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THE DEUTERON OPTICAL POTENTIAL IN THE ADIABATIC APPROXIMATION

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Abstract: The deformation of the deuteron in the presence of a target nucleus is studied in the adiabatic approximation. Its effect on the deuteron optical potential is discussed. The contribution of deuteron break-up to this potential is also calculated.

1. Introduction

The elastic scattering and polarization data for deuterons are not sufficient to determine uniquely the optical potential parameters of the deuteron. The existence of families of parameters which give equally good fits to the experimental data [1]) is now well known. In the case of nucleon scattering the known negative energy extrapolation of the optical potential (the shell model potential) eliminates this ambiguity. For deuterons, and in general for composite particles, such limits have not been definitely determined yet. However, it is expected, on the basis of first-order perturbation theory for the internal motion of the composite particle, that the potential should have a strength whose value is given by the number of nucleons in the composite particle times the nucleon potential strength [2, 3]).

In this paper we have investigated further this conclusion for deuterons. In order to carry out the calculations we made use of the adiabatic treatment for the internal motion of the deuteron. Though we have no definite guarantee that this approximation is valid for the treatment of the problem, we believe it to be accurate enough to reproduce the essential features of the deuteron optical potential.

2. The Method

The starting point of our considerations is the Schroedinger equation

$$\left(-\frac{\hbar^2}{2m}(\nabla_1^2+\nabla_2^2)+V_n(\boldsymbol{r}_1)+V_p(\boldsymbol{r}_2)+v(\boldsymbol{r}_1-\boldsymbol{r}_2)\right)\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2)=E\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2). \quad (1)$$

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The function $\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2)$ describes the state of the neutron-proton pair under their mutual interaction $v(\boldsymbol{r}_1-\boldsymbol{r}_2)$ and their interaction with the target nucleus, represented by $V_n(\boldsymbol{r}_1)$ and $V_p(\boldsymbol{r}_2)$ the respective neutron and the proton potentials. A central potential was used for $v(\boldsymbol{r}_1-\boldsymbol{r}_2)$ which correctly predicts the triplet scattering length and effective range. The potentials $V_n(\boldsymbol{r}_1)$ and $V_p(\boldsymbol{r}_2)$ were identified with the known phenomenological optical potentials respectively for neutrons and protons [4]). Eq. (1) therefore describes with suitable boundary conditions at infinity, the motion of a deuteron scattered by $V_p(\boldsymbol{r}_2)$ and $V_n(\boldsymbol{r}_1)$. Two mechanisms are responsible for the absorptive interaction of the deuteron with the target nucleus. First, the imaginary parts of $V_p(\boldsymbol{r}_2)$ and $V_n(\boldsymbol{r}_1)$ are responsible for the absorption of the deuteron due to the individual collisions of the neutron and proton with the target nucleus. Second, $V_n(\boldsymbol{r}_1)$ and $V_p(\boldsymbol{r}_2)$ represent forces that can cause the deuteron break-up. In choosing $V_n(\boldsymbol{r}_1)$ and $V_p(\boldsymbol{r}_2)$ one is confronted with two classes of potentials due to the fact that both surface and volume absorptive potentials can describe the mechanism of the interaction of nucleons with the target nucleus. We have chosen surface absorptive potentials. The results obtained using volume absorption can easily be understood from the results discussed here.

Initially, in order to obtain the deuteron potential from eq. (1) we will make use of a perturbation expansion of $\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2)$. There are two alternative choices of base vectors for the expansion. One may use the eigenstates of the single-particle Hamiltonians

$$H_n = -\frac{\hbar^2}{2m}\nabla_1^2 + V_n(\boldsymbol{r}_1),$$

$$H_p = -\frac{\hbar^2}{2m}\nabla_2^2 + V_p(\boldsymbol{r}_2),$$

as the base vectors. Such a procedure turned out to be very cumbersome as the deuteron state is not described in a simple manner in terms of the eigenfunctions of H_n and H_p. The other alternative, which is the one adopted in this paper, chooses the eigenfunctions of the Hamiltonian for the relative motion of the pair as the base vectors for the expansion.

Let us introduce the centre-of-mass ($\boldsymbol{R}$) and relative ($\boldsymbol{r}$) coordinates:

$$\boldsymbol{R} = \tfrac{1}{2}(\boldsymbol{r}_1+\boldsymbol{r}_2), \qquad \boldsymbol{r} = \boldsymbol{r}_1-\boldsymbol{r}_2.$$

Eq. (1) then takes the form

$$\left(-\frac{\hbar^2}{4m}\nabla_R^2 - \frac{\hbar^2}{m}\nabla_r^2 + V_n(\boldsymbol{R}+\tfrac{1}{2}\boldsymbol{r}) + V_p(\boldsymbol{R}-\tfrac{1}{2}\boldsymbol{r}) + v(\boldsymbol{r})\right)\Psi(\boldsymbol{R}, \boldsymbol{r}) = E\psi(\boldsymbol{R}, \boldsymbol{r}). \qquad (2)$$

Let us also introduce the base vectors $\varphi_i(\boldsymbol{r})$, which are eigenfunctions of the follow-

ing equation:

$$\left(-\frac{\hbar^2}{m}\nabla_r^2+v(r)\right)\varphi_i(r)=\varepsilon_i\varphi_i(r). \tag{3}$$

This equation has only one bound state which is the deuteron state, labelled $\varphi_0(r)$, with eigenvalue $\varepsilon_0 = -2.226$ MeV. All the other states form the continuum of the spectrum. We will use periodic boundary conditions in the volume Ω for the states $\varphi_k(r)$ of the continuum.

Projecting eq. (2) over each of the base vectors one obtains

$$\left[-\frac{\hbar^2}{4m}\nabla_R^2+\langle 0|V_n+V_p|0\rangle\right]\Phi_0(R)=(E-\varepsilon_0)\Phi_0(R)-\sum_k\langle 0|V_n+V_p|k\rangle\Phi_k(R), \tag{4}$$

$$\left[-\frac{\hbar^2}{4m}\nabla_R^2+\langle k|V_n+V_p|k\rangle\right]\Phi_k(R)=(E-\varepsilon_k)\Phi_k(R)-\sum_{k'}\langle k|V_n+V_p|k'\rangle\Phi_{k'}(R) -\langle k|V_n+V_p|0\rangle\Phi_0(R), \tag{5}$$

where we have set

$$\langle 0|V_n+V_p|k\rangle=\langle k|V_n^*+V_p^*|0\rangle^* =\int d^3r\,\varphi_0^*(r)[V_n(R+\tfrac{1}{2}r)+V_p(R-\tfrac{1}{2}r)]\varphi_k(r), \tag{6}$$

and similarly for $\langle 0|V_n+V_p|0\rangle$ and $\langle k|V_n+V_p|k'\rangle$.

We also introduced

$$\Phi_0(R)=\int d^3r\,\varphi_0^*(r)\Psi(r,R), \tag{7}$$

$$\Phi_k(R)=\int d^3r\,\varphi_k^*(r)\Psi(r,R). \tag{8}$$

As one is interested in the elastic scattering of deuterons, one must take the boundary conditions at infinity for $\Phi_0(R)$ as a plane wave plus outgoing scattered waves. For $\Phi_k(R)$ the conditions are only outgoing scattered waves at infinity.

The summation over k in eq. (4) corresponds to the contribution of the break-up of the deuteron to its elastic channel. Neglecting this term in eq. (4) one obtains for the deuteron optical potential

$$V(R)=\langle 0|V_n+V_p|0\rangle\equiv V^{(1)}(R), \tag{9}$$

which is the approximation used by Rook [3]).

The term $\langle k|V_n+V_p|k\rangle$ in eq. (5) is of the order of $1/\Omega$ and therefore is negligible in comparison with the kinetic energy term. Neglecting also the summation over k' in eq. (5), one obtains:

$$\Phi_k(R)=\int d^3R'\langle R|\frac{1}{E-\varepsilon_k+\frac{\hbar^2}{4m}\nabla_R^2+i\delta}|R'\rangle\langle k|V_n+V_p|0\rangle\Phi_0(R'), \tag{10}$$

as an approximation for $\Phi_k(R)$. Substituting eq. (10) into eq. (4) one obtains as the resulting optical potential for the deuteron:

$$\langle R|V|R'\rangle = V^{(1)}(R)\delta(R-R')+\langle R|\Delta V^{(2)}|R'\rangle, \tag{11}$$

where

$$\langle R|\Delta V^{(2)}|R'\rangle = \sum_k \langle 0|V_n+V_p|k\rangle\langle R| \frac{1}{E-\varepsilon_k+\frac{\hbar^2}{4m}\nabla_R^2+i\delta} |R'\rangle\langle k|V_n+V_p|0\rangle. \tag{12}$$

The term $\langle R|\Delta V^{(2)}|R'\rangle$ represents the correction to Rook's potential calculated in second-order perturbation. Here, the perturbative interaction is clearly $V_n(r_1)+V_p(r_2)$ One of the main purposes of this work is to approximate the resulting non-local potential for the deuteron by a local one. Let us suppose there is an equivalent local potential for $\langle R|V'|R'\rangle$. Then, $\Phi_0(R)$ would satisfy the equation

$$\left[-\frac{\hbar^2}{4m}\nabla_R^2+V(R)\right]\Phi_0(R) = (E-\varepsilon_0)\Phi_0(R). \tag{13}$$

If one assumes that the matrix elements $\langle k|V_n+V_p|0\rangle$ have a slow dependence on R to justify their commutations with ∇_R^2, then one is justified in making the substitution

$$\frac{\hbar^2}{4m}\nabla_R^2 \to V(R)+\varepsilon_0-E,$$

as given by eq. (13), or better to write

$$\langle R| \frac{1}{E-\varepsilon_k+\frac{\hbar^2}{4m}\nabla_R^2+i\delta} |R'\rangle \approx \frac{\delta(R-R')}{\varepsilon_0-\varepsilon_k+V(R)}. \tag{14}$$

Eq. (12) then gives

$$\langle R|\Delta V^{(2)}|R'\rangle = \Delta V^{(2)}(R)\delta(R-R'), \tag{15}$$

where

$$\Delta V^{(2)}(R) = \sum_k \frac{\langle 0|V_n+V_p|k\rangle\langle k|V_n+V_p|0\rangle}{\varepsilon_0-\varepsilon_k+V(R)}. \tag{16}$$

The approximation involved in eq. (14) is best known as the adiabatic approximation. We shall give now a direct definition of the adiabatic potential $V(R)$ without using perturbation theory. One writes

$$\Psi(r, R) = \varphi(r, R)\chi(R), \tag{17}$$

where $\Psi(R, r)$ is the solution of eq. (2). The function $\varphi(r, R)$ represents the deformed deuteron in the neighbourhood of the target field and, by the adiabatic assumption,

is made to correspond to the eigenstate of the following equation:

$$\left[-\frac{\hbar^2}{m}\nabla_r^2+v(r)+V(\boldsymbol{R}+\tfrac{1}{2}\boldsymbol{r})+V_{\mathrm{p}}(\boldsymbol{R}-\tfrac{1}{2}\boldsymbol{r})\right]\varphi(\boldsymbol{r},\boldsymbol{R})=[\varepsilon_0+V(\boldsymbol{R})]\varphi(\boldsymbol{r},\boldsymbol{R}), \tag{18}$$

which continuously goes into $\varphi_0(r)$ for large values of R. The eigenvalue of eq. (18) depends on the adiabatic parameter $\boldsymbol{R}$ and $V(R)$, determined by eq. (18), is the deuteron optical potential as will be shown.

Substituting eq. (17) into eq. (2) and making use of eq. (18), one obtains

$$\left[-\frac{\hbar^2}{4m}\nabla_R^2+V(\boldsymbol{R})\right]\varphi(\boldsymbol{r},\boldsymbol{R})\chi(\boldsymbol{R})=(E-\varepsilon_0)\varphi(\boldsymbol{r},\boldsymbol{R})\chi(\boldsymbol{R}). \tag{19}$$

In general eq. (19) has no solution for $\chi(\boldsymbol{R})$. However, it is only required that the equation be satisfied for its projection on the deuteron state. Multiplying both sides of eq. (19) by $\varphi_0^*(r)$ and integrating over the relative coordinate, one obtains

$$\left[-\frac{\hbar^2}{4m}\nabla_R^2+V(\boldsymbol{R})\right]a_0(\boldsymbol{R})\chi(\boldsymbol{R})=(E-\varepsilon_0)a_0(\boldsymbol{R})\chi(\boldsymbol{R}), \tag{20}$$

where

$$a_0(\boldsymbol{R})=\int \mathrm{d}^3r\,\varphi_0^*(r)\varphi(\boldsymbol{r},\boldsymbol{R}). \tag{21}$$

Therefore, $\Phi_0(\boldsymbol{R})$, which has already been defined, is given by

$$\Phi_0(\boldsymbol{R})=a_0(\boldsymbol{R})\chi(\boldsymbol{R}),$$

and $V(\boldsymbol{R})$ is the adiabatic optical potential for the deuteron. It is a simple matter to verify that $V(\boldsymbol{R})$, given by eq. (18), when calculated by a perturbation expansion with $V_{\mathrm{n}}+V_{\mathrm{p}}$ as the perturbative interaction, gives the same results as eqs. (9) and (16).

There are two major sources of error in the adiabatic approximation. The first comes from the incorrect treatment of the pole in eq. (12) and therefore the real break-up of the deuteron is not taken into consideration. In sect. 5 the contribution from the break-up is studied. The second comes from the approximation of non-local potentials by local ones. In the appendix a discussion of this kind of error and its magnitude is presented.

The rest of this section introduces the specific approximations made in the treatment of eq. (18).

To simplify the procedure, it was assumed that $V_{\mathrm{n}}=V_{\mathrm{p}}$ and initially all spin dependence of $V_{\mathrm{n}}(r_1)$ could be neglected.

Let us write

$$\varphi(\boldsymbol{r},\boldsymbol{R})=\sum_{l=0}^{l=\infty}\frac{u_l(r,R)}{r}\,Y_l^0(\theta)$$

for the decomposition of $\varphi(\boldsymbol{r}, \boldsymbol{R})$ in spherical harmonics of the angle θ between $\boldsymbol{r}$ and $\boldsymbol{R}$.

Substituting the above equations into eq. (18) and projecting it over each l-state, one obtains the following equations

$$\left[-\frac{\hbar^2}{m}\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{\hbar^2 l(l+1)}{mr^2}+v(r)+V_{ll'}(r, R)\right] u_l(r, R)$$
$$= (\varepsilon_0+V(R))u_l(r, R)-\sum_{l'\neq l} V_{ll'}(r, R)u_{l'}(r, R), \quad (22)$$

where the potentials

$$V_{ll'} = 2\pi\int_{-1}^{+1} Y_l^{0*}(\theta)[V_{\mathrm{n}}(R+\tfrac{1}{2}r)+V_{\mathrm{p}}(R-\tfrac{1}{2}r)]Y_{l'}^0(\theta)\mathrm{d}(\cos\theta) \quad (23)$$

were introduced.

The function $\varphi(\boldsymbol{r}, \boldsymbol{R})$ which is pertinent to the problem, must be an eigen-s-state of eq. (22) i.e., a state predominantly $l = 0$. The first term in eq. (22) contributing to a non-spherical deformation of the deuteron is $V_{02}(r, R)$ which causes a quadrupole deformation. In the following such terms will be neglected and only the reduced equation

$$\left[-\frac{\hbar^2}{m}\frac{\mathrm{d}^2}{\mathrm{d}r^2}+v(r)+V_{00}(r, R)\right] u(r, R) = (\varepsilon_0+V(R))u(r, R) \quad (24)$$

will be considered, which is equivalent to having set

$$\varphi(\boldsymbol{r}, \boldsymbol{R}) = \frac{1}{\sqrt{4\pi r^2}}u(r, R).$$

Introducing the notation

$$\Delta V(R) = V(R)-V^{(1)}(R), \quad (25)$$

$$\Delta V_{00}(r, R) = V_{00}(r, R)-V^{(1)}(R), \quad (26)$$

eq. (24) takes the form

$$\left[-\frac{\hbar^2}{m}\frac{\mathrm{d}^2}{\mathrm{d}r^2}+v(r)+\Delta V_{00}(r, R)\right] u(r, R) = (\varepsilon_0+\Delta V(R))u(r, R). \quad (27)$$

The term $\Delta V(R)$ is the contribution to the deuteron potential which is specifically related to its deformation. The potential $\Delta V_{00}(R, r)$ is the interaction that causes this deformation.

3. The First Calculation

First the eigenvalue $\Delta V(R)$ of eq. (27) was investigated by perturbation theory. For $V_{\mathrm{n}}(r)$ the potential

$$V_{\mathrm{n}}(r) = -Uf(r, r_{\mathrm{on}}, a_{\mathrm{on}})-iWg(r, r_{\mathrm{wn}}, a_{\mathrm{wn}}) \quad (28)$$

was employed.

The form factors used were

$$f(r, r_{on}, a_{on}) = \frac{1}{1+e^{(r-r_{on}A^{\frac{1}{3}})/a_{on}}}, \tag{29}$$

$$g(r, r_{wn}, a_{wn}) = e^{-\frac{1}{4}(r-r_{wn}A^{\frac{1}{3}})^2/a^2_{wn}}. \tag{30}$$

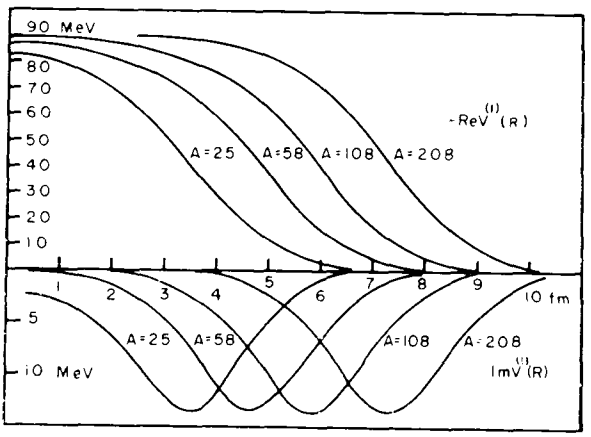


Fig. 1. The real and imaginary parts of $V^{(1)}(R)$ given by eq. (9) for different values of the mass number A of the target nucleus as indicated on the figure. The horizontal scale is R in units of fm. For the internal wave function of the deuteron the ground state solution of eq. (3) for the square well potential of eq. (32) has been used.

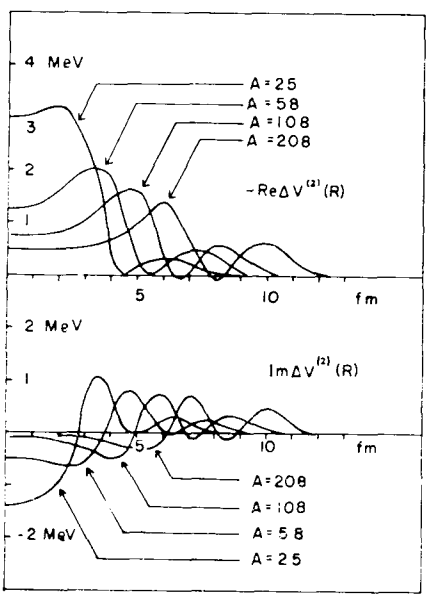


Fig. 2. The second-order perturbation corrections (real and imaginary parts) for the same nuclei as those of fig. 1. The intermediate states were solutions of eq. (3) for the square well potential given by eq. (32). The horizontal scale is R in units of fm.

The same geometrical parameters or the equivalent of those used by Perey [4]) were used:

$$r_{on} = r_{wn} = 1.25 \text{ fm},$$
$$a_{on} = 0.65 \text{ fm}, \qquad a_{wn} = 0.47 \text{ fm}. \tag{31}$$

The real and imaginary strengths were set equal to $U = 45$ MeV and $W = 11$ MeV.

The results of our calculation are shown in figs. 1 and 2. Fig. 1 shows the real and imaginary parts of $V^{(1)}(R)$ defined by eq. (9). For $\varphi_0(r)$ we used the ground state solution of eq. (3) taking $v(r)$ as the square well potential:

$$v(r) = -36.7 \text{ MeV} \qquad \text{for} \quad r < 2.0 \text{ fm},$$
$$= \quad 0 \qquad\qquad\quad \text{for} \quad r > 2.0 \text{ fm}. \tag{32}$$

If one gives $V^{(1)}(R)$ the same form as eq. (28) one gets

$$V^{(1)}(R) = -U_d f(R, r_{od}, a_{od}) - iW_d g(R, r_{wd}, a_{wd})$$

with the geometrical parameters

$$r_{od} = r_{wd} = 1.22 \text{ fm},$$
$$a_{od} = 0.85 \text{ fm}, \qquad a_{wd} = 0.75 \text{ fm}. \tag{33}$$

First it should be observed that the diffuseness parameters are larger due to the large size of the deuteron. Second, due to the curvature of the nuclear surface, one obtains a slightly smaller radius parameter.

The strength of the real part is equal to twice the corresponding strength for nucleons interacting with heavy nuclei ($A = 208$) and slightly smaller (84 MeV) for light nuclei ($A = 25$). The imaginary part does not show the same behaviour, since its strength (13.4 MeV) is almost equal to that for the nucleons (11 MeV).

Fig. 2 shows the results of the second-order perturbation calculation. The solutions of eq. (3) for the same square well potential given by eq. (32) were used for the intermediate states.

The striking result of this calculation is the smallness of $\Delta V^{(2)}(R)$ compared to $V^{(1)}(R)$. The general form of $\Delta V^{(2)}(R)$ does not change appreciably with mass number but it cannot be represented by the standard forms of eqs. (28)–(30).

In the appendix, the contribution from the non-local nature of the real part of the deuteron potential has been estimated. One finds that the non-local contribution is equivalent to a potential of the order of $\Delta V^{(2)}(R)$ but of opposite sign. Therefore one concludes that the second-order effects are, to a great extent, negligible and from the discrete families of potentials observed by Perey and Perey [1]), one should consider that family in which the strength of the real part is near the sum of the strengths for nucleons.

It is interesting to observe that if one had used volume absorptive potentials for $V_n(r_1)$ and $V_p(r_2)$, the conclusions would have been the same concerning the correc-

tions to $V^{(1)}(R)$ but the imaginary part of the deuteron potential would have been nearly equal to the sum of the imaginary parts of its nucleon potentials.

4. Spin-Orbit Potential

The smallness of the second-order term in the perturbation expansion of the deuteron optical potential has an immediate effect on the spin-dependent part of the potential. Watanabe [2]) has shown that the nucleon-nucleus spin-orbit interaction, in first-order perturbation also gives only a spin-orbit term for the deuteron potential. Therefore, one is forced to conclude that the higher-order terms which contain a tensorial component [5]) play a small role in deuteron scattering and polarization data.

Assuming the nucleon-nucleus spin-orbit potential to be of the form [5])

$$V_{\sigma\cdot l}(r) = \left(\frac{\hbar}{m_\pi c}\right)^2 V_{\text{s.o.}}\, h(r)\boldsymbol{\sigma}\cdot\boldsymbol{l},$$

one obtains for the deuteron spin-orbit potential

$$V_{S\cdot L}(R) = 2\left(\frac{\hbar}{m_\pi c}\right)^2 V_{\text{s.o.}}\, h_{\text{d}}(R)\boldsymbol{S}\cdot\boldsymbol{L}, \tag{34}$$

where $\boldsymbol{S}$ is the spin operator for the deuteron and $\boldsymbol{L}$ its orbital angular momentum operator.

The form factor $h_{\text{d}}(R)$ is given by

$$h_{\text{d}}(R) = \int \mathrm{d}^3 r |\varphi_0(r)|^2 h(|\boldsymbol{R}+\tfrac{1}{2}\boldsymbol{r}|)\left(1+\frac{1}{2}\frac{\boldsymbol{r}\cdot\boldsymbol{R}}{R^2}\right). \tag{35}$$

The Thomas form factor

$$h(r) = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} f(r, r_{\text{on}}, a_{\text{on}}) \tag{36}$$

was used with $f(r, r_{\text{on}}, a_{\text{on}})$ given by eq. (29) and r_{on} and a_{on} by eq. (31). The form factor for the deuteron spin-orbit potential $h_{\text{d}}(R)$, given by eq. (35), came out to be a function with the same form as eq. (36) except for the geometrical parameters which turned out to be the same as those given for r_{od} and a_{od} of eq. (33). Therefore, it is interesting to observe that the effect of averaging over the deuteron ground state yields the same geometrical parameters for both the central and spin-orbit terms, thus maintaining the relation used for nucleons (eq. (36)) between these two terms.

5. Second Calculation

Eq. (28) was solved numerically for $A = 58$ in order to verify the smallness of the second-order corrections just reported and, by an appropriate change of boundary

conditions, to estimate the contribution from the break-up of the deuteron. The problem was simplified by considering only the real part of the nucleon optical potential which was taken to be of the same form as for the first calculation but with a strength of 50 MeV. For $v(r)$, the more realistic Yukawa potential

$$v(r) = -V_0 \frac{e^{-r/a}}{r/a}$$

was used with $V_0 = 42.6$ MeV and $a = 1.58$ fm.

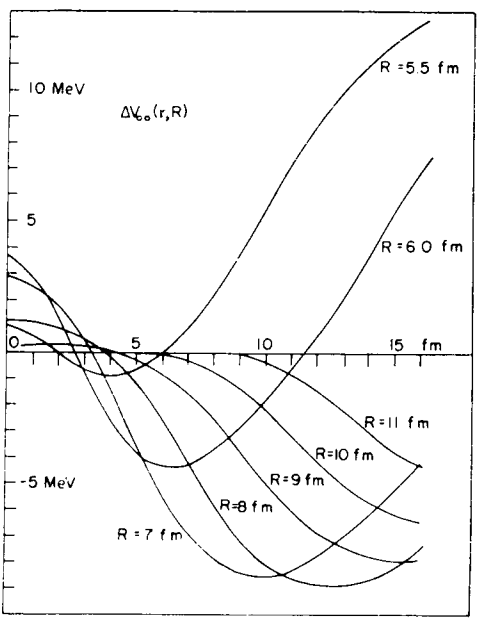


Fig. 3. The deformation potential $\Delta V_{00}(r, R)$ defined by eq. (26) for different values of R as indicated. The horizontal scale is r in units of fm.

Fig. 3 shows ΔV_{00} as a function of r for different values of R. For $R > 5.5$ fm the attractive part of ΔV_{00} moves away from $r = 0$ as R increases, and the nuclear interaction with the deuteron practically vanishes for the nucleus under consideration at $R = 10$ fm. For $R < 5.5$ fm, ΔV_{00} looks very much like the curve obtained for $R = 5.5$ fm shown in fig. 3 but with the position of the barrier moved from the origin and its height increasing more rapidly as R decreases. This barrier is the surface of the nucleus seen from inside. It should be mentioned that at $R = 5.5$ fm, the deuteron feels the weakest disruptive force. This is the reason for the minima in $\Delta V^{(2)}(R)$ shown in fig. 2 for $A = 58$. The potential $\Delta V_{00}(r, R)$ acts differently on the deuteron accordingly as $R < 5.5$ fm (the inside region) or as $R > 5.5$ fm (the region outside the nucleus). For the inside region the surface of the nucleus compresses the deuteron decreasing its size. For the outside region the effect is the opposite; the near-by nucleus

induces the break-up of the deuteron. In the adiabatic approximation this fact is reflected in the large size which the deformed deuteron acquires when it is close to the outer region of the nucleus. In order to include the possibility of a break-up of the deuteron and therefore, to make the adiabatic approximation more realistic, the de-

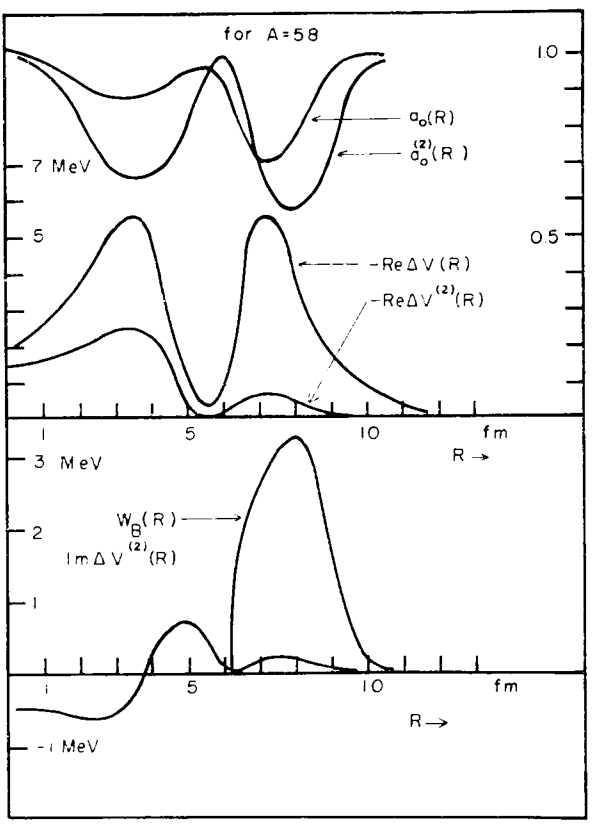


Fig. 4. The two uppermost curves represent $a_0(R)$. The one given by eq. (21) in second-order perturbation theory is labelled $a_0^{(2)}(R)$ and by direct numerical integration of eq. (27) is labelled $a_0(R)$. The vertical scale is on the right side of the figure. The other four curves are the real and imaginary parts of the correction to $V^{(1)}(R)$. When calculated in second-order perturbation theory they are labelled $\mathrm{Re}\,\Delta V^{(2)}(R)$ and $\mathrm{Im}\,\Delta V^{(2)}(R)$ and when by direct integration of eq. (27) they are labelled $\mathrm{Re}\,\Delta V(R)$ and $W_B(R)$. The horizontal scale for all curves is R in units of fm. The vertical scale for these four curves is on the left side of the figure.

formed deuteron was treated as a decaying state of the bound pair of nucleons. This change is attained by modifying the boundary condition that the solution $u(r, R)$ of eq. (27) must satisfy. We have set

$$\left[\frac{\partial u(r, R)/\partial r}{u(r, R)}\right]_{r=c} = ik(R), \tag{37}$$

which is the appropriate boundary condition for decaying states. The quantity $k(R)$ defined by

$$\frac{\hbar^2}{m} k^2(R) = \mathrm{Re}\, \Delta V(R) + \varepsilon_0 - \mathrm{Re}\, \Delta V_{00}(r, R), \tag{38}$$

is the wave number for the relative motion of the pair when their distance is larger than C. Due to the condition expressed by eq. (37), whenever the value of $k(R)$ given by eq. (38) is real, the eigenvalue of eq. (18) is complex even though ΔV_{00} is real. The imaginary part $W_B(R)$ of this eigenvalue is responsible for the absorption due to deuteron break-up †.

The choice of the value of C at which the condition given by eq. (37) is imposed, is somewhat arbitrary. It should be mentioned that the adiabatic approximation is obtained from the boundary condition above by taking the limit for large values of C. A too small value for C (smaller than the deuteron mean square radius), is physically unjustifiable. It is important also to mention that the general behaviour of $W_B(R)$ as exhibited in fig. 4 is essentially independent of the value of C as long as it is kept within a reasonable range.

For $R < 5.5$ fm, independent of the value taken for C, $W_B(R) = 0$ is always obtained, as eq. (38) gives only imaginary values for $k(R)$. The value of C was set equal to 10 fm which made the calculation for $R < 5.5$ fm the same as the adiabatic approximation. This already indicates that break-up of deuterons in the inner region of the target nucleus is negligible. It might seem paradoxical to find that deuterons are stable inside nuclei. However, this fact should not be related to the approximations involved in solving eq. (1) but rather to the more fundamental fact that eq. (1) does not include the exclusion principle effects which are relevant for the discussion of the deuteron stability in nuclear matter.

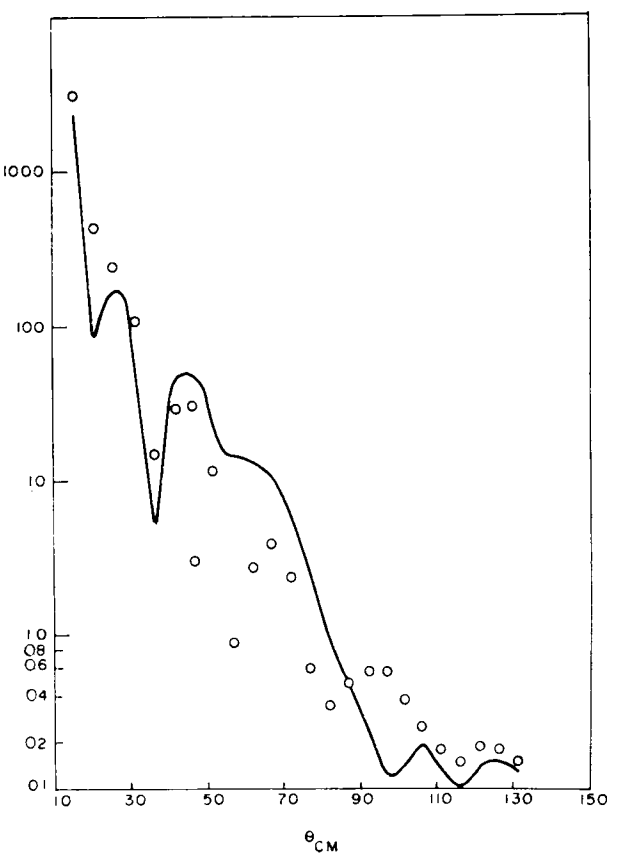
For values of $R > 10$ fm, as already mentioned, the deuteron is essentially free and $\mathrm{Re}\Delta V(R)$ and $W_B(R)$ are practically zero.

For 5.5 fm $< R <$ 10 fm, where the break-up of the deuteron contributes to its optical potential, C also was set equal to 10 fm. This value for C was chosen such as to emphasize the break-up contribution. It is expected therefore that the magnitude of $W_B(R)$ as calculated is somewhat exaggerated.

Fig. 4 shows a summary of our results and permits a comparison of the two calculations. First one notices that the general behaviour of $\mathrm{Re}\Delta V(R)$ and $\mathrm{Re}\Delta V^{(2)}(R)$ is similar but quantitatively $\mathrm{Re}\Delta V(R)$ is larger than $\mathrm{Re}\Delta V^{(2)}(R)$, especially in the region outside the nucleus. Nevertheless, it is still small compared with $\mathrm{Re}V^{(1)}(R)$. Comparing the imaginary parts one sees that the larger contribution comes from $W_B(R)$ which strictly speaking is a non-adiabatic correction to $\Delta V(R)$. One observes

† One might wonder why a direct calculation of the break-up contribution to the optical potential was not made by calculating the residue in eq. (12). The reason is because the contribution from eq. (12) has the form of a non-local potential and it would have been necessary to solve the problem of approximating this contribution by a local potential. The procedure described above is the manner by which we calculate the local potential contribution of deuteron break-up to its optical potential.

that W_B is peaked at 8 fm, which is outside the nucleus. This fact is certainly in agreement with the stripping mechanism at low energies where the stripping radius is usually very large. It is also in agreement with a more recent trend in elastic deuteron scattering analysis where a larger radius for the imaginary part of the potential seems to be necessary †.



$\sigma(\theta)$ in mb/sr. o-experimental points, —-theoretical curve.

Fig. 5. The elastic cross section $\sigma(\theta)$ of deuterons on ^{58}Ni at 27.5 MeV incident energy. The open circles are the experimental points of Testoni, Mayo and Rosenblatt. The solid curve is the theoretical prediction given by $V^{(1)}(R)$. The vertical scale is $\sigma(\theta)$ in units of mb/sr and the horizontal scale the angle of scattering $\theta_{c.m.}$ in the centre of mass of the system.

On the upper part of fig. 4 is shown $a_0(R)$ given by eq. (21) for both calculations; $a_0(R)$ has a very weak dependence on R except in the region outside the nucleus where the instability of the deuteron occurs. In the appendix it is shown that the weaker the dependence of $a_0(R)$ on R the better is the justification for the adiabatic

† We wish to thank Dr. P. E. Hodgson for a private communication of this result.

approximation. Therefore one sees that the approximation is quite good in theinner region of the nucleus.

6. Conclusions

The over-all conclusion that we can draw from our results is that the corrections to $V^{(1)}(R)$ due to the deformation of the deuteron are small.

Fig. 5 illustrates this fact, where one has plotted the experimental points of Testoni, Mayo and Rosenblatt [6]) for the elastic scattering of deuterons by ^{58}Ni at 27.5 MeV, together with the theoretical prediction (solid curve) of the cross sections given by $V^{(1)}(R)$. One observes that at this energy $V^{(1)}(R)$ reproduces most of the features of the experimental data as one would expect on the basis of our analysis. This fact enables us to draw a few conclusions of practical interest for optical model analysis. First, the strength of the deuteron central potential must be almost equal to the sum of the strengths of the neutron and proton central potentials. The uncertainty in this statement lies in the fact that these strengths for nucleons, are energy-dependent. It is consistent with the adiabatic approximation to say that each nucleon has half of the energy of the incident deuteron, therefore giving a criteria for the choice of the energy at which to choose the nucleon central potential strength. This result is sufficient to select one from the many discrete families of acceptable potentials found by Perey and Perey [1]). Second, the presence of tensorial terms in the spin-dependent part of the potential as suggested by Satchler [5]) should be negligible. Moreover, the spin-orbit term for the deuteron has the same geometrical parameters as the central term, thereby exhibiting the same relation as that used for nucleons. Third, the contribution of the break-up of the deuteron is such as to justify a larger radius parameter for the imaginary part of its optical potential.

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Appendix

ERROR INVOLVED IN THE APPROXIMATION OF EQ. (13)

For simplicity V_n and V_p are assumed to be real potentials.

Let us introduce the notation ,

$$e = \varepsilon_0 - \varepsilon_k + V(R),$$

$$e' = E - \varepsilon_k + \frac{\hbar^2}{4m} \nabla_R^2,$$

$$\delta = e - e'.$$

Then

$$\frac{1}{e'} = \frac{1}{e-\delta} = \frac{1}{e} + \frac{1}{e}\delta\frac{1}{e} + \frac{1}{e}\delta\frac{1}{e}\delta\frac{1}{e} + \ldots$$

Thus the error made in the approximation under consideration to the first order of the quantity δ, is given by

$$\delta V = \sum_{k} \langle 0|V_n + V_p|k\rangle \frac{1}{e}\left(-\frac{\hbar^2}{4m}\nabla_R^2 + V(R) + \varepsilon_0 - E\right)\frac{1}{e}\langle k|V_n + V_p|0\rangle,$$

which can be put in the more compact form

$$\delta V = \sum_{k} \langle 0|V_n + V_p|k\rangle \frac{1}{e}\left[-\frac{\hbar^2}{4m}\nabla_R^2, \frac{1}{e}\langle k|V_n + V_p|0\rangle\right], \tag{A.1}$$

where the square brackets in the above expression signify the commutator of the quantities involved.

The quantity $a_0(R)$, defined by eq. (12), in second-order perturbation, satisfies the equation

$$a_0^2(R)\left(1 + \sum_{k} \langle 0|V_n + V_p|k\rangle \frac{1}{e^2}\langle k|V_n + V_p|0\rangle\right) = 1.$$

So, let us define

$$b_0 = \frac{1}{a_0^2} - 1 = \sum_{k} \langle 0|V_n + V_p|k\rangle \frac{1}{e^2}\langle k|V_n + V_p|0\rangle. \tag{A.2}$$

After some algebraic manipulation, one obtains

$$\left[-\frac{\hbar^2}{4m}\nabla_R^2, b_0\right] = 2\delta V + \sum_{k}\left[\left[-\frac{\hbar^2}{4m}\nabla_R^2, \frac{1}{e}\langle 0|V_n + V_p|k\rangle\right], \frac{1}{e}\langle k|V_n + V_p|0\rangle\right].$$

Neglecting the commutator in the right-hand side of the above equation, one finally arrives at

$$\delta V = -\frac{\hbar^2}{8m}[\nabla_R^2, b_0] = -\frac{\hbar^2}{8m}\frac{d^2 b_0}{dR^2} - \frac{\hbar^2}{4m}\frac{db_0}{dR}\frac{1}{R}\frac{\partial}{\partial R}R. \tag{A.3}$$

The presence of a derivative operator in eq. (A.3) is the consequence of the non-local nature of the original potential. In order to estimate the error committed by neglecting δV, one has first to extract from it the contribution that actually produces a modification on the resulting scattering amplitude. The wave function $\Phi_0(R)$ describing the deuteron elastic scattering now satisfies the equation

$$\left(-\frac{\hbar^2}{4m}\nabla_R^2 + V(R) - \frac{\hbar^2}{8m}\frac{d^2 b_0}{dR} - \frac{\hbar^2}{4m}\frac{db_0}{dR}\frac{1}{R}\frac{\partial}{\partial R}R\right)\Phi_0(R) = (E - \varepsilon_0)\Phi_0(R),$$

where $V(R)$ is the potential in the adiabatic approximation.

Introducing the partial wave decomposition of $\Phi_0(R)$

$$\Phi_0(R) = \sum_{l=0} \frac{u_l(R)}{R} Y_l^0(\theta),$$

one obtains

$$\left(\frac{d^2}{dR^2} - \frac{l(l+1)}{R^2} - \frac{4m}{\hbar^2} V(R) + \frac{1}{2}\frac{d^2 b_0}{dR^2} + \frac{db_0}{dR}\frac{d}{dR}\right) u_l(R) = K^2 u_l(R), \qquad (A.4)$$

where

$$K^2 = \frac{4m}{\hbar^2}(E - \varepsilon_0).$$

Introducing the transformation

$$u_l(R) = \alpha(R)\omega_l(R) \qquad (A.5)$$

and choosing $a(R)$ such as to cancel the first-order derivatives in the resulting equation for $\omega_l(R)$, one arrives at

$$\left(\frac{d^2}{dR^2} - \frac{l(l+1)}{R^2} - \frac{4m}{\hbar^2} V(R) + \frac{1}{\alpha}\frac{d\alpha}{dR}\frac{db_0}{dR} + \frac{1}{\alpha}\frac{d^2\alpha}{dR^2} + \frac{1}{2}\frac{d^2 b_0}{dR^2}\right) \omega_l(R) = K^2 \omega_l(R), \qquad (A.6)$$

with

$$\frac{1}{\alpha}\frac{d\alpha}{dR} = -\frac{1}{2}\frac{db_0}{dR}. \qquad (A.7)$$

Eq. (A.7) can easily be integrated and one finds

$$\alpha(R) = e^{-\frac{1}{2}b_0(R)}.$$

Therefore, both $u_l(R)$ and $\omega_l(R)$ have the same scattering amplitude. The contribution of δV to the deuteron scattering is given therefore only by

$$\delta V = -\frac{\hbar^2}{4m}\left(\frac{1}{2}\frac{d^2 b_0}{dR^2} + \frac{1}{\alpha}\frac{d^2\alpha}{dR^2} + \frac{1}{\alpha}\frac{d\alpha}{dR}\frac{db_0}{dR}\right), \qquad (A.8)$$

which is easily obtained from eq. (A.6). Substituting eq. (A.7) into eq. (A.8) one obtains

$$\delta V = \frac{\hbar^2}{4m}\left(\frac{1}{2}\frac{db_0}{dR}\right)^2.$$

The correction δV can be related to $a_0(R)$ as it is known that

$$\frac{db_0}{dR} = -\frac{2}{a_0^3(R)}\frac{da_0}{dR}.$$

Thus we have

$$\delta V = \frac{\hbar^2}{4m}\left(\frac{1}{a_0^3(R)}\frac{\mathrm{d}a_0}{\mathrm{d}R}\right)^2.$$

Clearly, if $a_0(R)$ is a constant then the adiabatic approximation is exact and in particular $\delta V = 0$.

On fig. 4 the value of $a_0(R)$ is plotted for $A = 58$. As δV is proportional to $(\mathrm{d}a_0/\mathrm{d}R)^2$, it has an oscillatory character. A rough estimate of $\delta V(R)$ at its peak ($R \approx 8$ fm) gives a value of 5 MeV for $\delta V(R)$. As δV has the opposite sign from $\Delta V^{(2)}(R)$, its contribution has an over-all effect of cancelling the contribution of $\Delta V^{(2)}(R)$.

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