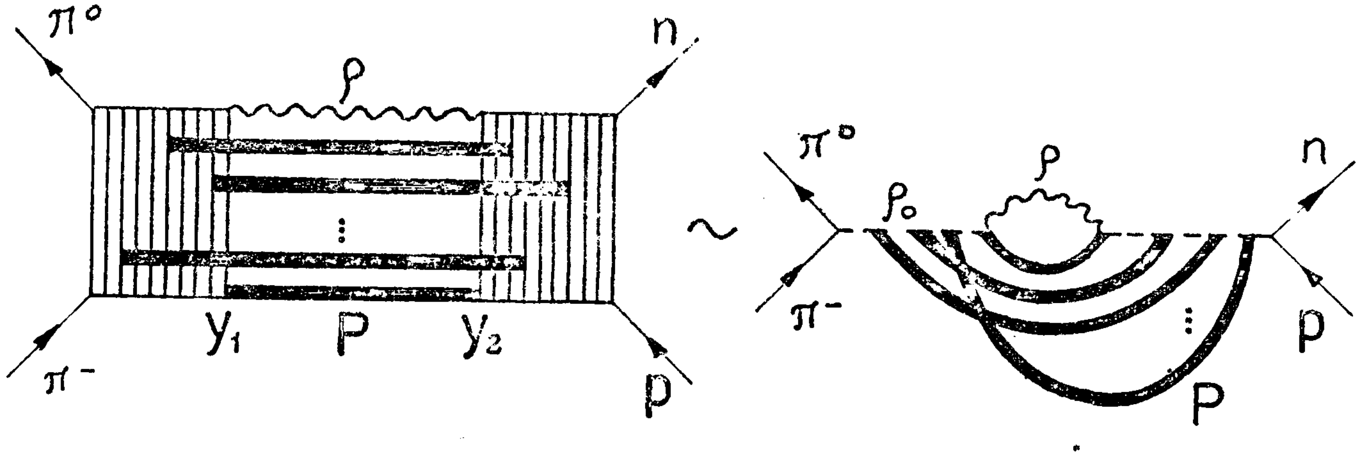


Fig. 6

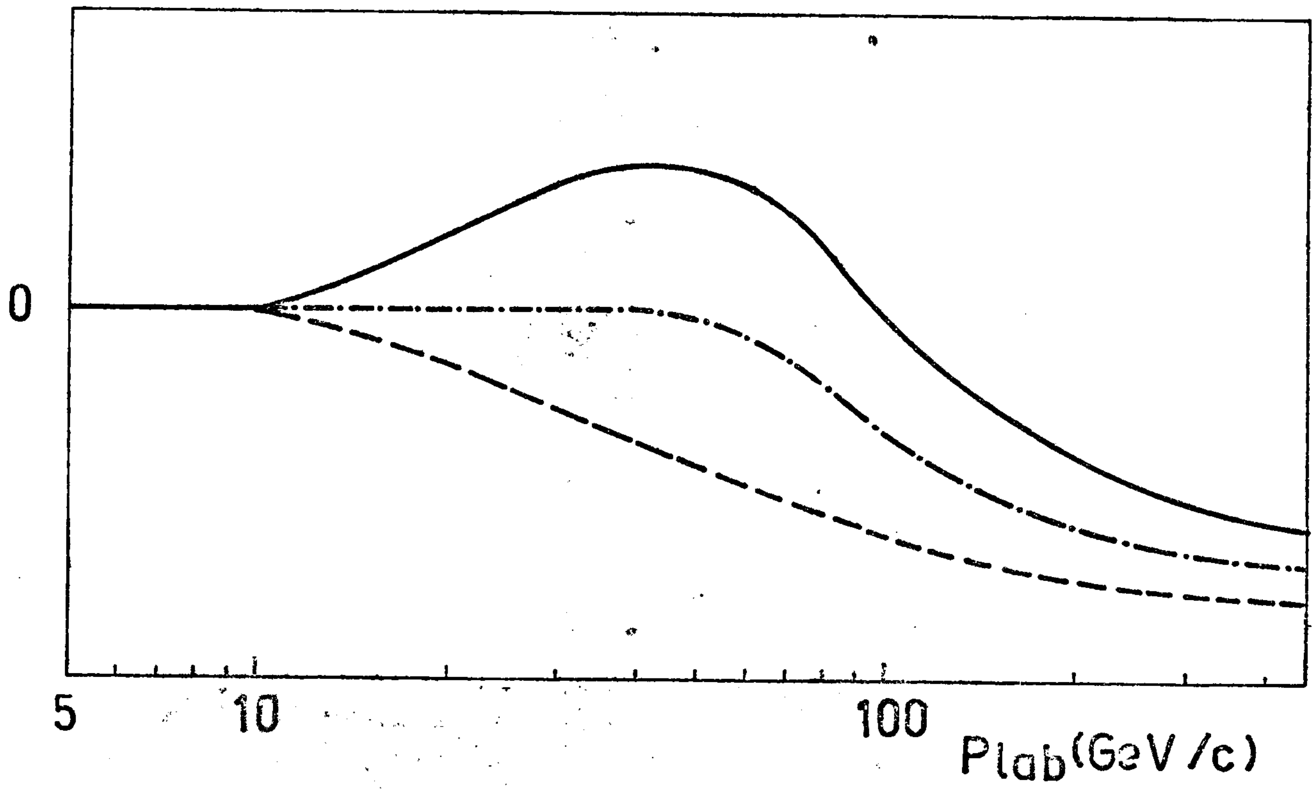


Fig.7