ANYONS IN SPIN LIQUIDS

K. Hallberg and C. A. Balseiro

Centro Atómico Bariloche and Instituto Balseiro

8400 Bariloche, Argentina

Abstract

We use a variational procedure to show that starting with the flux phase state of a Heisenberg model, holes added to the system behave as anyons. The hole distorts the gauge flux in its surroundings. The amount of flux carried by the hole depends on the ratio between the magnetic (J) and the kinetic (t) parameters. For $J/t \to 0$ the quasiparticles are fermions while for $J/t \to \infty$ they are semions.

PACS number(s): 75.10.Jm, 74.65+n

Immediately after the discovery of high Tc superconductivity in the cuprate materials, Anderson proposed that superconductivity could be due to the spin liquid nature of the Cu-O planes of these compounds[1]. Since then a lot of theoretical activity has been devoted to the difficult task of constructing a consistent theory of superconductivity in spin liquid (SL) systems[2-4].

Different approaches have been used. First the RVB state with magnetic excitations (spinons) with spin 1/2 and charge 0 obeying Fermi statistics and charge excitations (holons) with spin 0 and charge e obeying Bose statistics[5]. The natural consequence of these ideas is that superconductivity is due to the Bose condensation of the charge excitations. In this theory the flux quantum should be h/e rather than $h/2e$ as is experimentally observed. Various modifications of the theory have been proposed to overcome this difficulty.

A different approach has been proposed by Laughlin and collaborators[6]. They mapped a spin model of a 2-D triangular lattice onto a Bose gas with short range repulsive interactions in the presence of a strong external magnetic field[7]. These results make evident some analogies between the SL and the quantized Hall effect (QHE). Moreover, the Kalmeyer and Laughlin wave function[7] can be adapted to describe the SL state of a 2-D square lattice[8].

Since the excitations in the QHE have been shown to be anyons[9], quasiparticles obeying fractional statistics[10], the analogies between QHE and SL suggested that excitations in SL could also obey fractional statistics. These ideas triggered extensive studies on the ideal gas of anyons. Recent analytical and numerical works suggest that a gas of anyons is superconducting at low temperatures[11]. However the energy of the SL phase obtained with the Kalmeyer and Laughlin type of wave functions in systems with short range interactions is not particularly good, much better energies are obtained with other types of trial wave functions. This fact could leave some grounds of doubt on whether this analogy is the correct way to follow to obtain quantitative results.

The mean field theory of the SL phase of Heisenberg type Hamiltonians developed by Affleck and Marston[12] and independently by Kotliar[13] have shown to be another good alternative to describe the SL state. The flux phase states seem to be good trial wave functions and it has been recently argued that these flux phases in doped systems are superconducting[3]. There are some similarities between flux phases and anyon superconductivity, in both pictures there is a spontaneous generation of a strong magnetic field.

A complete theory of anyonic superconductivity for the cuprate materials should begin with 2-D spin models, in this framework it should obtain fractional statistic excitations and then derive superconductivity for the anyon gas. Although the last step (superconductivity of the anyon gas) has been extensively studied it is not clear yet how to build fractional statistic excitations starting from the SL state.

In this Letter we use a variational procedure to show that starting with the flux phase state, holes added to the system will behave as anyons. The idea is quite simple: in analogy with polarons or bags we show that a hole distorts the background in its neighborhood. In the present case an added hole changes the gauge flux in its surroundings and in this way we are able to build an excitation which consists of a flux line attached to the hole. A collection of such excitations will obey fractional statistics, the statistic being parametrized by the amount of flux carried by each hole. We show that this flux depends on the ratio between the magnetic (J) and kinetic (t) parameters. For $J/t \to 0$ the quasiparticles are fermions while for $J/t \to \infty$ they are semions (half-statistics quasiparticles).

Consider the nearest neighbor t-J model given by

$$H = -t \sum_{ij\sigma} c^{+}_{i\sigma} c_{i\sigma} + J \sum_{ij} \mathbf{S}_i . \mathbf{S}_j \qquad (1)$$

with the constraint of no double occupation.

This Hamiltonian was first derived starting from the Hubbard model in the large U limit (where $J=4t^2/U$). Although this derivation implies $J \ll t$ we will consider the whole parameter range.

In the undoped materials like $La_2Cu\,O_4$, the $Cu\,O_2$ planes are antiferromagnetic and insulating. The magnetic properties of these systems can be described using a Heisenberg model with one spin per unit cell, *ie.* the second term of Hamiltonian (1). Even in 2-D the ground state of a Heisenberg model probably has long range antiferromagnetic order. Presumably doping, which introduces holes in the magnetic planes, destroys the long range order stabilizing a SL phase. How doping destroys magnetism to give a SL phase is a difficult problem. In any case it is interesting to start with the undoped SL phase. A good understanding of this phase will allow us to give a reasonable description of the doped systems.

A good ansatz for the SL wave function is a generalization of the Gutzwiller wave function which is equivalent to the flux phase states described in refs.12 and 13. This type of trial functions are built by projecting out all double on-site occupations from uncorrelated electrons :

$$|\Psi\rangle = P_d \prod_{\nu=1,\sigma} c^{+}_{\nu\sigma} |0\rangle \qquad (2)$$

where P_d is the projection operator and $c^{+}_{\nu\sigma}$ creates one electron on a lattice of N sites with quantum numbers ν,σ and energy $E_{\nu,\sigma}$. The flux phases are obtained by considering tight binding electrons moving in a magnetic field perpendicular to the lattice. This fictitious field, which is associated with local gauge symmetries, is a variational parameter. For the undoped system one should consider a half filled electron band.

In what follows we present results obtained using two different approaches: i) exactly projecting the wave function of the uncorrelated electrons in a small cluster and, ii) using the Gutzwiller approximation to evaluate expectation values. In the Gutzwiller approximation the total energy of the undoped system is given by

$$E_J = g < J \sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j >_0 \qquad (3)$$

where $<\ldots>_0$ indicates the expectation value in the unprojected wave function and the classical factor g in the present case is g=4.

In this approximation then it is easy to show that the magnetic energy is simply given by

$$E_J = - \frac{3J}{16} g \left(\sum_{\nu,\sigma}^{occ} E_{\nu\sigma} \right)^2 \qquad (4)$$

The magnetic energy is then proportional to the square of the total energy of the uncorrelated system. It has recently been shown[14] that the energy of N_e uncorrelated electrons in a lattice as a function of an external magnetic field has an absolute minimum when the flux per plaquette is given by $\phi = n\, \phi_0 / 2$ where $n = N_e / N$ is the electron density per site and ϕ_0 the flux quantum. The Gutzwiller approximation for the undoped system implies that the gauge flux per plaquette is $\phi_0 / 2$. By exactly projecting out the double occupied sites on a finite square lattice we have recently shown that this is an intrinsic property of the Gutzwiller ansatz and not a consequence of the approximation involved in Eq.(3). Recent quantum Monte Carlo studies also support these results[15].

If holes are added to the lattice then there are two contributions: the magnetic energy E_J (that tends to reduce the flux per plaquette) and the kinetic energy proportional to t.

We study states in which a single hole is added to the system. We show that it is energetically favourable to change the flux locally in the vicinity of the hole. To do so we divide the problem into two steps: first we study the problem of a fixed hole and then include the effect of its dynamics.

Consider a fixed hole in a lattice which plays the role of a vacancy. To minimize the magnetic energy we found convenient to change the magnetic flux in the four plaquettes surrounding the vacancy. We consider states with $\phi = \phi_0 / 2 - \phi' / 4$ in the neighborhood of the hole and $\phi = \phi_0/2$ elsewhere. We compute the magnetic energy by exactly projecting the uncorrelated wave function in a small lattice (4x4). The results of the energy E_J as a function of ϕ' are shown in figure 1 for two different lattices: one with open boundary conditions in both x and y directions and the other with periodic boundary conditions only in the x direction.

The difference between the two cases studied indicates that the sample is too small and the energy depends strongly on the boundary conditions. However the cualitative behaviour is the same in both cases. The absolute minimum of E_J is obtained when the magnetic flux attached to the hole is $\phi' \cong \phi_0/2$. Unfortunatelly we cannot study larger lattices with this procedure. In any case we can compare this result with the one obtained with the Gutzwiller approximation. In figure 2 we present the results of the energy E_J as a function of ϕ' for a bigger lattice (11x11) as obtained with this approximation. In this case the results are not very sensitive to the boundary conditions and the results are qualitatively similar to those obtained in the previous case, the absolute minimum being located in $\phi' = \phi_0/2$. The magnetic energy is gauge independent, however periodic boundary conditions in the x direction can be used only if the Landau gauge is used.

The extra flux ϕ' increases the correlation $\langle S_i . S_j \rangle$ in the surroundings of the hole. In fig. 3 we present a scheme of the lattice with the spin-spin correlations of each bound.

Now we discuss the effect of the hopping. For this purpose, we define a set of wave functions $\Psi(\phi', R_i)$ describing the hole localized at site i with coordinate R_i. The wave functions

$\Psi(\phi',R_i)$ and their diagonal energy $E_{\phi'}$ depend on the flux ϕ' attached to the hole. The hopping term changes the coordinate of the hole leaving the flux line in its original position. The effective hopping of the quasiparticle (hole plus flux line) betwen neighbouring sites i and j is given by

$$t^{ij}_{eff}(\phi') = t < \Psi(\phi',R_i) \mid \psi^{(j)}(\phi',R_i) > \qquad (5)$$

where $|\psi^{(j)}(\phi',R_i)>$ is the wave function corresponding to the hole at site i and the flux line centered at site j. These matrix elements are gauge dependent, in particular the Landau gauge breaks the symmetry of the lattice: $|t^{ij}_{eff}(\phi')|$ depends on the direction of the hopping. In the symmetric gauge clearly the lattice symmetry is preserved. As in the ordinary flux phase both the gauge field and the gauge are to be taken as variational parameters. We evaluate the matrix elements using different gauges and the two schemes described above: the exactly projected states in a small cluster and in the Gutzwiller approximation. In all cases the results are again qualitatively the same. The modulus of the effective hopping $|t^{ij}_{eff}(\phi')|$ is maximum for $\phi'=0$ and has a minimum for $\phi'=\phi_0$. Projecting the Hamiltonian in the space defined by the set $\{ \Psi(\phi',R_i)\}$ we obtain

$$\tilde{H} = \sum_i E_{\phi'} \, a^+_i a_i + \sum_{ij} (t^{ij}_{eff}(\phi') \, a^+_i a_j + hc) \qquad (6)$$

where a^+_i creates a quasiparticle at site i. The energy of a single quasiparticle is then

$$E = E_{\phi'} - 4 \,\alpha \,|t^{ij}_{eff}(\phi')| \qquad (7)$$

where α is a number of the order of 1 and depends on the phases of $t^{ij}_{eff}(\phi')$. This energy is minimized with respect to ϕ' . The two contributions to the energy compete with each other shifting the absolute minimum from $\phi' \cong \phi_0/2$ when $J/t >> 1$ to $\phi' \cong 0$ when $J/t << 1$ as shown in fig. 4. A collection of these quasiparticles will behave as a gas of anyons being fermions for $J/t \to 0$ and semions for $J/t \to \infty$. The present calculations consider a single hole and consequently our conclusions are valid only for a dilute gas of quasiparticles (low doping). Effective interactions between the quasiparticles could arise by changing the flux in the surroundings of two closed holes like in bipolarons or bags with two excitations. These effects will be presented elsewhere.

In summary we have shown that starting with the flux phases, complex excitations can be obtained by changing the gauge flux in the neighbourhood of a hole. The magnetic energy tends to generate a flux line attached to the hole carrying a flux $\phi' = \phi_0/2$. In this case the average flux per plaquette in the lattice is again $\bar{\phi}=n\,\phi_0/2$ as is the case of the ordinary flux phase. The difference is that in the present case the change in the gauge flux is concentrated around the hole. The kinetic energy tends to keep a flux $\phi_0/2$ in every plaquette ($\phi' = 0$). The competition between these two contributions defines the statistics of the collective excitations which changes from fermionic to semionic as J/t increases from zero to infinite.

References

1. P.W.Anderson, Science **235**, 1196 (1987)

2. G.Baskaran and P.W.Anderson, Phys.Rev.B **37**, 580 (1988)

3. P.Lederer, D.Poilblanc and T.M.Rice, Phys.Rev.Lett.**63**, 1519 (1989)

4. Y.Chen, F.Wilczek, E.Witten and B.I.Halperin, to be published.

5. S.A.Kivelson, D.S.Rokhsar and J.P.Sethna, Phys.Rev.B **35**, 8865 (1989)

6. R.B.Laughlin, Science **242**, 525 (1988); V.Kalmeyer and R.B.Laughlin, Phys.Rev.B **39**, 11879 (1989)

7. V.Kalmeyer and R.B.Laughlin, Phys.Rev.Lett. **59**, 2095 (1987)

8. Z.Zou, B.Doucot and B.S.Shastry, Phys.Rev.B **39**, 11424 (1989)

9. F.D.M.Haldane, Phys. Rev. Lett. **51**, 605 (1983) and B.I.Halperin, Phys.Rev.Lett. **52**, 1583 (1984)

10. F.Wilczek, Phys.Rev.Lett.**48**, 1144 (1982); *ibid* **49**, 957 (1982).

11. G.S.Canright, S.M.Girvin and A.Brass, Phys.Rev.Lett.**63**, 2295 (1989); A.Fetter, C.Hanna and R.B.Laughlin, Phys.Rev.B **39**, 9679 (1989).

12. I.A.Affleck and J.B.Marston, Phys.Rev.B **37**, 3774 (1988).

13 G.Kotliar, Phys.Rev.B **37**, 3664 (1988).

14. I.Hasegawa, P.Lederer, T.M.Rice and P.B.Wiegmann, Phys.Rev.Lett. **63**, 907 (1989).

15. S.Liang and S.Trivedi, Phys.Rev.Lett. **64**, 232 (1990).

Figure Captions:

1) Magnetic energy E_J (in units of J) as a function of the flux attached to the hole obtained by exactly projecting the uncorrelated wave function in a small lattice (4x4); a) with periodic boundary conditions (bc) in the x direction; b) with open bc in the x and y directions. Note that both, the shape and the amplitude depend on the bc.

2) Magnetic energy E_J (in units of J) as a function of the flux attached to the hole obtained using the Gutzwiller approximation in a 11x11 site cluster with open boundary conditions (bc). The difference between using open bc or periodic bc in the x direction is at most 3% .

3) Correlation functions for the spins around the hole. The spin-spin correlation has the square symmetry of the lattice and in the figure we show only one sector of the 11x11 cluster. The big black circle indicates the position of the hole and the numbers in the links indicate the values of $\langle S_i . S_j\rangle|_{\phi'=.5} - \langle S_i . S_j\rangle|_{\phi'=0}$. Zeros indicate that this difference is smaller than 10^{-3}.

4) The flux ϕ' that minimizes the total energy as a function of the ratio J/t for a 11x11 cluster, α=1 and a symmetric gauge.

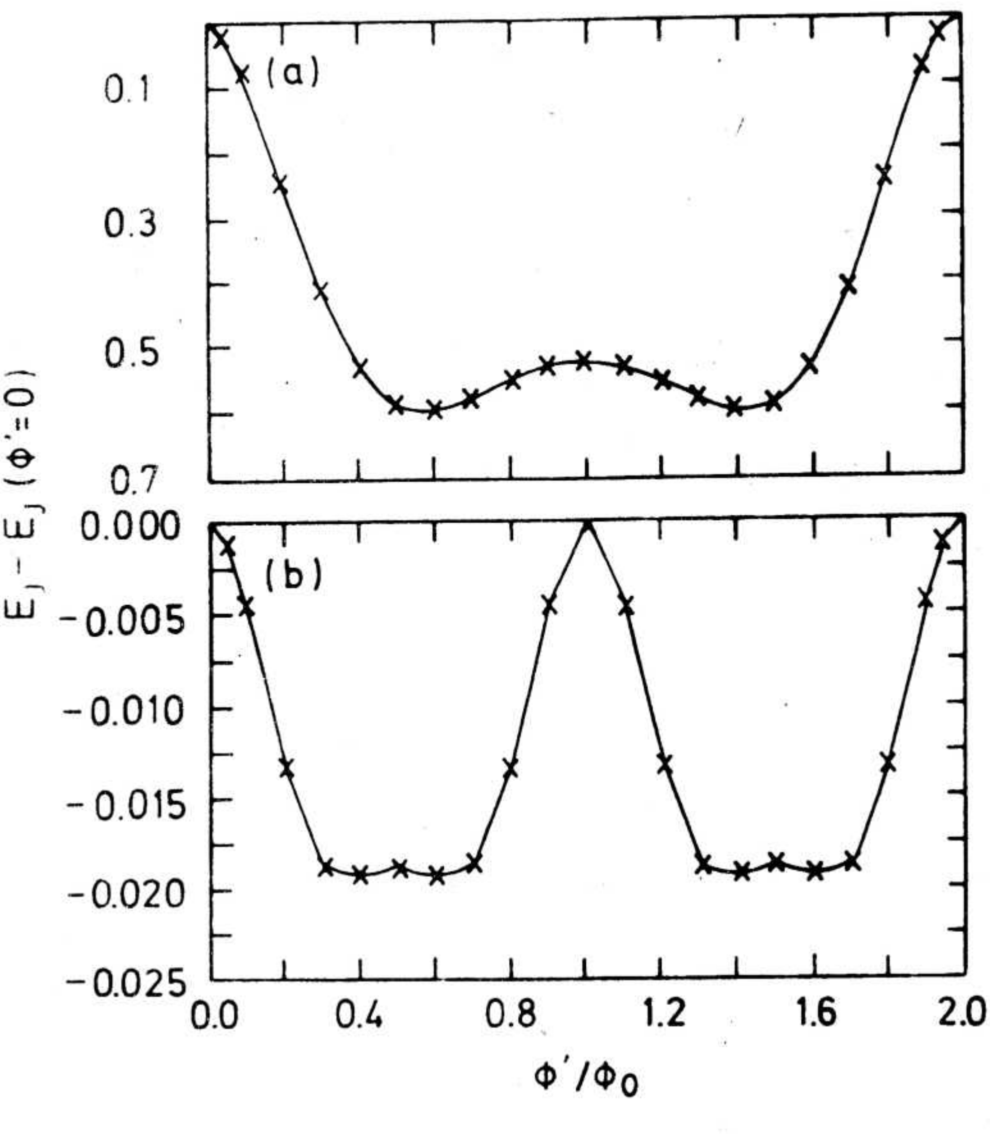
(a)
(b)
0.1
0.3
0.5
0.7
0.000
-0.005
-0.010
-0.015
-0.020
-0.025
0.0
0.4
0.8
1.2
1.6
2.0
$E_j - E_j\,(\Phi'=0)$
Φ'/Φ_0

Figure 1

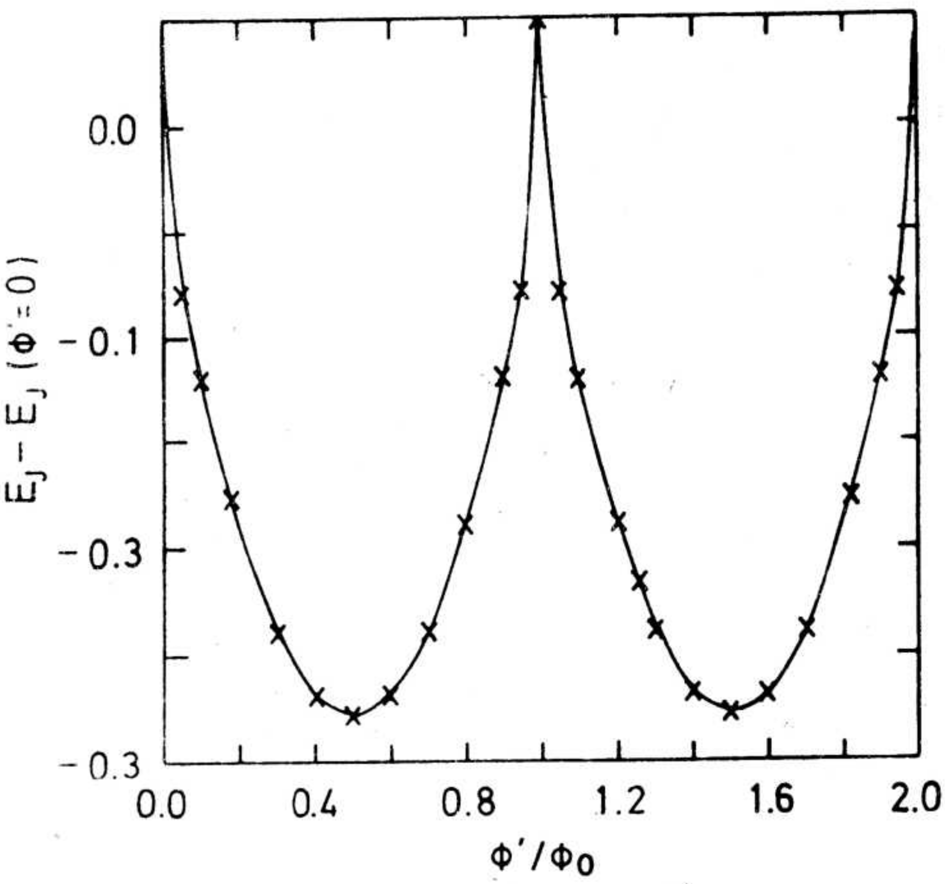

Figure 2

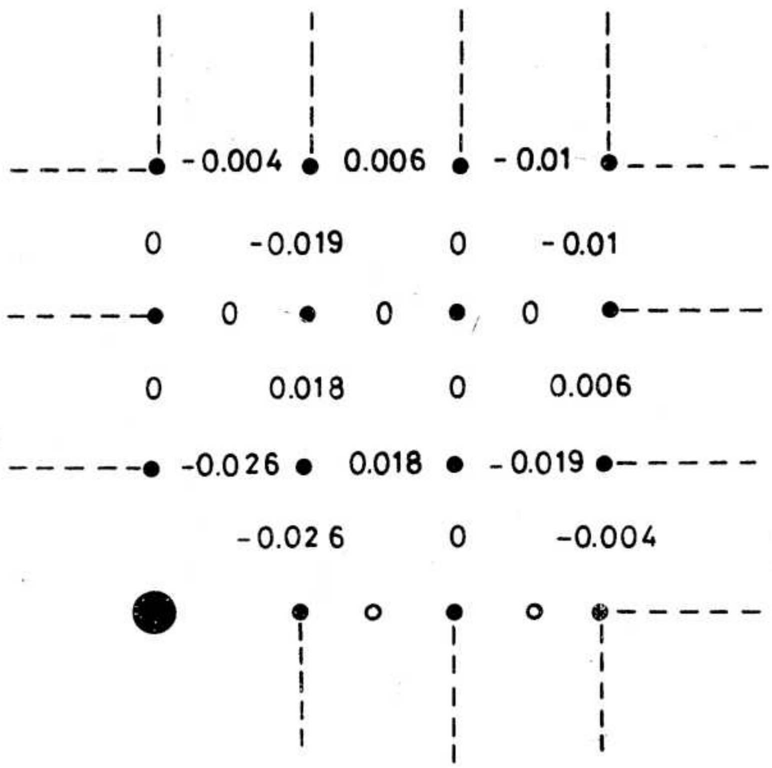
-0.004
0.006
-0.01
0
-0.019
0
-0.01
0
0
0
0
0.018
0
0.006
-0.026
0.018
-0.019
-0.026
0
-0.004

Figure 3

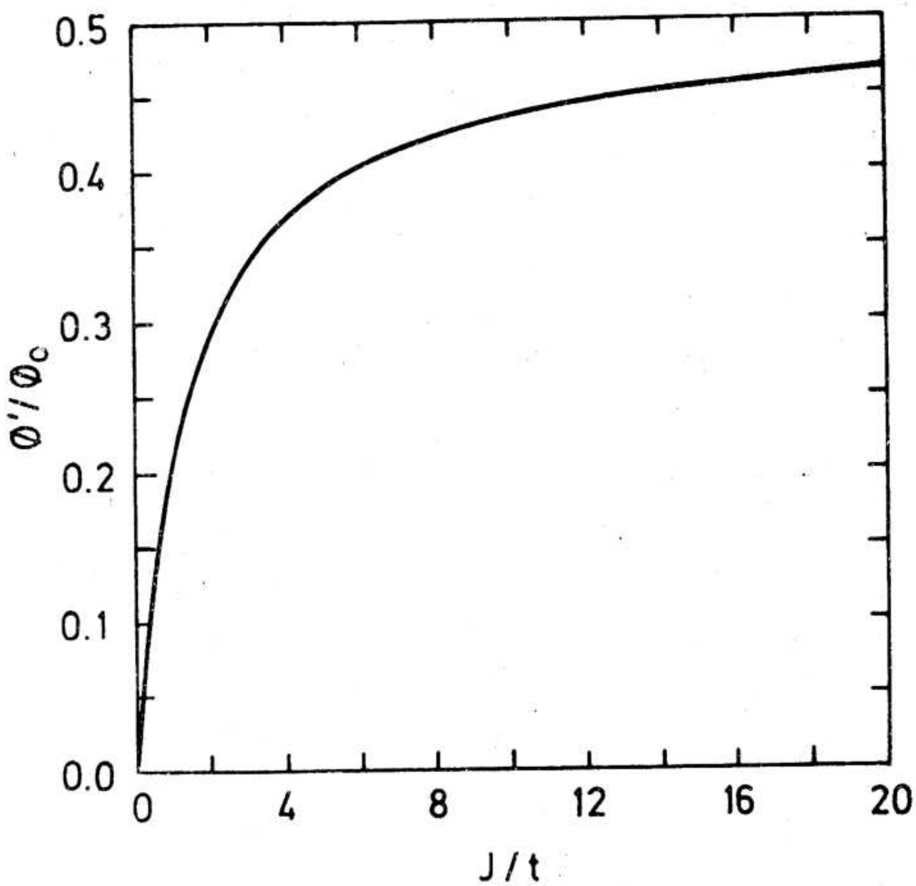

Figura 4