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Relation between the Algebra of Currents
and their Definition in Chiral Gauge Theories

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Abstract

We consider anomalous commutators between the charge density and any component of the gauge current in chiral gauge theories. This is done using the BJL definition of commutators, and without quantizing the gauge field. We study the relation between the vacuum expectation value (VEV) of these commutators for the case of consistent and covariant gauge currents. We obtain a simple relation valid for any dimension and gauge group. This relation shows that the difference between the VEV of the commutators for the consistent and covariant gauge currents depends only on the difference between the VEV of the corresponding currents.

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I. Introduction

The breakdown of classical symmetries by quantization is manifested in quantum field theory by the existence of anomalies. In an anomalous gauge theory (AGT) the current coupled to the gauge field, the gauge current, is not covariantly conserved when one considers the gauge field as external. A theory of fermions where its chirality components are differently coupled to a gauge field is an AGT unless its fermion content is carefully adjusted. Whether a chiral gauge theory (CGT) with arbitrary fermion content can be consistently quantized or not is still an open question. In 1+1 space time dimensions we have a CGT, the chiral Schwinger model, that can be consistently quantized.[1] For other CGT's the

problem consists in finding a consistent quantization method. In this respect, since we are dealing with gauge theories, it seems important to understand the structure of their constraints.[2] Such a question is not trivial due to the fact that anomalies change the classical canonical algebra to a different anomalous algebra.

One important ingredient of this understanding are the commutators between charge densities. These commutators have been calculated by various methods; one of them is the BJL limit.[3] In this respect it is important to remember that given a classical quantity involving products of fields in the same point one gets in general many quantum composite operators associated to that quantity. For the case of the gauge current in CGTs, two of such operators have been considered, the so called "consistent" and "covariant" currents. A precise and simple relation has been found[4] between the vacuum expectation values (VEV) of these currents in an external gauge field.

The purpose of this work is to look for an analogous relation between the VEVs of the commutators for consistent currents and covariant currents, defined via the BJL method. We also deal with the influence of regularization on the results for these commutators.

This is done by deriving Ward identities for the temporal ordered product (TOP) of two consistent and two covariant currents, and then looking to the restrictions that these identities impose on the difference of the above mentioned commutators.

This paper is organized as follows. Section II deals with the relation between the consistent covariant gauge currents. In section III the Ward identities and the transformation properties of the TOP of two consistent and two covariant currents are derived. Section IV gives the desired relation between commutators using the results of the previous section and also considers the dependence on the regularization of these commutators. Section V presents our conclusions.

II. The relation between the consistent and covariant gauge currents.

Let us consider a non abelian chiral gauge theory in an even dimensional spacetime. Regarding the gauge field as external, the Lagragian density for this theory is,

$$\mathcal{L} = i\bar{\psi}\, D\, \psi = i\,\left[\psi\, \partial\, \psi + J^{\mu}_{a}\, A^{a}_{\mu}\right] \tag{2.1}$$

and the notation we use in (2.1) is the following,

$$D = \partial + AP_{L}\ ,\quad A_{\mu} = A^{a}_{\mu}\,\lambda^{a},\quad \lambda^{a} = -\,(\lambda^{a})^{t},\quad \left[\lambda^{a},\ \lambda^{b}\right] = f^{abc}\lambda^{c}$$

$$A = A_{\mu}\,\gamma^{\mu},\quad P_{L} = \frac{1}{2}\,(1+\gamma^{5}),\quad \left\{\gamma^{\mu},\gamma^{\nu}\right\} = 2g^{\mu\nu},\quad \left\{\gamma^{\mu},\ \gamma^{5}\right\} = 0,\quad \gamma^{5}=\gamma^{5t}. \tag{2.2}$$

The classical current $J^{\mu}_{a}(x)$ appearing in (2.1) is,

$$J^{\mu}_{a}(x) = \bar{\psi}(x)\,\gamma^{\mu}\,P_{L}\,\lambda_{a}\psi(x) \tag{2.3}$$

For further use and in order to fix the notation we recall the main steps of the derivation that relates the VEVs of the consistent and covariant gauge currents.[4]

We start considering the generating functional,

$$Z[A] = \int \mathcal{D}\bar{\psi}\;\mathcal{D}\psi\;\exp\left\{i\int d^{4}x\,\mathcal{L}\right\} \tag{2.4}$$

and the effective action,

$$W[A] = -\ln Z[A] \tag{2.5}$$

Denoting by J^{μ}_{a} the operator of the consistent gauge current (CGC), and by J^{μ}_{a} its VEV, we have the definition,

$$J^{\mu}_{a}(x) = \langle 0|J^{\mu}_{a}(x)|0\rangle \equiv \langle J^{\mu}_{a}(x)\rangle = \frac{\delta W[a]}{\delta A^{a}_{\mu}(x)} \tag{2.6}$$

The lack of gauge invariance of $W[A]$ leads to the anomalous divergence of the CGC,

$$T_{\Lambda}W[A] = \int d^{4}x\,\frac{\delta W}{\delta A^{a}_{\mu}(x)}\,[A]\;T_{\Lambda}A^{a}_{\mu}(x) = \int d^{4}x\;J^{\mu}_{a}(x)\;D_{\mu}\Lambda^{a}(x)$$

$$= -\int d^{4}x\;\Lambda^{a}(x)\;D_{\mu}J^{\mu}_{a}(x) = \int d^{4}x\;\Lambda^{a}(x)\;G_{a}(x) \tag{2.7}$$

i.e.,

$$D_{\mu}J^{\mu}_{a}(x) + G_{a}(x) = 0 \tag{2.8}$$

The Wess-Zumino consistence condition[5] states that,

$$\left[T_{\Lambda},\ T_{\Lambda'}\right]\,W[A] = T\,[\Lambda,\ \Lambda']\;W\,[A] \tag{2.9}$$

which in view of (2.7) tell us that,

$$\int d^{4}x\,\left[\Lambda'^{a}(x)\;T_{\Lambda}\,G_{a}(x) - \Lambda^{a}(x)\;T_{\Lambda'}G_{a}(x)\right] = \int d^{4}x\;[\Lambda,\Lambda']^{a}\;Ga(x)\ . \tag{2.10}$$

In order to obtain the gauge transformation law for this current, we also consider the condition,

$$\left[\delta_{B},\ T_{\Lambda}\right]\,W\,[A] = \delta[B,\Lambda]\;W[A] \tag{2.11}$$

where δ_{B} means a general variation of the gauge field, i.e.,

$$\delta_{B}\;A^{a}_{\mu}(x) = B^{a}_{\mu}(x) \tag{2.12}$$

The transformation law that results from (2.11) is,

$$\int d^4x\, B^a_\mu(x)\, T_\Lambda J^\mu_a(x) = \int d^4x\, B^a_\mu(x)\left[J^\mu(x),\, \Lambda(x)\right]_a + \int d^4x\, \Lambda^a(x)\delta_B G_a(x), \tag{2.13}$$

where we see that the last term breaks the covariant transformation law.

The covariant gauge current ($\tilde{C}GC$) operator is defined as,

$$\tilde{J}^\mu_a = J^\mu_a + \hat{X}^\mu_a \quad . \tag{2.14}$$

where $\hat{X}^\mu_a$ is an operator whose transformation properties are such that cancel the anomalous gauge variation of J^μ_a. If we consider only VEV's of the operators, we obtain,

$$\tilde{J}^\mu_a = J^\mu_a + X^\mu_a \tag{2.15}$$

where X^μ_a must have the transformation law,

$$\int d^4x\, B^a_\mu(x)\, T_\Lambda X^\mu_a(x) = \int d^4x\, B^a_\mu(x)\, \left[X^\mu(x), \Lambda(x)\right]_a - \int d^4x\, \Lambda^a(x)\delta_B G_a(x). \tag{2.16}$$

Then, by (2.13), (2.15) and (2.16) we have,

$$T_\Lambda \tilde{J}^\mu_a\,(x) = \left[\tilde{J}^\mu(x),\, \Lambda(x)\right]_a \quad . \tag{2.17}$$

Equation [2.16] can be solved for X^μ.[4] The anomalous divergence of the CGC has been evaluated.[6] The CGC can be modified by adding counterterms to W[A].[7] If we adjust them so that the CGC have the minimal anomalous divergence, we obtain,

$$G_a(x) = -\frac{1}{24\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[\lambda^a\partial_\mu(A_\nu\partial_\rho A_\sigma + \frac{1}{2}A_\nu A_\rho A_\sigma)\right] \tag{2.18}$$

$$X^\mu_a(x) = \frac{1}{48\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[\lambda^a\partial_\mu(A_\nu F_{\rho\sigma} + F_{\rho\sigma}A_\nu - A_\nu A_\rho A\sigma)\right] \tag{2.19}$$

Now we look for the effect of the counterterms in the relation (2.15). If we start with the current of minimal anomalous divergence and change it by a counterterm in W[A],

$$J^\mu_a(x) \longrightarrow J^\mu_a(x) + \delta J^\mu_a(x) \tag{2.20}$$

then by (2.8) the anomaly (2.18) will be changed to,

$$G_a(x) \longrightarrow G_a(x) + \delta G_a(x) \tag{2.21}$$

where,

$$\delta Ga(x) = -\,D_\mu\delta J^\mu_a(x) \tag{2.22}$$

then, by [2.16], there will be a change in $X^\mu_a(x)$,

$$X^\mu_a(x) \longrightarrow X^\mu_a(x) + \delta X^\mu_a(x) \tag{2.23}$$

Hence, by the relation (2.15) we obtain,

$$\tilde{J}^\mu_a(x) = J^\mu_a(x) + \left[X^\mu_a(x) + \delta J^\mu_a(x) + \delta X^\mu_a(x)\right] \tag{2.24}$$

and using the fact that there is a unique solution X^μ_a for (2.16), we see from (2.24) that,

$$\delta J^\mu_a(x) + \delta X^\mu_a(x) = 0 \tag{2.25}$$

i.e., under a change in the regularization of W[A] both J^μ_a and X^μ_a will be altered, but $\tilde{J}^\mu_a(x)$ will remain the same.

Now we look for a relation between the expectation values of the currents, but in presence of external fermionic sources.

We start defining the generating functional,

$$Z[A,\eta,\bar{\eta}] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \, \exp\left\{ i \int d^4x \left[\mathcal{L} + \bar{\eta}\psi + \bar{\psi}\eta \right] \right\} \quad . \qquad (2.26)$$

with $\mathcal{L}$ given by [2.1].

Then the expectation value of the CGC is,

$$J^{\mu}_{a}(A,\eta,\bar{\eta}) = - Z^{-1}[A,0,0] \, \frac{\delta}{\delta A^{a}_{\mu(x)}} \, Z[A,\eta,\bar{\eta}] \quad ; \qquad (2.27)$$

which can be evaluated doing the quadratic integral in (2.26), leading to,

$$J^{\mu}_{a}(A,\eta,\bar{\eta}) = \left[J^{\mu}_{a}(A) + i \int d^4y \, d^4z \, \bar{\eta}(y) \, \frac{\delta}{\delta A^{a}_{\mu}(x)} D^{-1}(y,z) \, \eta(z) \right]$$

$$\exp\left\{ -i \int d^4y \, d^4z \, \bar{\eta}(y) \, D^{-1}(y,z)\eta(z) \right\} \qquad (2.28)$$

where,

$$D(x,y) = \delta(x-y) \, D(x) \qquad (2.29)$$

and $J^{\mu}_{a}(A)$ is the VEV defined in (2.6). The inverse of D, as usual, can be calculated by a perturbation expansion.

For the $\tilde{C}GC$ we adopt the following definition,

$$\tilde{J}^{\mu}_{a}(A,\eta,\bar{\eta}) = \lim_{\varepsilon \to 0} Z^{-1}[A,0,0] \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \, \bar{\psi}(x-) \, \gamma^{\mu} \, P_L \, \frac{1}{2} \left\{ \lambda^{a}, \phi(x-,x+) \right\} \psi(x+)$$

$$\exp\left\{ i \int d^4x \left[\mathcal{L} + \bar{\psi}\eta + \bar{\eta}\psi \right] \right\} \quad . \qquad (2.30)$$

where,

$$x\pm = x \pm \frac{1}{2} \varepsilon \quad . \qquad (2.31)$$

$\phi(x-,x+)$ is the corresponding phase factor that assures the gauge invariance of the regularization[8]; and a symmetrical limit is understood.

Ed. (2.24) can be rewritten as,

$$\tilde{J}^{\mu}_{a}(A,\eta,\bar{\eta}) = - \lim_{\varepsilon \to 0} \mathrm{Tr} \left[\gamma\mu \, P_L \, \frac{1}{2} \left\{ \lambda^{a}, \; \phi(x-,x+) \right\} S(x+,x-) \right] \qquad (2.32)$$

where S is the fermion propagator in an external A-field and in presence of the fermionic sources.

Taking the symmetrical limit in (2.32) we obtain,

$$\tilde{J}^{\mu}_{a}(A,\eta,\bar{\eta}) = \left[\tilde{J}^{\mu}_{a}(A) + i \int d^4y \, d^4z \, \bar{\eta}(y) \, \frac{\delta}{\delta A^{a}_{\mu}(x)} \, D^{-1}(y,z)\eta(z) \right]$$

$$\exp\left\{ -i \int d^4y \, d^4z \, \bar{\eta}(y) \, D^{-1}(y,z) \, \eta(z) \right\} \qquad (2.33)$$

Then, combining [2.28] and [2.33], we obtain,

$$\tilde{J}^{\mu}_{a}(A,\eta,\bar{\eta}) - J^{\mu}_{a}(A,\eta,\bar{\eta}) \equiv X^{\mu}_{a}(A,\eta,\bar{\eta}) = X^{\mu}_{a}(A) \times$$

$$\times \exp\left\{ - i \int d^4y \, d^4z \, \bar{\eta}(y) \, D^{-1}(y,z)\eta(z) \right\} \quad , \qquad (2.34)$$

i.e., the difference between the VEV's of the currents in the presence of fermionic sources is simply related to the difference when the sources are turned off.

III. Ward identities and transformation properties of the TOPs.

The connected part of the TOP of two consistent currents is obtained deriving twice the functional W[A],

$$\langle J^{\mu}_{a}(x) \, J^{\nu}_{b}(y) \rangle = - \frac{\delta^2 W[A]}{\delta A^{a}_{\mu}(x) \delta A^{b}_{\nu}(y)} \quad . \qquad (3.1)$$

In order to obtain a Ward identity for this TOP, we rewrite eq. (2.13) using the following relations,

$$\int d^4x\, B^a_\mu(x)\, T_\Lambda J^\mu_a(x) = - \int d^4x d^4y\, B^a_\mu(x)\, \Lambda^b(y) \left[D^y_\nu\right]^b_d \left(\frac{\delta^2 W}{\delta A^a_\mu(x) \delta A^d_\nu(y)} \right), \tag{3.2}$$

$$\int d^4x\, B^a_\mu(x) \left[J^\mu(x),\ \Lambda(x) \right]_a = \int d^4x d^4y\, B^a_\mu(x)\, \Lambda^b(y)\, \delta(x-y)\, f^{abc}\, J^\mu_c(x), \tag{3.3}$$

$$\int d^4x\, \Lambda^a(x)\, \delta_B G_a(x) = \int d^4x d^4y\, B^a_\mu(x)\, \Lambda^b(y) \frac{\delta G_b(y)}{\delta A^a_\mu(x)} \quad , \tag{3.4}$$

then, from (2.13), (3.2), (3.3) and (3.4), we obtain,

$$\left[D^x_\mu\right]^a_d <J^\mu_d(x)\ J^\nu_b(y)> = \delta(x-y)\ f^{abc}\ J^\nu_c(x) + \frac{\delta G_a(x)}{\delta A^b_\nu(y)} \tag{3.5}$$

Now, we derive the transformation law for the TOP of two CGC. For this purpose, note that starting with the relation,

$$<J^\mu_a(x)\ J^\nu_b(y)> = - \frac{\delta J^\nu_a(x)}{\delta A^b_\nu(y)} \tag{3.6}$$

we see that,

$$T_\Lambda <J^\mu_a(x)\ J^\nu_b(y)> = - \left[T_\Lambda\ \frac{\delta}{\delta A^b_\nu(y)} \right] J^\mu_a(x) - \frac{\delta}{\delta A^b_\nu(y)}\ T_\Lambda J^\mu_a(x) \tag{3.7}$$

and by (2.13),

$$T_\Lambda <J^\mu_a(x)\ J^\nu_b(y)> = <\left[T_\Lambda J^\mu_a(x)\right] J^\nu_b(y)> + <J^\mu_a(x) \left[T_\Lambda J^\nu_b(y)\right]> + \int d^4z\ \Lambda^c(z) \frac{\delta^2 G^c(z)}{\delta A^a_\mu(x) \delta A^b_\nu(y)} \tag{3.8}$$

If the anomaly is given by (2.18), along but direct calculation shows that,

$$\frac{\delta^2 G^c(z)}{\delta A^\mu_a(x) \delta A^\nu_b(y)} = 0 \tag{3.9}$$

then, in the same case we obtain,

$$T_\Lambda <J^\mu_a(x)\ J^\nu_b(y)> = <\left[T_\Lambda J^\mu_a(x)\right] J^\nu_b(y)> + <J^\mu_a(x) \left[T_\Lambda J^\nu_b(y)\right]>, \tag{3.10}$$

i.e., the regularization of W[A] that leads to the minimal consistent anomaly also assures the transformation law (3.10). In general there will be extra terms in the rhs of eq. (3.10), so we can say that the minimal regularization leads to a "minimal" transformation law for the TOP.

For the connected part of the TOP of $\tilde{C}$GC, we adopt the definition,

$$<\tilde{J}^\mu_a(x)\ \tilde{J}^\nu_b(y)> = \lim_{\varepsilon_1 \to 0}\ \lim_{\varepsilon_2 \to 0}\ <\ \tilde{J}^\mu_a(x|\varepsilon_1)\ \tilde{J}^\nu_b\ (y|\varepsilon_2)> \tag{3.11}$$

where

$$\tilde{J}^\mu_a(x|\varepsilon) = \frac{1}{2}\ \bar{\psi}(x-)\ \gamma^\mu\ P_L \left\{ \lambda_a,\ \phi(x-,x+) \right\} \psi\ (x+) \quad . \tag{3.12}$$

and symmetrical limits are understood.

Doing a gauge transformation in the fermionic fields of (2.32), and employing a gauge covariant regularization for the anomalous Jacobian[9] we obtain the Ward identity satisfied by the TOP of $\tilde{C}$GC,

$$\left[D^{x}_{\mu}\right]^{a}_{d} \langle \tilde{J}^{\mu}_{a}(x)\, \tilde{J}^{\nu}_{b}(y)\rangle = \delta(x-y)\, f^{abc}\, J^{\nu}_{c}(x) \tag{3.13}$$

For the transformation law of this TOP, we perform a gauge transformation in the fermionic <u>and</u> gauge fields in the rhs of (3.12) (before taking the symmetrical limit), obtaining,

$$T_{\Lambda} \langle \tilde{J}^{\mu}_{a}(x)\, \tilde{J}^{\nu}_{b}(y)\rangle = \left\langle \left[T_{\Lambda} \tilde{J}^{\mu}_{a}(x)\right] \right\rangle + \left\langle \tilde{J}^{\mu}_{a}(x) \left[T_{\Lambda} \tilde{J}^{\nu}_{b}(y)\right] \right\rangle \tag{3.14}$$

IV. Relation between different commutators

For the sake of completeness we give here the definition of the BJL limit. If one wishes to know the equal time commutator (ETC) between the operators $A(o,\bar{x})$ and $B(o,\bar{o})$, one starts with any matrix element of the TOP,

$$t(x) = \langle \alpha | TA(x)\; B(o) | \beta \rangle \tag{4.1}$$

then, one defines the Fourier transform,

$$T(q) = \int d^4x\; e^{iq.x}\; t(x) \tag{4.2}$$

and the matrix element of the ETC between the same states will be defined by,

$$\langle \alpha | \left[A(o,\bar{x}),\; B(o)\right] | \beta \rangle = \oint \frac{dq^{o}}{(2\pi)} \int \frac{d^3q}{(2\pi)^3}\; e^{i\bar{q}.\bar{x}}\; T(q) \quad , \tag{4.3}$$

where the 9^{o}- integral is taken over a circumference of infinite radius in the complex q^{o}- plane.

Note that the q^{o}- integral will get contributions only from the singularities in the q^{o}- dependence of $T(q)$. It is easy to see that these singularities correspond to non local terms in $t(x)$.[10]

Now we look for the relation between the ETC of covariant and consistent gauge currents. We define,

$$X^{\upsilon\nu}_{ab}(x) = \langle \tilde{J}^{\mu}_{a}(x)\; \tilde{J}^{\nu}_{b}(o)\rangle - \langle J^{\mu}_{a}(x)\; J^{\nu}_{b}(o)\rangle \tag{4.4}$$

which by (3.5) and 3.13) will satisfy,

$$(D_{\mu})^{a}_{d}\; X^{\mu\nu}_{db}(x) = \delta(x) f^{ab}\; X^{\nu}_{c}(x) - \frac{\delta G_{a}(x)}{\delta A^{b}_{\nu}(0)} \quad . \tag{4.5}$$

We will call $\bar{X}^{\mu\nu}_{ab}(x)$ to the difference between "complete" TOPs (i.e., not onl the connected part),

$$\begin{aligned} \bar{X}^{\mu\nu}_{ab}(x) &= \langle 0|T\; \tilde{J}^{\mu}_{a}(x)\; \tilde{J}^{\nu}_{b}(o)|0\rangle - \langle 0|T\; J^{\mu}_{a}(x)\; J^{\nu}_{b}(o)|0\rangle \\ &= \theta(xo) \left[\langle 0|\tilde{J}^{\mu}_{a}(x)\; \tilde{J}^{\nu}_{b}(o)|0\rangle - \langle 0|J^{\mu}_{a}(x)\; J^{\nu}_{b}(o)|0\rangle\right] + \\ &\quad + \theta(-xo) \left[\langle 0|\tilde{J}^{\nu}_{b}(o)\; \tilde{J}^{\mu}_{a}(x)|0\rangle - \langle 0|J^{\nu}_{b}(o)\; J^{\mu}_{a}(x)|0\rangle\right] \end{aligned} \tag{4.6}$$

Taking the covariant derivative on both sides of (4.6),

$$\begin{aligned} \left[D_{\mu}\right]^{a}_{d} \bar{X}^{\mu\nu}_{db}(x) &= \delta(xo) \left\{\langle 0| \left[\tilde{J}^{o}_{a}(x), \tilde{J}^{\nu}_{b}(o)\right] |0\rangle - \langle 0| \left[J^{o}_{a}(x),\; J^{\nu}_{b}(o)\right] |0\rangle\right\} \\ &\quad + \langle o|T\; D_{\mu}\tilde{J}^{\mu}_{a}(x)\; \tilde{J}^{\nu}_{b}(o)|0\rangle - \langle 0|T\; D_{\mu}J^{\mu}_{a}(x)\; J^{\nu}_{b}(o)|0\rangle, \end{aligned} \tag{4.7}$$

and noting that,

$$X^{\mu\nu}_{ab}\,(x) = \bar{X}^{\upsilon\nu}_{ab}(x) - \tilde{J}^{\mu}_{a}(x)\; \tilde{J}^{\nu}_{b}(o) + J^{\mu}_{a}(x)\; J^{\nu}_{b}(0) \quad . \tag{4.8}$$

we obtain,

$$\left[D_{\mu}\right]^{a}_{d} X^{\mu\nu}_{db}(x) = \delta(xo) \left\{\langle 0| \left[\tilde{J}^{o}_{a}(x),\; \tilde{J}^{\nu}_{b}(0)\right] |0\rangle - \langle 0| \left[J^{o}_{a}(x),\; J^{\nu}_{b}(o)\right] |0\rangle\right\}$$

$$+\left[\langle 0|\,T\, D_\mu \tilde{J}^\mu_a(x)\ \tilde{J}^\nu_b(o)\,|0\rangle - \left[D_\mu \tilde{J}^\mu_a(x)\right]\tilde{J}^\nu_b(o)\right] -$$

$$-\left[\langle 0|T\, D_\mu J^\mu_a(x)\ J^\nu_b(o)\,|0\rangle - \left[D_\mu J^\mu_a(x)\right]J^\nu_b(o)\right] \qquad (4.9)$$

Furthermore, one can see that,

$$\langle 0|T\, D_\mu \tilde{J}^\mu_a(x)\ \tilde{J}^\nu_b(0)\,|0\rangle = \left[D_\mu \tilde{J}^\mu_a(x)\right]\tilde{J}^\nu_b(0)$$

$$\langle 0|\ T\, D_\mu\ J^\mu_a(x)\ J^\nu_b(0)\,|0\rangle = \left[D_\mu J^\mu_a(x)\right]J^\nu_b(o) \qquad , \qquad (4.10)$$

as follows from the Ward identity,[9]

$$\langle D_\mu \tilde{J}^\mu_a(x) + \tilde{G}_a(x) + i\,\psi(x)\,P_R\lambda_a\eta(x) - i\bar{\eta}(x)\lambda_a P_L\psi(x)\rangle = 0 \quad , \quad (4.11)$$

which implies that inserting a current operator the last two terms give zero by charge conservation (an identical proof holds for the CGC).

Thus we obtain,

$$\left[D_\mu\right]^a_d X^{\mu\nu}_{db} = \delta(xo)\left[\langle 0|\left[\tilde{J}^o_a(x),\ \tilde{J}^\nu_b(o)\right]|0\rangle - \langle 0|\left[J^o_a(x),\ J^\nu_b(o)\right]|0\rangle\right] \qquad (4.12)$$

with the lhs given by (4.5).

But (2.16) allows us to show that,

$$\left[D_\mu\right]^a_d X^{\mu\nu}_{db}(x) = \left[\left[D^\nu_\mu\right]^b_d \frac{\delta X^\nu_a(x)}{\delta A^d_\mu(y)}\right]_{y=0} \qquad (4.13)$$

so we have obtained the relation between the ETCs in terms of the difference of the VEVs;

$$\delta(xo)\left[\langle 0|\left[\tilde{J}^o_a(x),\ \tilde{J}^\nu_b(o)\right]|0\rangle - \langle 0|\left[J^o_a(x),\ J^\nu_b(o)\right]|0\rangle\right] =$$

$$\left[\left[D^\nu_\mu\right]^b_d \frac{\delta X^\nu_a(x)}{\delta A^d_\mu(y)}\right]_{y=0} \qquad (4.14)$$

Let us now consider the effect of a change in the regularization, on the ETC of CGC. A change in the regularization will produce a variation ΔW in W[A],

$$W[A] \longrightarrow W[A] + \Delta\, W[A] \qquad , \qquad (4.15)$$

which will be a local polynomial in the gauge fields and their derivatives.[7] Then, by (3.1),

$$\langle J^\mu_a(x)\ J^\nu_b(o)\rangle \to \langle J^\mu_a(x)\ J^\nu_b(o)\rangle + \Delta\langle J^\mu_a(x)\ J^\nu_b(0)\rangle \quad , \qquad (4.16)$$

where the variation will also be a local polynomial.

As the TOP changes by a local polynomial, its Fourier transform will change by a polynomial in q, which will not contribute to the BJL.

V. Conclusions

In section IV we have shown tht the ETC between CGCs does not depend upon the regularization of the effective action. As the ETC between $\tilde{C}$GCs can be obtained from ETC between CGCs (eq. (4.14)), we see that the former ETC will depend on the regularization, because X^μ_a does. This, together with eq. (2.25) tell us that the VEV of the CGCs will be regularization independent (RI); and the VEV of the $\tilde{C}$GC will be RI but their ETC will be regularization dependent (RD).

Regarding the transformation properties of the TOPs, we hae shown that the TOP of $\tilde{C}$GC is covariant and that this transformation property is RI, on the contrary the TOP of CGC is not covariant and this transformation property is RD.

As a final remark, note that the above statements do not depend on the space time dimensionality nor on the gauge group.

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