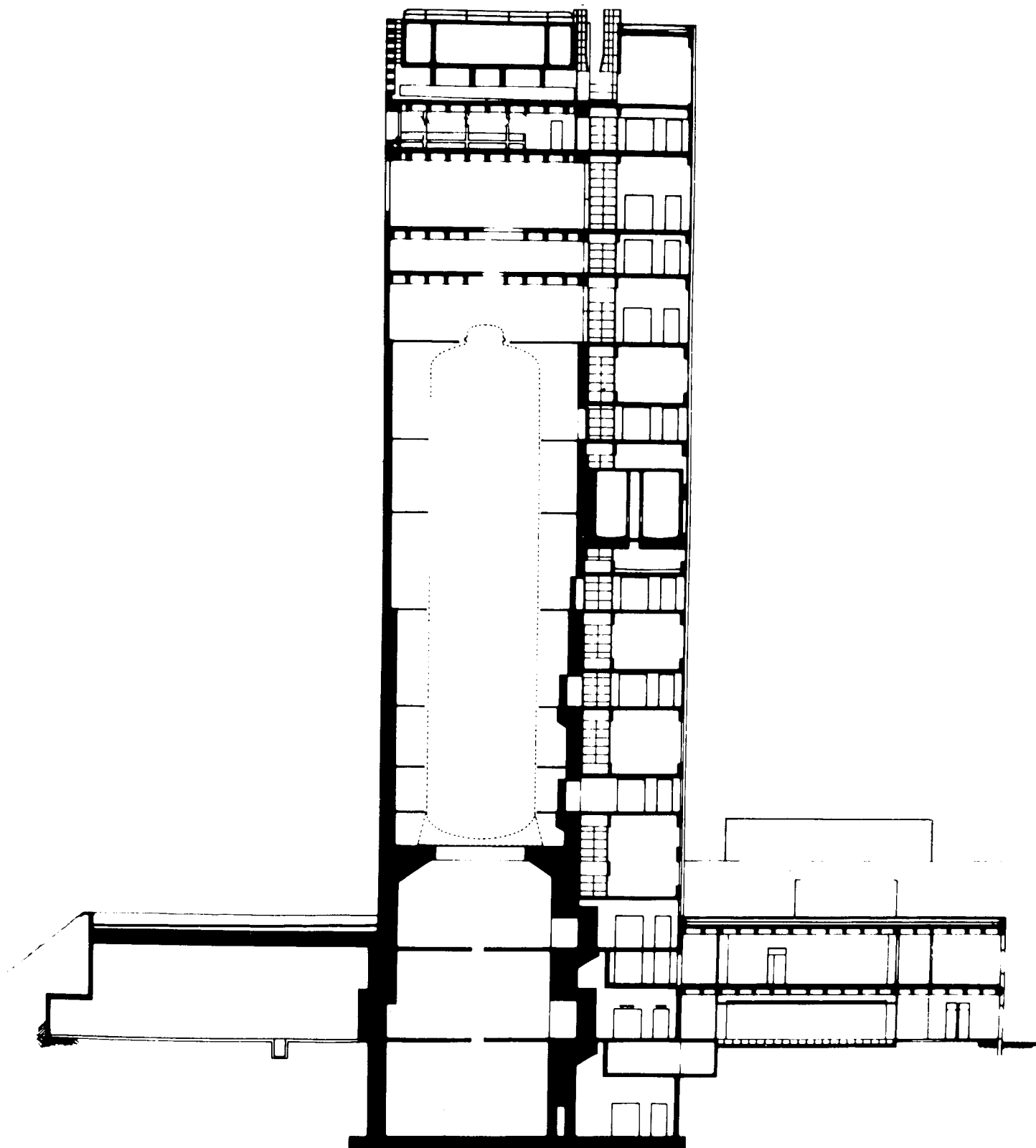


COMISION NACIONAL DE ENERGIA ATOMICA  
TANDAR

V NUCLEAR PHYSICS WORKSHOP

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CNEA - NT 20/82

# V Nuclear Physics Workshop

Chapadmalal, Pcia. de Buenos Aires  
12 - 18 April, 1982

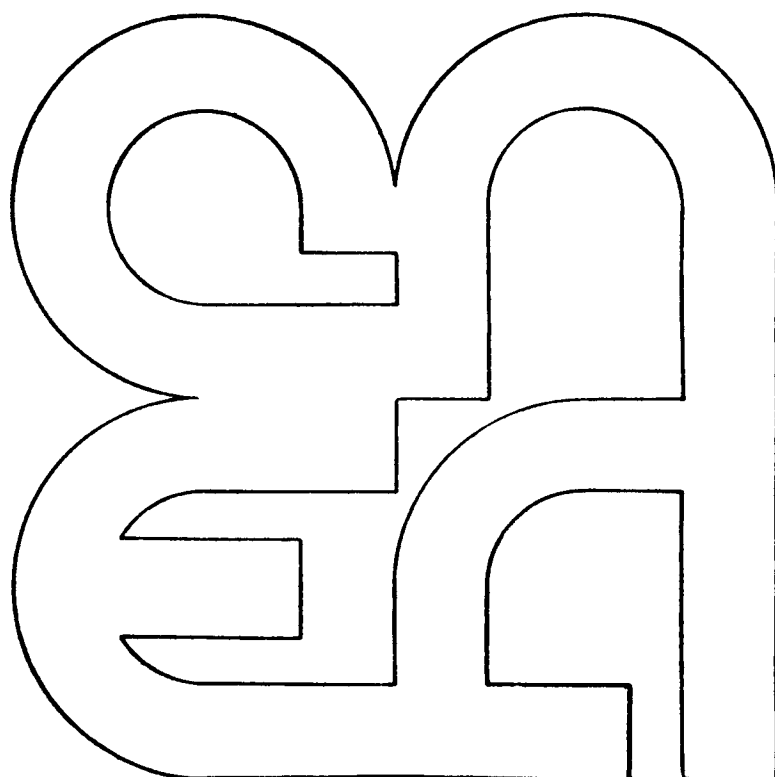
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Dirección de  
Investigación y Desarrollo

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Investigaciones

BUENOS AIRES - ARGENTINA  
1982





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## INTRODUCTION

This volume contains the proceedings of the "Fifth Nuclear Physics Workshop" held from April the 12<sup>th</sup> to the 18<sup>th</sup> at Chapadmalal, Prov. of Buenos Aires. This meeting marks the end of a period starting in 1978, that was invested in the construction and assembly of the TANDAR laboratories. A new stage now begins in which the facilities will be put in operating conditions and in which the fruits of the effort of the past years will start to be harvested.

In this occasion the Federal Government supported the meeting by declaring it of "national interest". The National Research Council and the Organization of American States contributed with important grants that made possible to enlarge the scope of our previous meetings by hosting a large delegation of Latinamerican colleagues from Brazil, Chile, México and Venezuela and by scheduling parallel sessions in Solid State Physics. The invited speakers were Drs. H.Mang, J.Ball, J.Axe and R.Levine from Germany, U.S.A. and Israel . The meeting took place during the first stages of a conflict in the South Argentine Atlantic, a consequence of which other invited British colleagues, choosed to postpone their visit. Their contributions have been included in the present volume.

Only now after five years we can judge positive the balance of the past meetings. Similarly only in a few years from now, we will be able to evaluate the new exchange and collaboration plans that have been suggested during this meeting.

This time the standard acknowledgements must be somewhat more extended. Not only Mrs. M.T.Carmuega has edited this proceedings with her usual efficiency but we have also to acknowledge the invaluable help of Mrs. L.González and Messers J.Prieto, L.Rodríguez, J.Vidallé and H.Soler in most of the organizing problems of this meeting that gained a lot in complexity without losing its efficiency.

R.P.J.Perazzo  
O.Dragún

Organizing Committee

## DISCURSO DE APERTURA A CARGO DEL PRESIDENTE DE CNEA

Vicealmirante Carlos Castro Madero

Cumplo con verdadero placer en dar la bienvenida a un grupo tan numeroso como calificado de científicos provenientes tanto de otras instituciones del País, como de Europa, Estados Unidos y Latinoamérica.

Es esta la quinta de una serie de reuniones que comenzaron en 1978 a poco más de un mes de haber firmado el contrato para la compra del acelerador que Uds. visitarán esta tarde. A lo largo de los pasados cuatro años éstas ajustaron el plan de sus actividades a la marcha del proyecto que ha entrado ahora en su etapa final de realización. Este es pues el momento adecuado para programar el detalle de los planes de investigación y fijar los lineamientos para los acuerdos de colaboración e intercambio científico, temas estos centrales de la Reunión que hoy comienza.

El proyecto TANDAR no hubiera podido nacer y avanzar al paso que lo ha hecho sin apoyarse en una arraigada tradición en el desarrollo de ciencia básica en nuestra institución. A muy poco de nacer, más de treinta años atrás, la CNEA concretó la compra del acelerador Cockroft-Walton y el Síncrociclotrón, equipos ambos que contribuyeron eficazmente a impulsar la física nuclear del país. Un testimonio de la continuidad que hemos propiciado -y que es imprescindible para estas actividades- fue dado por el hecho que el día en que celebrábamos los veinticinco años de vida de estos equipos, se iniciaban los movimientos de tierra de los nuevos laboratorios del TANDAR.

La CNEA siempre reconoció el valor de la ciencia básica tanto en la formación de recursos humanos como en su carácter catalizador y ejemplarizante para la incorporación y desarrollo de técnicas de avanzada. La investigación científica nos ha servido, finalmente, para extender a otras áreas las pautas de rigor y excelencia que aquella usa para medir su propia producción.

El reconocimiento de estos hechos fue en gran medida responsable de haber podido luego transitar con firmeza las otras etapas del desarrollo tecnológico y la aplicación industrial.

A lo largo de este complejo proceso de crecimiento todas las actividades de la Casa sin excepción se han enriquecido de su estrecha interacción con las otras. Pudimos así sortear exitosamente numerosas fronteras tecnológicas que encontramos tendidas en torno de países de elevado nivel de desarrollo y que limitan el libre flujo de conocimientos de fructífera aplicación en el ámbito del desarrollo social.

Es por todas estas razones que deseamos que los beneficios que deriven del proyecto TANDAR sean compartidos no solo por los especialistas de otras instituciones de la Argentina sino también por los de naciones hermanas del continente que aquí se encuentran tan dignamente representados. Las nuevas instalaciones han sido encaradas para que se

conviertan en un medio efectivo para tal fin, con toda la infraestructura que les posibilite actuar como un verdadero laboratorio nacional con sus puertas abiertas a científicos visitantes de todas partes.

Es en este contexto que deseo agradecer el apoyo que nos ha otorgado el Poder Ejecutivo declarando esta Reunión de Interés Nacional, así como al CONICET que prestó su auspicio no solo a esta sino a las anteriores cuatro Conferencias de este ciclo. Un especial reconocimiento nos merece la ayuda brindada por la OEA que ha posibilitado la concurrencia de una numerosa delegación latinoamericana, ayudándonos a concretar un primer paso en nuestra política de puertas abiertas para el aprovechamiento de los nuevos laboratorios. Deseamos pues que esta Reunión sea el comienzo de una colaboración a la que la CNEA está dispuesta a prestar su máximo estímulo.

Sólo me resta aprovechar esta oportunidad para desear a todos Uds. una estimulante reunión de trabajo así como una placentera estadía en Buenos Aires y Chapadmalal.



## INTRODUCCION

Este volumen contiene los anales de la "Va. Reunión de Trabajo de Física Nuclear y Uso de Aceleradores" que tuvo lugar desde el 12 al 18 de abril de 1982 en Chapadmalal, Pcia. de Buenos Aires.

Esta Reunión señala el fin de un ciclo que comenzó en 1978 y que ha de identificarse con el montaje y construcción de los laboratorios del TANDAR. Ahora comienza la nueva etapa de su puesta en funcionamiento, y en la que se comenzará a cosechar el fruto del esfuerzo realizado.

Para esta ocasión se contó con el apoyo del Gobierno Nacional que declarara esta Reunión de Interés Nacional, del CONICET y de la OEA. De este modo fue posible extender la perspectiva de nuestras reuniones anteriores incluyendo una nutrida delegación de colegas de Brasil, Chile, México y Venezuela e incorporando además sesiones paralelas en Física del Sólido. Los conferenciantes invitados fueron los Dres. H.Mang, J.Ball, J.Axe y R.Levine, de Alemania, Estados Unidos de América e Israel . Esta Reunión tuvo lugar en los momentos iniciales del conflicto en el Atlántico Sur Argentino a consecuencia del cual otros colegas británicos invitados al evento optaron por posponer su venida. Sus contribuciones han sido incluidas en el presente volumen.

Sólo ahora, después de cinco años, podemos aventurar un balance positivo de las pasadas reuniones. Del mismo modo, sólo dentro de algún tiempo podremos justipreciar los nuevos horizontes de colaboración que se han abierto con esta conferencia.

Los agradecimientos de rigor deben ser esta vez más amplios. No sólo María Teresa Carmuega ha editado estos anales con su habitual eficiencia sino que además, debemos contabilizar la inapreciable colaboración de la Sra. L.González y los Sres. J.Prieto, L.Rodríguez, J.Vidallé y H.Soler en muchos de los aspectos organizativos de una Reunión que ganando complejidad no perdió eficiencia.

R.P.J.Perazzo  
O.Dragún

Comité Organizador

NUCLEAR PHYSICS PROGRAMME

|       | 12 Mo           | 13 Tu   | 14 Wed                                   | 15 Thu | 16 Fr | 17 Sat            | 18 Sun |
|-------|-----------------|---------|------------------------------------------|--------|-------|-------------------|--------|
| 9.00  |                 | BUS     | Mang                                     | Axe    | Ball  | Levine            | RETURN |
| 10.30 |                 | TRIP    | C O F F E E                              |        |       | Sala              | TO     |
| 11.00 |                 |         | Ball                                     | Sarac. | Mang  | Bes               | BS.AS  |
| 12.30 | BANK            |         | L U N C H                                |        |       |                   |        |
| 14.00 | LUNCH<br>CNEA   |         | I N F O R M A L<br>D I S C U S S I O N S |        |       |                   |        |
| 15.30 | VISIT<br>TANDEM |         | S/S                                      | S/S    | FREE  | Perez<br>Ferreira |        |
| 17.00 | SITE            |         | C O F F E E                              |        |       |                   |        |
| 17.30 |                 | REGIST. | S/S                                      | S/S    |       | Mariscotti        |        |
| 19.00 |                 |         |                                          |        | FREE  |                   |        |
| 20.30 |                 |         | D I N N E R                              |        |       | CLOSING<br>DINNER |        |

S/S: Thirty minutes seminars

## Contributed Seminars

### Wednesday:

- 15.30 A.Szanto de Toledo, M.M.Coimbra, "A search for the low-angular momentum cut-off in the  $^{12}\text{C}+^{16}\text{O}$  fusion"
- 16.00 S.Hernández, "Some conflicting aspects of collisional nuclear dynamics".
- 16.30 R.Donangelo, "Mass distributions and reaction times for the fast fusion processes".
- 17.00 Coffee Break
- 17.30 A.O.Gattone, O.Dragún and A.M.J.Ferrero, "One and two step processes contribution to the cross section in the preequilibrium regime"
- 18.00 O.Dragún, H.Massmann and G.Ramírez, "Excitation of the octupole vibration in  $^{154}\text{Sm}$ ".
- 18.30 D.R.Bes, O.Dragún and E.Maqueda, "Wigner versus Heisenberg processes in pair-exchange reactions".

Chairman: Jorge Flores Valdés

### Thursday:

- 15.30 F.A.Brieva, "Calculation of transition operators from realistic internucleon forces".
- 16.00 R.Lichentaler, A.Lepine, "Elastic Scattering for Heavy Ion Systems with alpha structure".
- 16.30 A.J.Kreiner, "Quantal description of the discontinuity in Mallmann's plot".
- 17.00 Coffee Break
- 17.30 D.Digregorio, A.J.Fendrik and A.J.Kreiner, "Structure of  $^{186}\text{Ir}$  and general conditions for decoupling in doubly odd nuclei".
- 18.00 J.Dukelsky, P.Federman, R.P.J.Perazzo and H.Sofía, "An interacting quartet-boson model".
- 18.30 G.G.Dussel, R.J.Liotta and R.P.J.Perazzo, "Cluster of nucleons as elementary modes of excitation in nuclei".

Chairman: Lutz Döhnert

### Friday:

- 14.30 R.M.Amazawa, N.Rao, W.Seale, "Lifetimes in  $^{91}\text{Zr}$ ".
- 15.00 M.D.Ferrareto and S.Paciornik, "The SAO PAULO Pelletron data acquisition system concepts".

Chairman: Herbert Massmann

### Saturday:

- 15.00 E.Pérez Ferreira, "A description of the TANDAR facilities"
- 15.30 M.A.J.Mariscotti, "Some Ideas of External Users"
- 16.30 Coffee Break
- 17.00 D.R.Bes, "Concluding Remarks"

Chairman: Edgardo Ventura

SOLID STATE PROGRAMME

|       | 12 Mo                                     | 13 Tu       | 14 Wed                                                  | 15 Thu                         | 16 Fr  | 17 Sat            | 18 Sun |         |
|-------|-------------------------------------------|-------------|---------------------------------------------------------|--------------------------------|--------|-------------------|--------|---------|
| 9.00  |                                           |             | Axe I                                                   | Axe                            |        | ---               | RETURN |         |
| 10.00 | WELCOME                                   |             | C O F F E E                                             |                                |        |                   |        | TRIP TO |
| 10.15 |                                           |             | ---                                                     | ---                            | Axe IV | ---               |        |         |
| 11.15 |                                           |             | C O F F E E                                             |                                |        |                   |        |         |
| 11.30 |                                           |             | ---                                                     | ---                            | ---    | ---               |        |         |
| 12.30 |                                           |             | ---                                                     | ---                            | ---    | ---               | BS.AS. |         |
| 14.00 | LUNCH AT CNEA L U N C H                   |             |                                                         |                                |        |                   |        |         |
| 15.30 | VISIT<br><br>TO<br><br>TANDEM<br><br>SITE |             | Axe II                                                  | Axe III                        | Axe V  |                   |        |         |
| 16.30 |                                           |             | S/S<br>Parente                                          | F.Valdes<br>Bonadeo<br>Fulfaro | FREE   |                   |        |         |
| 16.45 |                                           |             | C O F F E E                                             |                                |        |                   |        |         |
| 17.45 |                                           |             | Wainer<br>Nunes                                         | Inf.Disc.<br>Theory            |        |                   |        |         |
| 18.00 |                                           |             | I N F O R M A L<br><br>D I S C U S S I O N S    F R E E |                                |        |                   |        |         |
| 20.30 |                                           | D I N N E R |                                                         |                                |        | CLOSING<br>DINNER |        |         |

S/S: Thirty minutes seminars

# TABLE OF CONTENTS

## I. INVITED TALKS.

|      |                                                                                   |     |
|------|-----------------------------------------------------------------------------------|-----|
| I.1  | "High Spins in Nuclear Physics"                                                   | 1   |
|      | Hans Mang .....                                                                   |     |
| I.2  | "A Study of High-energy Light Particle Emission from Heavy Ion Induced Reactions" | 29  |
|      | Jim Ball .....                                                                    |     |
| I.3  | "A Status on the Holified Heavy Ion Research Facility"                            | 67  |
|      | Jim Ball .....                                                                    |     |
| I.4  | "Time Dependent Variational Description of alpha-alpha scattering"                | 103 |
|      | M.Saraceno .....                                                                  |     |
| I.5  | "Surprisal Analysis of Energy Disposal in Heavy Ion Transfer Reactions"           | 133 |
|      | R.Levine .....                                                                    |     |
| I.6  | "Concluding Remarks"                                                              | 141 |
|      | D.R.Bes .....                                                                     |     |
| I.7  | "Description of the TANDAR Facilities"                                            | 145 |
|      | E.Pérez Ferreira .....                                                            |     |
| I.8  | "Some Ideas on External Users"                                                    | 155 |
|      | M.Mariscotti .....                                                                |     |
| I.9  | "Excitation of Giant Resonances Via Direct Reactions"                             | 161 |
|      | Fred E. Bertrand .....                                                            |     |
| I.10 | "The Experimental Tools to Exploit the Daresbury NSF"                             | 225 |
|      | William Gelletly .....                                                            |     |
| I.11 | "Relations between the Simultaneous and Sequential Transfer of Two Nucleons"      | 297 |
|      | G.R.Satchler .....                                                                |     |
| I.12 | "Folding Models for Elastic and Inelastic Scattering"                             | 323 |
|      | G.R.Satchler .....                                                                |     |

## II. CONTRIBUTED SEMINARS

|      |                                                                                                  |     |
|------|--------------------------------------------------------------------------------------------------|-----|
| II.1 | "A search for the low-angular cutoff in the $^{12}\text{C}+^{16}\text{O}$ fussion"               | 359 |
|      | A.Szanto de Toledo, M.M.Coimbra, N.Carlin F°, T.M.Cormier, M.Herman, B.Lin and P.M.Stwerka ..... |     |
| II.2 | "Some conflicting aspects of collisional nuclear dynamics"                                       | 362 |
|      | C.O.Dorso, E.S.Hernández and H.G.Solari.....                                                     |     |

|       |                                                                                                                                                |     |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| II.3  | "One and two step processes contribution to the cross section in the preequilibrium regime"<br>A.O.Gattone, O.Dragún and A.M.J.Ferrero .....   | 366 |
| II.4  | "Excitation of the octupole vibration in $^{154}\text{Sm}$ "<br>O.Dragún, H.Massmann and Giancarlos Ramírez.....                               | 368 |
| II.5  | "Wigner versus Heisenberg processes in pair-exchange reactions"<br>D.R.Bes, O.Dragún and E.E.Maqueda .....                                     | 370 |
| II.6  | "Calculation of transition operators from realistic internucleon forces"<br>F.A.Brieva .....                                                   | 372 |
| II.7  | "Elastic scattering for heavy ion systems with alpha structure"<br>R.Lichtenhaler F°, A.Lépine, A.C.C.Villari, W.Mittig and V.J.G.Porto .....  | 375 |
| II.8  | "Elastic scattering for heavy ion systems near the Coulomb barrier"<br>A.C.C.Villari, A.Lépine, W.Mittig, R.Lichtenhaler and V.J.G.Porto ..... | 377 |
| II.9  | "Quantal description of the discontinuity in Mallmann's plot"<br>A.J.Kreiner .....                                                             | 379 |
| II.10 | "Structure of $^{186}\text{Ir}$ and general conditions for decoupling in doubly odd nuclei"<br>D.Di Gregorio, A.J.Fendrik and A.J.Kreiner..... | 382 |
| II.11 | "An interacting quartet-boson model"<br>J.Dukelsky, P.Federman, R.P.J.Perazzo and H.M.Sofía .....                                              | 384 |
| II.12 | "Cluster of nucleons as elementary modes of excitation in nuclei"<br>G.G.Dussel, R.J.Liotta and R.P.J.Perazzo .....                            | 387 |
| II.13 | "Lifetimes in $^{91}\text{Zr}$ "<br>R.M.Anazawa, M.Rao, W.Seale and R.Ribas .....                                                              | 392 |
| II.14 | "Energy loss of Ag ions in solids"<br>M.Rao, W.Seale and R.Ribas .....                                                                         | 392 |
| II.15 | "The upgrade of SADE"<br>M.D.Ferraretto and S.D.Paciornik .....                                                                                | 393 |
| II.16 | "Core polarization effects on the magnetic form factor of $^{17}\text{O}$ " .....                                                              | 396 |
| III.  | PARTICIPANTS .....                                                                                                                             | 399 |

## INVITED TALKS



# HIGH SPINS IN NUCLEAR PHYSICS

PROF. HANS MANG

PHYSIK-DEPARTMENT DER  
TECHNISCHEN UNIVERSITÄT MÜNCHEN, W.GERMANY



## 1. Theory

### 1a) The Inglis cranking model

In 1954 D. Inglis<sup>1)</sup> suggested a simple model for rotating nuclei, which soon proved highly successful in describing many phenomena. The model is based on the observation that for a rotating deformed nucleus the single particle model should be a valid approximation only in the rotating frame of reference. This implies the following new single particle Hamiltonian

$$H'_{sp} = H - \vec{\omega} \cdot \vec{J} \quad (1)$$

Here  $\vec{\omega}$  is a constant angular velocity. Its direction is perpendicular to any axis of symmetry. It has become a convention to take the z-axis as the axis of symmetry and the x-axis as the axis of rotation. In the early days of the model  $-\omega J_x$  was treated as a perturbation which leads to the famous "Inglis Formula" for the moment of inertia. Nowadays  $\omega J_x$  is treated exactly and this is called the rotating shell-model<sup>2)</sup>.

It is easy to obtain a selfconsistent version of the Inglis model. The single particle Hamiltonian  $H_{s.p.}$  is replaced by the exact Hamiltonian  $H$  and the resulting Schrödinger equation is solved approximately in the single particle approximation. From a mathematical point of view the angular velo-

city  $\omega$  is a parameter of the theory and not a truly measurable quantity as in classical mechanics. The reason is obvious. In classical mechanics one has as one of Hamilton's equations

$$\frac{\partial H}{\partial I} = \omega \quad (I = \text{angular momentum}) \quad (2)$$

The quantal analog would be

$$\frac{\partial \langle H \rangle}{\partial \langle J \rangle} = \langle \omega \rangle \quad (3)$$

(Ehrenfest theorem)

However, equ.3 does not make sense because  $J$  is quantized as is the energy. In order to relate  $\omega$  to experimental data the following procedure is applied.

$$\omega(I-1) = \frac{E(I) - E(I-2)}{\sqrt{I(I+1)} - \sqrt{(I-2)(I-1)}} \quad (4)$$

$$\Theta = \frac{\sqrt{I(I-1)}}{\omega(I-1)}$$

This clearly implies that the equation

$$\delta \langle H - \omega J_x \rangle = 0 \quad (5)$$

must be solved under the constraint

$$\langle J_x \rangle = \sqrt{I(I+1)} \quad (6)$$

Because only then  $\langle H \rangle$  can be interpreted as the energy of the state with angular momentum I

$$E_I = \langle H \rangle_{\langle J_x \rangle} = \sqrt{I(I+1)} \quad (7)$$

The constraint (equ.6) replaces rigorous angular momentum conservation by a "quantisation" of the expectation value  $\langle J_x \rangle$ . Indeed a solution  $|\Phi\rangle$  of eqs.5 and 6 will in general not be an eigenstate of  $\hat{J}^2$  and  $\hat{J}_z$ . In short the cranking model is a semi-classical approximation.

It is well known that a semi-classical approximation is no longer valid when fluctuations of important operators like  $J$  ( $\hat{J} - \langle \hat{J} \rangle$ )<sup>2</sup> = ( $\Delta J$ )<sup>2</sup> are rapidly varying. In some cases it is even sufficient that fluctuations become large, fortunately not in the case under consideration. In addition to this criticism one would like to derive the cranking model from first principles, that is Schrödingers equation, and in this way be able to estimate or even calculate corrections.

#### 1b) Derivation of the cranking model.

The starting point is the equation

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0 \quad (8)$$

For  $|\Phi\rangle$  a general single particle wavefunction (HFB) is

used<sup>3)</sup>. A straightforward solution of equ.8 would in general yield a deformed solution, that is, it violates angular momentum conservation. One should therefore use

$$|\psi_I^M\rangle = \hat{P}_I^M |\Phi\rangle \quad (9)$$

where  $P_I^M$  is a "projection operator" onto eigenstates of  $\hat{J}^2$  and  $\hat{J}_z$ <sup>4)</sup>. One such operator is given by

$$\begin{aligned} \hat{P}_I^M &= \sum_K \int \hat{D}_{MK}^I(\Omega) \hat{R}(\Omega) d\Omega \\ \hat{R}(\Omega) &= \exp(i\alpha \hat{J}_z) \exp(i\beta \hat{J}_y) \exp(i\gamma \hat{J}_z) \end{aligned} \quad (10)$$

Insertion of (9) and (10) into (8) yields after some trivial manipulations

$$\frac{\langle \psi_I^M | H | \psi_I^M \rangle}{\langle \psi_I^M | \psi_I^M \rangle} = \frac{\sum_{KK'} \int \hat{D}_{KK'}^I \langle \Phi | H \hat{R}(\Omega) | \Phi \rangle d\Omega}{\sum_{KK'} \int \hat{D}_{KK'}^I \langle \Phi | \hat{R} | \Phi \rangle d\Omega} \quad (11)$$

The crucial quantities are  $\langle H \hat{R} \rangle$  and  $\langle \hat{R} \rangle$ . One is able to show<sup>4,5)</sup> that  $\langle H \hat{R} \rangle$  can be expressed as follows:

$$\langle \Phi | H \hat{R} | \Phi \rangle = [a + b L_x + c L^2 + \dots] \langle \Phi | \hat{R} | \Phi \rangle \quad (12)$$

$L_x, L_y, L_z$  are angular momentum operators in the Euler angles  $\alpha \beta \gamma$ . Equ.12 is exact if the sum on the right hand side is not truncated. Truncating after quadratic terms is justified provided the nucleus is well deformed and almost axially

symmetric.

$$\begin{aligned} \langle \Delta \hat{J}_x^2 \rangle + \langle \hat{J}_y^2 \rangle &\gg 1 \\ \langle \hat{J}_z^2 \rangle &\ll I(I+1) \end{aligned} \quad (13)$$

In order to obtain the cranking model I shall now also neglect the term  $cL^2$ . Furthermore I shall use the relations

$$\begin{aligned} L_i \langle \Phi | \hat{R} | \Phi \rangle &= \langle \Phi | \hat{R} J_i | \Phi \rangle \\ i &= x, y, z \end{aligned} \quad (14)$$

Equations for a and b are now obtained by setting  $\Omega = 0$  in equ.12 and then also in the equation which is obtained by operating with  $L_x$  on equ.12<sup>+</sup>.

$$\begin{aligned} \langle \Phi | H | \Phi \rangle &= a + b \langle \Phi | \hat{J}_x | \Phi \rangle \\ \langle \Phi | H \hat{J}_x | \Phi \rangle &= a \langle \Phi | \hat{J}_x | \Phi \rangle + b \langle \Phi | \hat{J}_x^2 | \Phi \rangle \end{aligned} \quad (15)$$

The solution is

$$\begin{aligned} a &= \langle \Phi | H | \Phi \rangle - \frac{\langle \Phi | H \Delta \hat{J}_x | \Phi \rangle}{\langle \Phi | \Delta \hat{J}_x^2 | \Phi \rangle} \langle \Phi | \hat{J}_x | \Phi \rangle \\ b &= \frac{\langle \Phi | H \Delta \hat{J}_x | \Phi \rangle}{\langle \Phi | \Delta \hat{J}_x^2 | \Phi \rangle} ; \Delta \hat{J}_x = \hat{J}_x - \langle \Phi | \hat{J}_x | \Phi \rangle \end{aligned} \quad (16)$$

<sup>+</sup>Note that  $\langle J_y \rangle = \langle J_z \rangle = 0$

$$\frac{\langle \psi_I^M | H | \psi_I^M \rangle}{\langle \psi_I^M | \psi_I^M \rangle} = a + b \frac{\sum_{KK'} \int \tilde{D}_{KK'}^I L_x \langle \Phi | \hat{R} | \Phi \rangle d\Omega}{\sum_{KK'} \int \tilde{D}_{KK'}^I \langle \Phi | \hat{R} | \Phi \rangle d\Omega}$$

The second inequality of (13) guarantees that<sup>+</sup>

$$\begin{aligned} \sum_{KK'} \int (L_x \tilde{D}_{KK'}^I)^* \langle \Phi | \hat{R} | \Phi \rangle d\Omega &\approx \\ &\approx \sqrt{I(I+1)} \sum_{KK'} \int \tilde{D}_{KK'}^I \langle \Phi | \hat{R} | \Phi \rangle d\Omega \end{aligned} \quad (17)$$

or

$$\begin{aligned} E_I &= \langle \Phi | H | \Phi \rangle + \omega(\Phi) (\sqrt{I(I+1)} - \langle \Phi | \hat{J}_x | \Phi \rangle) \\ \omega(\Phi) &= \frac{\langle \Phi | H \Delta \hat{J}_x | \Phi \rangle}{\langle \Phi | \Delta \hat{J}_x^2 | \Phi \rangle} \end{aligned} \quad (18)$$

A variation of  $E_I$

$$\frac{\delta E_I}{\delta \Phi} = 0$$

yields

$$\begin{aligned} \delta \langle \Phi | H | \Phi \rangle &= 0 \\ \langle \Phi | \hat{J}_x | \Phi \rangle &= 0 \end{aligned} \quad (19)$$

---

<sup>+</sup>Only small values of  $K, K'$  contribute to the sum because of the structure of  $|\Phi\rangle$  which contains only components of small  $K$  values.



The proof of the set of equations is rather simple

$$\begin{aligned} \delta E_I = & \delta \langle \Phi | H | \Phi \rangle - \omega \delta \langle \Phi | \hat{J}_x | \Phi \rangle \\ & + (\sqrt{I(I+1)} - \langle \Phi | \hat{J}_x | \Phi \rangle) \cdot \delta \omega = 0 \end{aligned} \quad (20)$$

This relation has to hold for any variation  $|\delta\phi\rangle$ . A special variation is given by

$$|\delta\phi_s\rangle = \eta \Delta \hat{J}_x |\Phi\rangle \quad |\eta| \ll 1 \quad (21)$$

Upon inserting  $|\delta\phi_s\rangle$  one obtains

$$\begin{aligned} & \langle \Phi | H \Delta \hat{J}_x | \Phi \rangle - \omega \langle \Phi | \Delta \hat{J}_x^2 | \Phi \rangle \\ & + (\sqrt{I(I+1)} - \langle \Phi | \hat{J}_x | \Phi \rangle) \delta \omega_s = 0 \end{aligned} \quad (22)$$

which is equivalent to

$$(\sqrt{I(I+1)} - \langle \Phi | \hat{J}_x | \Phi \rangle) \delta \omega_s = 0 \quad (23)$$

because of the definition of  $\omega$  in equ.18. It is a trivial matter to convince oneself that  $\delta \omega_s$  is in general not equal to zero and hence  $\sqrt{I(I+1)} - \langle \hat{J}_x \rangle$  must be equal to zero.

The set of equations 19 formulates the self-consistent cranking model. These equations can be solved either by introducing a Lagrangian multiplier whose value is determined by the constraint  $\langle \hat{J}_x \rangle = \sqrt{I(I+1)}$  or more conveniently by a method of steepest descent<sup>6)</sup>. The latter has the additional advantage that it also works in those cases in which  $\langle \hat{J}_x \rangle$  is not a single-valued function of  $\omega$ .

Concerning fluctuations it is straightforward but tedious to carry through an analogous calculation. I shall give only the result for the energy  $E_I$

$$\begin{aligned}
 E_I = & \langle H \rangle - \frac{1}{2\Theta_Y} \langle \Delta \hat{J}_x^2 + \hat{J}_y^2 \rangle \\
 & + \omega \left( \sqrt{I(I+1) - \langle \hat{J}_z^2 \rangle} - \langle \hat{J}_x \rangle \right) \\
 & + \frac{1}{2\Theta_Y} \left( \sqrt{I(I+1) - \langle \hat{J}_z^2 \rangle} - \langle \hat{J}_x \rangle \right)^2
 \end{aligned} \tag{24}$$

with

$$\frac{1}{\Theta_Y} = \frac{\langle (H - \langle H \rangle) (\Delta \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2) \rangle}{\langle \Delta \hat{J}_x^2 \rangle^2 + \langle \hat{J}_y^2 \rangle^2 + \langle \hat{J}_z^2 \rangle^2 + \frac{1}{2} \langle \Delta \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2 \rangle}$$

It is obvious that the variation of this expression is again straightforward but will produce a formidable number of different terms. The variational equation cannot be converted in such a simple form as equ.19. A close inspection of equ.24 however, shows that the following is a rather good first approximation

$$\begin{aligned}
 \delta \left( \langle \Phi | H | \Phi \rangle - \frac{1}{2\Theta_Y} \langle \Phi | \Delta \hat{J}_x^2 + \hat{J}_y^2 | \Phi \rangle \right) &= 0 \\
 \sqrt{I(I+1) - \langle \hat{J}_z^2 \rangle} &= \langle \Phi | \hat{J}_x | \Phi \rangle
 \end{aligned} \tag{25}$$

#### (1c) Transition rates and moments.

In order to calculate transition rates and moments one has to start again with the wavefunction  $|\psi_I^M\rangle = P_I^M |\Phi\rangle$  <sup>7)</sup>. One has to perform analogous manipulations as for the expectation value of H. The difference lies in the fact that H is a scalar two-body operator whereas the transition operators are one-

body tensor operators. Of course, one can use an expansion analogous to the one given in equ.12. Even then, however, the norm-matrix  $n_{kk}^I = \int \hat{D}_{kk}^{*I} \langle \Phi | \hat{R} | \Phi \rangle d\Omega$  does not disappear from the final result because of the tensor properties of the operators. Therefore  $n(\Omega) = \langle \Phi | \hat{R}(\Omega) | \Phi \rangle$  has to be calculated. A good approximation is provided by

$$n(\Omega) = \exp \left( \sum_{L=0,2} \sum_{MM'} C_{MM'}^L D_{MM'}^L(\Omega) \right) \quad (26)$$

This expression would be exact, were the sum on L not truncated at L=2. With the help of equ.26 the norm-matrix  $n_{kk}^I$  can be given in a closed form as a rapidly converging sum over Bessel Functions. Details can be found in ref.7. The reduced matrix element of a tensor operator of rank  $\lambda$  is given in equ.27.

$$\begin{aligned} \langle I \| T^\lambda \| I' \rangle &= \sum_{\kappa \kappa' \bar{\kappa} \mu} \frac{(2I+1)(2I'+1)}{16 \pi^2} \\ &\left\{ (-1)^{I-\kappa} \begin{pmatrix} I & \lambda & I' \\ \kappa & \mu & \kappa' \end{pmatrix} \int \hat{D}_{\bar{\kappa} \kappa'}^{*I'} \langle \Phi | T_\mu^\lambda \hat{R} | \Phi' \rangle d\Omega \right. \\ &\left. + (-1)^{I-\kappa'} \begin{pmatrix} I & \lambda & I' \\ \bar{\kappa} & \mu & \kappa' \end{pmatrix} \int \hat{D}_{\kappa \bar{\kappa}}^{*I} \langle \Phi | \hat{R} T_\mu^\lambda | \Phi' \rangle d\Omega \right\} \quad (27) \\ \langle \Phi | T_\mu^\lambda \hat{R} | \Phi' \rangle &= [a_\mu^\lambda + \sum_k b_{\mu k}^\lambda L_k + \dots] \langle \Phi | \hat{R} | \Phi' \rangle \\ \langle \Phi | \hat{R} T_\mu^\lambda | \Phi \rangle &= [\tilde{a}_\mu^\lambda + \sum_k \tilde{b}_{\mu k}^\lambda \tilde{L}_k + \dots] \langle \Phi | \hat{R} | \Phi \rangle \end{aligned}$$

Note the symmetrisation with respect to initial and final states.

Finally I would like to mention an important and helpful property of the expansion of equ.12 in the case of transition rates as

well as moments. It turns out that the first term corresponding to  $a_{\mu}^{\lambda}$  is always dominant, the reason being that the second term is very small because of the constraint and the third one is smaller by a factor of  $\frac{1}{\langle \Delta J^2 \rangle}$ .

Before going on to applications of the theory a few words about projection of particle number are in order. This problem is definitely much simpler. The projector has the form

$$\hat{P}_N = \frac{1}{2\pi} \int_0^{2\pi} d\varphi e^{i(\hat{N}-N)\varphi} \quad (23)$$

The energy  $E_N$  is given by

$$E_N = \frac{\langle \Phi | H \hat{P}_N | \Phi \rangle}{\langle \Phi | \hat{P}_N | \Phi \rangle} \quad (29)$$

$\delta E_N = 0$  leads to

$$\frac{\langle \Phi | \alpha_{\nu} \alpha_{\mu} H \hat{P}_N | \Phi \rangle}{\langle \Phi | \hat{P}_N | \Phi \rangle} - E_N \frac{\langle \Phi | \alpha_{\nu} \alpha_{\mu} \hat{P}_N | \Phi \rangle}{\langle \Phi | \hat{P}_N | \Phi \rangle} = 0 \quad (30)$$

for all  $\{\nu, \mu\}$

These equations can be solved without any approximations<sup>8)</sup>.

Technical details can be found in ref.<sup>8)</sup>.

## 2. Applications

### 2a) Energy levels

As examples I shall discuss four nuclei all from the rare earth region  $^{158}\text{Dy}$ ,  $^{164}\text{Dy}$ ,  $^{164}\text{Er}$ ,  $^{180}\text{Hf}$  and  $^{182}\text{W}$ . Measured and

and calculated energy levels as well as deformations, alignments and energy gaps are given in Figs.1 to 9. Several sets of calculations have been performed. All are based on the "pairing plus quadrupole model" of Baranger and Kumar. The simplest calculation is the one with the selfconsistent cranking model as given in equation 19. Two versions have been used, one with monopole pairing only and one with quadrupole pairing added. The addition of quadrupole pairing had the effect that no longer a core moment of inertia was needed to obtain the correct moment of inertia at low angular momentum ( $I=2,4,6$ ). Another calculation used the energy expression of equation 24. In both cases particle number conservation was treated in the BCS-approximation. Finally there is a calculation for  $^{182}\text{W}$  done with exact particle number projection before variation but within the framework of the self-consistent cranking model.

From a comparison of these results with existing experimental data one concludes that qualitative agreement is always obtained. The more sophisticated calculations, i.e. the ones with number projection and with  $E_I$  of equ.24, generally improve agreement with experimental data. Of course, this is only so if one compares to simple cranking calculations performed with the same set of parameters. It should be noted that up to now all calculations with quadrupole pairing have been performed only in the simple cranking approximation. A comparison of the back-bending plots in Fig.2 clearly shows that the extra terms in  $E_I$  of equ.24 must contribute quite substantially to the average potential and in particular to the pair field. Fig.1 demonstrates

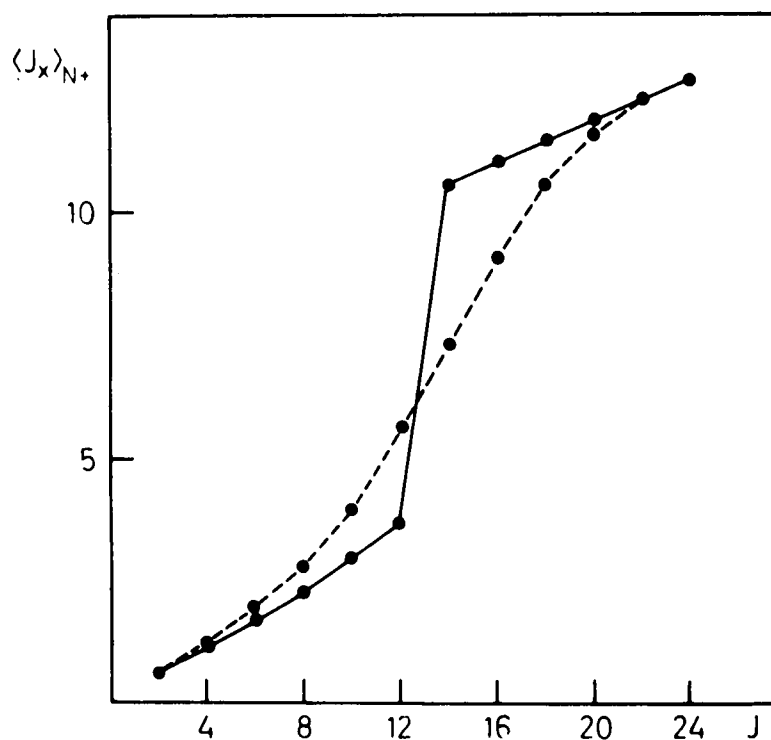
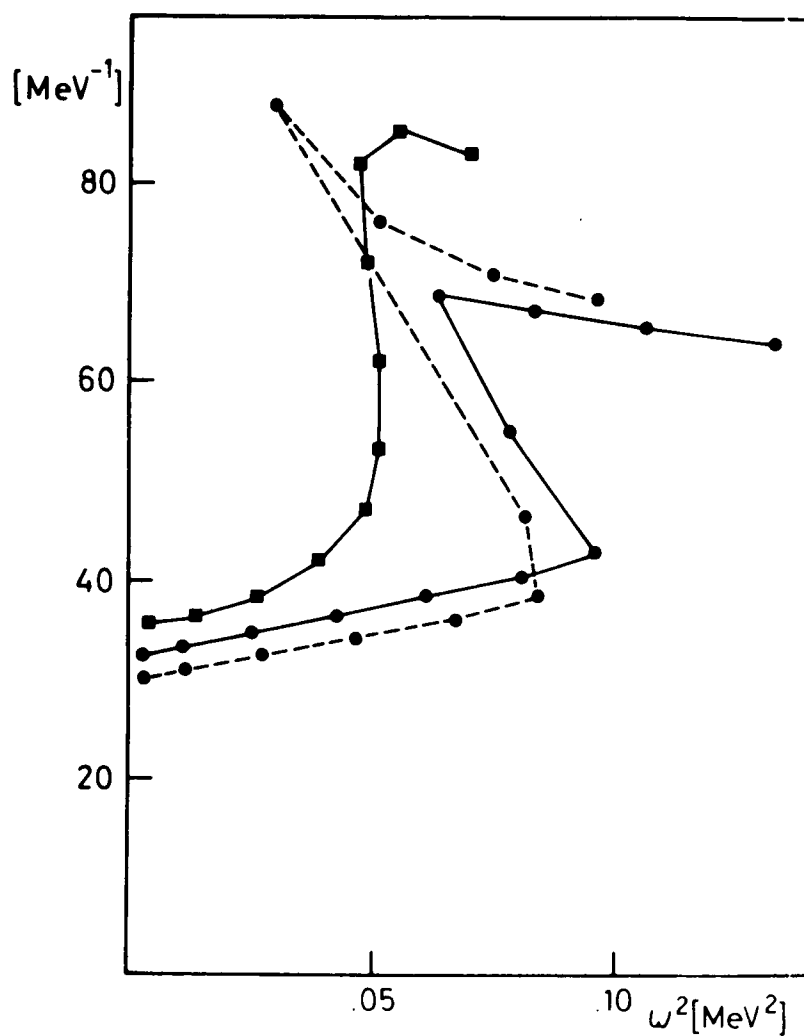


Figure 1:

Contribution to  
 $\langle J_x \rangle$  by neutrons  
of positive  
parity versus J.

Figure 2:

Backbending plot  
for  $^{164}\text{Er}$   $\blacksquare$   
cranking;  
 $\bullet$ ----- $\bullet$  cranking  
with fluctuations  
included;  
 $\bullet$ — $\bullet$  experiment-  
al data.



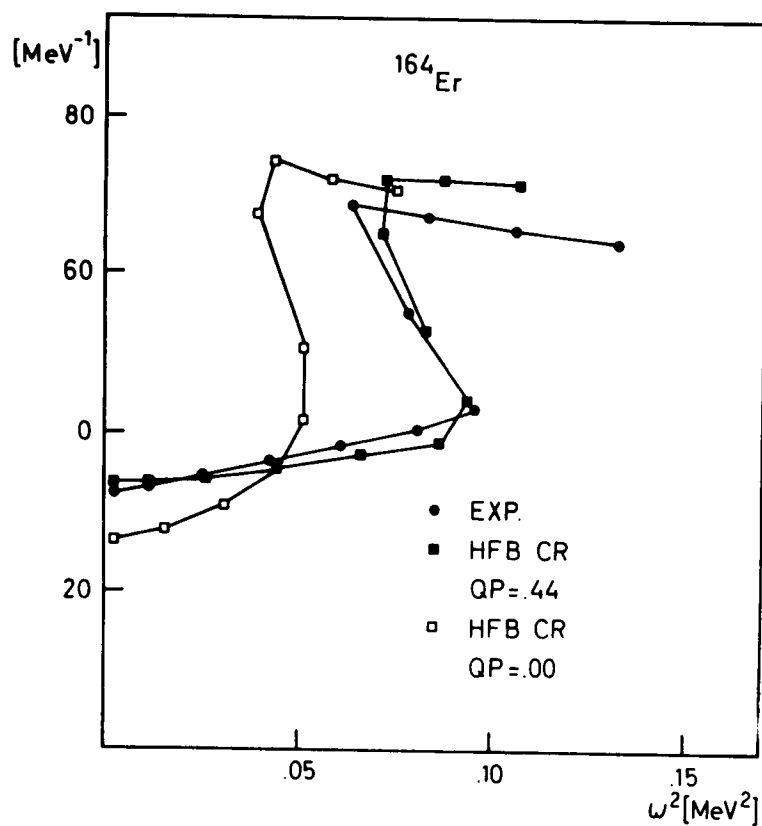


Figure 3

Backbending plot for <sup>164</sup>Er.

- cranking, monopole pairing only;
- cranking, ratio of quadrupole to monopole pairing .44;
- experimental data.

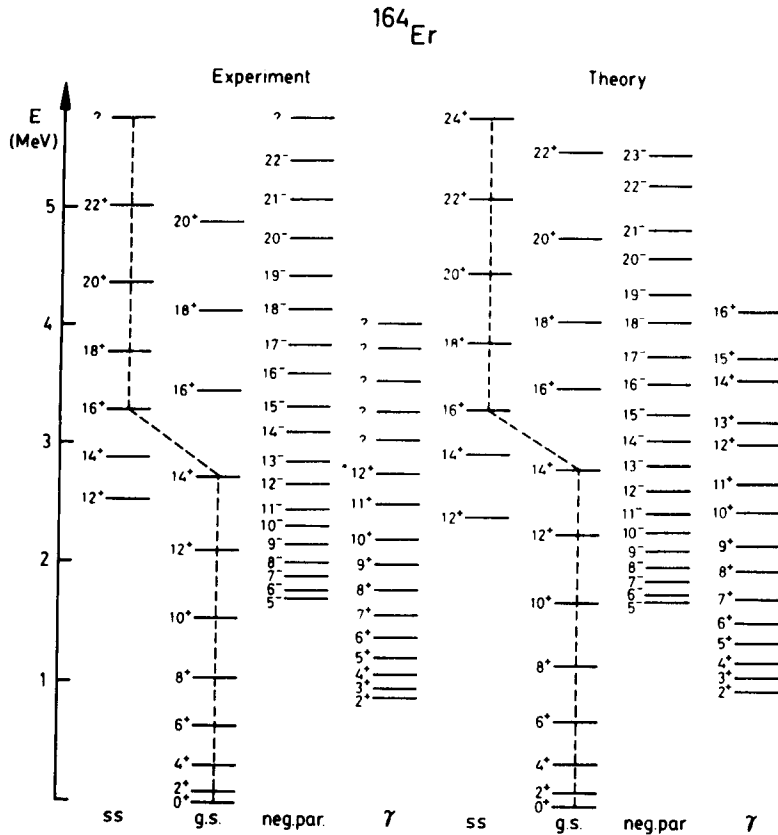


Figure 4

Energy levels of  $^{164}\text{Er}$ . Levels of the gamma-band are based on the random phase approximation<sup>7)</sup>.



that the extra terms hinder alignment and thus produce a much more sudden transition from the non-aligned to the aligned configuration. In the same way particle number projection keeps pairing correlations strong and also hinders alignment with very much the same effect. Fig.3 and 4 show that even on the basis of the conventional cranking model good agreement with experimental data can be obtained if the parameters of the forces are slightly adjusted<sup>7)</sup>. The interesting feature of the calculations for  $^{180}\text{Hf}$  and  $^{182}\text{W}$  is the occurrence of an extremely sharp band-crossing between  $I=22$  and 26. The exact crossing point is, of course, model dependent. The crossing band has an oblate almost axially symmetries deformation. The rotation is not around the axis of symmetry. In the crossing region both bands show neutron and proton alignment, but the oblate band has the larger total alignment, a fact which compensates for the smaller moment of inertia of an oblate rotor in comparison to a prolate rotor of the same deformation. Unfortunately there are no experimental data up to now which could confirm or disprove these calculations.

## 2b Transition rates and moments

I'll restrict myself to BE2-values and magnetic moments and discuss three nuclei for which some experimental data are available ( $^{158}\text{Dy}$ ,  $^{164}\text{Dy}$  and  $^{164}\text{Er}$ ). They should exhibit a quite different behaviour if neutron alignment is the process which dominates the behaviour of states with angular momentum  $I$  less than 24. The reason for such an expectation is that  $^{158}\text{Dy}$  shows

a strong upbending which indicates a rapid transition from a normal to an aligned band. This in turn implies a decrease in BE2-values of the Yrast-band while intraband transitions should increase markedly. Most easily such a behaviour is understood if one interprets the whole process as a mixing of a non-aligned and an aligned band. In case of no mixing, intraband transitions would completely dominate whereas for very strong mixing interband transitions would be equally strong. Indeed there is a case in  $^{164}\text{Er}$  in which the ratio between interband and intraband transitions is approximately .5. The situation is even more clear in the case of magnetic moments. It is obvious that aligned neutrons contribute little (or even a negative amount) to magnetic moments but a very substantial part to the total angular momentum. Hence one expects a decreasing gyromagnetic ratio if neutrons align. None of the experimental results contradict this prediction and some even support it. Figs.10 and 11 give information on calculated and experimental results obtained for  $^{158}\text{Dy}$  and  $^{164}\text{Dy}$ . Unfortunately, experimental error bars and theoretical uncertainties are so that one cannot claim more than qualitative agreement.

Finally Fig.12 shows calculated g-factors for  $^{164}\text{Er}$ . The calculations are the ones which led to the "alignment" and back-bending plots of Figs.1 and 2. The differences in the g-factors simply reflect the differences of "alignment" as depicted in Fig.1. The abrupt change in the g-factor is a clear fingerprint of the fluctuation terms in equ.24 without them one will never

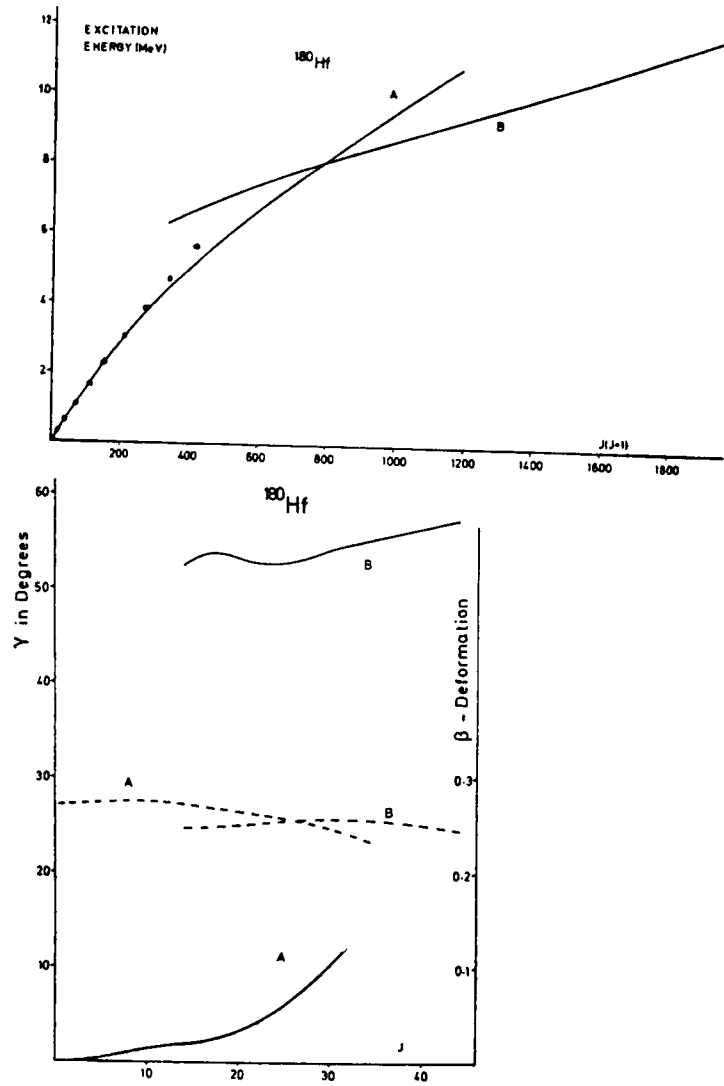


Figure 5:

Energies and deformations for  $^{180}\text{Hf}$   
(cranking, monopole pairing only).

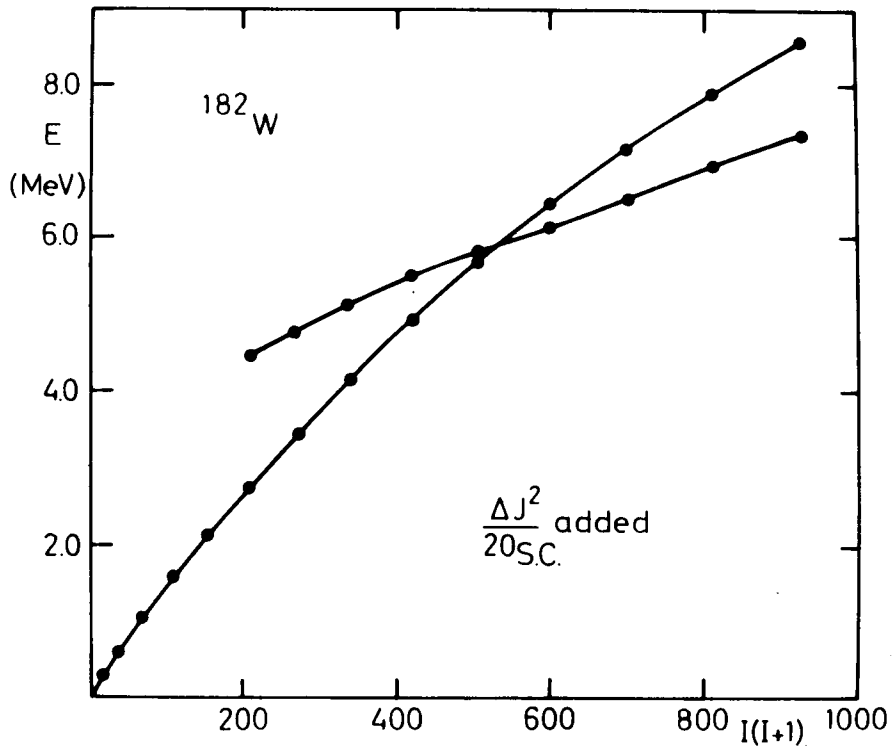


Figure 6:  
Bandcrossing in  $^{182}\text{W}$  (Energies versus  $I(I+1)$ )

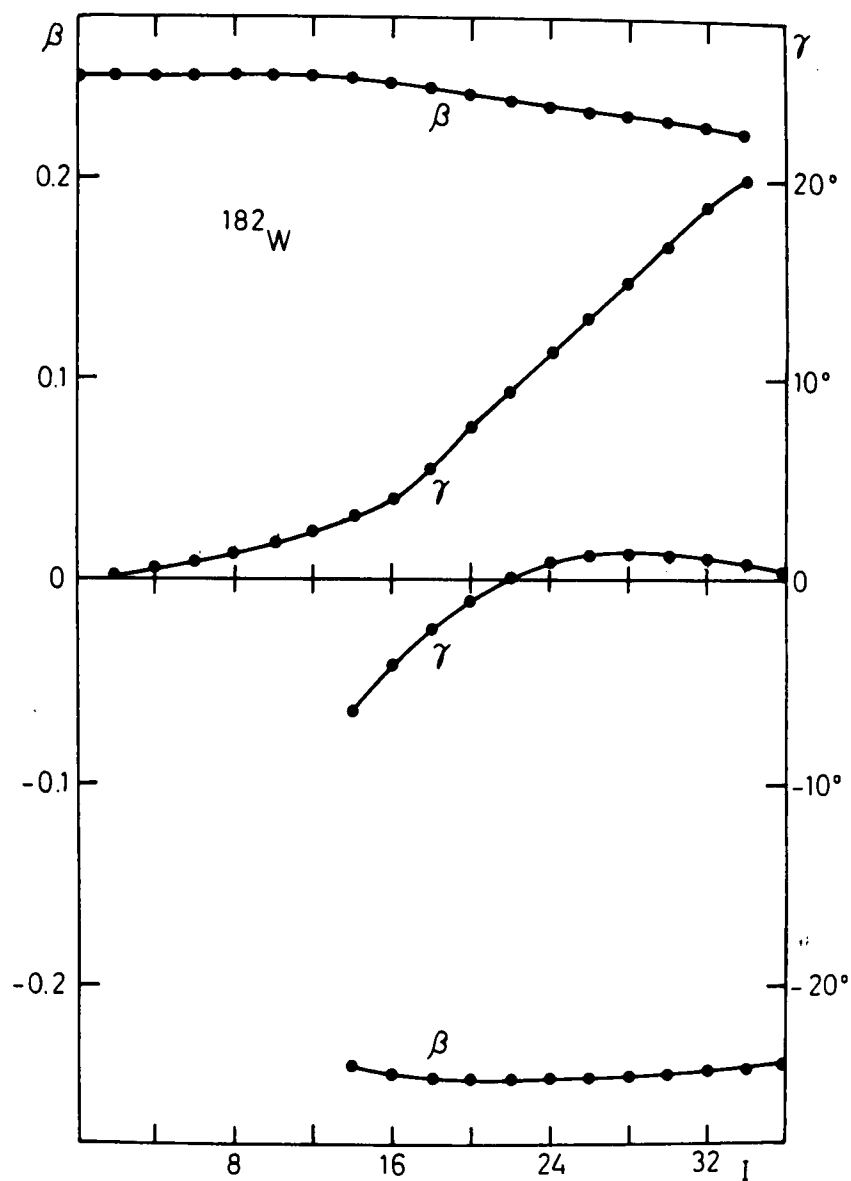


Figure 7  
Deformations versus  $I$  for  $^{182}\text{W}$ .

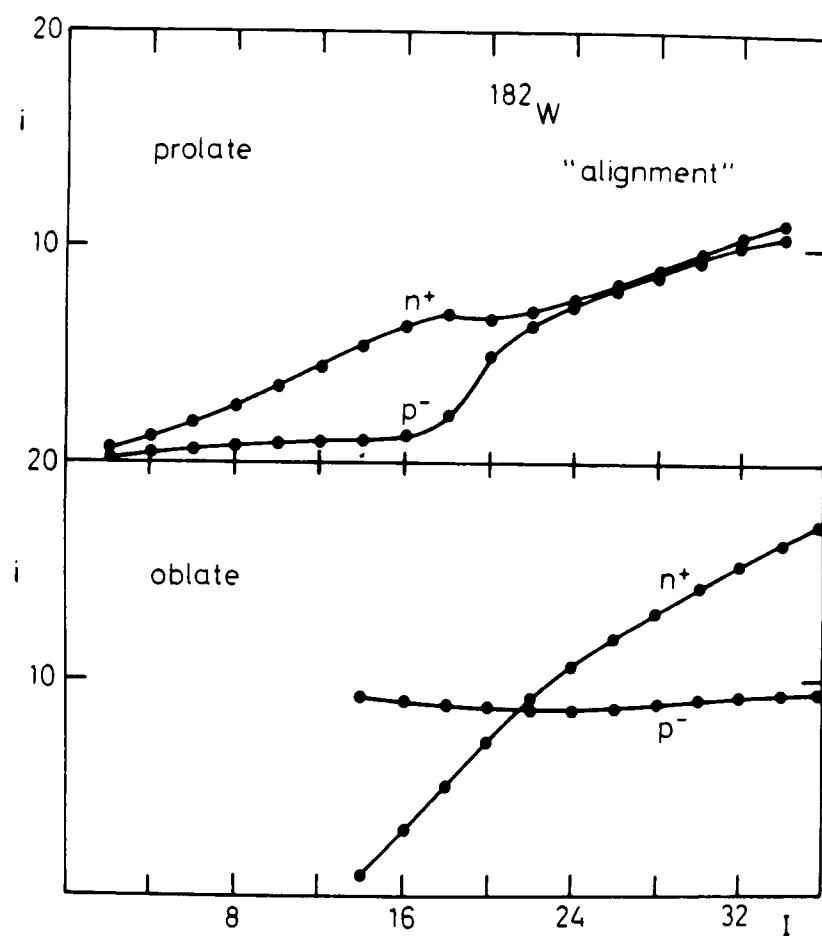


Figure 8:

"Alignment" of neutrons and protons  
in the bands of  $^{182}\text{W}$ .

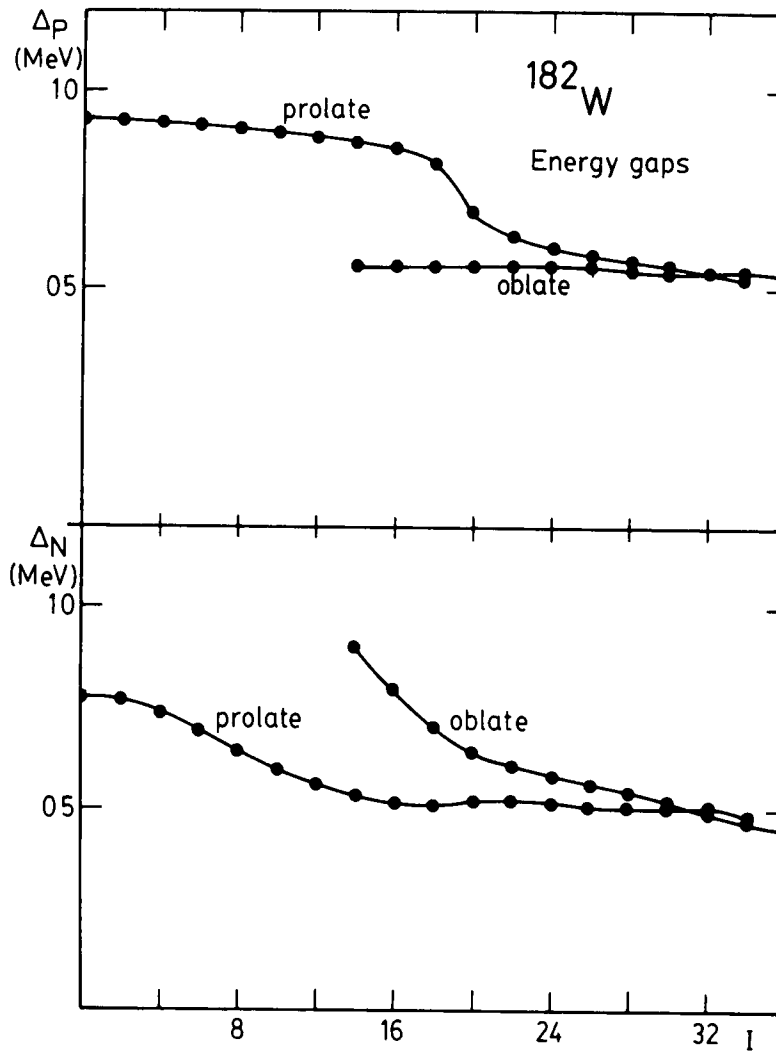


Figure 9:  
Energy-gaps versus  $I$  for  $^{182}\text{W}$ .

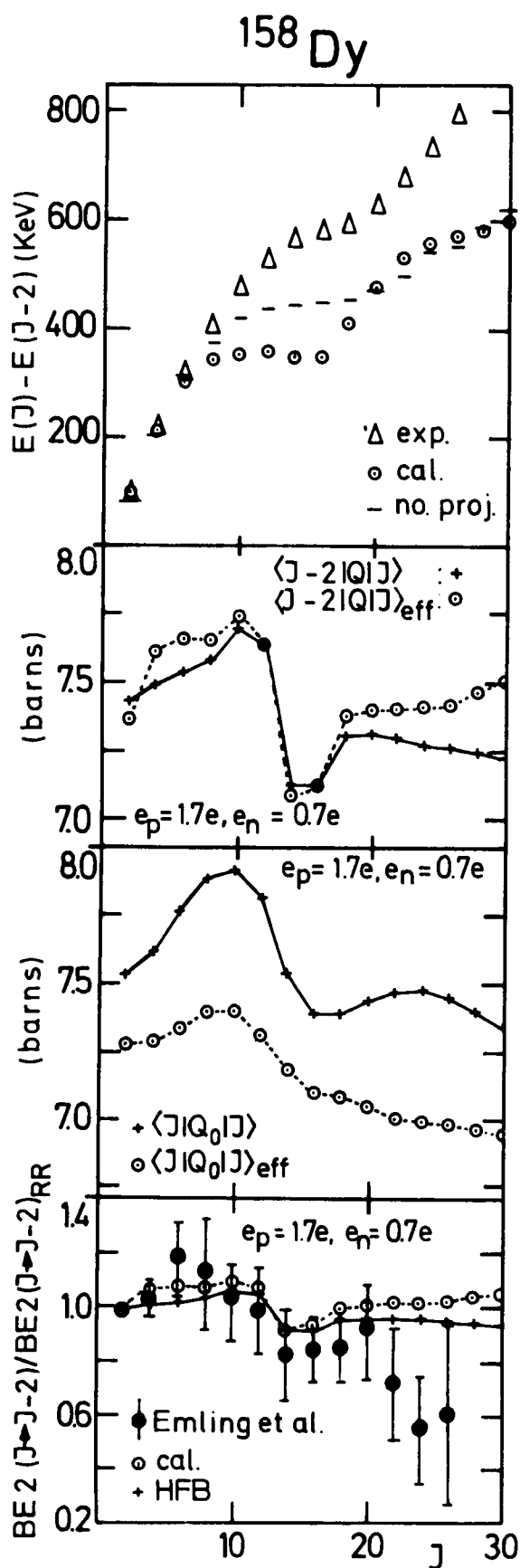


Figure 10:

Transition  
rates and  
moments for  
 $^{158}\text{Dy}^9$ .



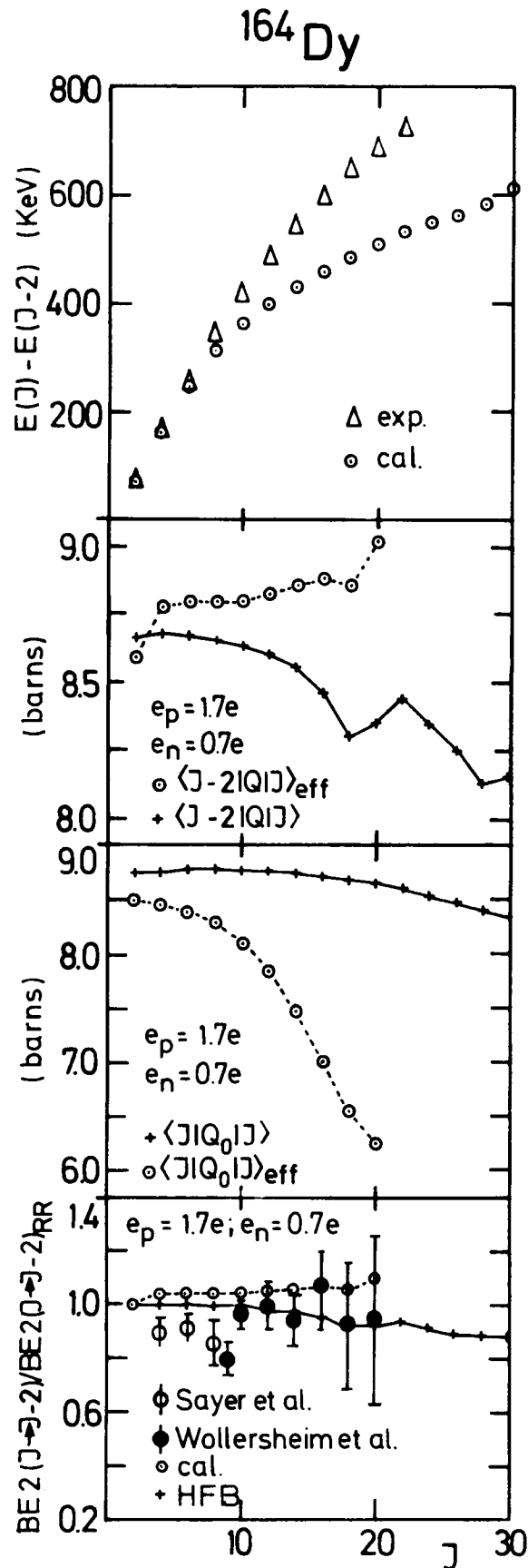


Figure 11:  
Transition  
rates and  
moments for  
 $^{164}\text{Dy}^9$ .

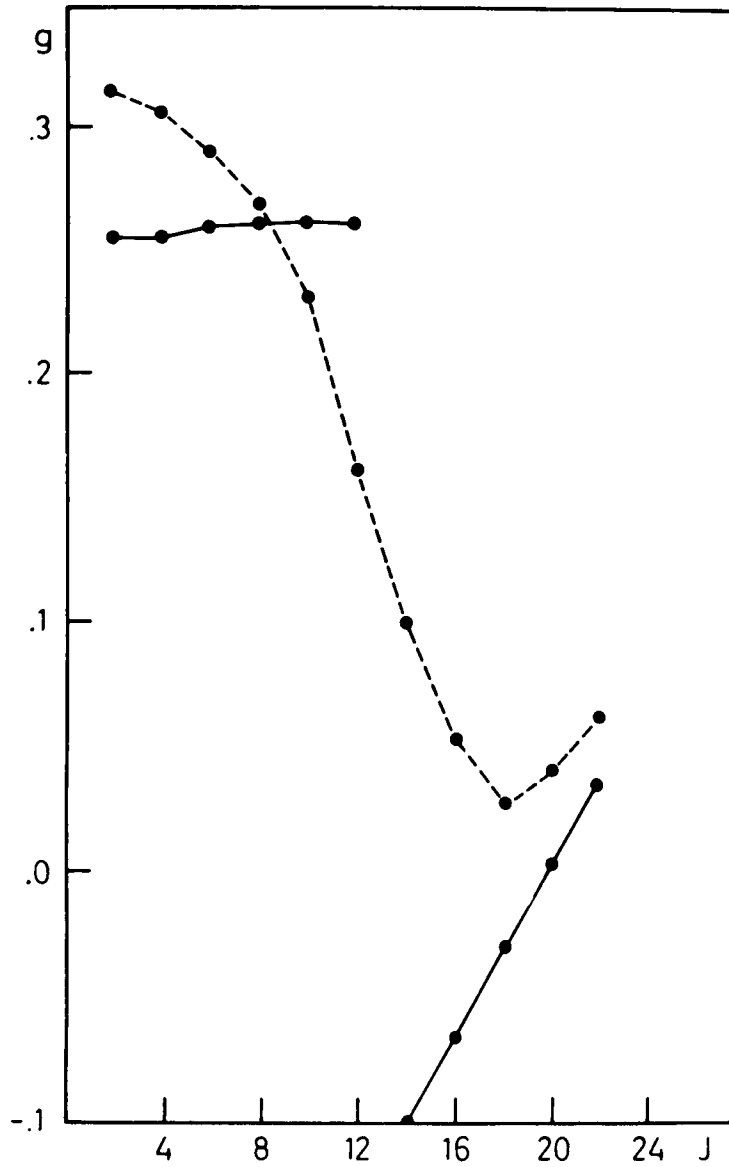


Figure 12:

g-factors for  $^{164}\text{Er}$  versus  $I$ ;  
•---• cranking, •—• with fluctuations  
included.

obtain such a sharp transition even if one is able to reproduce the energy levels fairly well.

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A Study of High-Energy Light Particle Emission  
from Heavy Ion Induced Reactions

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Introduction

An exciting aspect of the heavy ion beams provided by accelerator facilities now or soon to be available for research is the opportunity to study nuclear matter under stresses that cannot be produced with any other probes. At the lower projectile energies the nuclear structure and reaction dynamic effects of the high angular rotation stresses are being studied to better define the collective and single particle behavior of nuclear matter. As we push to higher energies, we introduce the possibility of compressional forces that may produce nuclear densities greater than those normally present in the nucleus of an atom. It is with such studies that we can hope to derive information about the behavior of the nuclear-matter equation of state.

At collision energies of one to two times the Coulomb barrier energy, the relative velocities are less than the typical Fermi velocities of the bound nucleons. In this domain, we expect the mean field properties of the nuclei to dominate the reaction process. At the other extreme, at relativistic collision energies, we expect the reaction to proceed as an incoherent superposition of individual two-particle collisions. To derive information on the behavior of the nuclear equation of state, we must understand the processes which lead to this transition from mean field to independent particle dominance of the nuclear reaction.

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\*Operated by the Union Carbide Corporation with the U.S. Department of Energy.

If high nuclear-matter densities are produced during reactions in this transition region, the lifetime of this hot, dense aggregation will be very short - the order of the reaction time. Thus, whatever we can deduce about this state of nuclear matter will have to be reconstructed from our examination of the nuclear debris remaining after the reaction.

One of the early surprises in examining reaction products from heavy ion reactions at 10 MeV/nucleon and above was the large yield of light particles emitted and the high energies to which the spectra of these particles extended. At Oak Ridge, a group of us have been involved actively in studying light particle emission from heavy ion reactions since about 1976. This paper is intended as a progress report on that work and a look at our present state of understanding about this phenomenon. This summary relies heavily on the work, in particular, of Ron Auble, Russell Robinson, and Clyde Fulmer of the Oak Ridge group.

#### A Bit of History

In 1976, after we had commenced construction of the new 25MV tandem facility at Oak Ridge, we began to examine possible future large booster accelerators that would provide significant increases in ion energies. As part of that study we had to examine shielding requirements for experiment areas where one could safely stop heavy ion beams with energies from 100 to 200 MeV/nucleon. The amount of shielding required would be, of course, mandated by the flux of fast neutrons produced by stopping such beams. Early estimates generally involved assumptions that the neutron spectra would cover energies up to that corresponding

to the energy/nucleon of the incident projectile. To better quantify the neutron spectra, computations were made using an intra-nuclear cascade-evaporation code that had been modified to treat incident heavy ions.<sup>1</sup> The calculated neutron spectra were found to extend to energies significantly higher than the energy per nucleon introduced by the incident projectile. Because of the impact on cost of shielding structures, it was deemed important to test the predictions of this calculation.

Because there were no 100 MeV/nucleon heavy ion beams available at that time, the cascade-evaporation calculations were repeated for incident 16 MeV/nucleon carbon ions. This ion-energy combination represented the highest energy/nucleon available to us with ion beams from the Oak Ridge Isochronous Cyclotron utilizing its internal Penning source. Surprisingly, the calculations predicted that the spectra of neutrons and protons emitted would extend to several times the energy/nucleon of the incident carbon ions.

Since there was little difference between predicted spectra of neutrons and protons, we chose to perform the easier measurement of the charged-particle spectra.<sup>2</sup> The results, shown in Fig. 1, confirmed the existence of light particles emitted with very high energies. There were, to us, two additional surprises in these results: First, the similar cross sections observed for emission of protons and the composite  $Z=1$  particles - deuterons and tritons. This is in sharp contrast to the more familiar order of magnitude differences seen in direct reactions and low-energy compound nuclear reactions. Second, the large magnitude of the observed cross sections for energetic particles. The proton spectra, as shown in Fig. 2, have an instrumental cut-off at

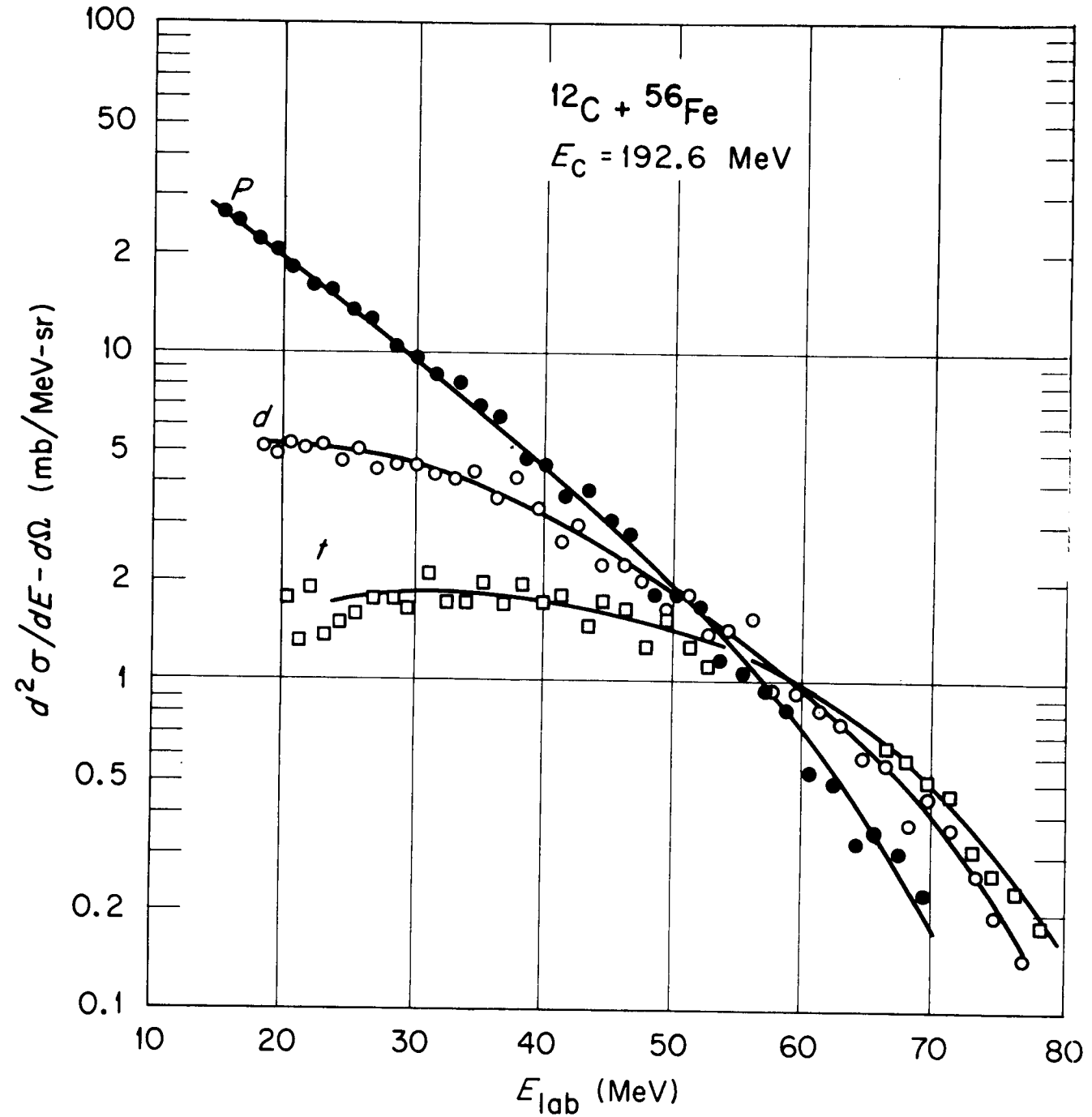


Fig. 1. Energy spectra observed at a laboratory angle of  $15^\circ$  for  $Z=1$  particles emitted following the reaction of 16 MeV/nucleon  $^{12}\text{C}$  ions with a target of  $^{56}\text{Fe}$ .



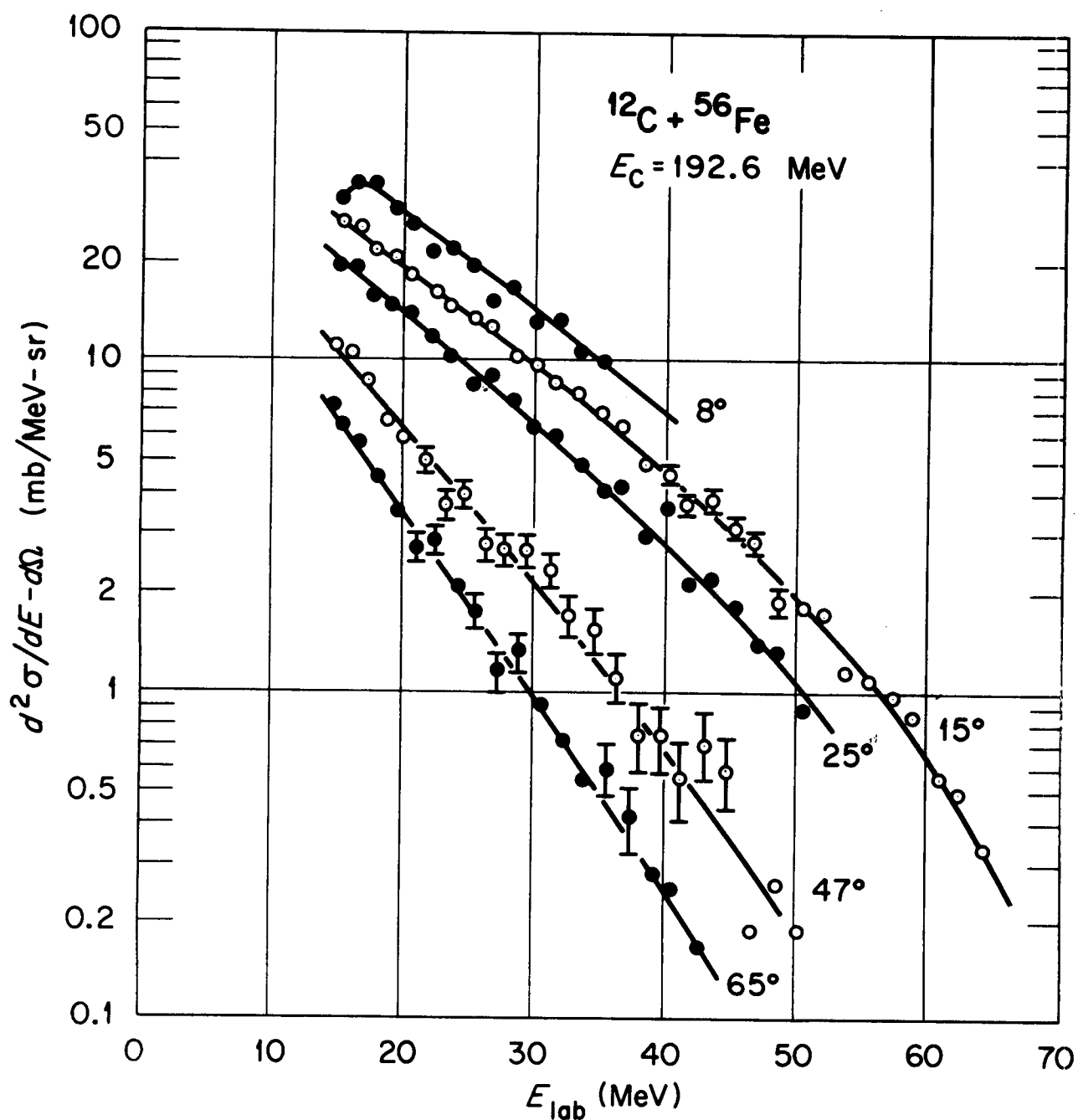


Fig. 2. Energy spectra of protons emitted at several angles from the reaction of 16 MeV/nucleon  $^{12}\text{C}$  with  $^{56}\text{Fe}$ .

about 15 MeV. The observed integrated yield above this energy is about 0.8 barns. Such a large cross section implies that processes leading to the emission of these light particles must play a major role in the reaction mechanism at these energies.

Thus, what had begun as a quick experiment to answer a practical question resulted in raising some very interesting questions about the physics of these reactions.

#### Reactions Below 20 MeV/Nucleon

The experiment discussed above was not the first identification of possible non-equilibrium emission of light particles from heavy-ion induced reactions. In 1961, Britt and Quinton reported a forward-peaked component in proton spectra from reactions at 10.5 MeV/nucleon.<sup>3</sup> They proposed that this direct component was produced by breakup of the incident particle during a grazing collision with the target. However, in their experiment, the high energy nature of the proton spectrum was not identified. Since the breakup mechanism will produce laboratory energies for the light particles that will not differ greatly from the energy/nucleon of the incident ion, this mechanism cannot account for the results obtained with  $^{12}\text{C}$  ions at 16 MeV/nucleon where the proton spectra extend almost to 5 times this energy.

The results of the cascade-evaporation calculation discussed in the previous section are compared to the 16 MeV/nucleon induced proton spectra in Fig. 3. Although somewhat deficient at the higher energies, the calculation gives a quite reasonable account of the data. How much

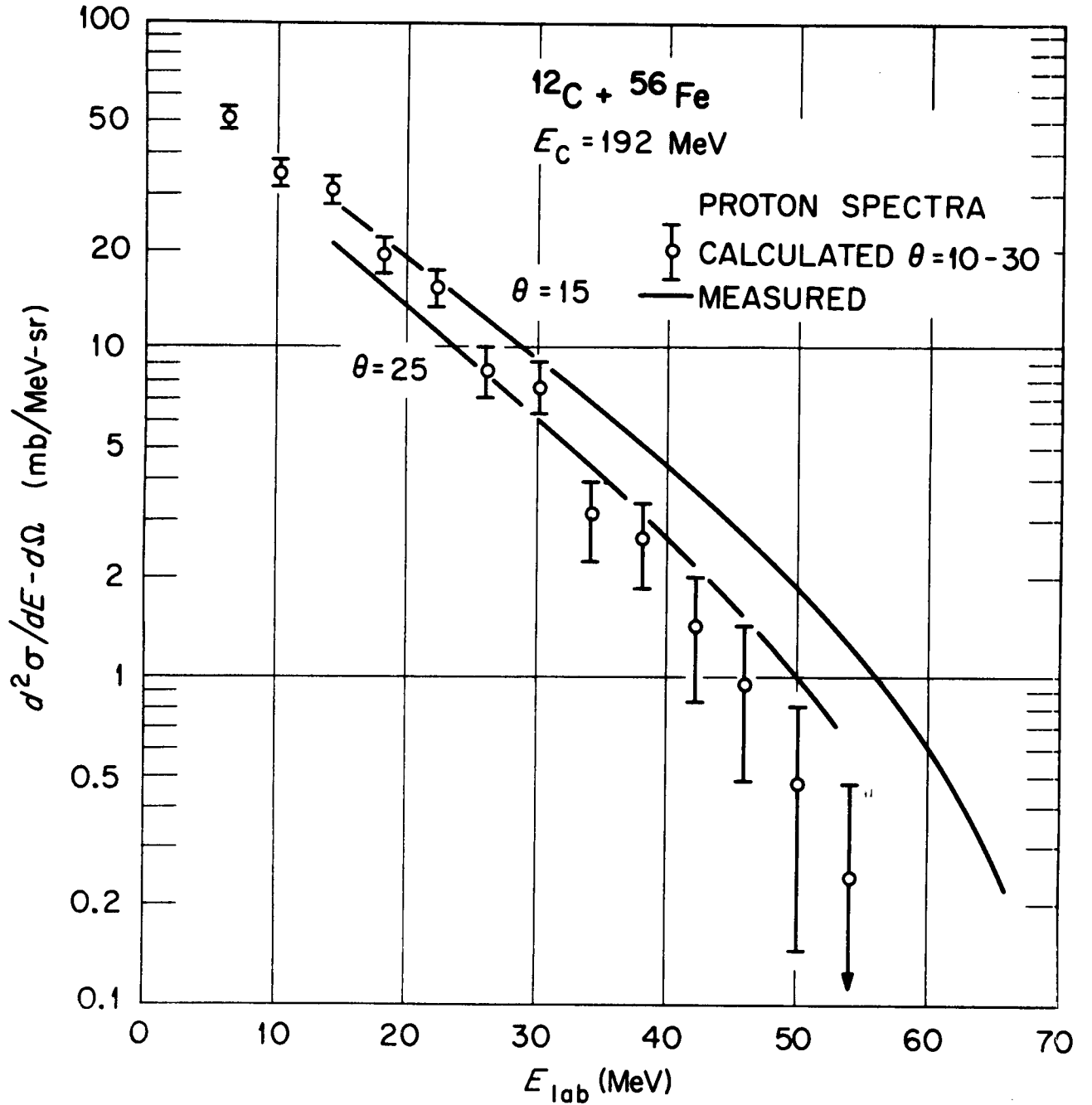


Fig. 3. Comparison of observed proton spectra with results of the cascade-evaporation calculation. The lines are those drawn through the experimental points on Fig. 2. The error bars on the calculated points represent the statistical uncertainties introduced by use of the Monte Carlo method of the calculations.

of this is fortuitous is difficult to say. In particular, the calculations employ the impulse approximation which is difficult to justify at such low energies. Even so, it seems worthwhile to use the calculations as a guide. An examination of the details of the calculations<sup>4</sup> shows that, at these energies, essentially all of the high energy light ions are produced by excitation of the projectile and subsequent evaporation from the excited projectile-like fragment. Such a mechanism had been proposed earlier<sup>5</sup> as a means to produce high energy light ions in heavy ion collisions. In this picture, the high-energy components of the light particle spectra are produced by the addition of the velocity of the evaporated particle in the rest frame of the moving projectile fragment to the laboratory velocity of this source.

Note that there is a subtle distinction here with the time scale of the reaction and the instant at which the light ions are produced. For the projectile breakup mechanism, the production takes place at the instant of projectile-target separation. For the projectile evaporation mechanism, the production takes place at some time following the separation. One inference would be that the particles formed by the latter process would carry very little information about the collision process. This point will be returned to below.

If projectile evaporation is an important mechanism for producing the high-energy light ions, then we should be able to detect a correlation between the fast light particles and their associated projectile-like fragments.

At Oak Ridge, Robinson and co-workers have performed such a measurement<sup>6</sup> for reactions induced by 16 MeV/nucleon  $^{12}\text{C}$  ions on a  $^{60}\text{Ni}$  target. This experiment involved 16 detectors and observed a limited angular distribution of emitted light ions in coincidence with 1) projectile-like fragments, 2)  $Z=1,2$  particles, 3)  $\gamma$ -ray multiplicities, and 4) discrete  $\gamma$ -rays. The experimental arrangement utilized four light ion detector telescopes at laboratory angles of  $17^\circ$ ,  $38^\circ$ ,  $-23^\circ$ , and  $-49^\circ$ . The telescope at  $17^\circ$  was preceded by a heavy ion telescope. Two  $\text{Ge(Li)}$  gamma-ray detectors were located in the scattering plane at  $90^\circ$  and  $-145^\circ$ . Four  $\text{NaI(Tl)}$  detectors above the scattering plane were used for determining gamma-ray multiplicities.

The angular yields of energetic alpha particles in coincidence with He, Li, and C fragments are shown in Fig. 4. For all gating fragments, the angular distribution of the alpha particles exhibits a single peak and appears symmetric about the beam axis. No obvious correlation with the gating detector is observed.

For protons, as shown in Fig. 5, the result is more complex. For the heavier fragments, a drop in the coincidence yield appears to correlate with the position of the gating detector. Thus it would appear that the heavy projectile-like fragments are acting as a shield--essentially the opposite effect we would expect from the picture of projectile evaporation.

The multiplicity of energetic protons and alpha particles that occur in coincidence with a projectile fragment is plotted in Fig. 6. Note, for example, that there is no sharp break in proton multiplicity between  $Z=5$  and 6. This indicates that few of the fast protons come

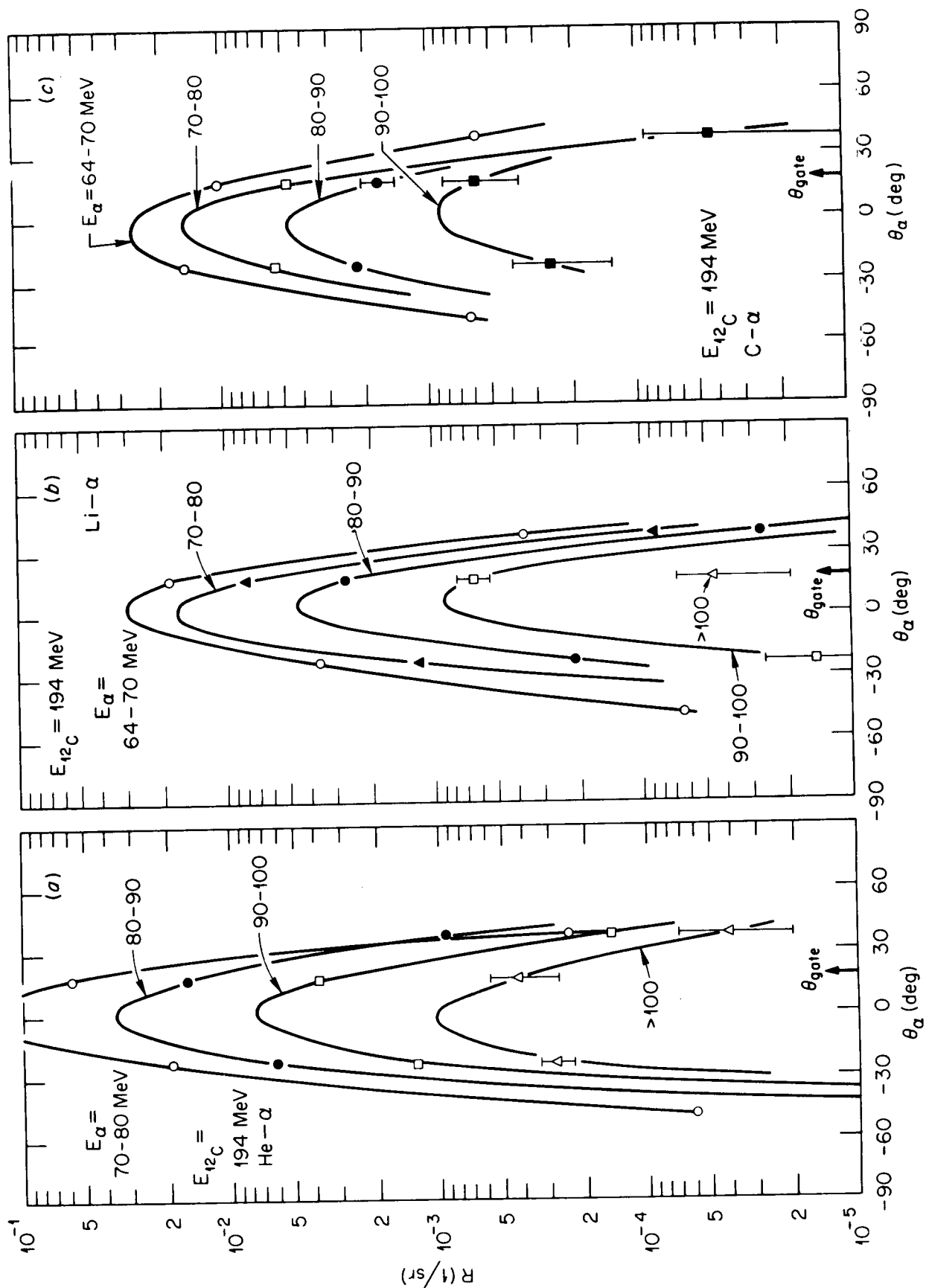


Fig. 4. Angular yields of the energetic alpha particles in coincidence with a) He, b) Li, and c) C from the interaction of 16 MeV/nucleon  $^{12}\text{C}$  with  $^{60}\text{Ni}$ . The coincident alpha particles are divided in the energy bins noted in the figure. The angle of the gating particle is  $17^\circ$ .

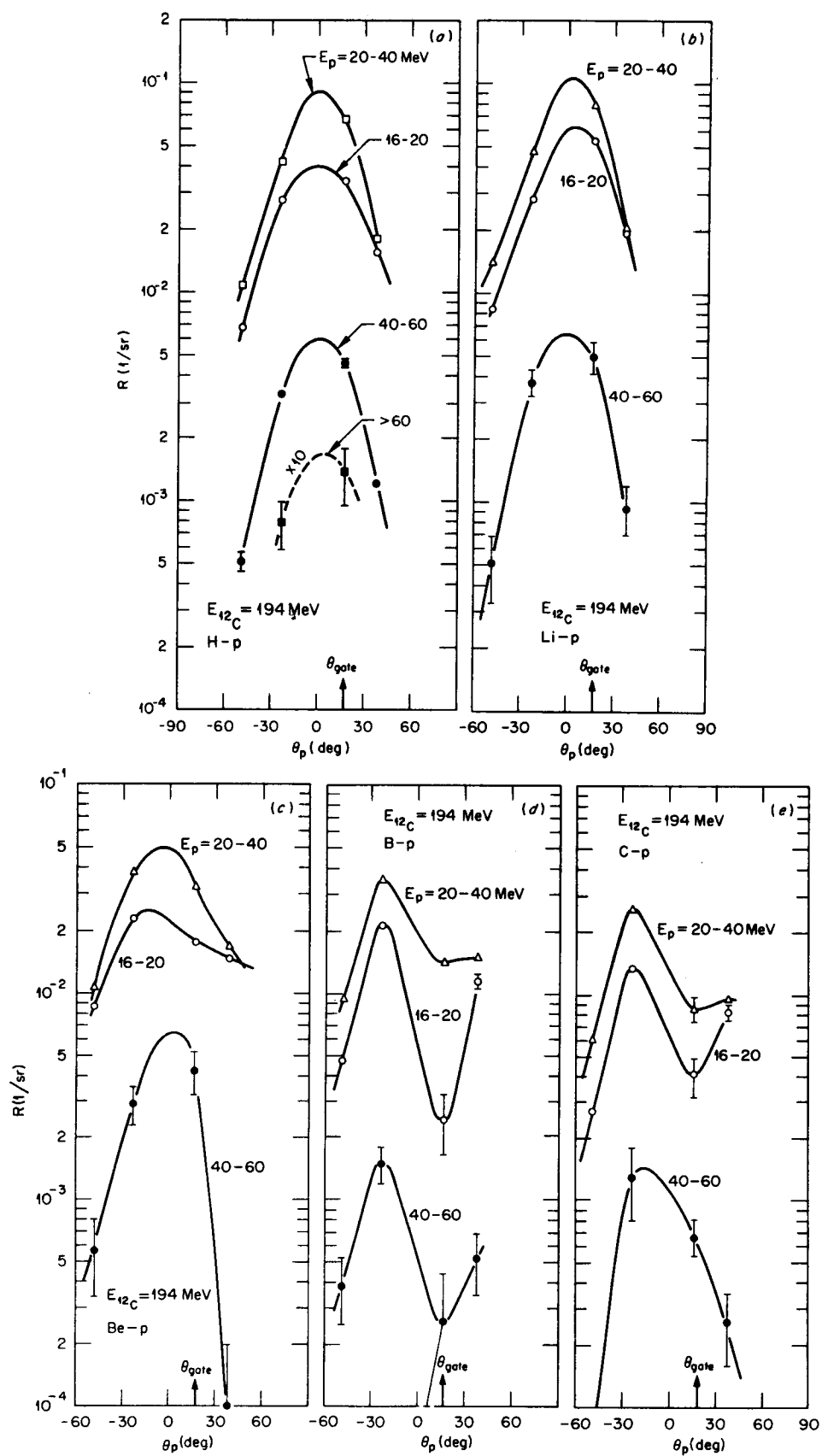


Fig. 5. Angular yields of energetic protons in coincidence with a) H, b) Li, c) Be, d) B, and e) C for the same conditions as the data of Fig. 4.

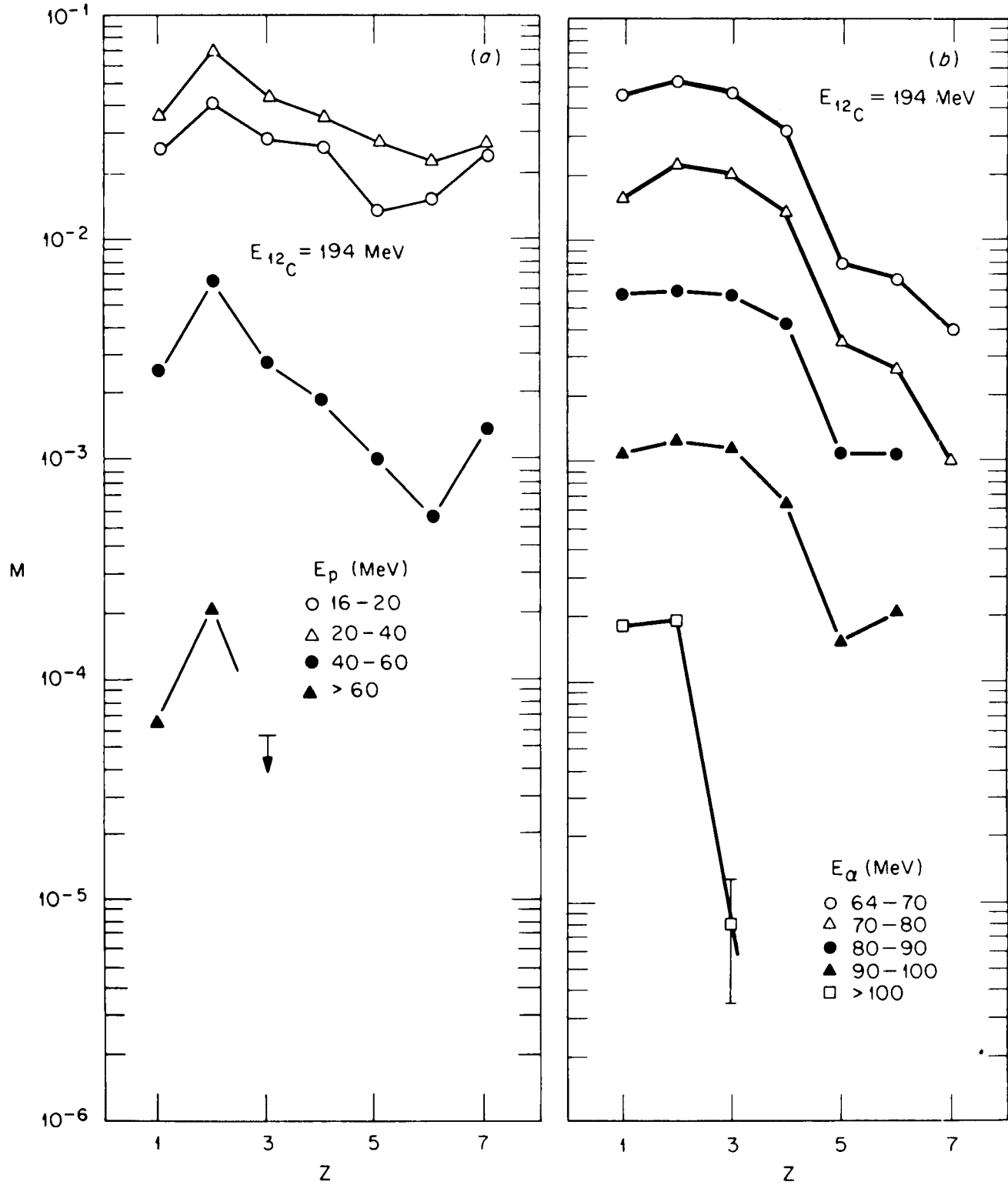


Fig. 6. The multiplicity of a) protons and b) alpha particles in coincidence with  $Z=1$  to  $7$  particles. The reaction is  $16 \text{ MeV/amu } ^{12}\text{C}$  on  $^{60}\text{Ni}$ .



from the simple process of the carbon ion breaking up into  $B + p$  without further breakup of the  $B$  fragment. The observation of alpha particles in coincidence with  $Z \geq 5$  fragments and protons with  $Z \geq 6$  fragments shows clearly that at least some energetic  $Z=1,2$  particles do not come from the projectile in a simple one-step process. Taken together, however, these results do not support the projectile evaporation picture as a dominant reaction process.

Another possibility is that, following evaporation of a light particle, the projectile fragment and target fuse. Such a process has been observed by the Texas A&M group and termed "massive transfer."<sup>7,8</sup> In the present experiment, the coincident gamma-ray spectra were used to identify the fusion residues. A plot of the yield of residual nuclei on an  $N$  versus  $Z$  map is shown in Fig. 7. Note that the distributions tend to center about the  $^{60}\text{Ni}$  target. It is interesting to compare this with similar measurements made with 11.3 MeV/nucleon carbon ions (Fig. 8). Here the centroid tends to be a few masses higher. The implication, then, is that as the reaction energy is increased, more particles are emitted from the reacting system. Note also that the maximum yield for any channel never exceeds about 5% to 8% of the total. The conclusion I would draw from these two figures is that while the process of "massive transfer" may be present, it does not play a dominant role in the reactions at these energies.

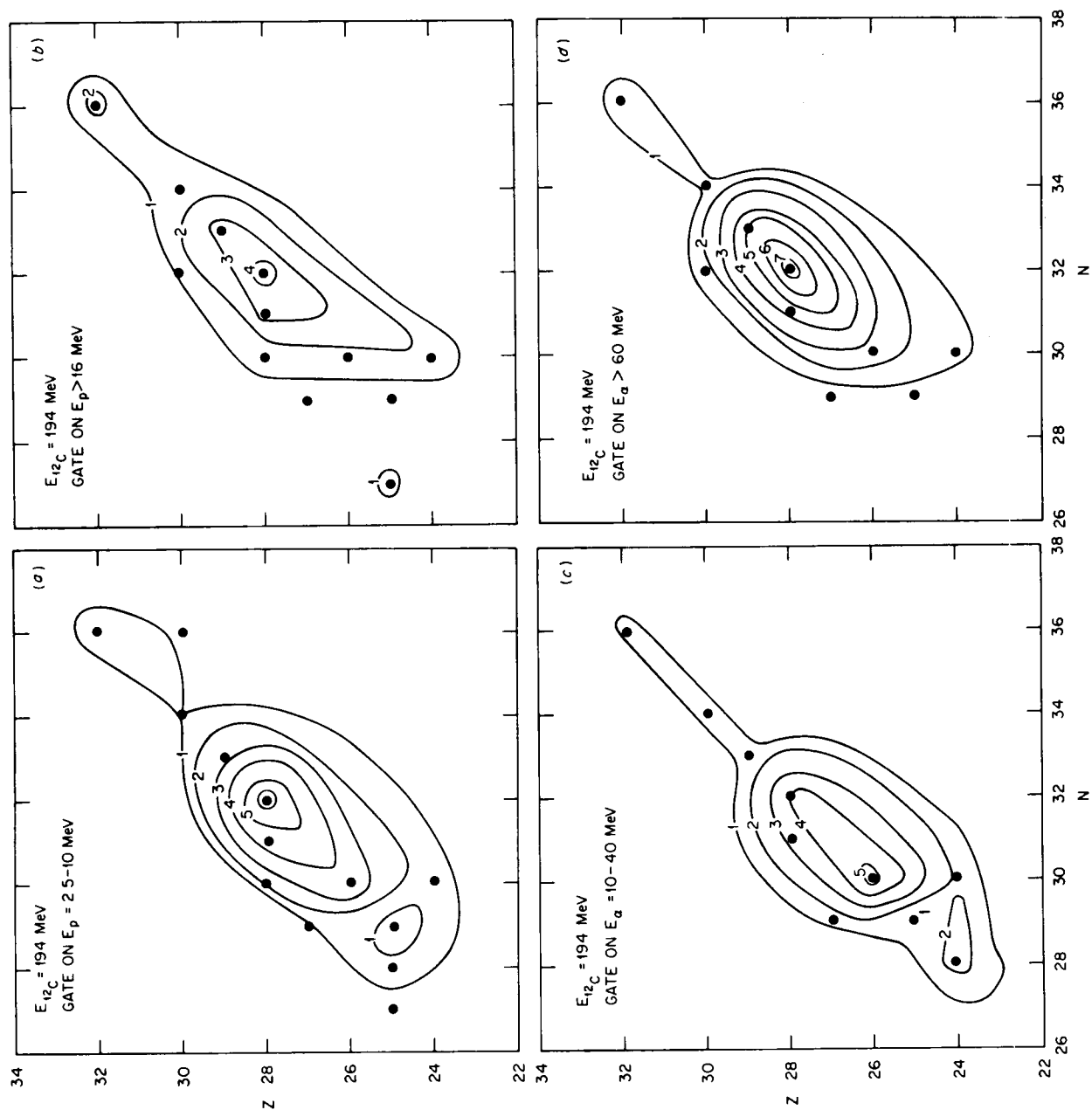


Fig. 7. Distribution of residual nuclei, for 16 MeV/amu  $^{12}\text{C}$  on  $^{60}\text{Ni}$ , in coincidence with a) 2.5 to 10 MeV protons, b)  $>16$  MeV protons, c) 10 to 40 MeV alpha particles, and d)  $>60$  MeV alpha particles. The contour lines correspond to percent of all coincident residual nuclei. The measured points on which these contours are based are indicated by the solid points.

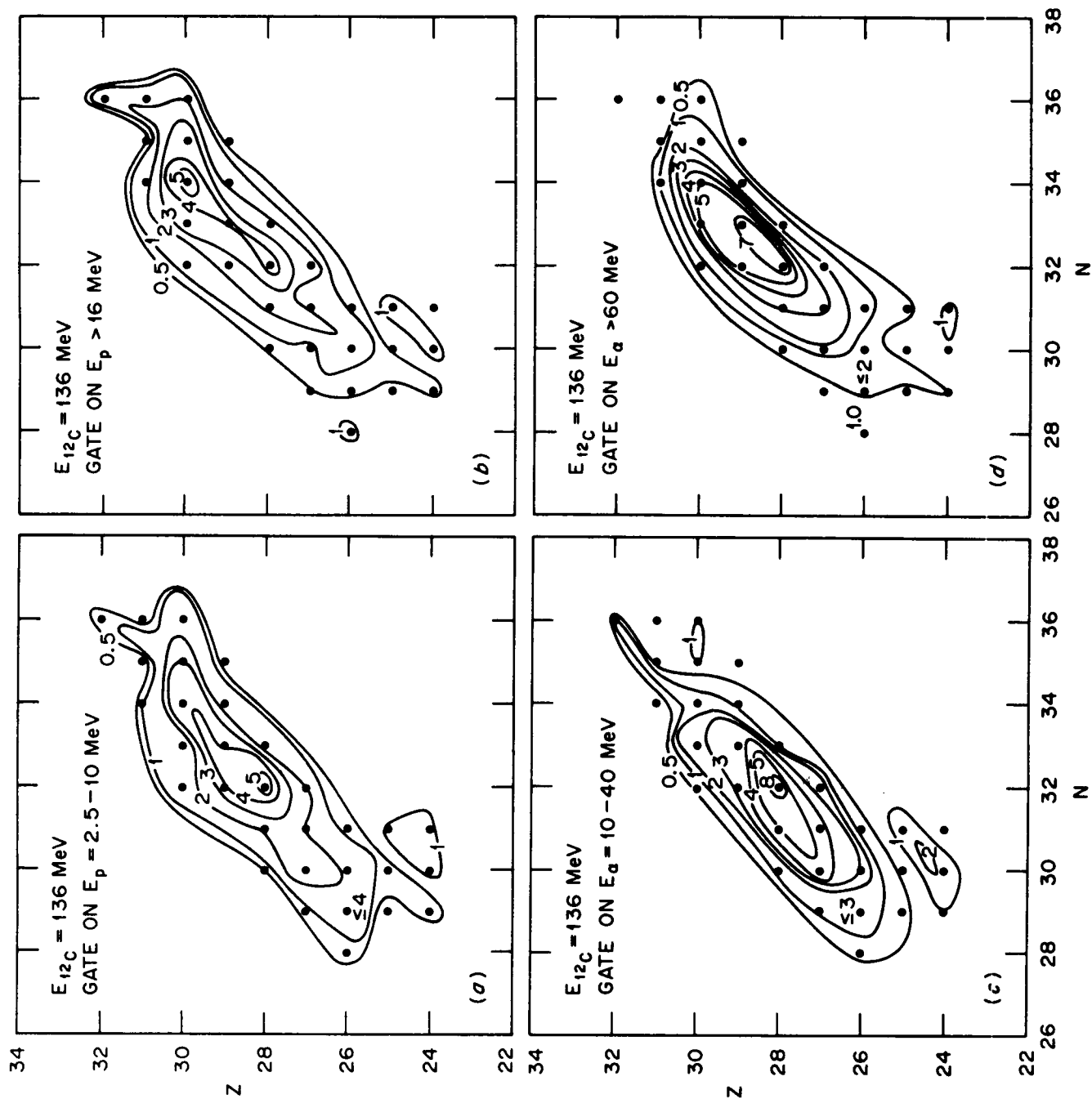


Fig. 8. Distribution of coincident residual nuclei, as shown in Fig. 7, except that the incident  $^{12}\text{C}$  projectile energy has been reduced to 11.3 MeV/amu.

### A Moving Source

One of the most successful models applied to the interpretation of relativistic heavy ion collisions involves the assumption of an excited assembly of nucleons moving at a velocity intermediate between those of the target and incident projectile.<sup>9,10</sup> Pictorially, such a source would be formed from the overlap region between the incident projectile and the target nucleus. Various versions have gone by such exotic names as "nuclear fireball," "firestreak," etc.

The resulting parameterization, that of an excited moving source emitting particles at some equilibrated temperature, has been interestingly applied to data in the 20 MeV/nucleon region.<sup>11,12</sup> An example<sup>13</sup> of such a treatment of data from  $^{12}\text{C}$ -induced reactions is shown in Fig. 9 for incident ion energies of 7.5 and 15.8 MeV/nucleon. A similar treatment of composite particle spectra observed from the experiment at 15.8 MeV/nucleon is shown in Fig. 10. These and data at 10 and 13 MeV/nucleon are summarized in Table I. The parameters deduced exhibit a marked consistency and show a rather smooth energy dependence.

### To Higher Energies

Given a parameterization, the moving source, that can be fit to data in the 10 to 20 MeV/nucleon energy region as well as to data<sup>9</sup> at relativistic energies, an obvious question is can anything be learned by following the systematics of this parameterization through the intermediate "transition" region. In particular, will there be any

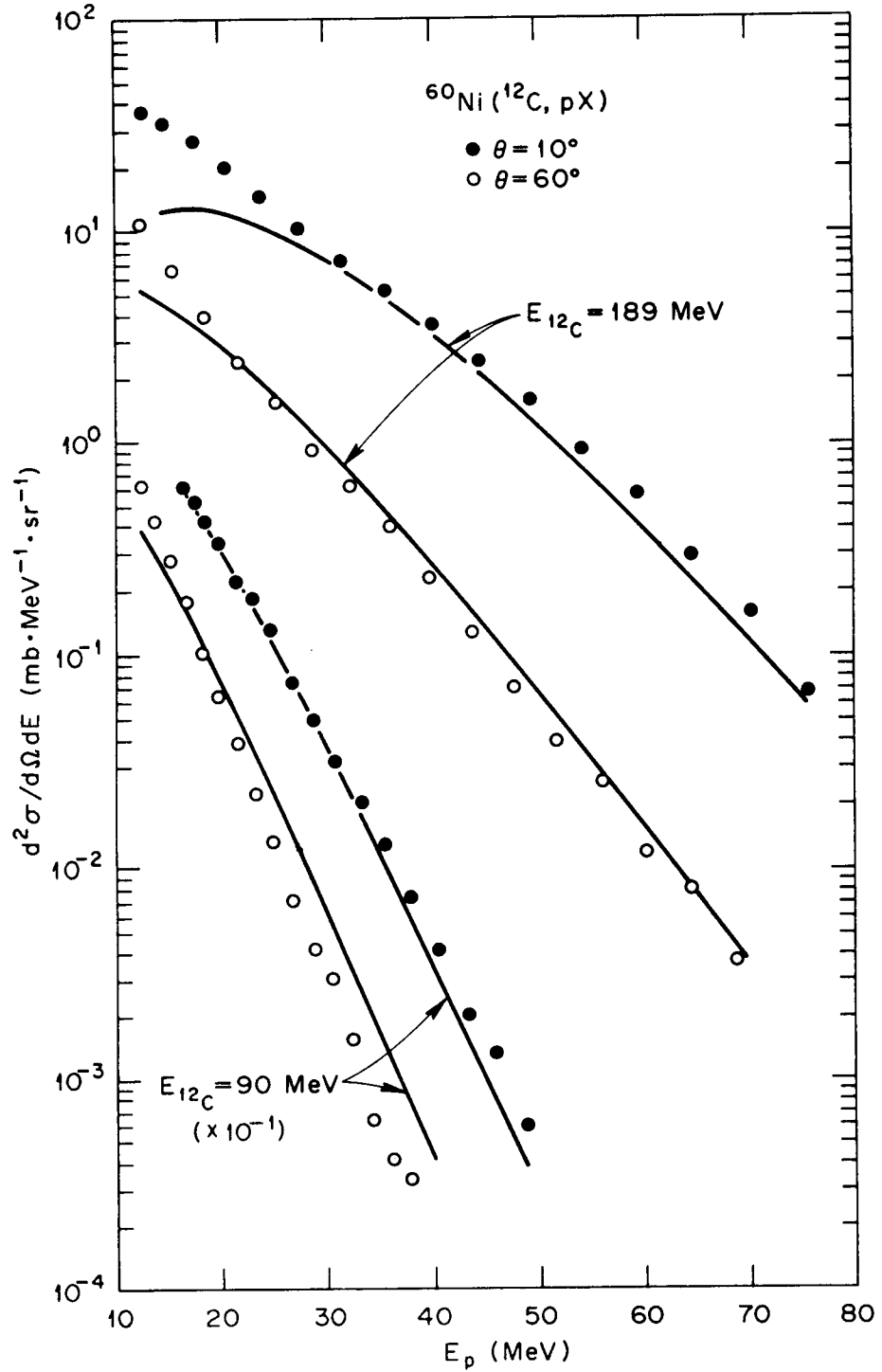


Fig. 9. Comparison of proton spectra at 7.5 and 15.8 MeV/amu incident energy (solid and open points) with fits obtained assuming light-ion emission from a moving, excited source (solid curves).

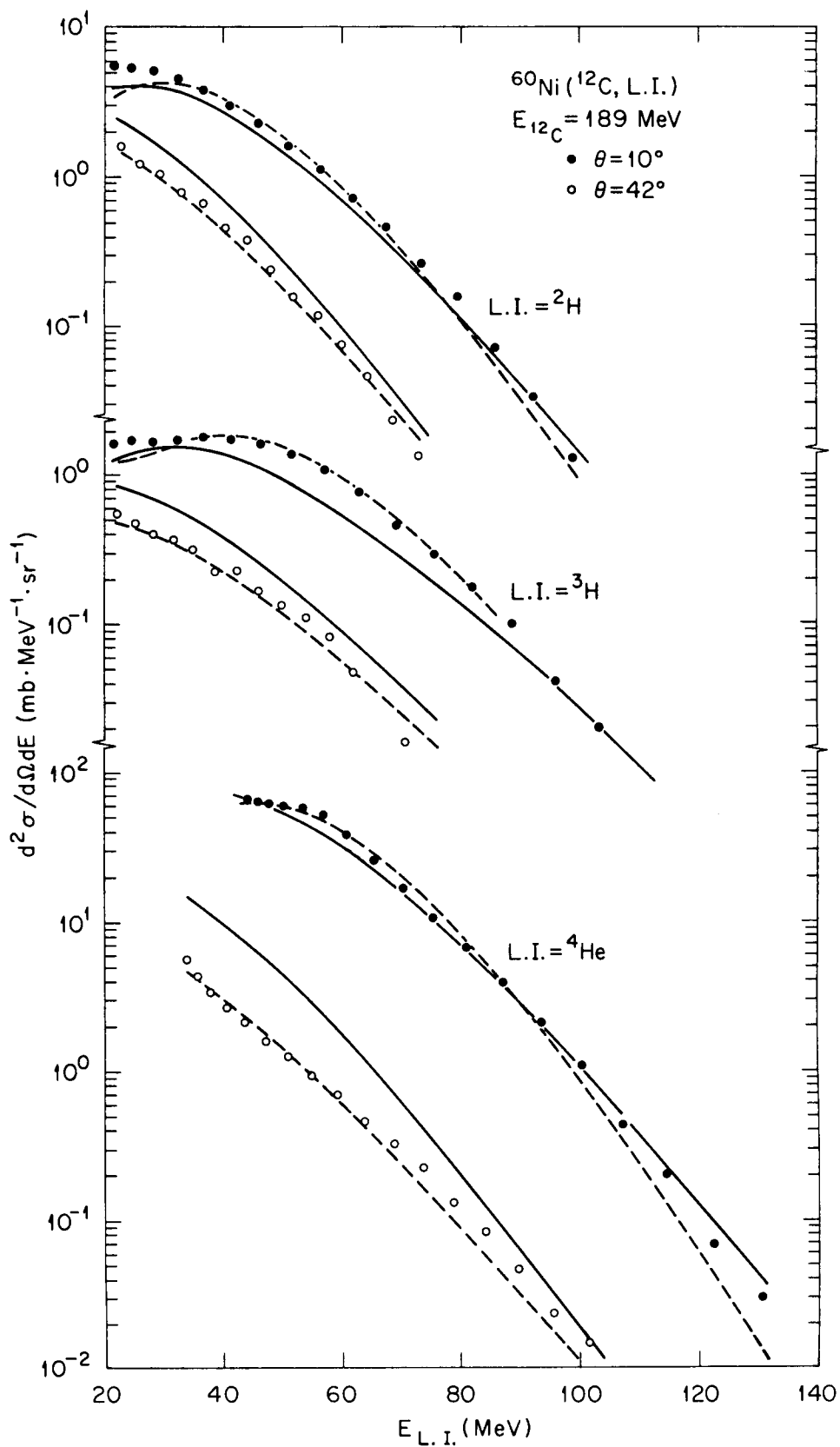


Fig. 10. Comparison of composite particle spectra with assumption of emission from a moving source.

Table I. Velocity ( $V_S/V_{PROJ}$ ) and temperature (T) parameters, with associated normalization factors (K), obtained by parameterizing light ion spectra from  $^{60}\text{Ni}(^{12}\text{C}, \text{L.I.})$  reactions in terms of a moving source.

| $E_{\text{Beam}}$<br>(MeV) | $V_S/V_{PROJ}$ | T(MeV) | K    |
|----------------------------|----------------|--------|------|
| PROTONS                    |                |        |      |
| 90                         | 0.53           | 2.9    | 5.7  |
| 123                        | 0.56           | 3.6    | 4.9  |
| 156                        | 0.60           | 4.3    | 3.7  |
| 189                        | 0.56           | 5.3    | 3.3  |
| DEUTERONS                  |                |        |      |
| 90                         | 0.47           | 3.0    | 0.68 |
| 123                        | 0.50           | 3.9    | 0.89 |
| 156                        | 0.53           | 4.5    | 1.2  |
| 189                        | 0.56           | 5.8    | 0.82 |
| TRITONS                    |                |        |      |
| 90                         | 0.42           | 2.7    | 0.44 |
| 123                        | 0.46           | 3.9    | 0.35 |
| 156                        | 0.54           | 4.9    | 0.34 |
| 189                        | 0.56           | 6.6    | 0.24 |
| ALPHAS                     |                |        |      |
| 90                         | 0.46           | 3.9    | 4.4  |
| 123                        | 0.51           | 4.0    | 15   |
| 156                        | 0.55           | 4.1    | 27   |
| 189                        | 0.54           | 5.0    | 14   |

discontinuities that could signal the onset of compressional modes, thermal saturation, etc.

To study light particle emission in this intermediate energy region, measurements were made using 50, 100, and 147 MeV/nucleon oxygen ion beams at the low-energy beam line of the LBL Bevalac.<sup>14</sup> An example of proton spectra observed from the nickel target is shown in Fig. 11. The data extend over an angular range from 6° to 115°. Alpha particle spectra observed from this same reaction are shown in Fig. 12. At forward angles, the particle spectra are seen to peak at an energy near the energy per nucleon of the projectile. This general behavior is exemplified by the 12-degree proton spectra taken at the three projectile energies as shown in Fig. 13.

Typical proton spectra at 100 MeV/nucleon for a range of targets are shown in Fig. 14. The spectral shapes are seen to be independent of target mass. Moving source parameterization of the data was obtained for the angular range from 24 to 115 degrees to reduce the influence of the fragmentation component. Velocity and temperature parameters were found to be insensitive to the target mass. A plot of these, including the lower energy <sup>12</sup>C data, is shown in Fig. 15. The insensitivity of the ratio of source velocity to projectile velocity and the smooth variation of the "temperature" parameter are quite striking. Certainly one does not find any evidence for discontinuities in such a parameterization through this energy region.

It may be worth remarking that the concept of a "temperature" as defined here is not clear. In particular, there is no a priori reason for assuming that the emission of the particles observed is from an



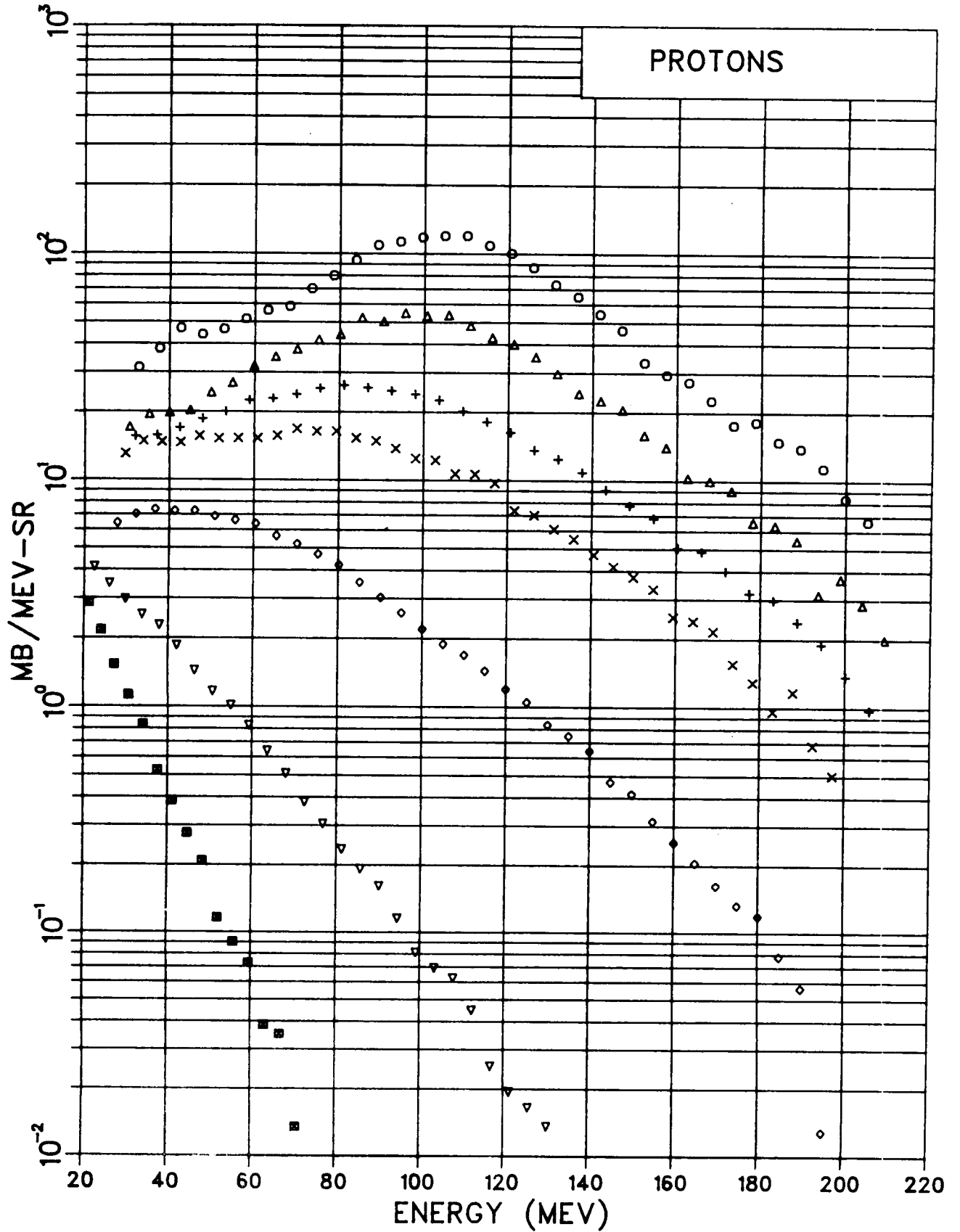


Fig. 11. Proton spectra observed from the reaction of 100 MeV/amu  $^{160}\text{Ni}$  ions with a natural nickel target. The angles of observation are (from top to bottom): 6, 12, 18, 24, 45, 85, and 115 degrees.

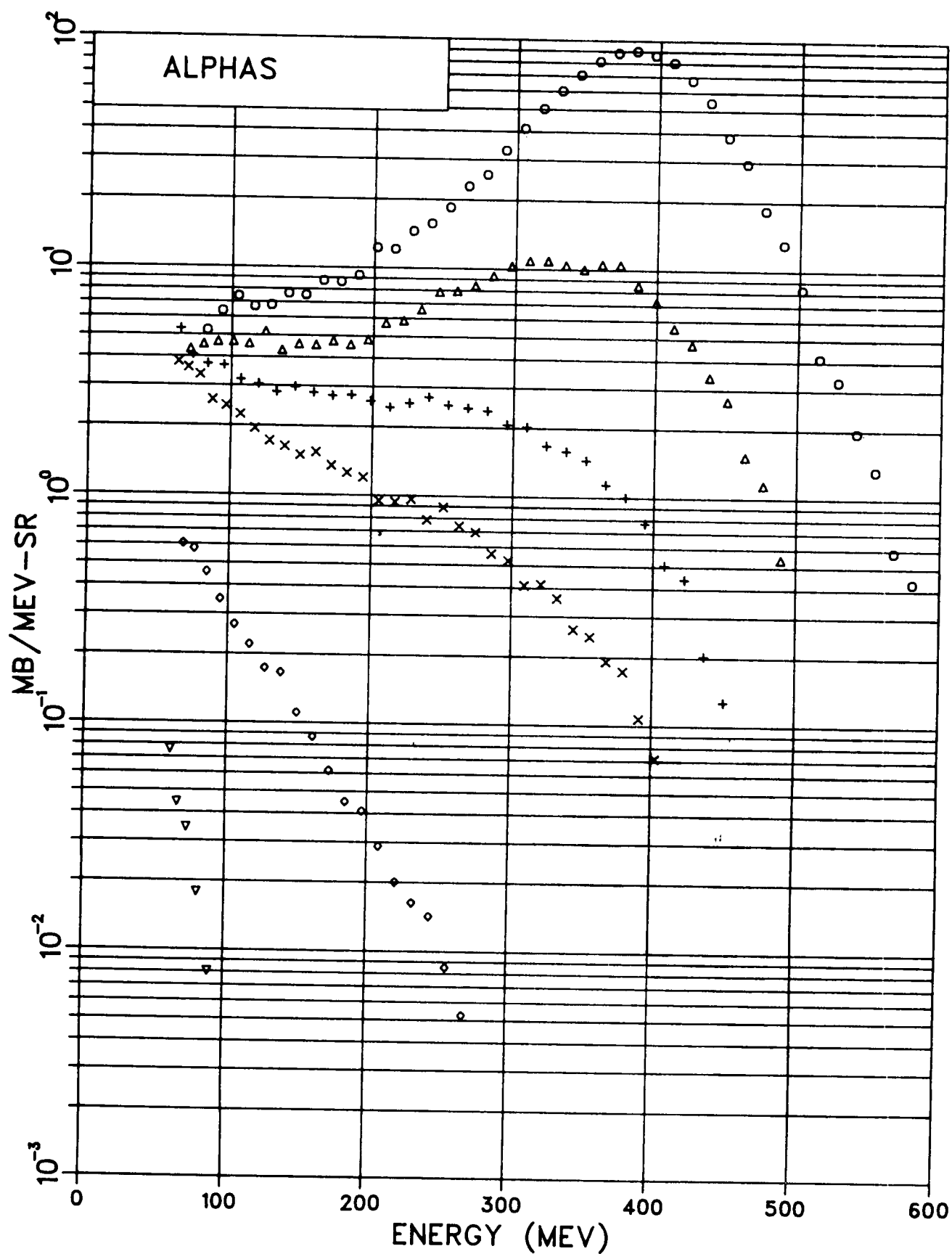


Fig. 12. Alpha particle spectra observed from the reaction of 100 MeV/amu  $^{16}\text{O}$  ions with a natural nickel target. The angles of observation are (from top to bottom): 6, 12, 18, 24, 45, and 85 degrees.

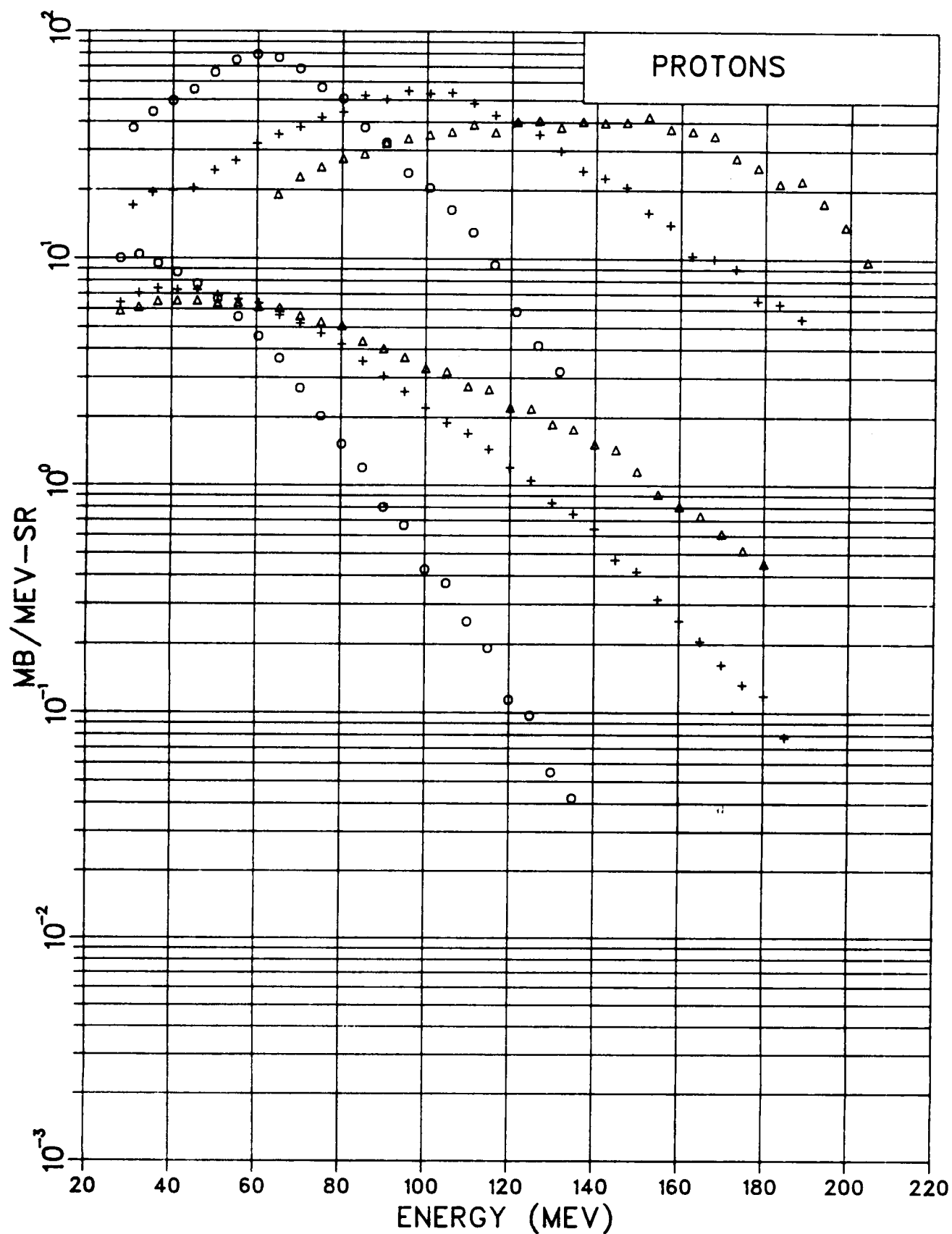


Fig. 13. Proton spectra at 160 energies of 51 (circles), 100 (crosses), and 147 MeV/amu (triangles) at scattering angles of 12 and 45 degrees.

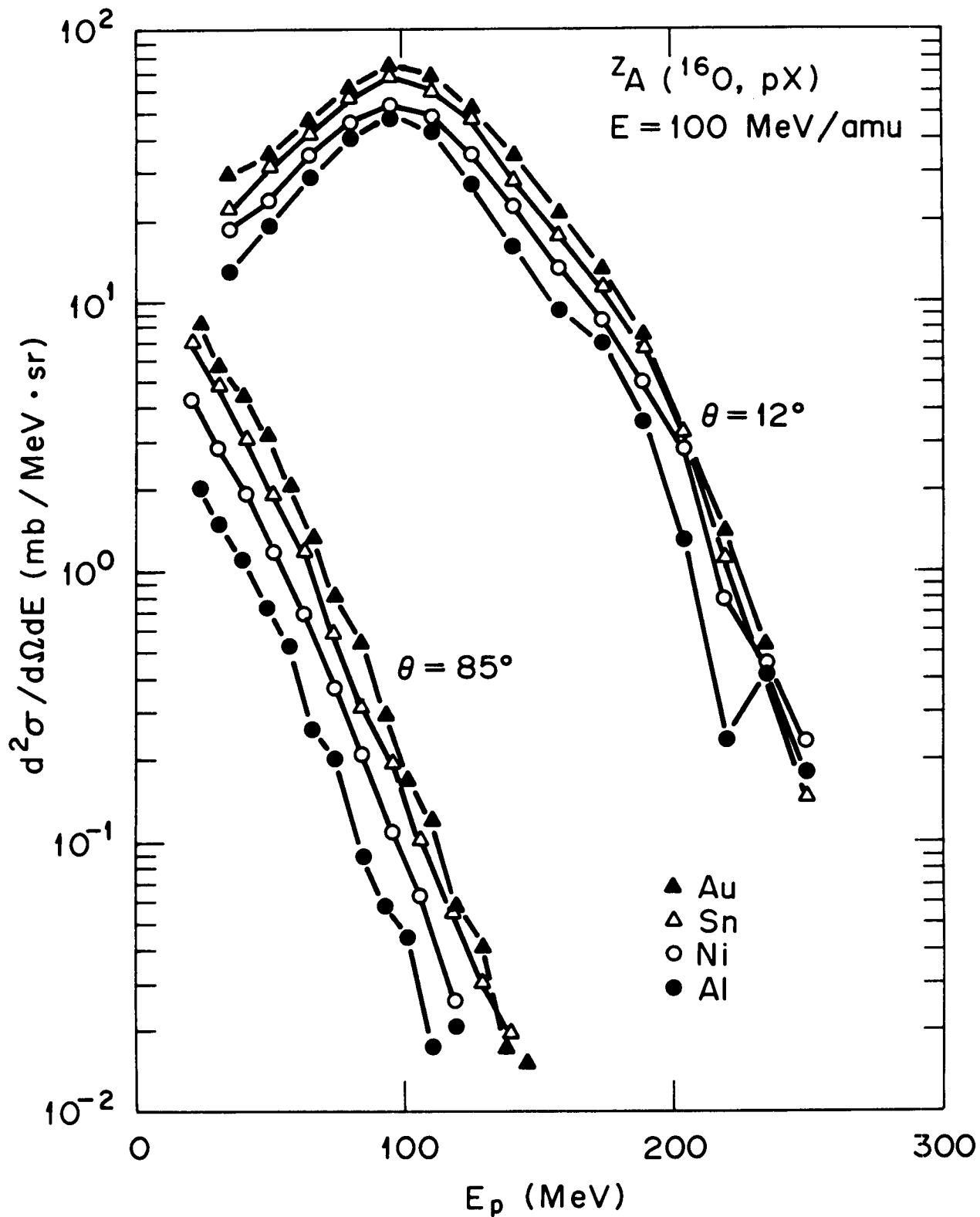


Fig. 14. Proton spectra from reactions induced by 100 MeV/amu  $^{16}\text{O}$  ions with various targets.

equilibrated source. More likely, we are observing the momentum distribution of an interaction region with a rest frame different from the projectile or the compound system. This point will be returned to in the last section.

There is another interesting feature that emerged from the 100 MeV/nucleon data. If the energy integrated yields at each angle are plotted as a function of target mass, the behavior as shown in Fig. 16 is found. At the forward angles, the yield is seen to vary as  $A^{1/3}$  and to change smoothly to an  $A^{2/3}$  dependence as the angle is increased. This suggests a change from predominately peripheral interactions at forward angles to predominately central collisions contributing to the larger angles. This may suggest that the most forward angle spectra, where the cross sections peak at or near energies corresponding to the projectile velocity, could be due primarily to projectile fragmentation.<sup>15</sup> A predominately thermal mechanism may be responsible for the particle spectra observed at larger angles. The moving source parameterization, shown in Fig. 15, did not include the most forward angles for the high energy data.

### Composite Particles

In examining the systematics of proton spectra for reactions induced at incident ion energies between 7.5 and 147 MeV/nucleon, no change in qualitative behavior has been observed that might signal a change in reaction dynamics over this "transition" region. It may be that we are not using an appropriate analysis to see signs of such a

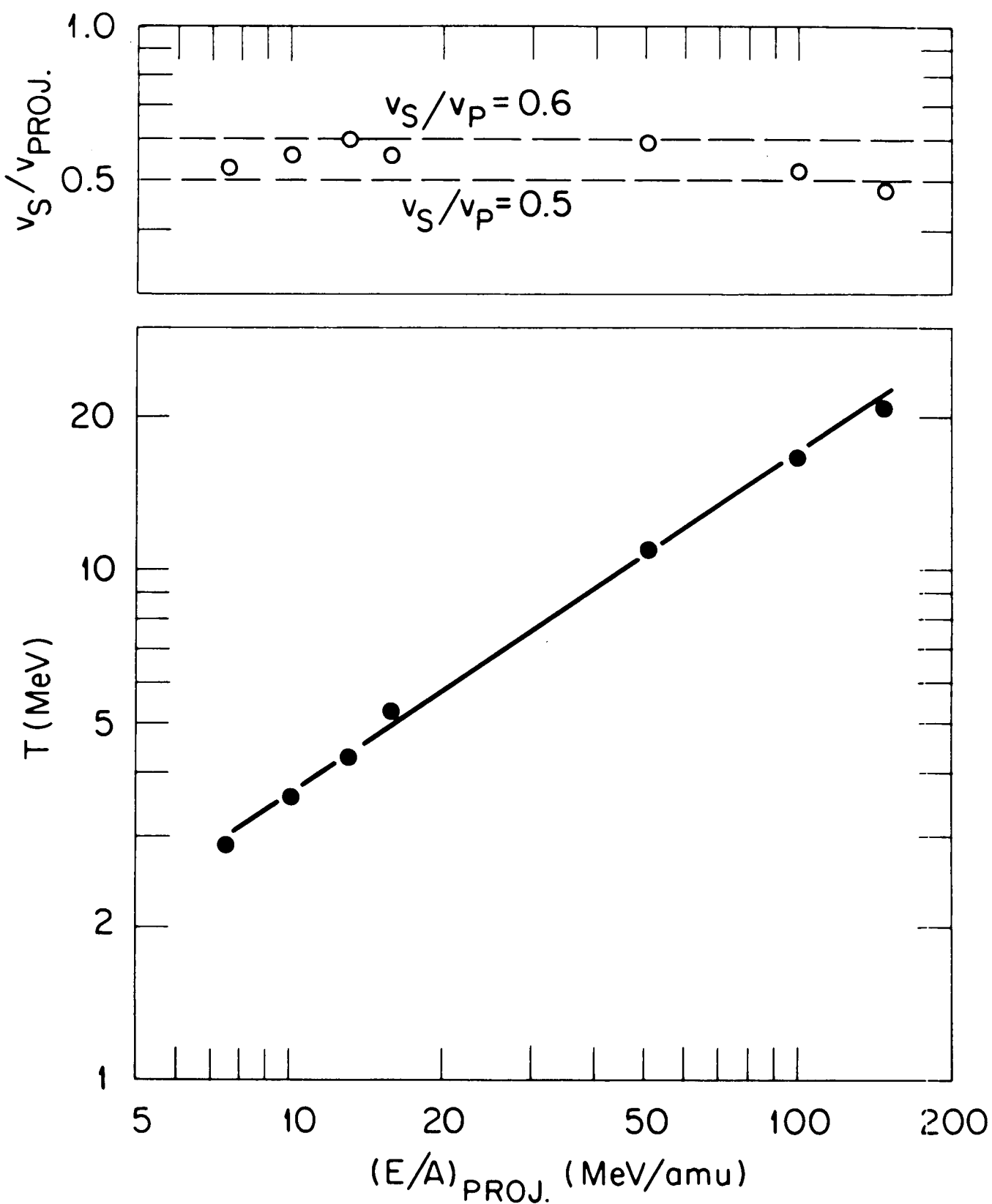


Fig. 15. Systematics of moving source parameters obtained from fitting proton spectra at incident projectile energies from 7.5 to 147 meV/amu.

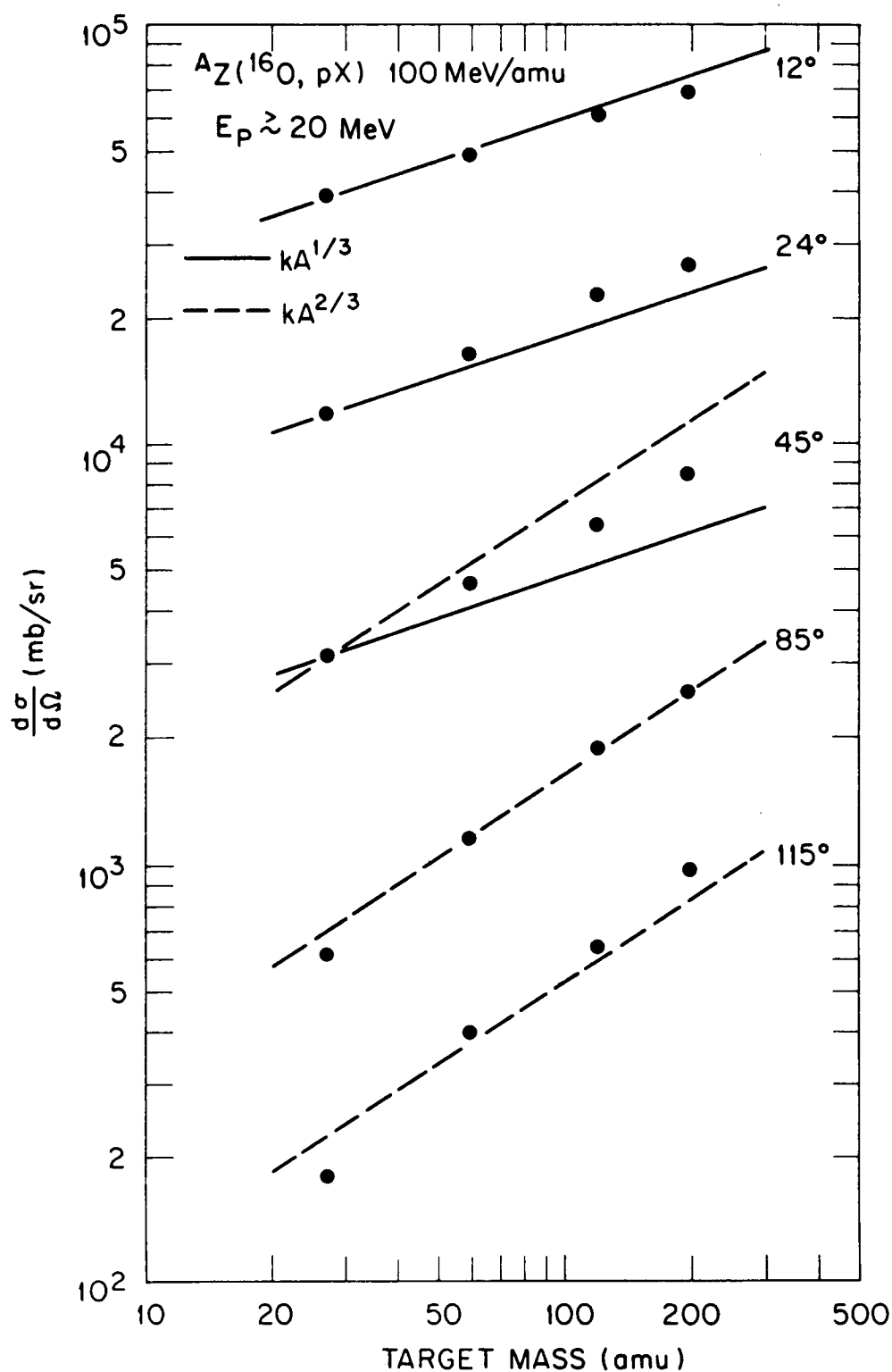


Fig. 16. Energy integrated yields of protons produced by the interaction of 100 MeV/amu  $^{16}\text{O}$  ions with targets of Al, Ni, Sn, and Au. The solid and dashed lines indicate the expected slopes for  $A^{1/3}$  and  $A^{2/3}$  dependence, respectively.

transition. Another possibility is that we are looking in the wrong place or at the wrong thing. Siemens and Kapusta have suggested<sup>16</sup> that the composite particles may carry important information about details of the collision process. In particular, the relative yields of composite particles to protons may carry information about the entropy associated with the hot, dense emitting source. An increase of entropy would be expected from the introduction of new degrees of freedom such as excitation of more mesonic degrees of freedom, or more exotic modes such as pion condensation.

At the lower energies (7.5 to 16 MeV/nucleon  $^{12}\text{C}$  ions), the integrated yield obtained<sup>13</sup> for emitted hydrogenic particles is shown in Fig. 17. The yields shown here are for particles whose laboratory velocity exceeds that of the incident projectile. While the proton yield remains quite constant, the deuteron and triton yields increase as if approaching some limiting value.

The same general features as have been discussed here for the Oak Ridge studies have also been observed in the studies of Awes et al., with oxygen ions at 8.75, 13.4, and 19.4 MeV/nucleon.<sup>17</sup> In that study, the yield of composite particles has been related to the yield of protons through a coalescence model.<sup>18,19</sup> Here the composite particles are assumed to be formed from an aggregation of independent nucleons which find themselves in the same region of phase space. This gives rise to a simple power law relating the complex particle spectra to the proton spectra. In general, rather good agreement is observed<sup>17</sup> although there is a disturbing variation in the deduced coalescence radii with projectile energy and projectile-target combination. One implication of this



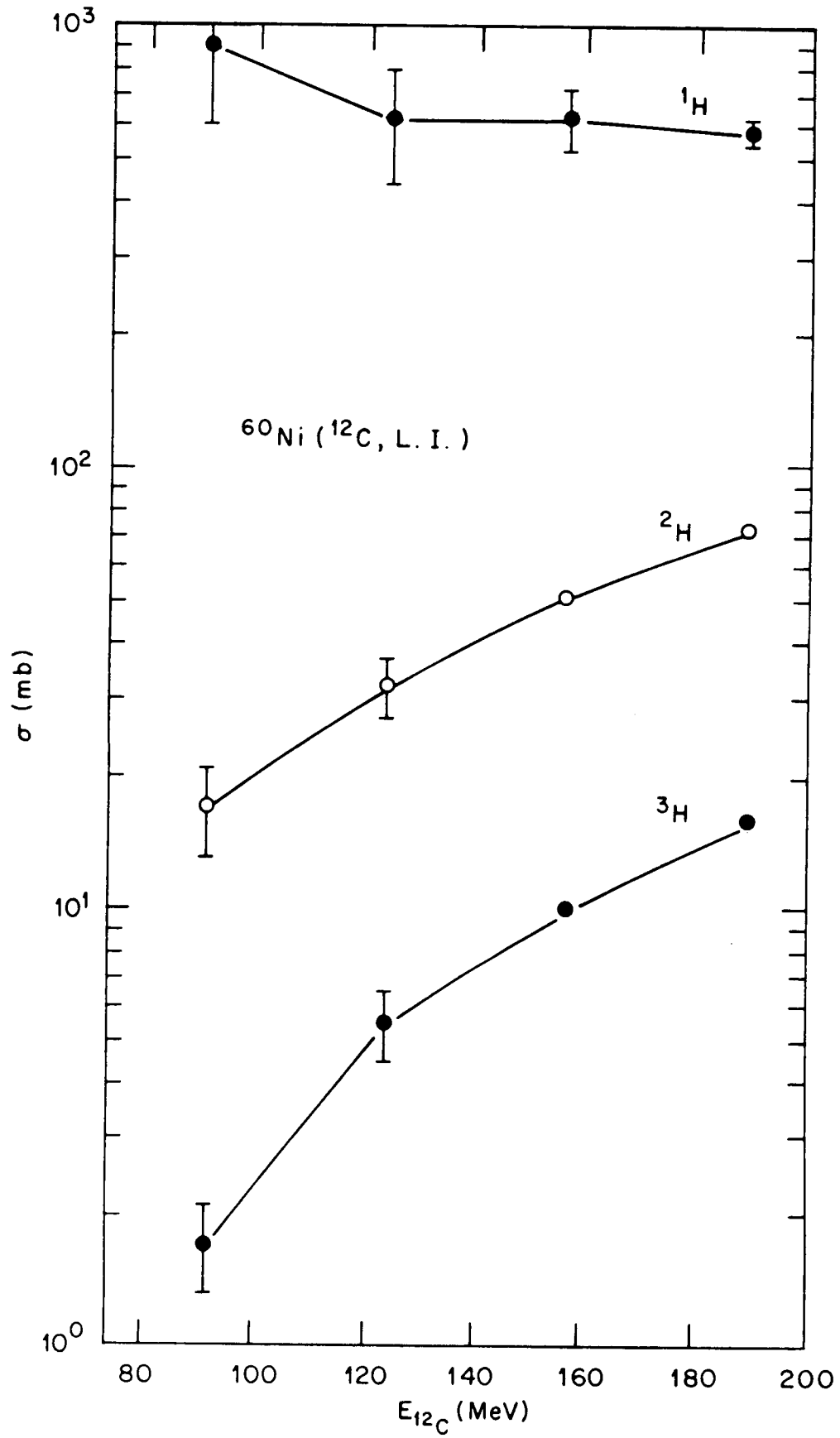


Fig. 17. Integral yields of  $Z=1$  ejectiles having velocities greater than that of the incident beam. The lines are drawn to guide the eye.

may be that the composite particles observed at these energies do not represent equilibrium yields formed during the initial collision but are modified in later stages of the interaction. An open question is how much information is retained that relates to the properties of an initial emitting source.

An example of composite particle spectra observed from 100 MeV/nucleon  $^{16}\text{O}$  on  $^{60}\text{Ni}$  is shown in Fig. 18. Again, at these forward angles the spectra peak at energies associated with the velocity of the incident projectile. A notable exception to this behavior is seen for the tritons. We have not yet compared these spectra with predictions of a coalescence model but obviously the tritons will show very poor agreement. A summary of "moving source" parameters extracted from the proton and composite particle spectra for the Bevalac experiment is given in Table II. The ratio of proton to deuteron to triton yields seem to have stabilized at these higher energies at about 1.0:0.3:0.1. As for the lower energies, the fits derived from this parameterization seem quite consistent and well-behaved. No evidence is observed for any discontinuities in the observed behavior of the composite particles.

#### Present Status

In this paper I have concentrated on the results from our program at Oak Ridge. I have not covered all of that work, but have tried to highlight its main thrust. There have also been major efforts in this field by groups at Texas A&M University, Lawrence Berkeley Laboratory, and Michigan State University. Some of these results have been

Table II. Moving source parameterization of light ion spectra from  $^{16}\text{O}$  induced reactions on a target of natural nickel. The angular range included in the fitting procedure was from 24 to 115 degrees for all particles except  $^3\text{He}$ , which included only 24 to 85 degrees.

| Detected Particle | $V_S/V_{\text{PROJ}}$ | T(MeV) | K     |
|-------------------|-----------------------|--------|-------|
| 51.8 MeV/nucleon  |                       |        |       |
| $^1\text{H}$      | 0.60                  | 11.1   | 3.4   |
| $^2\text{H}$      | 0.58                  | 14.8   | 0.72  |
| $^3\text{H}$      | 0.63                  | 13.8   | 0.30  |
| $^3\text{He}$     | 0.68                  | 15.4   | 0.16  |
| $^4\text{He}$     | 0.66                  | 14.8   | 0.80  |
| 100 MeV/nucleon   |                       |        |       |
| $^1\text{H}$      | 0.54                  | 17.2   | 1.25  |
| $^2\text{H}$      | 0.50                  | 22.6   | 0.33  |
| $^3\text{H}$      | 0.52                  | 21.4   | 0.12  |
| $^3\text{He}$     | 0.56                  | 25.2   | 0.053 |
| $^4\text{He}$     | 0.48                  | 21.9   | 0.13  |
| 147 MeV/nucleon   |                       |        |       |
| $^1\text{H}$      | 0.51                  | 22.0   | 0.84  |
| $^2\text{H}$      | 0.46                  | 28.4   | 0.22  |
| $^3\text{H}$      | 0.47                  | 28.1   | 0.067 |
| $^3\text{He}$     | 0.50                  | 32.3   | 0.029 |
| $^4\text{He}$     | 0.43                  | 31.0   | 0.048 |

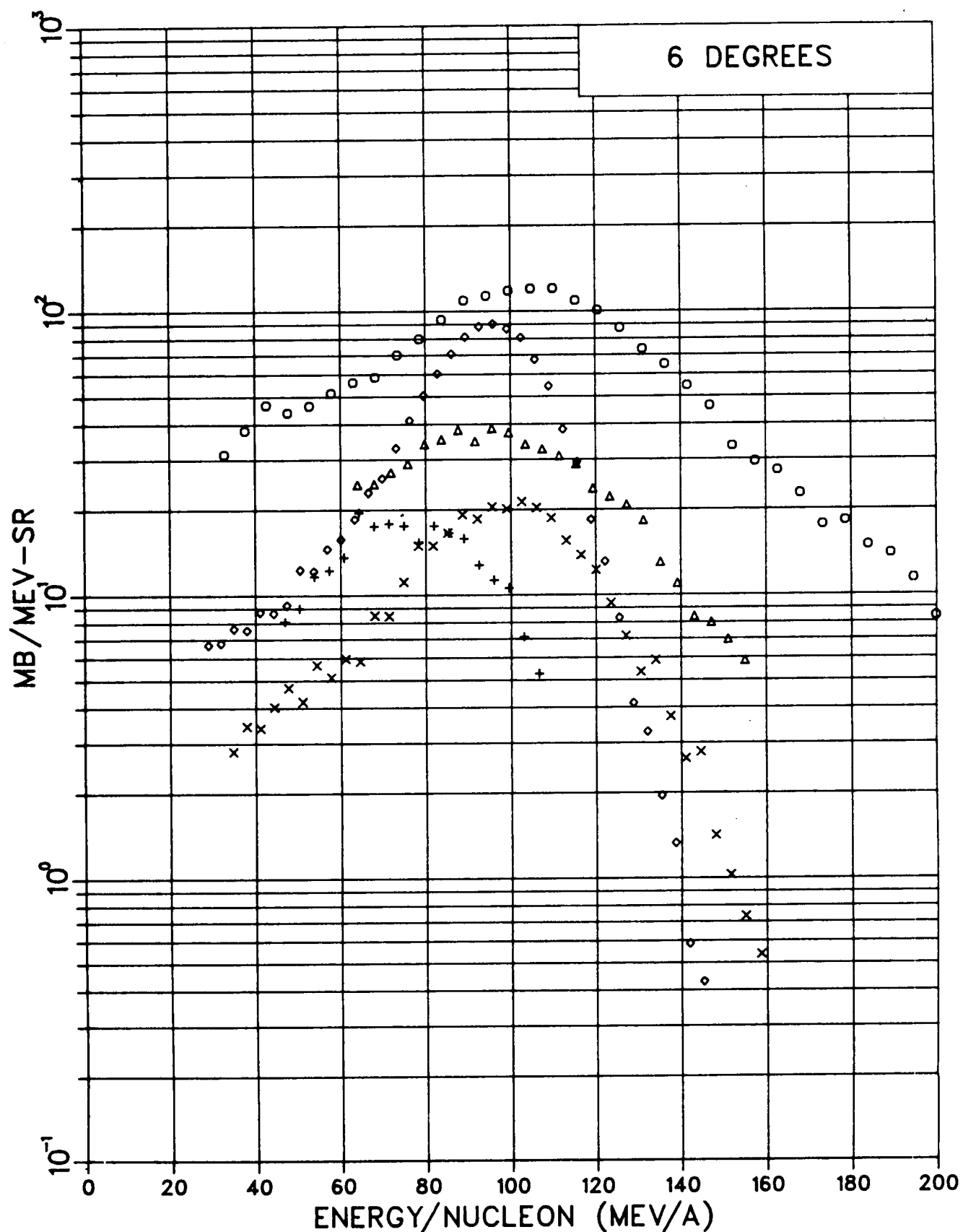


Fig. 18. Energy spectra observed at a laboratory angle of 6 degrees for protons (circles), deuterons (triangles), tritons (plusses), helium-3 (crosses), and helium-4 (diamonds) from the interaction of 100 MeV/amu  $^{160}$  ions with a nickel target. Spectra are plotted as a function of the energy per nucleon of the observed ejectile. Note the dissimilarity of the t and  $^3\text{He}$  spectra.

referenced here, but for additional information the reader is referred to the excellent review paper by Scott.<sup>20</sup>

In the present work, we have seen the interpretation of the origin of the high energy light ions evolve from a picture of projectile excitation and subsequent evaporation to one of pre-equilibrium emission. Our "time scale" for emission has thus moved from one that occurs following the initial collision to one that occurs at the very early stages of the collision.

At low energies we have seen the observed light ion spectra to be reasonably well reproduced by the assumption of emission from a heated source moving at a velocity corresponding to approximately half the projectile velocity. At higher energies, the larger angle data are also well reproduced by such a parameterization which extrapolates smoothly from the lower energies. At the smaller angles, however, the higher energy data shows a peaking around the projectile velocity characteristic of a projectile fragmentation process. This duality of processes at higher energies is consistent with the observed change of target mass dependence between small and large angles of observation leading to the conjectured change from peripheral to central collision dominance. One obvious next step is to utilize this angular dependence to try to disentangle these two processes and examine their individual behavior.

The moving source picture which has been used to parametrize the existing data has a certain appeal in its simplicity and ability to account qualitatively for a rather large range of data with a minimum of parameters. Obviously, this picture is too simplistic and merely a

convenience. We need to replace such a treatment with an approach which deals more directly with momentum distributions of particles in the interacting system. One such formulation has been proposed by Wong.<sup>21</sup> The momentum distribution is generalized in such a way that it gives the thermal distribution, if indeed thermal equilibrium is established. The light particle spectra are treated as arising from two distinct momentum distributions, a central distribution and a fragmentation distribution, so that the invariant cross section is written as:

$$E \frac{d^3\sigma}{dp^3} = F_c(p) + F_f(p)$$

In both portions of this distribution the transverse and longitudinal momentum components are treated separately. In addition, in analyzing the experimental data, it is found necessary to allow a "Wood-Saxon" shape for the momentum distribution of the central source. A Gaussian distribution is used for the fragmentation source. The resulting formulation has, then, eight parameters which are fit to the data. An example of the results of such a fit are shown in Fig. 19 for the protons observed from 100 MeV/nucleon <sup>160</sup> on nickel. Constraining the parameters to less than eight clearly compromises the quality of the fits achieved.

Although we have just begun analyzing the light particle inclusive spectra with this more complete parameterization, some interesting features seem to be emerging from the preliminary results. The velocity of the central source is seen to center about 0.55 V<sub>PROJ</sub>, in agreement with the simpler moving source picture. At 100 and 147 MeV/nucleon projectile energies the longitudinal momentum component of the central

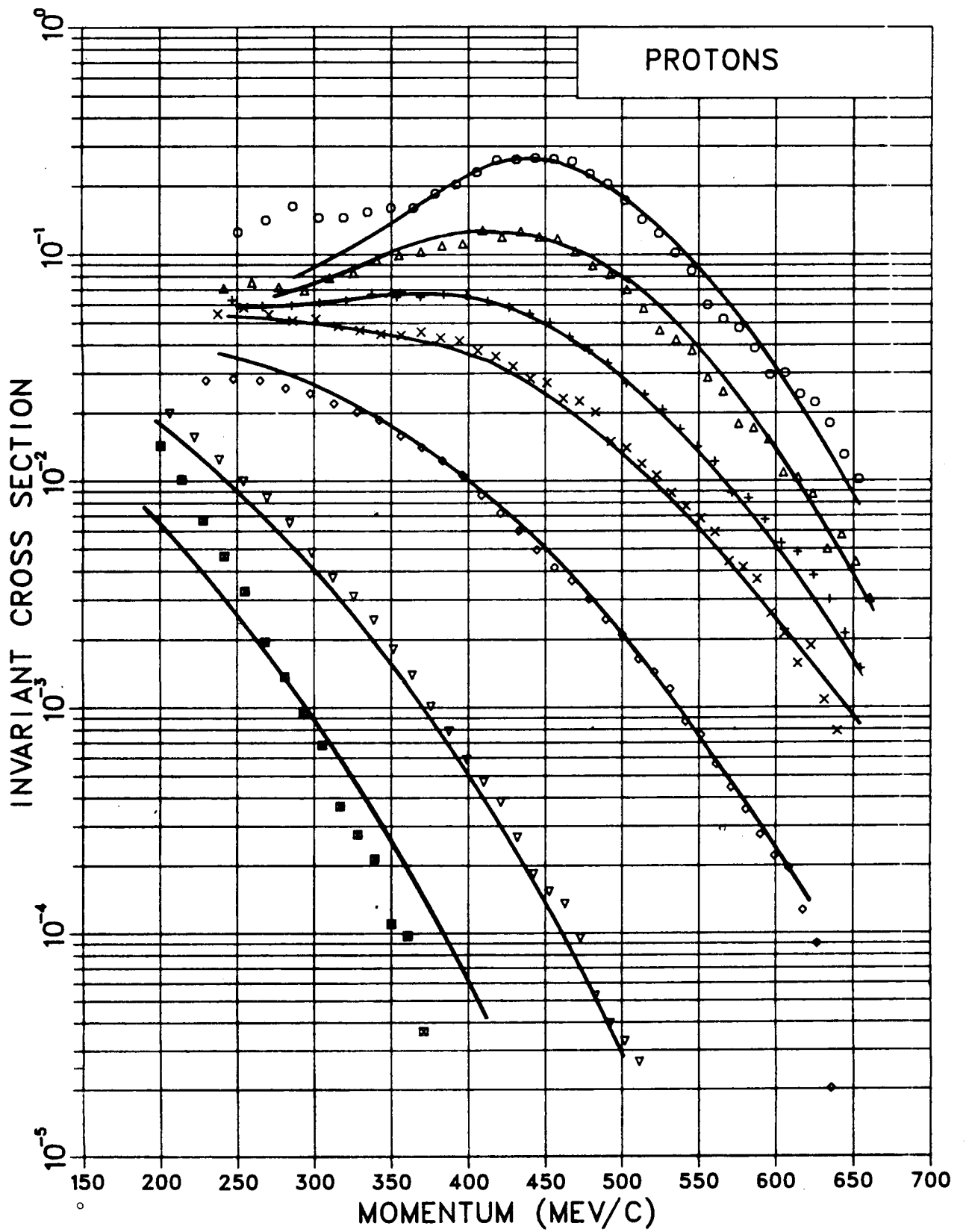


Fig. 19. Proton spectra of Fig. 11 (100 MeV/amu  $^{160}$  on natural nickel) fit with the two-source, unconstrained momentum formulation of Wong.<sup>21</sup>

source exceeds the transverse component. This is interpreted as indicating the particles are being emitted prior to equilibration of the source. Using the parameters derived from fitting the 100 MeV/nucleon data, integration may be performed to obtain the total yield from each source. The emission from the central source is found to vary as  $A^{2/3}$  of the target while that from the fragmentation source varies as  $A^{1/3}$  in good agreement with the interpretation given to Fig. 16.

If we are correct in our assumption that a significant number of the high energy light ions emitted from heavy ion reactions are created during the early states of the collision, then these particles should in fact carry information about the processes involved. The challenge remains for us to design the appropriate experiments and interpretations to understand the reaction processes involved and use this understanding to deduce such information.



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A STATUS REPORT ON THE HOLIFIELD HEAVY ION  
RESEARCH FACILITY

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I. Introduction

At Oak Ridge we are now in the final stages of completion of our heavy ion project. The facility, shown in Fig. 1, has as its centerpiece the 25MV Pelletron built by National Electrostatics Corporation. At the time of this writing (March 1982), the only tasks remaining for completion are the acceptance tests with beam to be performed at the design rating of 25MV on terminal. Along with the 20MV machine at Tokai and the 20MV machine at Buenos Aires, this trio of accelerators represents the new generation of Super-tandems presently in various stages of completion by NEC. In this paper, I shall review briefly some of the history of the work on the 25MV accelerator at Oak Ridge. In doing so, I will try to bring out those points that I feel are most illustrative of what may be expected as installation proceeds on the machine here in Argentina. Finally, as time permits, I'll discuss some of the other features of the Holifield Facility.

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\*Operated by Union Carbide with the U.S. Department of Energy.

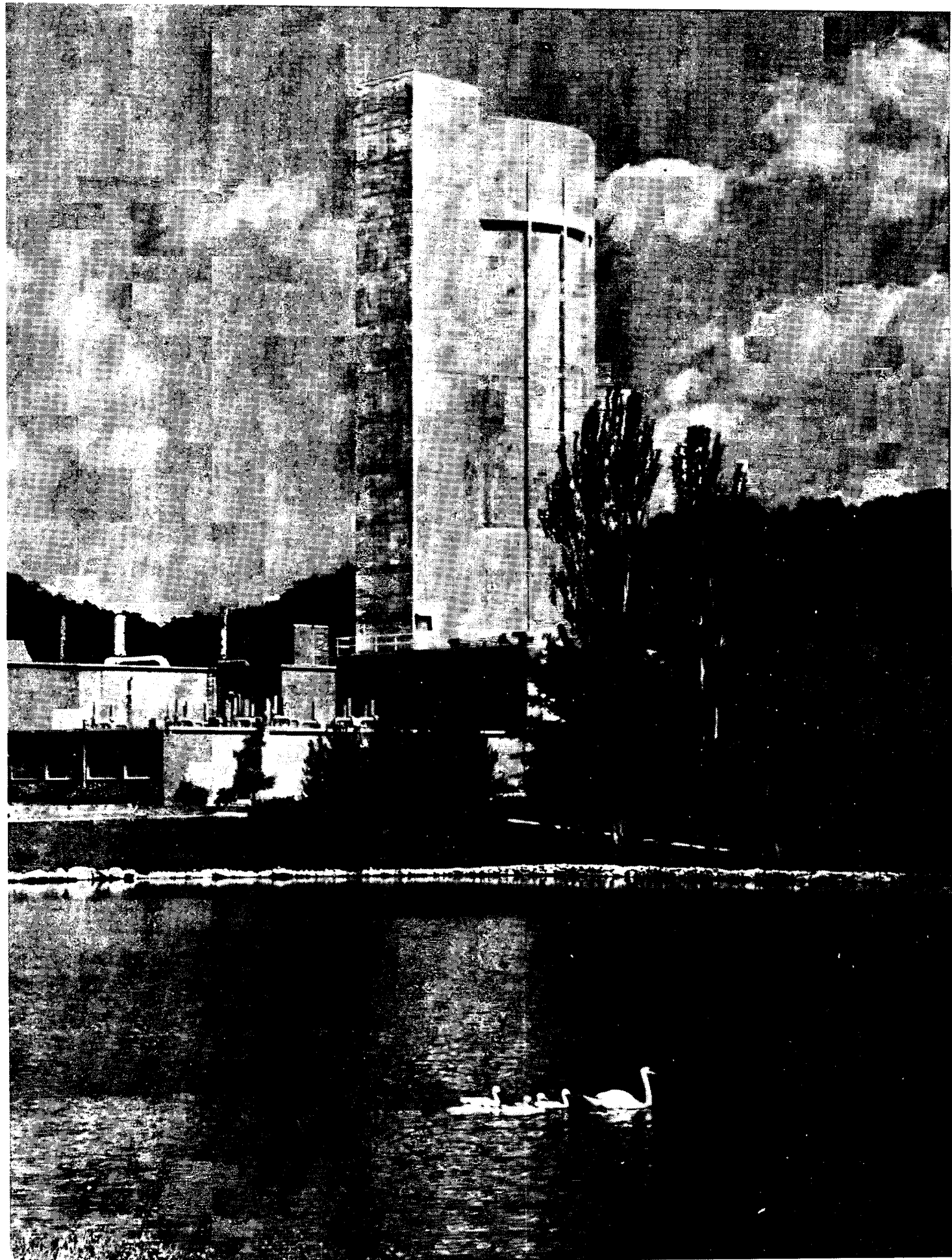


Fig. 1. The Holifield Heavy Ion Research Facility at Oak Ridge National Laboratory.

## II. The 25MV Pelletron

### A. The High Voltage Generator

We chose to have the tandem for Oak Ridge built in a "folded" configuration, unlike the conventional linear column structure of most tandem accelerators. The resultant structure is indicated schematically in Fig. 2. The column structure is 3.3m (11 ft) in diameter and 18.9m high, topped with a 4.8m-high terminal shell. The column contains 27 of the standard 0.6m, 1MV, NEC modules. These are separated by two major dead sections into 3 sets of 9 units each. In turn, each of these thirds is divided once by a minor dead section. Pumps and electron traps are provided in all dead sections.

The pressure vessel housing the column structure is approximately 10m in diameter and 30m high. It is designed for operating pressures up to about 0.7 MPa (100 psig).

A view of the 180-deg magnet in place atop the column is shown in Fig. 3. A view looking down on the completed column structure, after installation of the top portion of the terminal shell, is shown in Fig. 4. The installation of the column structure, including charging chains and power transmission shafts but not the acceleration tubes, was completed in April, 1979.

The following month, May 1979, voltage tests on the column structure were run. These went quite smoothly and very quickly--requiring only about four days. A summary of the results of these tests is presented graphically in Fig. 5. Note that, at the

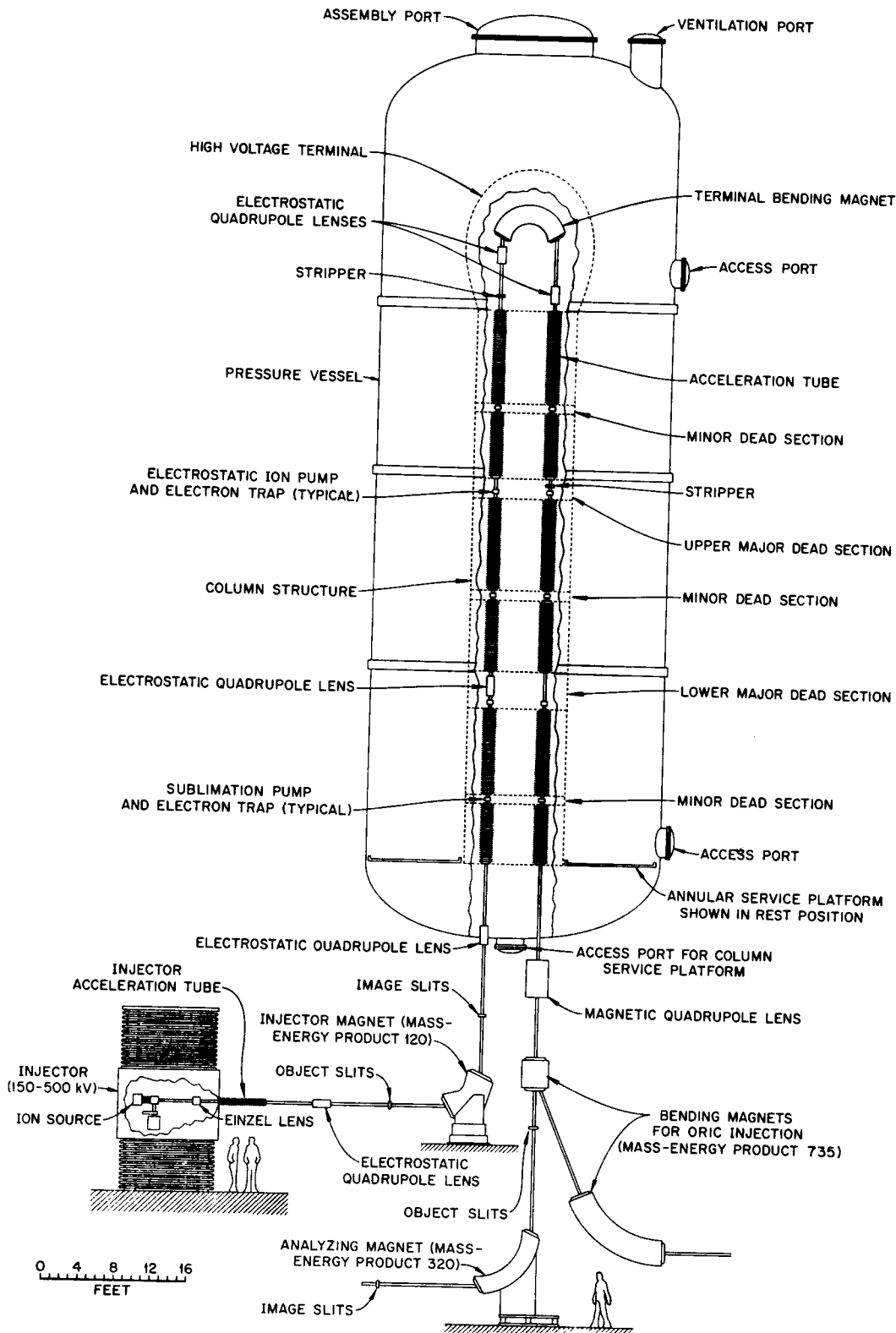


Fig. 2. Schematic view of the 25MV folded tandem accelerator.

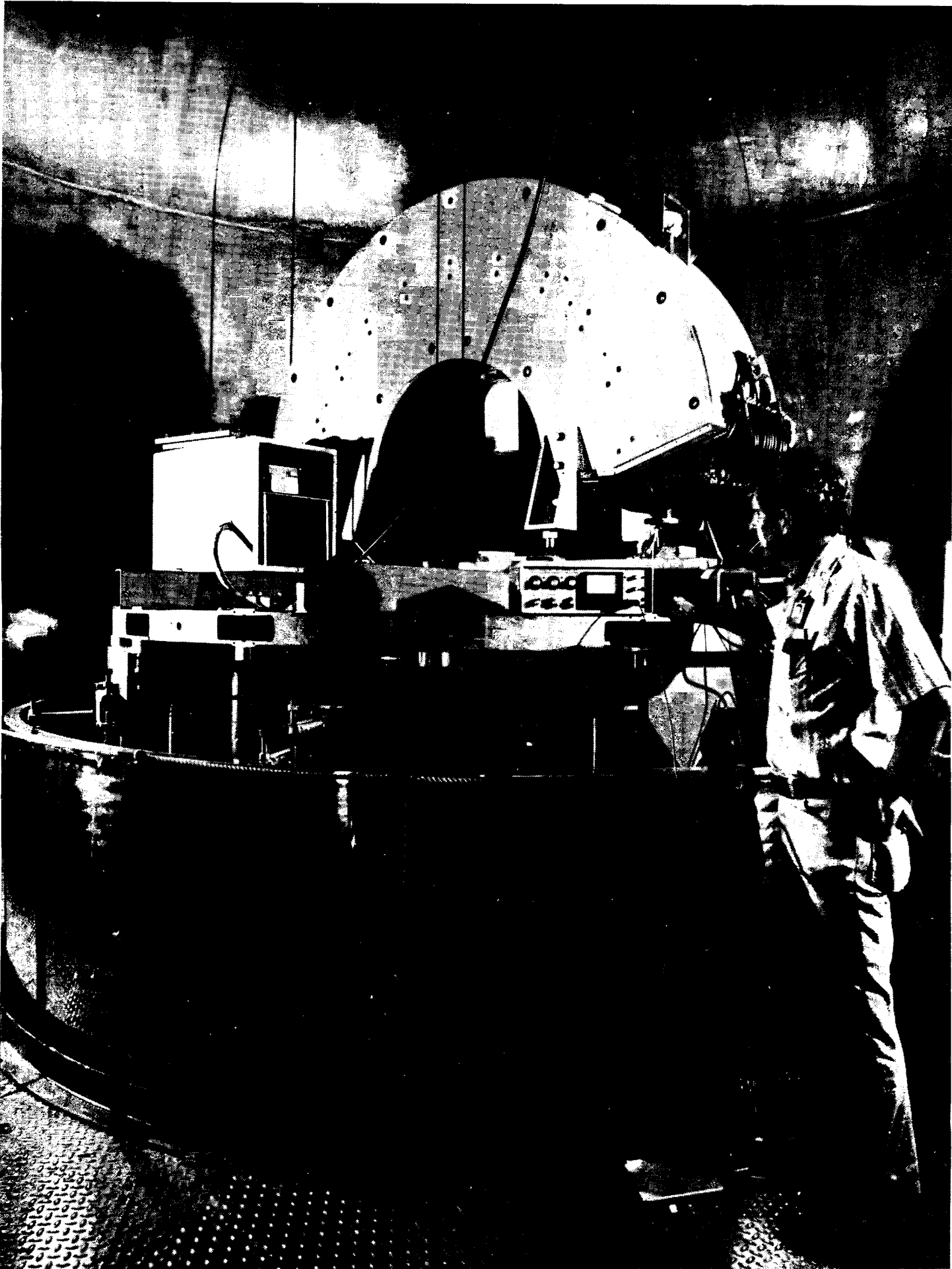


Fig. 3. The 180-degree magnet installed in the terminal of the 25MV Pelletron.

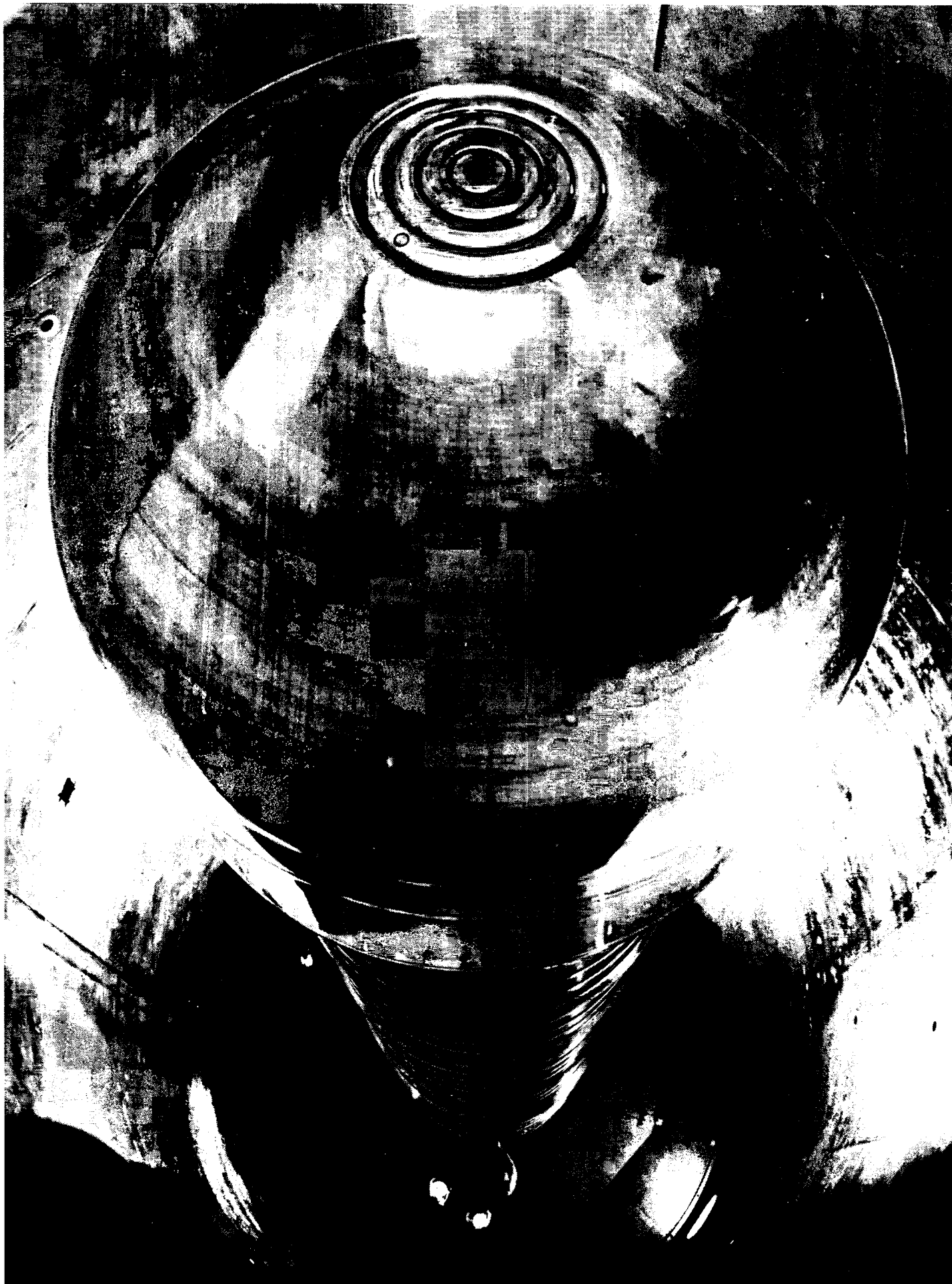


Fig. 4. A view looking down on the high voltage terminal and column of the 25MV Pelletron.



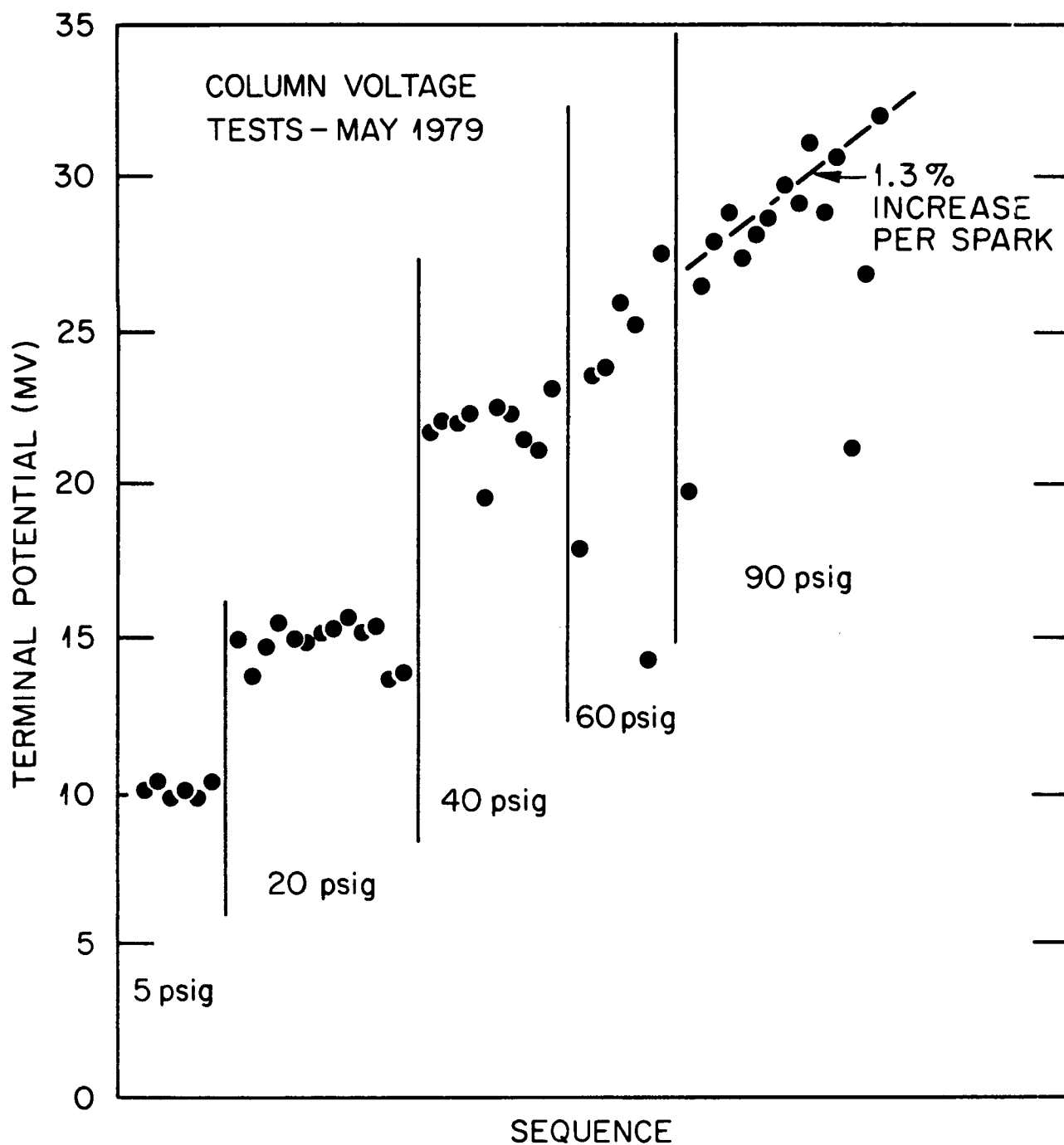


Fig. 5. Terminal voltage at breakdown as a function of spark sequence for the Oak Ridge column voltage tests. A Recent recalibration indicates the voltage scale shown on this figure is too high by about 8%.



Fig. 6. A terminal spark during column voltage tests.

higher pressures, the maximum breakdown voltage does not appear to level off with breakdown number. Instead, the voltage appears to be still rising toward the end of the test and exhibiting a conditioning behavior. The tests were terminated at this point because of uncertainties about the probability of spark-induced damage to the column components and since the voltage holding capabilities of the column had been sufficiently demonstrated.

Examination of the column following these tests revealed evidence of 42 spark "hits" that were rather uniformly distributed over the terminal shell and upper portion of the column. Areas of discoloration associated with these hits ranged up to about 2cm in diameter with some roughness of metal noticeable at the center of these spots. No evidence was observed that would suggest any "restriking" to the same spot due to this roughened surface. Our conclusion is that the column structure is able to withstand very well the effects of sparks at full design potential and beyond (Fig. 6).

#### B. Operation with Beam

The remaining components of the accelerator were installed in the column during the period from June to December, 1979. At this time, NEC began preparing the accelerator for tests with beam. However, a number of small, but time-consuming problems such as water leaks and vacuum leaks, delayed the first acceleration of beam through the machine until May, 1980. On May 12, a beam of oxygen ions was accelerated through the machine at a terminal

potential of 15.5MV. The next few weeks were spent in utilizing the beam to perform a careful calibration of the high-energy beam analyzing magnet. In July and August 1980, system acceptance tests were performed at 7.5MV and at 17.5MV on terminal. These tests are summarized in Table I.

Following these tests, major maintenance was performed on the accelerator to prepare it for tests at higher terminal potentials. During this period, NEC chose to replace the closed corona grading system with an open corona system. All subsequent operation of the machine has been with this open corona system. Conditioning of the accelerator was resumed in October, 1980, and proceeded until early December. It was discovered at that time that eight of the modular sections of acceleration tube had been damaged. This damage, in the form of electrode material deposited on the inner surface of the tube insulator, was attributed by NEC to improper spacing of the annular spark gaps at the end of the tube sections. The damaged tube sections were replaced and all of the end spark gaps were readjusted. No further damage of acceleration tubes has been experienced.

In January, 1981, we had a brief use of the accelerator to test our existing cyclotron as a booster accelerator. This will be described in more detail below. In the period February through April of 1981, NEC continued to work on conditioning the machine to higher gradients. Maximum potentials achieved during this period were in the range of 22MV.

Table I. Operation Chronology

|                    |                                                                                                                                                                                                                                                     |
|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| May 1979           | - Column voltage tests                                                                                                                                                                                                                              |
| May 1980           | - First beam through accelerator system,<br>1606+ at 15.5 MV on terminal.                                                                                                                                                                           |
| July-August 1980   | - System acceptance tests performed<br>127I10+, 7.5 MV, foil stripper, 10 pA for 10 min.<br>127I4+, 7.5 MV, gas stripper, 1 pA for 60 min.<br>127I6+, 17.5 MV, gas stripper, 1 pA for 60 min.<br>127I13+, 17.5 MV, foil stripper, 10 pA for 10 min. |
| January, 1981      | - Successful use of ORIC as a booster accelerator,<br>401 MeV 1608+.                                                                                                                                                                                |
| June-November 1981 | - Operation of facility for experimental programs                                                                                                                                                                                                   |
| March, 1982        | - Operation for experiments                                                                                                                                                                                                                         |
| May, 1982          | - Planned start of full-time operation for<br>experiments.                                                                                                                                                                                          |

In May, 1981, we reached an agreement with NEC for Oak Ridge to operate the accelerator for six months for the experimental program. During this period, beams were provided to some twelve experimental programs with the accelerator operating at terminal potentials up to 19MV. A summary is given in Table II.

In November, 1981, NEC returned to complete various mechanical tasks that remained inside the accelerator and on the gas handling system. Included in this work was installation of a complete set of new corona points. During this time, the acceleration tubes were not under vacuum for about six weeks. Various conditioning strategies were explored during January, 1982. The lowest third of the machine was subjected to glow discharge conditioning. The middle third was reconfigured by removing equipotential "stringers" to provide more decoupling of the acceleration tubes and the column. At this time, it is unclear whether any of these techniques will be beneficial.

This February, the accelerator was subjected to a very extensive bakeout. The present plans call for operation of the machine for experiments in March and for NEC to return in April to perform the final performance acceptance tests.

It is interesting to note that, even with the extensive February bake and the excellent acceleration tube base pressures achieved, the running period in March has been troubled by classical spark-induced deconditioning. This result only emphasizes the desirability of keeping the acceleration tubes under vacuum at all times. It seems clear that achieving the high potential capability

Table II. Experiments Run at HHIRF in the Period from June 17 to November 15, 1981

| Experiment                                                                                                     | Spokesman       | Run Number | Target Station     | Beam                             |         |
|----------------------------------------------------------------------------------------------------------------|-----------------|------------|--------------------|----------------------------------|---------|
|                                                                                                                |                 |            |                    | Type                             | E(MeV)  |
| TANDEM                                                                                                         |                 |            |                    |                                  |         |
| Coulomb Barrier Interactions of Ni+Ni: Elastic Transfer                                                        | K.A. Erb        | H023       | 1.6-m chamber      | <sup>60</sup> Ni                 | 175,225 |
| m-Substate Study of Discrepancies in Single Nucleon Transfer Reactions on <sup>208</sup> Pb                    | J.L.C. Ford     | H002       | Split-pole magnet  | <sup>11</sup> B, <sup>16</sup> O | 71      |
| Lifetimes in the Backbending Region of <sup>160</sup> Yb using ORPSS                                           | N.R. Johnson    | H017       | γ-ray spectrometer | <sup>48</sup> Ti                 | 204     |
| In-Beam γ-Ray Spectroscopy Following <sup>40</sup> Ca( <sup>32</sup> S) Reactions                              | A.V. Ramayya    | H001       | γ-ray spectrometer | <sup>32</sup> S                  | 120-160 |
| Multi-particle Processes in <sup>28</sup> Si + <sup>12</sup> C and <sup>28</sup> Si + <sup>24</sup> Mg Systems | R.W. Novotny    | O003       | 1.6-m chamber      | <sup>28</sup> Si                 | 120-150 |
| Potential Energy Surfaces for Rotating Nuclei from Fission Studies                                             | H.C. Britt      | H024       | 1.6-m chamber      | <sup>12</sup> C                  | 84-126  |
| Nuclear High-Spin Properties from Continuum γ-Ray Studies                                                      | I.Y. Lee        | 9A02       | γ-ray spectrometer | <sup>25,26</sup> Mg              | 115-132 |
| Incomplete Fusion and Deep-Inelastic Reactions of <sup>19</sup> F with <sup>150</sup> Nd                       | M. Jaaskelainen | H030       | Spin spectrometer  | <sup>19</sup> F                  | 162-190 |
| Survey of the Entry Region and Gamma Decay Properties Near Limiting Angular Momentum                           | M.L. Halbert    | H031       | Spin spectrometer  | <sup>50</sup> Ti                 | 230     |
| COUPLED                                                                                                        |                 |            |                    |                                  |         |
| Excitation of Giant Multipole Resonances through Inelastic Scattering of Heavy Ions                            | F.E. Bertrand   | H019       | 0.8-m chamber      | <sup>16</sup> O                  | 400     |

of these new tandems will require very thorough and very patient conditioning of the acceleration tubes.

### C. Ion Sources

A photograph of the injector platform for the Oak Ridge machine is shown in Fig. 7. The platform has been demonstrated successfully to its design rating of 500kV. In accelerator operation to date, typical operating voltage has tended to be of the order of 300kV.

Only one ion source is in place on the platform at a time. Each source is mounted on a self-aligning pump module to allow rapid replacement or change of source (1 to 2 hour change time). One such source module can be seen in the foreground of Fig. 7.

The ion source most utilized during the period of running for experiments has been the ORNL modified version of the Aarhus negative ion source. One of the exciting results from this early running experience is the ability to produce large currents of many ion species with this source. The source, as modified by Gerald Alton, is shown schematically in Fig. 8. The source is a radial extraction geometry plasma discharge source with a negatively charged ( $\sim 1000V$ ) sputter probe mounted near the plasma boundary. The source is equipped with a cesium oven which provides the electron donor and the sputtering agent to act on the probe material. Arc support gases may be introduced when needed to provide appropriate molecular ion species.



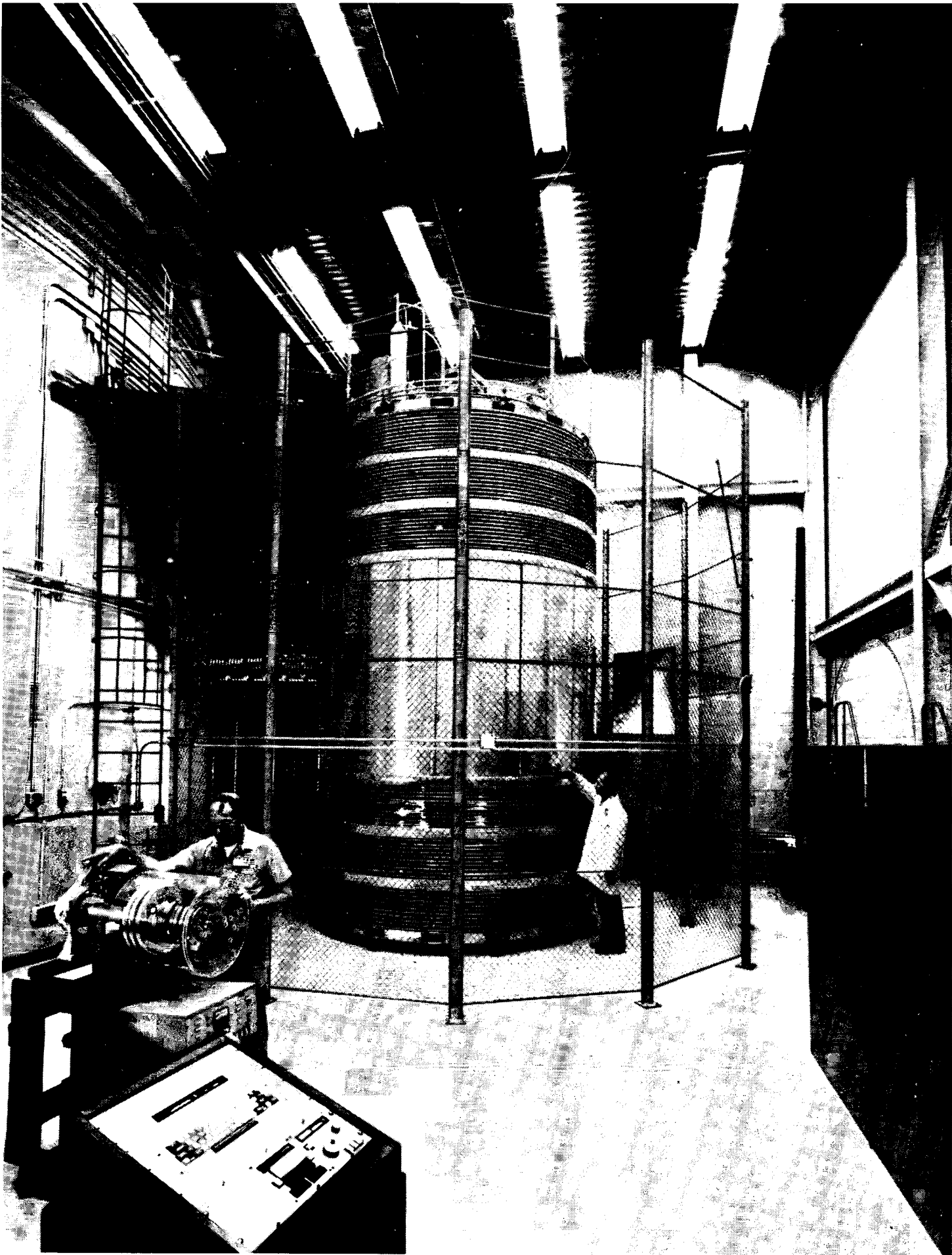


Fig. 7. The injector platform at the Holifield Facility.

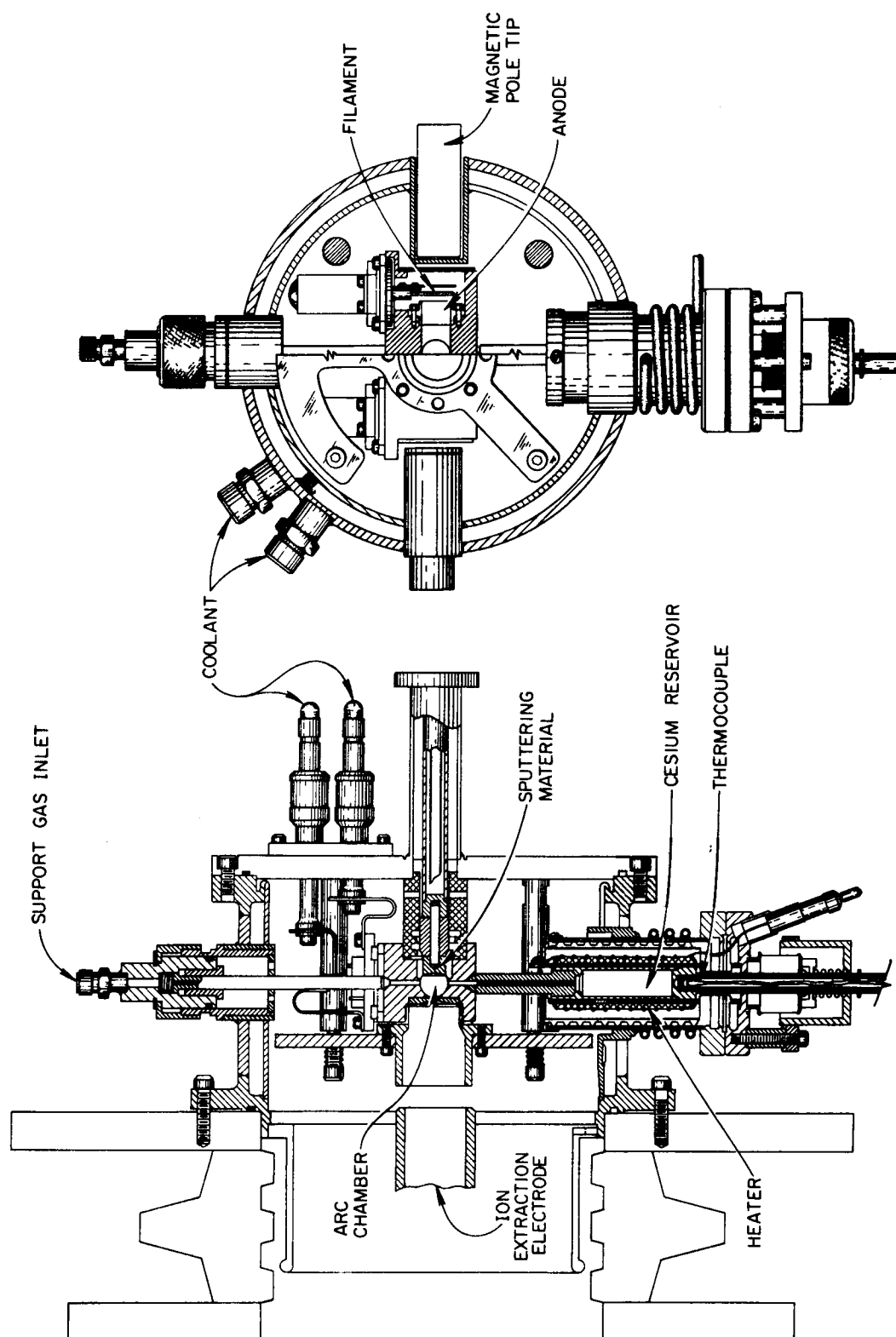


Fig. 8. The Oak Ridge version of the Aarhus negative ion source.

The range of ions produced with this source during our period of operation for experiments can be seen from Table II. Of particular significance was the production of large ( $>10$   $\mu\text{A}$ ) beams of  $\text{MgH}^-$  and  $\text{MgH}_3^-$  from this source when using a  $\text{MgCu}$  alloy probe and hydrogen support gas. Microampere beams of  $^{25}\text{Mg}$  and  $^{26}\text{Mg}$  were obtained from a natural magnesium sample. Application of this same technique to produce calcium beams has not yielded such large currents but the results look promising and this work is continuing.

### III. COUPLED ACCELERATOR OPERATION

As part of the present construction project, we have modified our existing cyclotron (ORIC) to provide a second stage of acceleration for ion beams from the Pelletron. The spatial relationship of the two accelerators is indicated in Fig. 9.

When the accelerators are operating in the coupled mode, the ion beam from the tandem is directed along a transfer line which terminates at the rear boundary of the ORIC rf resonator. The handling of the beam beyond this point is indicated in Fig. 10. An inflection magnet, located within the resonator, places the tandem-produced beam on a trajectory which is captured by the fringing field of the cyclotron magnet and brought tangent to an acceleration orbit. At this point, a special mechanism places a thin carbon foil to provide a sudden increase in the ion charge. The system is adjusted so that the ions stripped to the desired charge state are bent onto an acceleration orbit. From this point, the cyclotron accelerates the ion beam as in a conventional cyclotron.

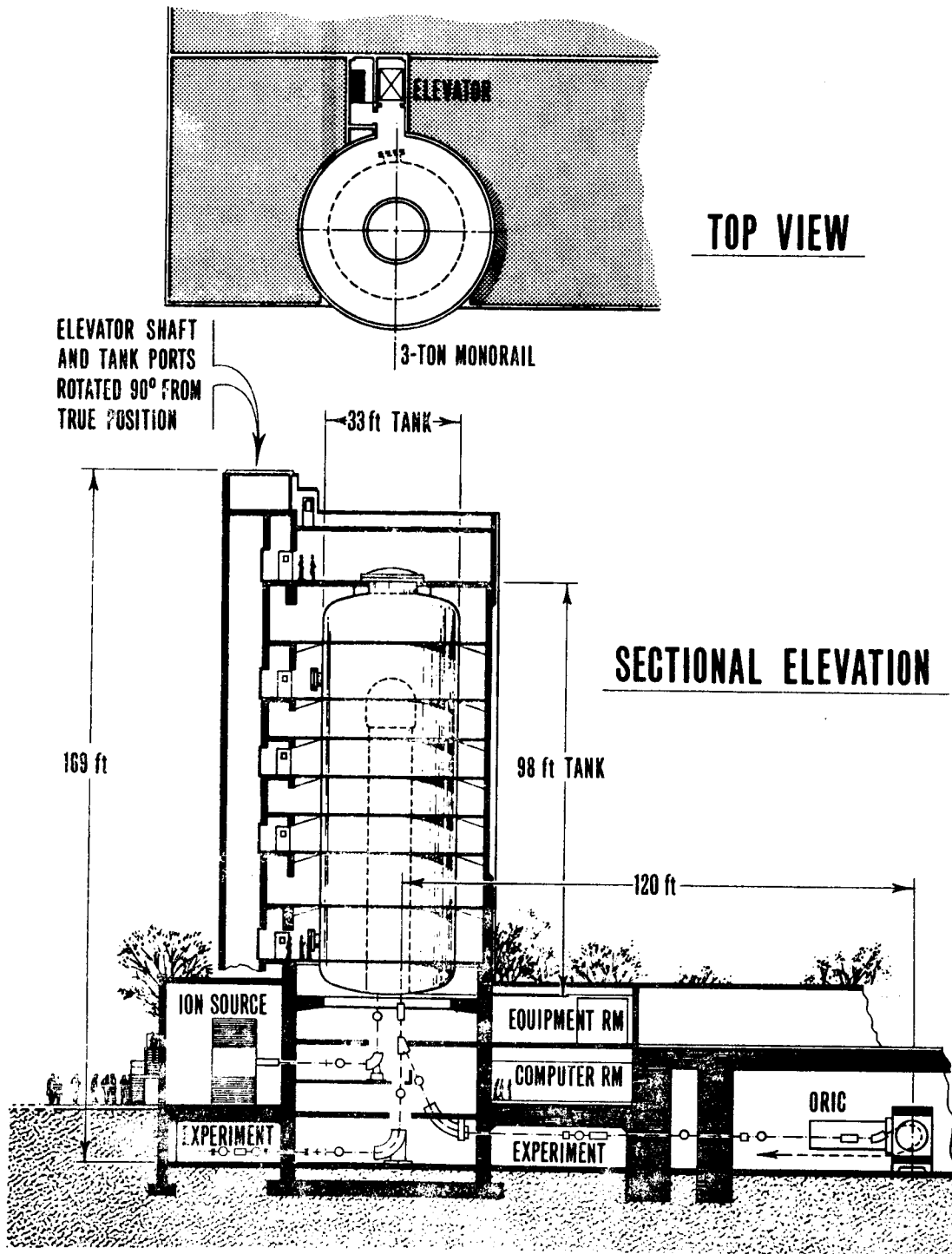


Fig. 9. Cross-sectional view of the Holifield Facility.

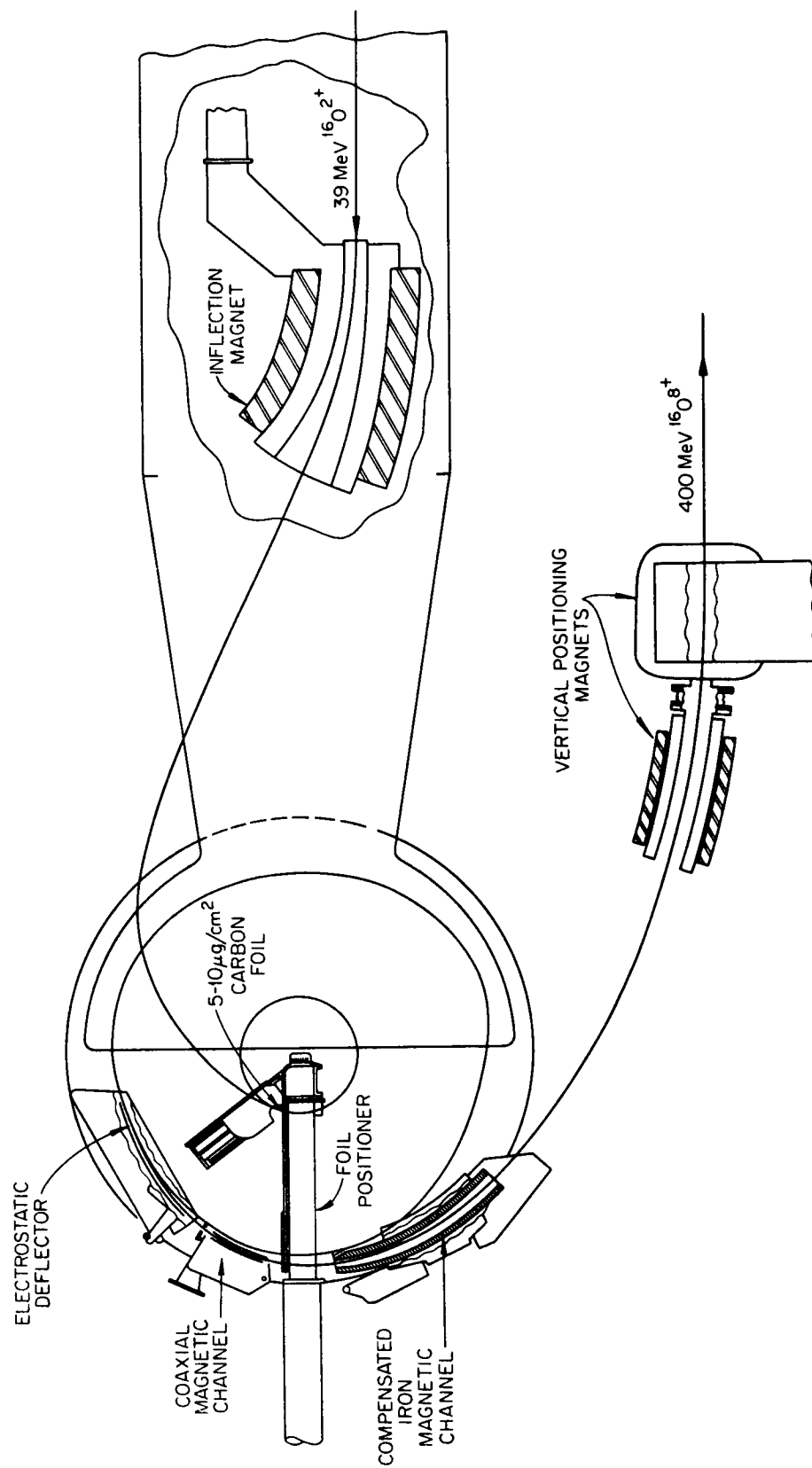


Fig. 10. The Oak Ridge cyclotron (ORIC) as modified for booster accelerator operation.

The conversion of ORIC to booster accelerator status required considerable reworking of the accelerator electrode, trimming capacitors, etc., to accommodate the required inflection orbits. During some of this period, a major program of detailed measurement of the ORIC magnetic field was accomplished. Setup of the ORIC for booster operation is done utilizing computer orbit calculations based on these measurements. To date, these have proved very reliable.

Initial experience with coupled accelerator operation was obtained during a series of brief tests in January, 1981. The first beam produced was a 324 MeV beam of oxygen ions. As the test proceeded, this energy was raised to 400 MeV (25 MeV/nucleon). This is the maximum energy/nucleon achievable with the ORIC booster.

A scattering spectrum taken during the initial coupled operation tests is shown in Fig. 11. We are particularly excited about the excellent energy resolution achieved. It appears that it is possible to couple the machines in such a way that the high beam quality of the tandem (particularly the brightness and energy resolution) is preserved.

When serving as an injector for the cyclotron, the tandem operates in a pulsed beam mode. A double-drift, harmonic buncher, designed by Bill Milner and Norval Ziegler, is installed on the low-energy beam line following the mass analyzing magnet. The system bunches about 50-60% of the DC beam of the tandem within the rf "window" required for injection into the cyclotron. Typical operation would be a  $1\frac{1}{2}$ -nsec-wide beam pulse every 100 nsec. Tuning of the buncher system to the cyclotron rf has proven to be straightforward with long-term stabilities demonstrated. Note that one of the experiments run during the June-November 1981 cycle used coupled operation.

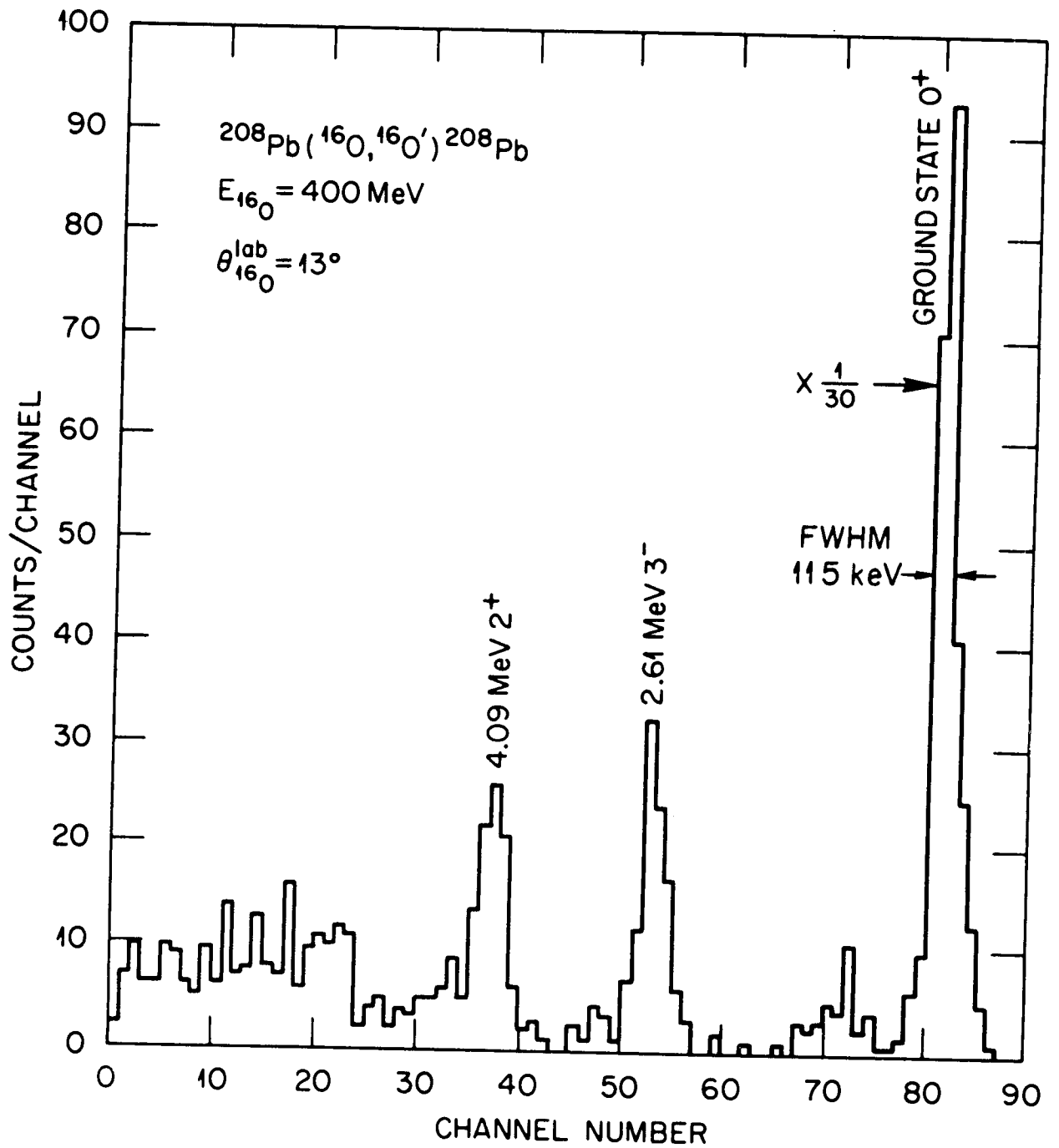


Fig. 11. A spectrum of oxygen ions scattered from a lead target taken during initial tests of coupled accelerator operation.

#### IV. THE EXPERIMENTAL FACILITY

It requires more than operating accelerators to provide a vigorous research facility. The experimental devices present at the facility will, to a large extent, determine the research possibilities even more than the accelerators. This section reviews briefly some of the major experimental devices available at the Holifield Facility and the types of programs to which they apply.

A layout of the Holifield Facility is shown in Fig 12. This floor plan shows rather graphically the effects of adding piecewise to what was originally a two-room cyclotron facility.

Shown on the figure are two in-beam gamma-ray spectroscopy facilities---each on a line from one accelerator. There are also two magnetic spectrographs. The smaller of these is served only by beams from the tandem. Note, however, that the larger spectrograph can be reached by beams from either accelerator. This is also true of the time-of-flight facility located in the same room. Neither of the magnetic spectrographs at Oak Ridge is a modern state-of-the-art device. However, we decided that the availability of these older devices allowed us the opportunity to delay on such an expensive new device until the research programs using spectrographs are better defined. The extremely high resolution beams obtained from our first coupled operation run suggest that this is an area we will need to readdress shortly.

The time-of-flight system, Fig. 13, is a device on which much of the fission-fusion work is performed. The trend on this device is toward larger and larger area detectors to examine as much of the reaction sphere as possible in coincidence with evaporation residues.



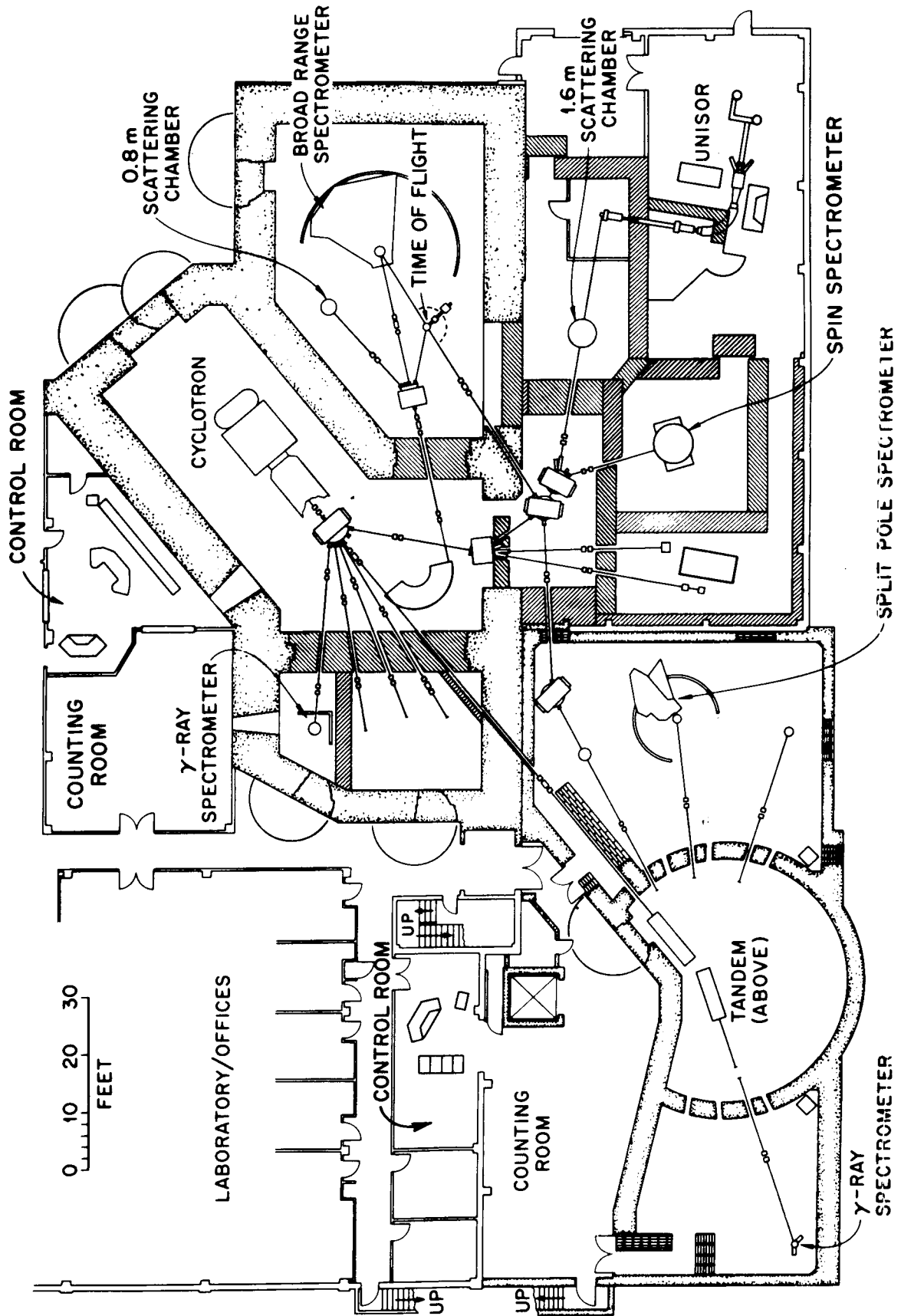


Fig. 12. Floor plan of the Holifield Facility.

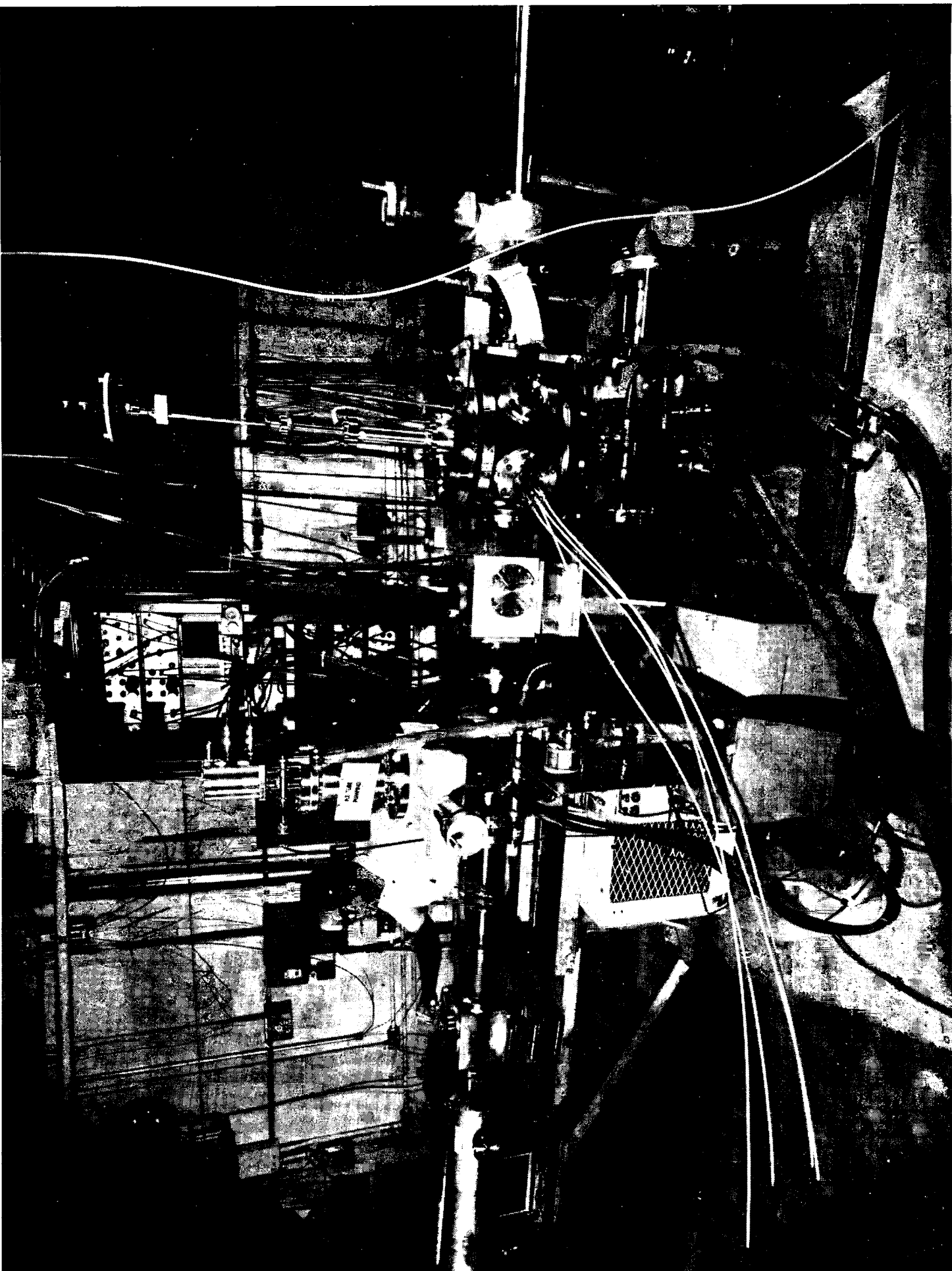


Fig. 13. The time-of-flight facility.

Our 1.6m scattering chamber is shown in Fig. 14 during setup of an experiment involving some 44 detectors in and out of the reaction plane.

A powerful research tool for nuclear spectroscopy is the on-line isotope separator, UNISOR. This device (Fig. 15) is operated by a consortium of 11 universities. The device has four active mass lines and often supports 3 to 4 simultaneous research efforts. At the present time a laser system is being installed on one of the mass lines. This will make possible fast-atomic-beam measurements to determine magnetic moments, electric moments, isomer shifts, etc., of very short-lived nuclear species far from the region of beta stability. UNISOR itself has been operating since 1973. The laser system is expected to be operational in the next few months.

Finally, one of our most spectacular pieces of new equipment, the spin spectrometer designed and built at Washington University under the direction of Demitrios Sarantites, is shown in Fig. 16. This closely packed array of 7" deep, equal-area NaI detectors is used to make total energy and gamma-ray multiplicity measurements on an event-by-event basis. For most experiments, two of the 72 elements are missing to allow an entrance and exit path for the ion beam.

Perhaps the spin spectrometer is an extreme example of the degree to which the data acquisition system of a facility must accommodate the tremendous extent of parameter space presented by the coming generation of highly complex experimental devices. In a typical experiment, in addition to the NaI detectors of the ball, there will often be a number of charge particle detector telescopes within the central scattering

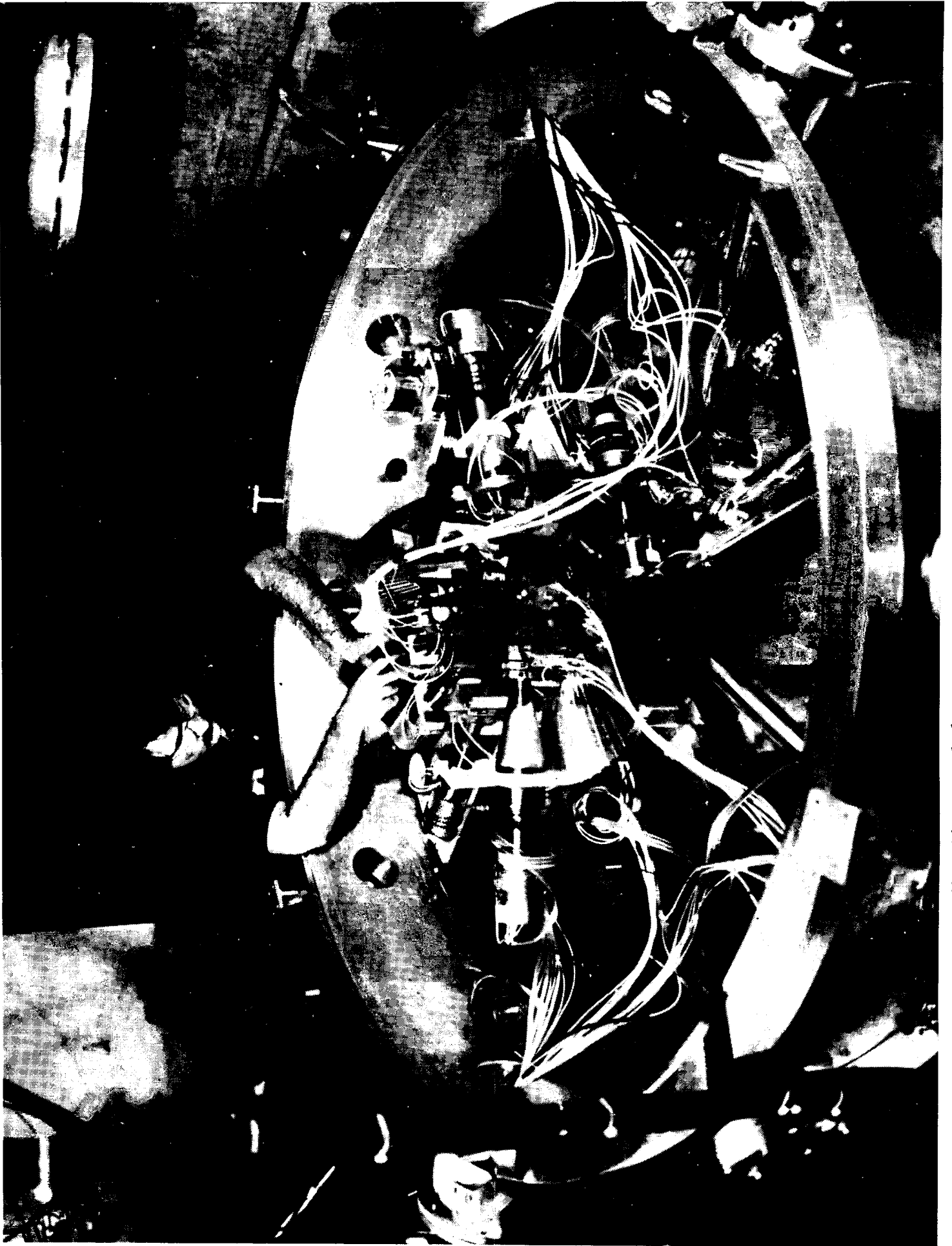


Fig. 14. The 1.6m general purpose scattering chamber.

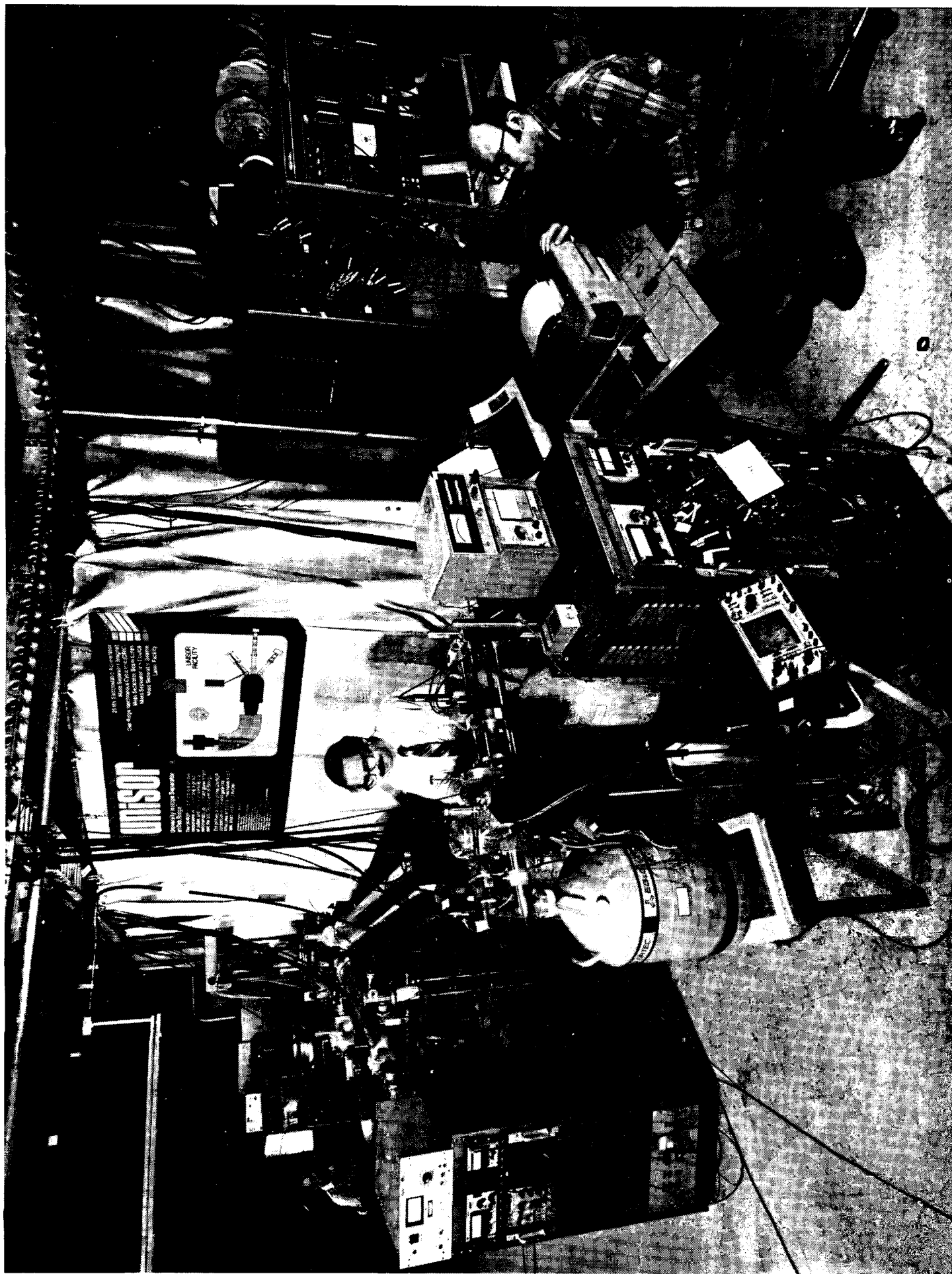


**Fig. 15.** The on-line isotope separator, UNISOR.

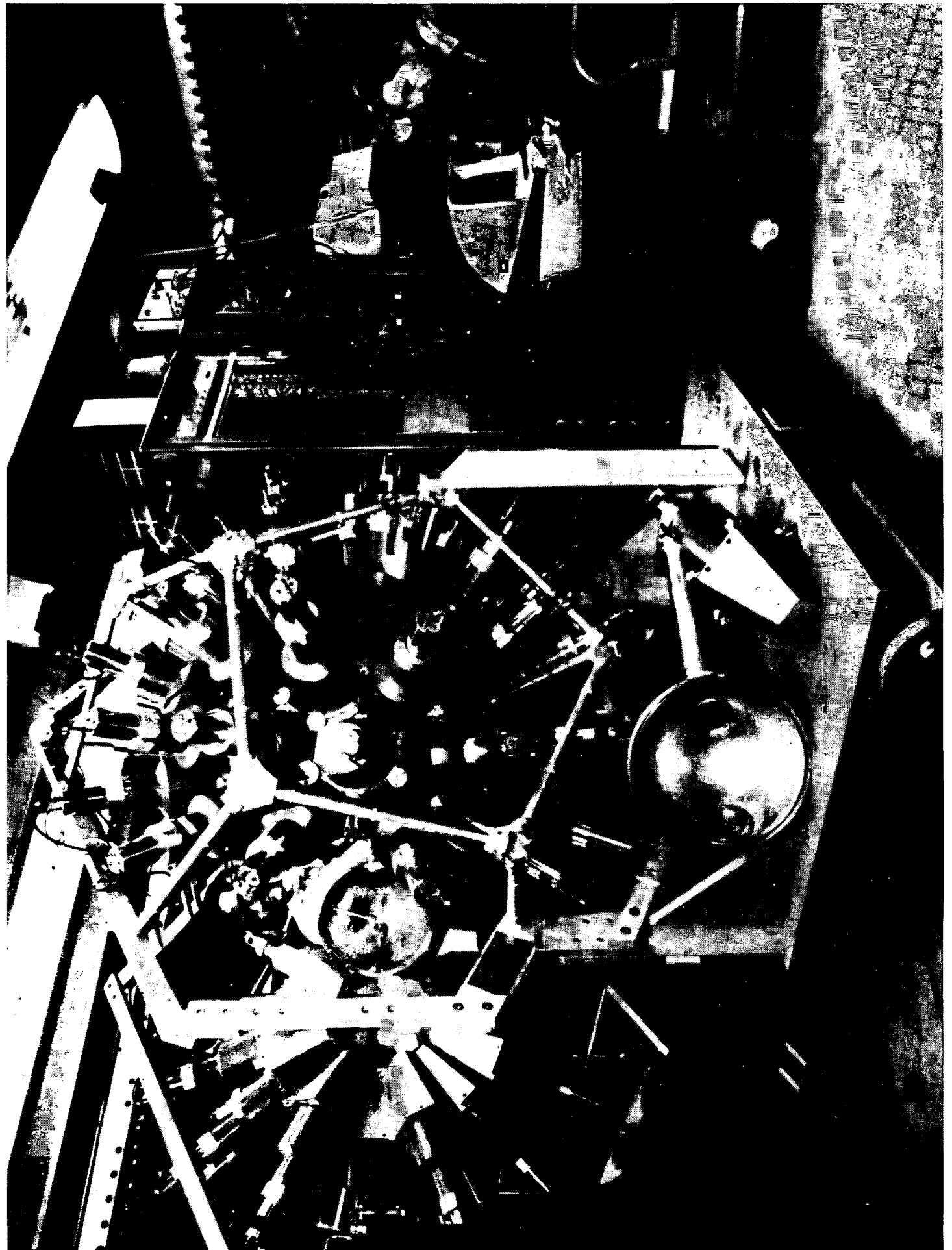


Fig. 16. The spin spectrometer.

chamber and sometimes neutron detectors outside the ball. Such experimental arrangements can present over 180 digitized outputs to the data acquisition system to be sorted for each event. In order to handle this sort of dimensionality, the system developed at the Holifield Facility has been executed fully in terms of CAMAC. This not only provides large expansion capabilities but allows us to benefit from design and development work done for high-energy physics.

Our data acquisition system is based on three independent Perkin-Elmer 3320 CPU's each with 1MB of fast memory. This year, two of these will be upgraded to 2MB. A schematic representation of one-third of our system is shown in Fig. 17. Each of these systems, including those on-line taking data, operate under a multi-tasking system which supports a number of concurrent tasks. This system, which has now been operational for about a year, has proven very effective during the first round of experiments.

## V. FUTURE PLANS

With the Holifield Facility set to become fully operational later this year, it may seem a bit premature to be looking toward major facility expansion. However, the realities of the funding process are such that we have to plan now for what we may hope to have operating by the beginning of the next decade.

When we are able to operate routinely with 25 MV on the terminal of the Pelletron, we will have the ion energy capabilities shown in Fig. 18. Our recent studies for facility expansion have centered on

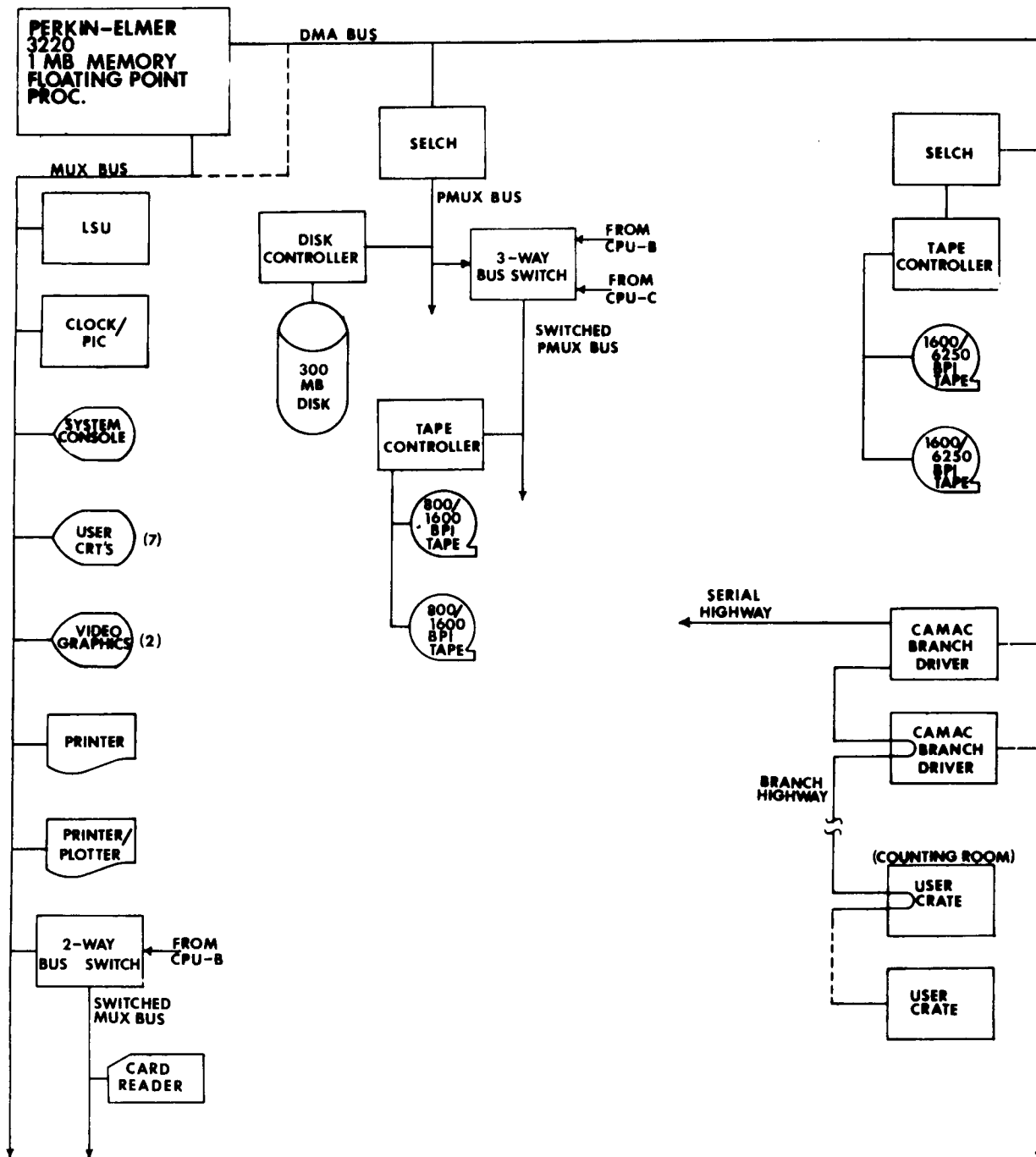


Fig. 17. Schematic representation of one-third of the HHIRF data acquisition system.



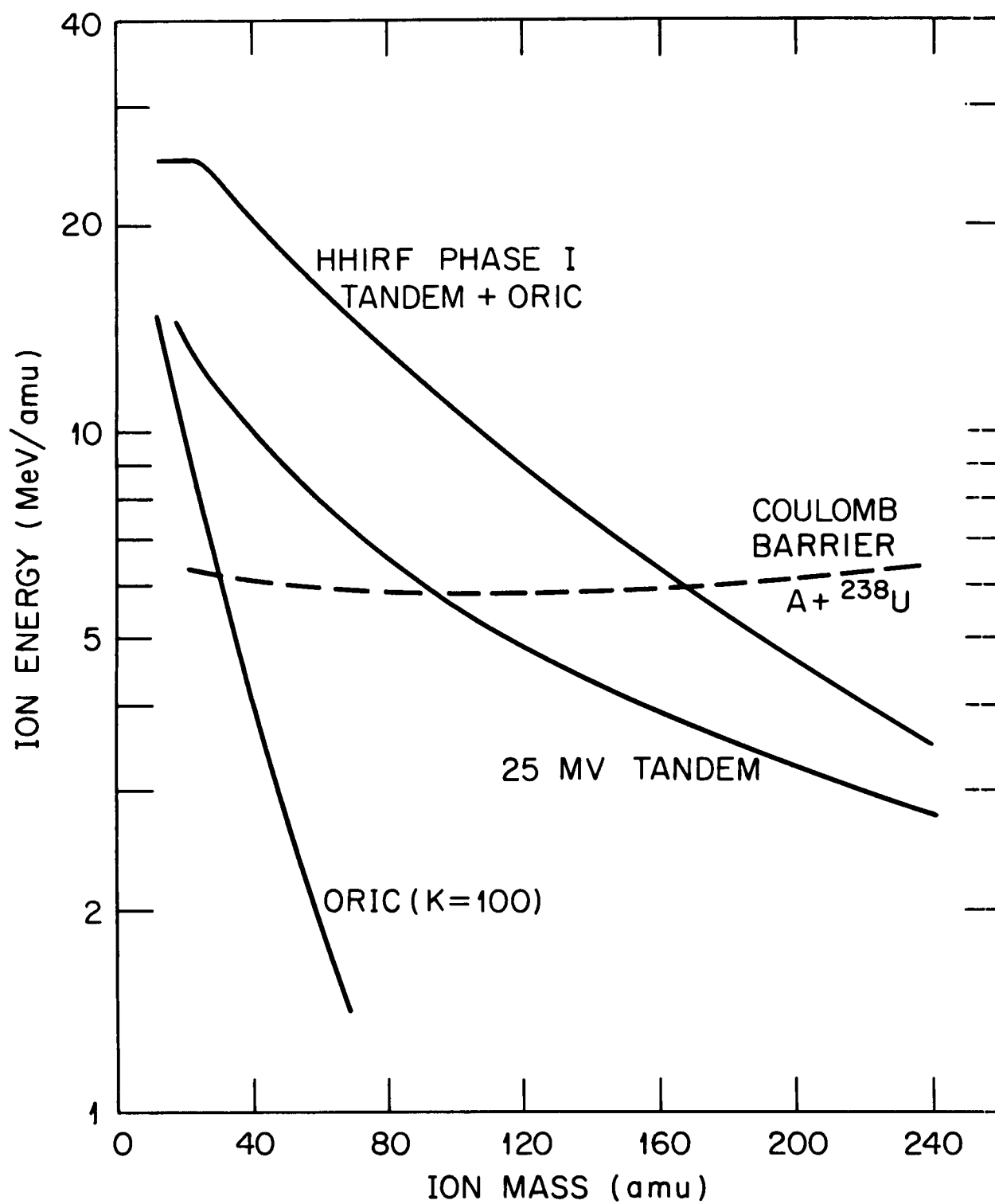


Fig. 18. Ion energy performance capabilities of the present phase of the Holifield Facility.

replacement of one existing booster cyclotron with a superconducting cyclotron patterned after the second stage being built for the double cyclotron facility at Michigan State University. Typically, such a booster would provide us more than an order of magnitude increase in available ion energy. The performance capabilities are shown in Fig. 19. Of particular interest to us with this system is the total amount of energy that may be introduced at the reaction site. There are now indications that there are some processes where the important parameter is this total energy and not the energy per nucleon. Recasting the previous figure in terms of total ion energy yields the performance curve shown in Fig. 20. Thus, it is seen that the proposed facility would be capable of introducing a very large energy impulse at the reaction site for a wide variety of ion-target combinations.

That, however, is something to consider for the long-term future. In the nearer term, our efforts will be directed toward working to achieve full performance from the 25MV Pelletron and to utilize these new ion beams for physics research. An important aspect of this will be the continued development of new experimental devices and adding new accelerator capabilities.

This year, one such new experimental device, a velocity filter/beam trap will be added on a new beam line from the tandem. This device, designed under the direction of Harold Enge and built at MIT, will enable fusion-type reaction studies to be performed with detection at zero degrees.

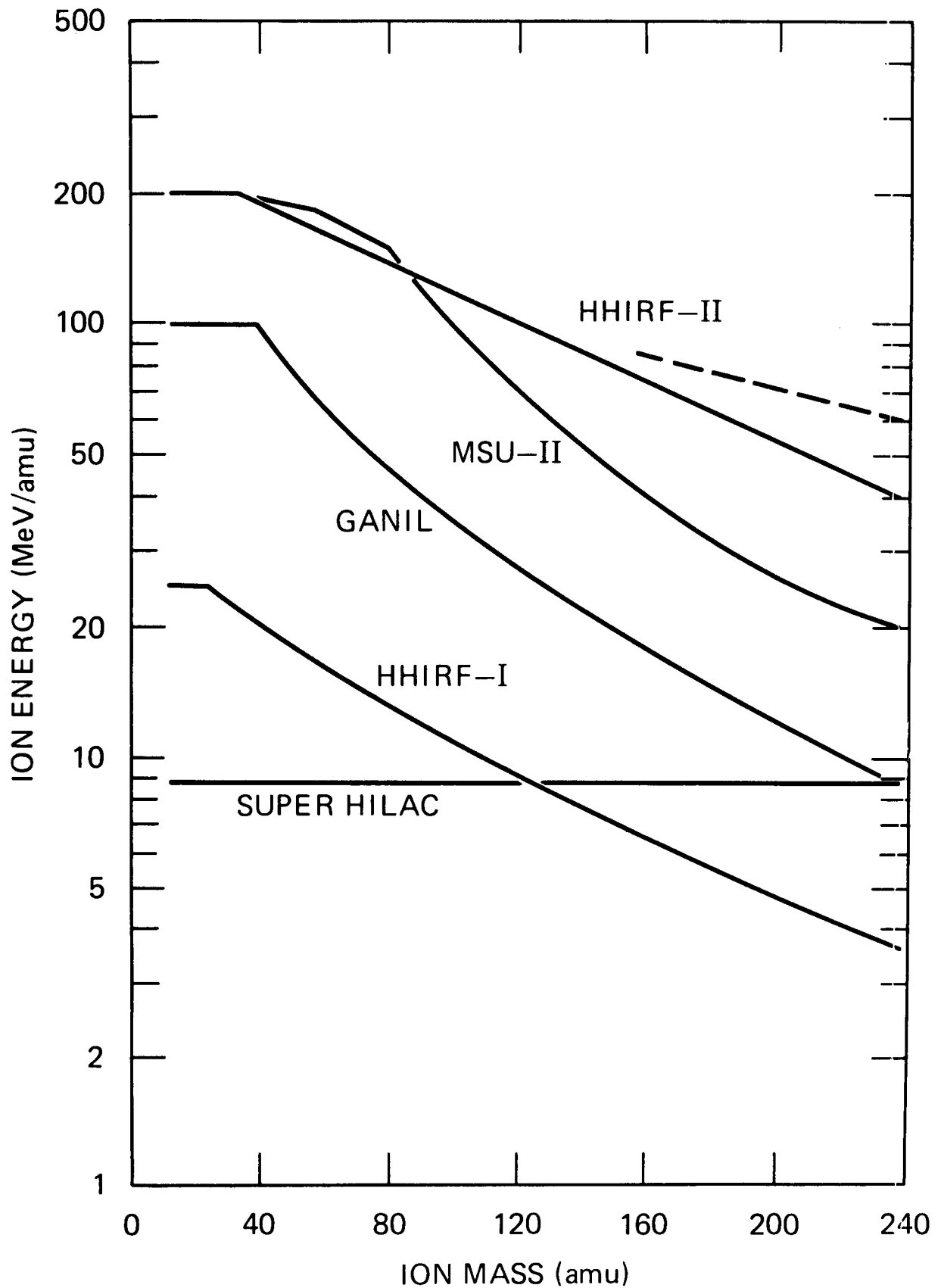


Fig. 19. Increased performance capabilities provided by the proposed new booster accelerator.

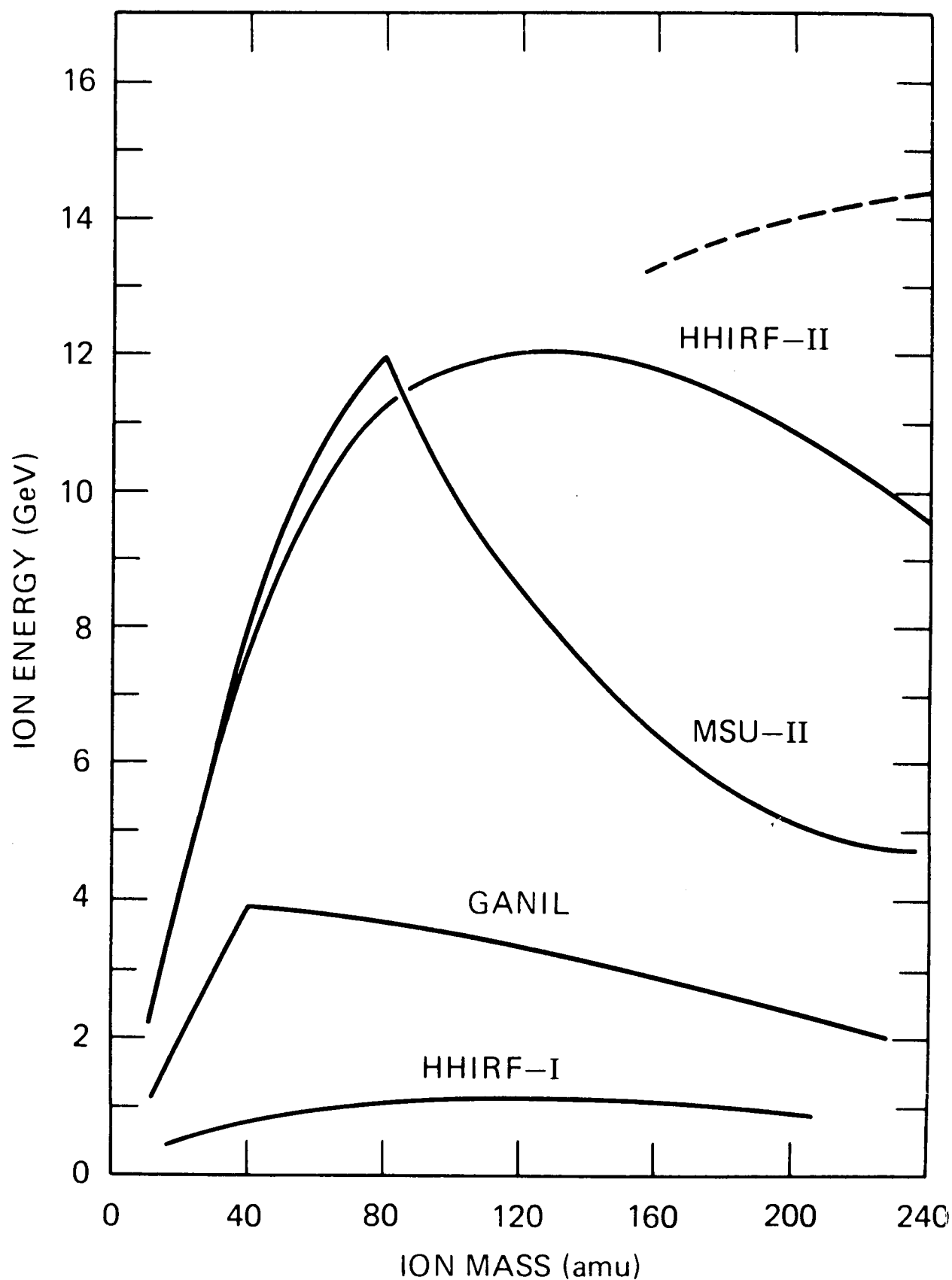


Fig. 20. Total energy performance available from the proposed facility upgrade.

Currently under design is a new subnanosecond beam pulser and chopper system for the tandem. The implementation of this system will include installation of a rebuncher cavity in the high voltage terminal of the tandem. This system, which we hope to have completed in about two years, will then add time-of-flight experimental capability to the tandem. This is particularly important for studies involving the emission of neutrons.

As we work on our accelerator and experimental programs over the next few years, we should expect that there will be many areas of common interest and concern between our facility at Oak Ridge and yours at Buenos Aires. We have enjoyed the participation of your staff members at Oak Ridge over the past several years and we look forward to an increasing collaboration between our facilities that will be to our mutual benefit.



# TIME DEPENDENT VARIATIONAL DESCRIPTION OF ALPHA-ALPHA SCATTERING<sup>+</sup>

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## 1. Introduction

The microscopic description of the scattering of nuclear fragments is rather complicated. For light fragments the methods that have been applied with success are the Resonating Group Method<sup>1,2)</sup> (RGM) and the Generator Coordinate Method<sup>3,4)</sup> (GCM). They are closely related and their main advantage is that the Pauli principle between fragments is preserved exactly. The computation of the overlap and energy kernels that provide the microscopic input to these type of calculations is at a very sophisticated stage and makes use of powerful mathematical techniques<sup>5,26)</sup> (Bargmann space, Group Theory). As the fragments become heavier the analytical computation of the kernels becomes more and more involved. More significantly the usual assumption that the internal structure of the fragments remains unchanged through the collision becomes invalid and the need to perform coupled channel GCM calculations arises<sup>7)</sup>. Both the RGM and the GCM provide as output fully quantum mechanical Information like relative motion wave functions and elastic phase shifts.

The other method that has been used is the Time Dependent Hartree Fock (TDHF) approach. This method, based on a time dependent variational principle in which the wave function is constrained at all times to be a Slater determinant, can be and has been applied to many reactions in all mass regions<sup>8)</sup>. It provides a reasonable description of many qualitative and also quantitative features of the scattering including the deep inelastic and fusion regions in heavy systems. The principal disadvant

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<sup>+</sup>This is a preliminary report on Research Program developed in collaboration with Profs. P.Kramer and F.Fernandez from the Institut für Theoretische Physik, Universität Tübingen, Germany.

ages of TDHF are the lack of a practical way of extracting quantum mechanical information from the calculations and their intrinsic numerical difficulty.

In this paper we propose a method to treat the scattering of composite fragments using the Time Dependent Variational Principle and apply it to the simple but still quite realistic case of alpha-alpha scattering.

The time dependent description of interacting alpha-clusters has been studied recently<sup>9,10)</sup>. However the emphasis has been in the classical description by initial value calculations much in the same spirit as in TDHF. In this work we shift the emphasis to the study of the symplectic structure associated to the norm kernel. We are thus able to incorporate the effects of the Pauli principle assigning them to a distortion of the relative motion phase space and to study quantitatively their influence on the scattering.

Instead of solving the equations of motion and displaying trajectories we show how meaningful quantum information (in this case the elastic phase shifts) can be extracted from the phase space diagrams. These phase shifts include the effects of the Pauli principle at a semiclassical level and compare very favorably with the full RGM calculation of Okai and Park<sup>11)</sup> at a fraction of the computational effort.

The method is readily extended to heavier fragments and makes use only of the diagonal part of the energy and normalization kernels in the Bargmann representation<sup>5)</sup>. General analytical expression for these kernels are available in explicit form for most binary combinations of fragments up to  $^{40}\text{Ca}$ - $^{40}\text{Ca}$ .

From the point of view of the TDVP, our method of computing the phase shifts extends to the continuum the requantization techniques proposed for TDHF quantized bound states for trajectories with periodic boundary conditions<sup>12,13)</sup>.



## 2.1 The Time Dependent Variational Principle

Recently, we have studied the structure and the geometry of the Time Dependent Variational Principle (TDVP) in quantum mechanics, especially in relationship with Lie Groups and symplectic structures on orbits of their coadjoint representations<sup>14)</sup>. In this section we review briefly the symplectic structure and the variational equations associated with states parametrized analytically, independently of the existence of such a group structure.

We assume a variational wave function  $|\Psi(\bar{z}_i)\rangle^*$  parametrized in terms of  $L$  complex parameters  $z_i$ ,  $i=1, \dots, L$ . The significance of the  $z_i$  parameters depends on the problem at hand and generally represent some geometric property whose approximate time dependence one wishes to extract. We will assume that the dependence of the wave function on the parameters is analytical. This requirement is in most cases violated if one wishes to use normalized wave functions and therefore we will not assume a definite normalization. Accordingly we modify the usual<sup>15)</sup> variational principle to make the action, and therefore the equations of motion, invariant under a change of phase or normalization of the wave function.

We define the action as

$$S = \int \mathcal{L}[z(t)] dt \quad (2.1)$$

where the lagrangian  $\mathcal{L}$  is

$$\mathcal{L} = \frac{i}{2} \frac{\langle \Psi | \dot{\Psi} \rangle - \langle \dot{\Psi} | \Psi \rangle}{\langle \Psi | \Psi \rangle} - \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

---

(\*) The bar denotes complex conjugation. The use of  $\bar{z}$  in the ket wave function is conventional and most of the time we will abbreviate  $|\Psi(\bar{z})\rangle = |z\rangle$ . In this section we set  $\hbar = 1$ .

Using the parametrized w.f. we can rewrite this as

$$\mathcal{L} = -\frac{i}{2} \sum_{i=1}^L (\dot{\bar{z}}_i \frac{\partial}{\partial \bar{z}_i} - \dot{z}_i \frac{\partial}{\partial z_i}) \ln \mathcal{N} - \mathcal{H} \quad (2.2)$$

The ingredients needed to compute the action are then the norm and energy kernels

$$\mathcal{N} = \langle z | z \rangle \quad (2.3)$$

$$\mathcal{H} = \frac{\langle z | H | z \rangle}{\langle z | z \rangle} \quad (2.4)$$

Variation of the action yields the equations of motion

$$i \sum_j g_{ij} \dot{\bar{z}}_j = \frac{\partial \mathcal{H}}{\partial z_i} \quad \text{and c.c.} \quad (2.5)$$

where  $g_{ij}$  is given by

$$g_{ij} = \frac{\partial^2}{\partial z_i \partial \bar{z}_j} \ln \mathcal{N} \quad (2.6)$$

The symplectic structure associated with the parametrization is revealed by defining a Poisson bracket for general functions  $F(z, \bar{z})$

$$\{F, G\} = \sum_{i,j=1}^L \frac{\partial F}{\partial z_i} \{z_i, \bar{z}_j\} \frac{\partial G}{\partial \bar{z}_j} - \frac{\partial G}{\partial z_i} \{z_i, \bar{z}_j\} \frac{\partial F}{\partial \bar{z}_j} \quad (2.7)$$

The fundamental brackets are given in terms of the inverse of  $g$

$$\{\bar{z}_i, z_j\} = g_{ij}^{-1} \quad (2.8)$$

$$\{z_i, z_j\} = \{\bar{z}_i, \bar{z}_j\} = 0$$

With these definitions the equations of motion take the symplectic form

$$i\dot{\bar{z}}_i = \{ \bar{z}_i, \mathcal{H} \} \quad \text{and c.c.} \quad (2.9)$$

This form implies energy conservation along the solution trayectories. Once the equations of motion are solved and a specific trayectory  $z_i(t)$  is found, the approximate solution to the time dependent Schrödinger equation is given by

$$|\Psi(t)\rangle = \exp(iS) \frac{|z(t)\rangle}{(\langle z|z\rangle)^{1/2}} \quad (2.10)$$

where  $S$  is the action (2.1) computed along the given trayectory.

The main advantage of defining the lagragian as in (2.2) is that the equations of motion are independent of the phase and normalization of the wave function. The proper phase and norm<sup>6)</sup> are restored by computing the action integral.

We should remark here that when  $g_{ij} = \delta_{ij}$  the symplectic structure (2.7) and the fundamental brackets (2.8) become the usual ones of ordinary classical mechanics in complex form. This happens when the overlap has the form

$$\mathcal{N} = \exp \sum_{i=1}^L z_i \bar{z}_i \quad (2.11)$$

In this case the action becomes

$$S = \int \left[ \frac{i}{2} \sum_{i=1}^L \dot{\bar{z}}_i z_i - \dot{z}_i \bar{z}_i - \mathcal{H} \right] dt \quad (2.12)$$

This form, when real and imaginary parts of  $z$  are defined as  $z_i = (1/2)^{-1/2} (q_i - ip_i)$ , reduces to the canonical action<sup>16)</sup> in real form

$$S = \int p_i dq_i - \mathcal{H} dt \quad (2.13)$$

## 2.2 The Trial Wave Function

The trial wave function that we will use in the TDVP is a Slater determinant built up from oscillator states centered at different positions. Therefore the basic assumption is that each of the fragment ground state can be well represented by a Slater determinant of four 0s oscillator states and that in the collision process they remain unexcited. This is the usual assumption in the RGM or GCM description of the scattering of composite particles.

Instead of using the single particle oscillator states in their  $r$ -representation we use the analytically simpler Bargmann representation and use the associated coherent states to construct the antisymmetrized trial wave function. Due to the simplicity of the fragments we will use rather pedestrian methods. More elaborate methods are available for more complicated cases<sup>5)</sup>.

We describe the ground state  $|\phi\rangle$  of a nucleus of  $N$  nucleons by a properly antisymmetrized wave function of single particle oscillator states of frequency  $\omega$ . The state  $|\phi\rangle$  is a wave packet whose mean center of mass (CM) coordinates are at the origin in phase space

$$\langle\phi|\underline{P}|\phi\rangle = \langle\phi|\underline{R}|\phi\rangle = 0$$

where

$$\underline{P} = \sum_{i=1}^N \underline{p}^i, \quad \underline{R} = N^{-1} \sum_{i=1}^N \underline{r}^i \quad (2.14)$$

A fragment with non-zero C.M. coordinates is constructed as a coherent state

$$|z\rangle = \exp(\bar{z} \cdot \underline{A}^+) |\phi\rangle \quad (2.15)$$

where

$$\underline{A}^+ = 1/2 (\underline{R} \underline{b}_N^{-1} - i \frac{\underline{b}_N}{\hbar} \underline{P}) \quad (2.16)$$

and where the complex vector  $\underline{z}$  is related to the CM mean values as

$$\frac{\langle \underline{z} | \underline{P} | \underline{z} \rangle}{\langle \underline{z} | \underline{z} \rangle} = \frac{i\hbar}{\sqrt{2} \underline{b}_N} (\underline{z} - \bar{\underline{z}}) \quad (2.17)$$

$$\frac{\langle \underline{z} | \underline{R} | \underline{z} \rangle}{\langle \underline{z} | \underline{z} \rangle} = \frac{\underline{b}_N}{\sqrt{2}} (\underline{z} + \bar{\underline{z}})$$

In (2.16) and (2.17)  $\underline{b}_N = \sqrt{\hbar (Nm \omega)^{-1}}$  is the oscillator length of a particle of mass  $Nm$  in a potential of frequency  $\omega$ .

For two fragments of mass number  $N_1$  and  $N_2$  the trial state is

$$|z_1, z_2\rangle = \mathcal{A} \exp(\bar{\underline{z}}_1 \cdot \underline{A}_1^+ + \bar{\underline{z}}_2 \cdot \underline{A}_2^+) |\phi_1\rangle |\phi_2\rangle \quad (2.18)$$

where  $\mathcal{A}$  is the antisymmetrizer. We can introduce relative and total CM coordinates by the transformation

$$\begin{aligned} \underline{A}_{CM}^+ &= (N_1 + N_2)^{1/2} (\sqrt{N_1} \underline{A}_1^+ + \sqrt{N_2} \underline{A}_2^+) \\ \underline{B}_{CM}^+ &= \mu^{1/2} \left( \frac{1}{\sqrt{N_2}} \underline{A}_2^+ - \frac{1}{\sqrt{N_1}} \underline{A}_1^+ \right) \end{aligned} \quad (2.19)$$

and

$$\begin{aligned} \bar{\underline{s}} &= \frac{1}{\sqrt{2}} (\underline{b}_\mu^{-1} \underline{R} + i \underline{b}_\mu \underline{P} / \hbar) \\ \bar{\underline{z}}_{CM} &= \frac{1}{\sqrt{2}} (\underline{b}_N^{-1} \underline{R}_{CM} + i \underline{b}_N \underline{P}_{CM} / \hbar) \end{aligned} \quad (2.20)$$

In (2.19) and (2.20)  $\underline{R}$  and  $\underline{P}$  are the relative coordinates of the centers of mass of the two fragments and

$$\underline{b}_\mu = \sqrt{\frac{\hbar}{\mu m \omega}} \quad (2.21)$$

is the oscillator length corresponding to the reduced mass number

$\mu = N_1 N_2 / (N_1 + N_2)$ . With this transformation the trial state is rewritten as

$$|z_{CM}, \underline{s}\rangle = \exp(\underline{z}_{CM} \cdot \underline{A}_{CM}^+) \mathcal{A} \exp \bar{\underline{s}} \cdot \underline{B}^+ |\phi_1 \phi_2\rangle$$

where we have used the fact that  $\underline{A}_{CM}^+$  is symmetric in the particle coordinates to commute it with the antisymmetrizer. Because of translational and Galileian invariance of the relative hamiltonian and because we are using single particle oscillator states we can drop the  $z_{CM}$  dependence altogether and deal only with the trial state

$$|\underline{s}\rangle = \mathcal{A} \exp \bar{\underline{s}} \cdot \underline{B}^+ |\phi_1 \phi_2\rangle \quad (2.22)$$

$\underline{B}^+$  is the creation operator for quanta of relative motion while  $\underline{s}$  is a complex vector related to the relative position and momentum of the fragments by (2.20)

The state  $|\underline{s}\rangle$  is built from non-orthogonal single particle states and is totally antisymmetric. It is parametrized analytically by the complex vector  $\underline{s}$  and it is not normalized. If we attempt normalization analyticity would be lost. In any representation  $|\underline{s}\rangle$  is a Slater determinant. Techniques for the explicit computation of the overlap and mean values of one and two body operators with these states are standard<sup>5)18)</sup>. The explicit expressions needed for the alpha-alpha case are given in the appendix.

Notice that the state  $|\underline{s}\rangle$  does not have a well defined quantum angular momentum but that a classical value can be extracted from the vector  $\underline{s}$  as

$$L' = \underline{R} \times \underline{P} = \hbar i \underline{\bar{s}} \times \underline{s} \quad (2.23)$$

The application of the general formulas of section 2.1 with the trial wave function  $|\underline{s}\rangle$  yields equations of motion for  $\underline{s}(t)$  which take the form

$$i \sum_j g_{ij} \dot{\underline{s}}_j = \frac{\partial \mathcal{H}}{\partial s_i} \quad \text{and c.c.}$$

where  $g_{ij}$  and  $\mathcal{H}$  are given explicitly in the appendix.

Although we could solve these equations (numerically) directly in terms of  $\underline{s}$  as is done in ref.<sup>9)</sup>, it is more convenient to first introduce canonical variables in such a way that all the usual results of classical two body scattering can be applied. The price to be paid (which turns out to be a bonus in terms of physical insight) is that the new phase space in which the motion occurs has a more complicated nature than ordinary phase space and its more distinct feature is the appearance of forbidden regions which can be directly linked to the Pauli principle. We devote the next section to the study of these features.

### 3. Transformation to canonical variables

We have seen that the variables  $\underline{R}$  and  $\underline{P}$  are the distance and relative momentum of the centers of mass of the two fragments. In the scattering of two structureless particles they form a canonical pair, i.e.,  $\{R_i, P_j\} = \delta_{ij}$  in classical mechanics or  $[iP_i, R_j] = \hbar \delta_{ij}$  in quantum mechanics. This is in turn reflected in the fact that their complex combinations analogous to (2.20) satisfy  $\{\bar{s}_i, s_j\} = \delta_{ij}$ . The deviations of the fundamental brackets from unity reflect the composite nature of the fragments and allow a study of the effects of the Pauli principle in classical phase space.

#### 3.1 The Pauli distortion factor

The overlap of the trial states for two alpha clusters is

$$\langle \underline{s} | \underline{s} \rangle = \exp \underline{s} \cdot \bar{\underline{s}} (1 - \exp - \frac{1}{2} \underline{s} \cdot \bar{\underline{s}})^4 \quad (3.1)$$

The fundamental bracket  $\{s_i, \bar{s}_j\}$  is not unity and is computed explicitly in the appendix. To find a transformation that turns  $\underline{s}$  into a canonical variable we go back to the action (without the energy term)

$$S_0 = \int dt \frac{i}{2} \sum_{i=1}^3 (\dot{\bar{s}}_i \frac{\partial}{\partial \bar{s}_i} - \dot{s}_i \frac{\partial}{\partial s_i}) \ln \mathcal{N}$$

Using (3.1) we find

$$S_0 = \int dt \frac{i}{2} (\dot{\bar{s}} \cdot \underline{s} - \dot{s} \cdot \bar{\underline{s}}) u$$

where

$$u(\underline{s}, \bar{\underline{s}}) = \frac{\partial}{\partial (\underline{s} \cdot \bar{\underline{s}})} \ln \mathcal{N} = 1 + \frac{2 \exp - \frac{1}{2} \underline{s} \cdot \bar{\underline{s}}}{1 - \exp - \frac{1}{2} \underline{s} \cdot \bar{\underline{s}}} \quad (3.3)$$



Then in terms of new variables  $\omega_i$

$$\omega_i(\underline{s}, \bar{\underline{s}}) = s_i u^{1/2} \quad \text{and c.c.} \quad (3.4)$$

the action is given by

$$S_0 = \int dt \frac{i}{2} (\dot{\bar{\omega}} \cdot \omega - \dot{\omega} \cdot \bar{\omega}) \quad (3.5)$$

It is straightforward, although rather tedious, to show that, using the bracket defined in (2.7) and with  $g^{-1}$  calculated explicitly in (A.2) we have

$$\begin{aligned} \{\bar{\omega}_i(\underline{s}, \bar{\underline{s}}), \omega_j(\underline{s}, \bar{\underline{s}})\} &= \delta_{ij} \\ \{\omega_i, \omega_j\} &= \{\bar{\omega}_i, \bar{\omega}_j\} = 0 \end{aligned} \quad (3.6)$$

These brackets become obvious when we consider that the action, in terms of  $\omega$ , takes the canonical form (2.12).

In the new variables the equations of motion become

$$i\dot{\bar{\omega}}_j = \frac{\partial \mathcal{H}}{\partial \omega_j} \quad \text{and c.c.} \quad (3.7)$$

where we have assumed  $\mathcal{H}$  given in terms of  $\omega$  by inversion of (3.4).

We can also define real quantities in analogy to  $\underline{R}$  and  $\underline{P}$  by

$$\underline{\omega} = \frac{1}{\sqrt{2}} (\underline{r} b_\mu^{-1} + i \frac{b_\mu}{\hbar} \underline{p}) \quad (3.8)$$

where  $\underline{r} = \underline{R} u^{1/2}$  and  $\underline{p} = \underline{P} u^{1/2}$  are canonical. In terms of these variables everything will look like ordinary classical mechanics. In fact the action will be

$$S = \int \frac{1}{2} (\underline{p} \cdot \dot{\underline{r}} - \dot{\underline{p}} \cdot \underline{r}) - \mathcal{H}(\underline{r}, \underline{p}) \quad dt \quad (3.9)$$

which leads to the equations of motion

$$\dot{\underline{r}} = \{ \underline{r}, \mathcal{H} \} \quad \dot{\underline{p}} = \{ \underline{p}, \mathcal{H} \} \quad (3.10)$$

with the brackets defined as

$$\{ r_i, p_j \} = \delta_{ij}$$

However the allowed ranges of the new variables have been changed reflecting the fact that the phase space has a more complicated structure.

The mapping (3.4) between the  $s_i$  and the  $\omega_i$  variables is not analytic and is not canonical. However it transforms the non-canonical variables  $s_i$  into canonical  $\omega_i$ . The existence of such a transformation is guaranteed locally for any symplectic manifold (this is known as Darboux Theorem<sup>19)</sup>). In (3.4) we have found a global transformation that defines canonical variables everywhere. We can thus study the mapping and in particular its boundaries. To this effect notice that the norm kernel only depends on the scalar quantity  $\underline{s} \cdot \bar{\underline{s}}$ . Its allowed range is

$$0 \leq \underline{s} \cdot \bar{\underline{s}} < \infty$$

The allowed range of the analogous quantity  $\underline{\omega} \cdot \bar{\underline{\omega}}$  is found by examining the function

$$\underline{\omega} \cdot \bar{\underline{\omega}} = \underline{s} \cdot \bar{\underline{s}} u(\underline{s} \cdot \bar{\underline{s}}) = \underline{s} \cdot \bar{\underline{s}} \left[ 1 + 2 \frac{\exp - \frac{1}{2} \underline{s} \cdot \bar{\underline{s}}}{1 - \exp - \frac{1}{2} \underline{s} \cdot \bar{\underline{s}}} \right] \quad (3.11)$$

This function, which for reasons that will become clear below will be called the Pauli distortion factor, is plotted in Fig.1 . We find immediately that its range of allowed values is

$$4 \leq \underline{\omega} \cdot \bar{\underline{\omega}} < \infty$$

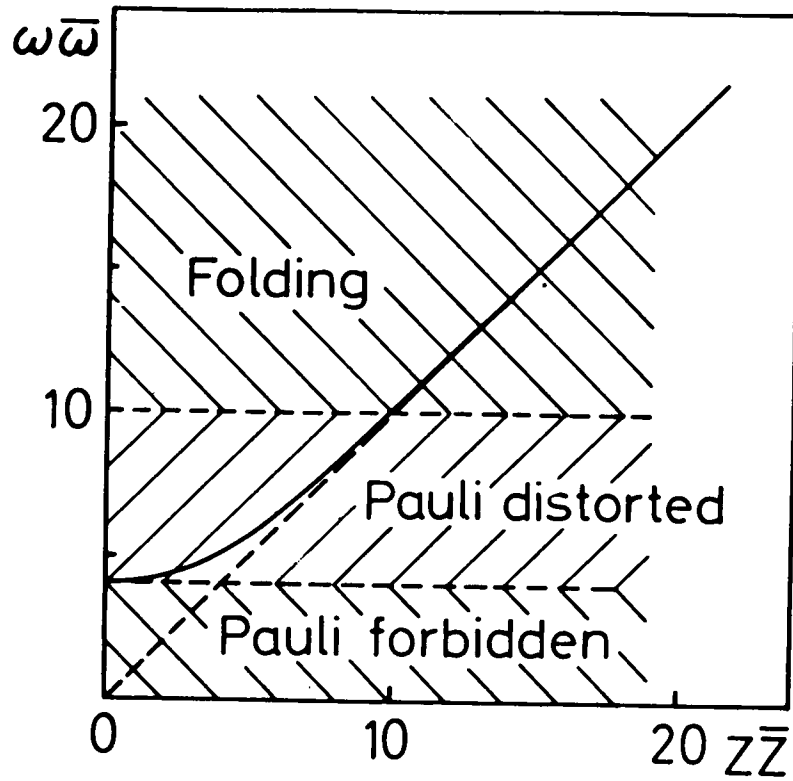


Figure 1:

Pauli phase space distortion factor for alpha-alpha scattering. The limit between "Pauli distorted" and "folding" has been taken (arbitrarily) at the point where  $\omega \cdot \bar{\omega}$  and  $s \cdot \bar{s}$  differ by 1%.

In Fig.1 we distinguish three regions:

- a)  $\omega \cdot \bar{\omega} < 4$  Forbidden region
- b)  $4 < \omega \cdot \bar{\omega} < 10$  Distorted region
- c)  $10 < \omega \cdot \bar{\omega}$  Folding region

The limit between b) and c) is somewhat arbitrary and here it has been taken where  $\omega \cdot \bar{\omega}$  differs from  $s \cdot \bar{s}$  by 1% . The reason for the name in c) is as follows. If we were to omit the antisymmetrizer in the trial wave function (2.22) the overlap would be  $\exp s \cdot \bar{s}$  and then  $\omega \cdot \bar{\omega}$  would equal  $s \cdot \bar{s}$  throughout their ranges. This has been drawn as a dotted line in Fig.1 . The deviation from this line reflects the effects of the Pauli principle. In the asymptotic region where  $\omega \cdot \bar{\omega} \simeq s \cdot \bar{s}$  the effects of the antisymmetrizer disappear and the trial wave function becomes a product of two determinants. In the appendix we show that in this limit we obtain the double folding approximation<sup>21)</sup>.

### 3.2 Radial coordinates

To see the significance of the quantity  $\omega \cdot \bar{\omega}$  and to exploit the rotational invariance of the hamiltonian we introduce radial variables. To this effect notice that as the fragments are spherical the energy will only depend on scalar combinations of  $s$ , namely  $s \cdot s$ ,  $\bar{s} \cdot \bar{s}$  and  $s \cdot \bar{s}$  . The same will happen upon transformation to the new variables  $\omega_1$  . Thus

$$\mathcal{H} = \mathcal{H}(\omega \cdot \bar{\omega}, \omega \cdot \omega, \bar{\omega} \cdot \bar{\omega}) \quad (3.12)$$

This implies that the angular momentum defined as

$$\underline{L} = \pi \underline{1} = \pi i \bar{\omega} \times \omega \quad (3.13)$$

is conserved. Notice that  $l^2$  is not quantized and can take any continuous value. Moreover, the relationship with the asymptotic angular momentum  $\underline{L}'$  defined in (2.23) is,

$$\underline{L} = u \underline{L}'$$

Therefore only in the folding region do the conserved and the asymptotic angular momentum coincide.

We now introduce radial canonical variables by the formulas

$$\begin{aligned} r &= (\underline{r} \cdot \underline{r})^{1/2} = b_\mu \left[ 1/2(\omega^2 + \bar{\omega}^2) + \underline{\omega} \cdot \underline{\bar{\omega}} \right]^{1/2} \\ k &= \frac{(\underline{r} \cdot \underline{p})}{\pi r} = \frac{1}{2ir} \left[ \omega^2 - \bar{\omega}^2 \right] \\ l^2 &= \pi^{-2} |\underline{r} \times \underline{p}|^2 = (\underline{\omega} \cdot \underline{\bar{\omega}}) - \omega^2 \bar{\omega}^2 \end{aligned} \quad (3.14)$$

With this transformation we can rewrite the hamiltonian (3.12) as

$$\mathcal{H} = \mathcal{H}(k, r, l^2) \quad (3.15)$$

which defines a reduced radial two body scattering at fixed angular momentum  $l^2$ . As the energy is conserved the phase space trayectories will be equipotentials in the  $k, r$  plane. Therefore to find  $k$  as a function of  $r$  there is no need to explicitly solve the equations of motion and find  $r(t)$ ,  $k(t)$  although this could be easily done numerically.

### 3.3 Effects of the Pauli principle in the radial phase space

We have reduced the equations of motion derived from the variational principle to what looks like an ordinary radial scattering problem in classical mechanics. There are, however, important new

features which are hidden in the transformation (3.4). First of all the hamiltonian is highly non local: by no means a kinetic energy quadratic in  $k$  can be separated. For the same reason the centrifugal barrier will not appear simply as an additive term proportional to  $l^2/r^2$  but there will be a complicated  $l^2$  dependence. But the most significant new feature is the appearance of an  $l$ -dependent forbidden zone. Its size and shape is given by  $\underline{\omega} \cdot \bar{\omega} = 4$  which translated to radial variables means

$$\underline{\omega} \cdot \bar{\omega} = 1/2 \left[ r^2/b_\mu^2 + b_\mu^2 (k^2 + l^2/r^2) \right] \leq 4$$

These are closed curves whose size decrease with increasing  $l$ . For  $l=4$  they shrink to a point and disappear altogether for larger  $l$ . These regions are shown as shaded areas in Fig.2 for  $l=0,2,4$ . Also shown as a dotted line is the limit of the folding region at  $\underline{\omega} \cdot \bar{\omega} = 10$ . Outside this region the effects of antisymmetrization are negligible.

At this point we should remark the close parallelism of our discussion with the usual results on the effects of the Pauli principle in terms of forbidden states of relative motion. These are obtained in RGM by diagonalizing the overlap kernel and obtaining the states with zero eigenvalue<sup>2)</sup>. When the trial single particle states are oscillator states of the same parameter, as in the present discussion, it is well known that the forbidden states of relative motion are also oscillator states with principal quantum number  $N < 4$ , independent of  $l$ . Therefore, considering that  $N=2n_r+1$ , there are two forbidden states for  $l=0$  ( $n_r=0,1$ ) one forbidden state for  $l=2$  ( $n_r=0$ ), and none for  $l \geq 4$ . If now we consider that semiclassically a quantum state occupies an area  $2\pi$  in phase space it is clear from the shaded regions in Fig.2 that this is exactly the situation in the modified phase space, but depicted semiclassically. The forbidden area for  $l=0$  is double the area for  $l=2$  and disappears for  $l=4$  and its size has the correct magnitude (notice that as we have only drawn the part  $k > 0$  of the radial phase space the area of a quantum state in these diagrams is  $\pi$ ).

The situation with respect to the Pauli distorted region is very similar. In the RGM calculation the non zero eigenvalues of the overlap kernel are given by<sup>23)</sup>

$$\lambda_N = 1 - \frac{4}{2^N} \quad N \geq 4$$

For  $N \gg 10$  they are unity (to within 1%) signifying that the effects of antisymmetrization have disappeared. The region  $4 \leq N < 10$  corresponds to the region of partial forbiddenness. Semiclassically we describe this region by the area inside the dotted line where the asymptotic phase space (spanned by variables  $s_1$ ) differs from the canonical phase space (spanned by variables  $\omega_1$ ) but is not forbidden.

It is not our intention at this point to study exhaustively the relationship between the diagonalization problem for the overlap kernel and the symplectic structure associated with it. As the simple illustration given above shows such a relationship exists and will be studied in detail in future publications. For the time being notice that all the information about the structure of the global change of variables that turns the symplectic manifold into a canonical phase space is contained in what we have called the Pauli distortion factor which can be computed easily from the diagonal part of the overlap kernel in the Bargmann representation. Its physical meaning is closely related to the mean number of oscillator quanta in the relative motion.

In Fig.2 we have plotted the phase space trajectories for the hamiltonian in terms of radial canonical variables. The two body interaction and the oscillator parameter are the same as used by Okai Park<sup>11)</sup>. The explicit expressions are given in the appendix. The Coulomb force has been included in the folding approximation only, thus neglecting antisymmetrization, to produce results comparable with the RGM computation of<sup>11)</sup>.

A word of caution should be raised here concerning the assignment of angular momentum. As the trial wave function violates rotational invariance it is not clear from our analysis how to select the proper  $l$ -values that have physical meaning. We have here used the Langer prescription which is the procedure used when applying the WKB methods in heavy ion scattering<sup>22)</sup>. By this prescription the quantum angular momentum is

$$L = L_{QM} + 1/2 \hbar$$

The low allowed values of  $L_{QM}$  are 0,2,4,6,... (odd waves are forbidden for alpha-alpha scattering). The use of the Langer prescription is here validated a posteriori by the excellent agreement with the quantum calculation in the next section. It is obvious that the correct procedure to assign the angular momentum would be to use a projected trial wave function in the variational principle. This would lead to a classical hamiltonian  $\mathcal{H}_L^p(k,r)$  in which the continuous dependence on  $l^2$  in (3.15) is replaced by a discontinuous dependence on the allowed  $L$ -values of the relative angular momentum. Because of the increased difficulty in using  $l$ -projected states in the variational principle it would be very desirable to study more in detail this question of the assignment of low  $l$ -values so as to be able to use this (or another) prescription with confidence.

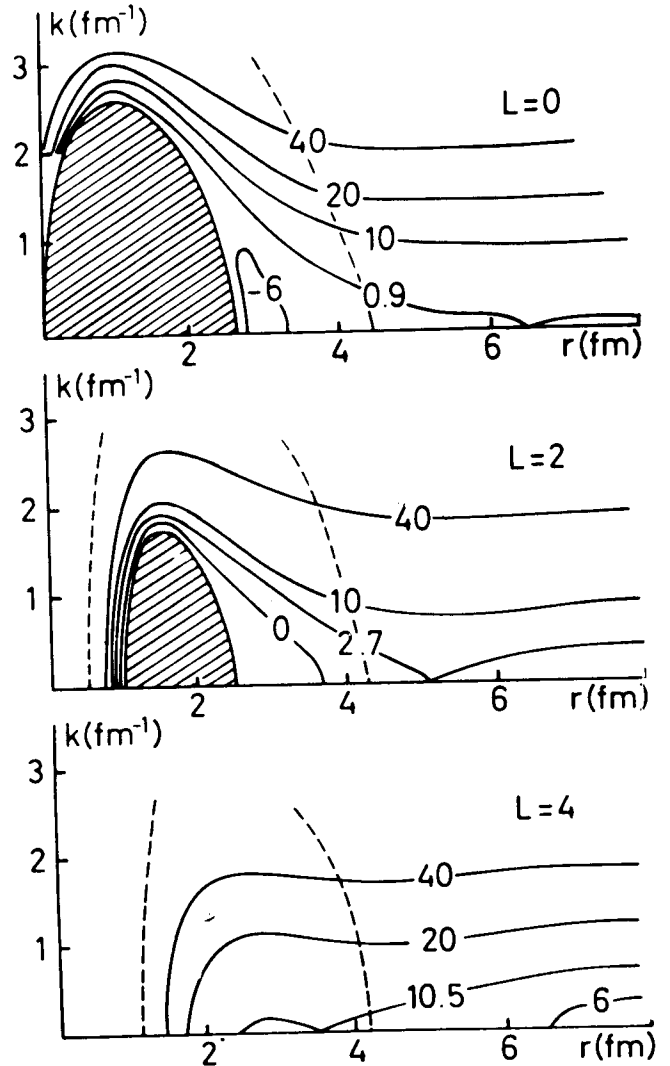
In Fig.2 the essential characteristics of the alpha-alpha scattering can be recognized. In all cases we have plotted the critical trajectory leading to the top of the Coulomb barrier. This trajectory leads to orbiting of the fragments at a fixed distance. Notice that the  $L=0$  and  $L=2$  barriers are well outside the distortion region indicated by the dotted line. This means that the barrier position and height is determined solely from the folding part of the potential. For  $L=4$



instead the critical trajectory enters the Pauli distorted region and is therefore affected by antisymmetrization. Another feature that is clearly brought out in Fig.2 is that the forbidden region affects the bound states of the system by excluding area in phase space that would otherwise be occupied by closed trajectories. This is the semiclassical statement of the results of the orthogonality condition model of Saito<sup>25)</sup> in which the effects of the Pauli principle are approximated by rendering the relative motion wave function orthogonal to the forbidden states.

Figure 2:

Radial phase space trajectories for alpha-alpha scattering. The oscillator parameter is  $b = 137$  fm. The trajectories are labeled by  $E_{CM}$  in MeV. The shaded region is Pauli forbidden and the dotted line gives the limit of the region of Pauli distortion as defined in Fig.1. In all cases the critical trajectory that corresponds to the top of the Coulomb barrier is shown. The angular momentum has been assigned by the Langer prescription  $L = \ell + 1/2$ .



#### 4. Computation of phase shifts

Semiclassically, in the WKB approximation, the nuclear phase shifts for two body scattering with a Coulomb plus nuclear potential are computed as<sup>24)</sup>

$$\delta_l(E) = \int_{r_0}^{\infty} k \, dr - \int_{r_c}^{\infty} k_c \, dr \quad (4.1)$$

where  $k$  and  $k_c$  are the wave numbers corresponding to the radial motion with and without the nuclear potential  $U(r)$ .

$$\begin{aligned} k(r) &= \hbar^{-1} [2\mu \{E - U(r) - V_c(r)\} - (1 + \frac{1}{2})^2 / r^2]^{\frac{1}{2}} \\ k_c(r) &= \hbar^{-1} [2\mu \{E - e^2 Z_1 Z_2 / r\} - (1 + \frac{1}{2})^2 / r^2]^{\frac{1}{2}} \end{aligned} \quad (4.2)$$

$r_0$  and  $r_c$  are the more outward turning points for the two trajectories, i.e. the points where  $k$  or  $k_c$  vanish.  $V_c$  is the Coulomb potential which at short distances can be different from the point Coulomb interaction  $e^2 Z_1 Z_2 / r$ . The centrifugal potential is computed for the classical angular momentum  $1 + \frac{1}{2}$ .

In figure 3 we have represented the phase shift (4.1) schematically as an area in radial phase space. Disregarding for the moment the forbidden region (B + C), it is the area bounded between the curves  $k(r)$  and  $k_c(r)$ .

We propose to compute approximately the phase shifts with a similar formula where  $k(r)$  is replaced by the trajectory obtained from the variational principle. Examples of such trajectories were given in figure 2. The function  $k(r)$  is defined implicitly for a given CM energy  $E$  and angular momentum  $L$  by

$$E = \mathcal{H}(k, r, (L + \frac{1}{2}n)^2)$$

where  $\mathcal{H}$  is given by (3.15).

In the computation of the area under  $k(r)$  the forbidden region should be excluded. The second term in (4.1) is not modified. The resulting phase shift is again best characterized as an area in phase space. In figure 3 it is the area A-C.

In figure 4 we show the results of this computation and compare with the RGM results of Okai and Park<sup>11)</sup> with the same two body force. The areas were evaluated by dividing the phase plane in a grid of  $\Delta k = 0.03 \text{ fm}^{-1}$  and  $\Delta r = 0.06 \text{ fm}$  and adding the areas  $\Delta k \Delta r$  that satisfied the condition of belonging to region A (counted positive) or C (counted negative). The accuracy of the procedure was checked in several instances by halving the size of the grid and verifying that the computed phase shifts did not change by more than one degree.

As expected the semiclassical phase shifts show a discontinuity at the energy corresponding to the top of the Coulomb barrier. This discontinuity is caused by the sudden penetration of the trajectory beyond the barrier radius as soon as the energy becomes larger than the barrier height. Quantum effects not included in the simple WKB formula (4.1), namely tunneling and reflection above the barrier, would smooth out this discontinuity when included. Both the magnitude and the position of the jump and the overall behaviour of the phase shifts is very similar to the full quantum calculation. In particular the repulsive effect of antisymmetry (the "Pauli hard core") is accounted for by the negative contribution (area C in figure 3) where the Coulomb trajectory enters the Pauli forbidden zone.

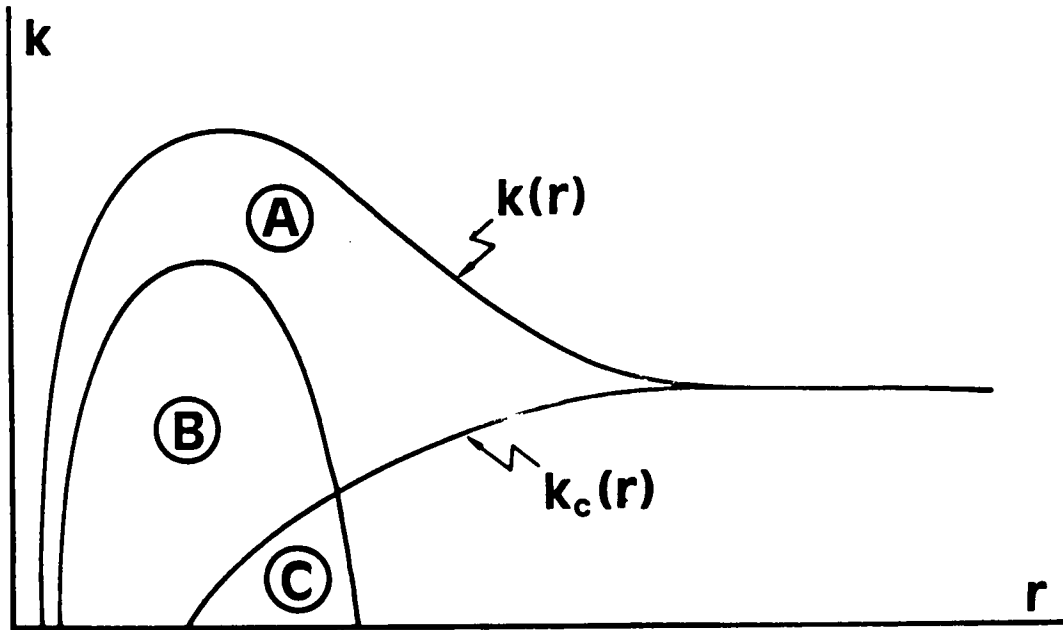


Figure 3:

Graphical computation of the semiclassical phase shift (schematic). The area B+C is forbidden.  $k(r)$  is the trajectory as given in Fig.2.  $k_c(r)$  is the Coulomb trajectory with the same energy and angular momentum. The phase shift is computed as the area A-C.

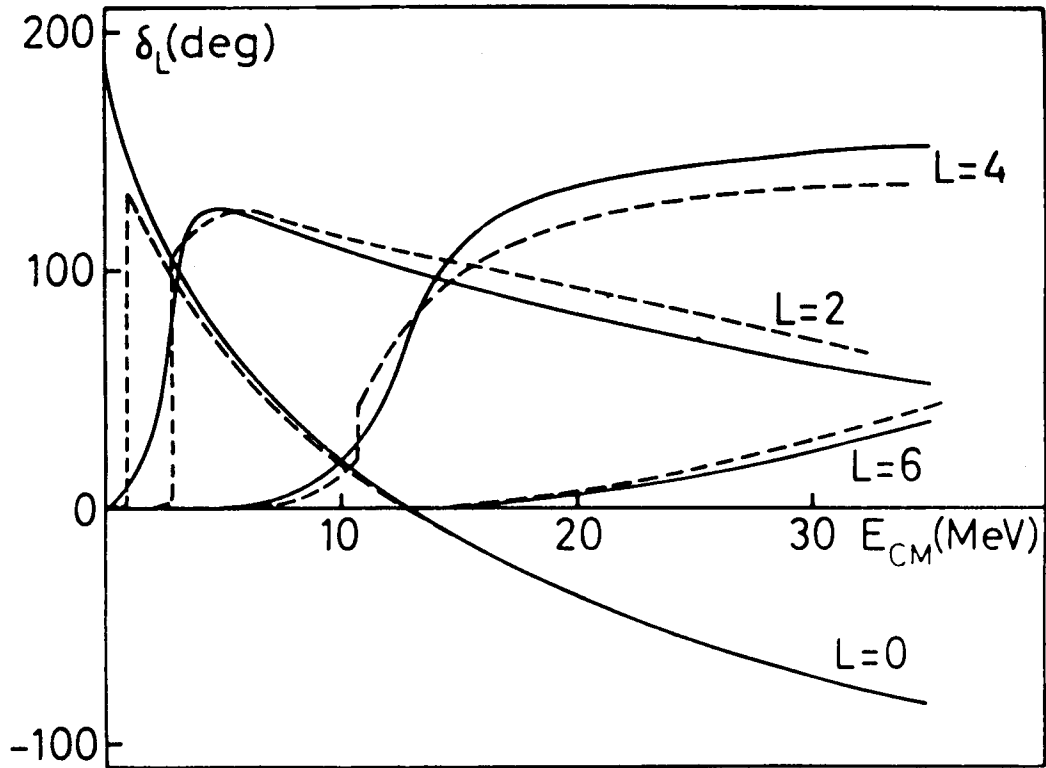


Figure 4:

Nuclear phase shifts for alpha-alpha scattering. Full line is the RGM calculation of Okai-Park<sup>11)</sup>. Dotted lines are the results of this paper.

## 5. Discussion and conclusions

It is not the intention of this paper to attempt a detailed comparison, or even a detailed fit, to experimental data. We have taken the same parameters of an existing calculation<sup>11)</sup> which is fairly realistic and compared with our variational calculation. The main results of this paper are twofold. First we have considered the modification of the structure of the phase space resulting from the requirements of antisymmetrization. The symplectic structure associated with the norm kernel has been seen to contain many of the features that arise in RGM or GCM calculations. In particular it provides a semiclassical description of the states forbidden by the Pauli principle. We found that the phase space is modified by the Pauli distortion factor  $u$  which is directly related to the norm kernel. This extremely simple result can be traced to the fact that the norm kernel only depends on  $\underline{s} \cdot \underline{\bar{s}}$  which in turn is a consequence of the fact that the alpha-particle is an SU(3) scalar. Thus we can state that the method used to transform to canonical variables in section 3 is directly applicable when the composite particles are SU(3) scalars. In calculations involving heavier fragments we expect that our method will provide a fairly accurate description of the effects of antisymmetrization on the folding potential. This can be done either with the full kernels or with a step by step inclusion of the effects of exchange of an increasing number of particles. In fact the decomposition of the overlap kernels according to the number of particles exchanged is well known<sup>5),26)</sup> and it would be possible to study how the symplectic structure changes all the way from a "flat" phase space when no exchange is included to the full effects of antisymmetry.

The second result that we want to stress is the fact that meaningful quantum Information on the scattering can be extracted from

the classical equations derived from the variational principle. This problem of requantizing the variational equations has been approached recently by many authors<sup>12),13),27)</sup> but only in connection with bound state problems in simple models. We have shown here that in a fairly realistic situation these requantization methods can be extended with success to scattering. This discussion is relevant to current TDHF calculations which deal mainly with scattering initial conditions. In fact we can view our approach as a constrained TDHF. This comes about because although we restrict the trial wave function to be a Slater determinant at all times we do not allow it to be the more general determinant. Thus all internal excitations of the fragments are frozen. In the alpha-alpha case this constraint is not very limiting and the effects of partially relaxing it by allowing breathing mode oscillations of the fragments have been analyzed recently<sup>9)</sup>.

It is interesting to notice that the methods used for bound state requantization can also be applied in this case. In fact periodic trajectories can be seen in figure 2 (i.e the small loop at -6 MeV in the  $L = 0$  phase space). However in this case these trajectories are too small to hold a quantum state so that the Bohr-Sommerfeld condition is not satisfied at negative energies. This is a pleasant result because  $^8\text{Be}$  has no bound state and it is strongly dependent on the fact that the forbidden area is not counted when applying the Bohr-Sommerfeld quantization condition. In other systems where bound states exist one can well imagine a uniform description of bound states and phase shifts within the same time dependent variational description.

## Appendix

We collect here the relevant formulas for the case of alpha-alpha scattering. They can be derived using the general methods of <sup>5)</sup>, or, for the simple case at hand also from <sup>18)</sup>.

### a) Trial state.

$$|s\rangle = \mathcal{A} \exp \bar{s} \cdot B^+ |\phi_1 \phi_2\rangle$$

where

$$B^+ = \frac{1}{2} \left\{ \sum_{i=1}^4 a_i^+ - \sum_{i=5}^8 a_i^+ \right\}$$

$$\bar{s} = \frac{1}{\sqrt{2}} (b_{\mu}^{-1} R_{\mu} + i b_{\mu} P_{\mu} / \hbar)$$

We use  $b=1.37$  fm in accordance with ref. <sup>11)</sup>. This value of  $b$  yields a rms radius of 1.45 fm for the alpha-cluster and an oscillator frequency  $\hbar\omega = 20.73$  MeV.

### b) Overlap and symplectic structure.

$$\langle s | s \rangle = \exp \bar{s} \cdot \bar{s} (1 - \exp - \frac{1}{2} \bar{s} \cdot \bar{s})^4 \quad (A.1)$$

The  $g_{ij}$  matrix defined in (2.6) is

$$g_{ij} = \frac{\partial^2}{\partial s_i \partial \bar{s}_j} \ln \mathcal{N} = u \delta_{ij} + \bar{s}_i s_j v$$

where  $u$  is defined in (3.3) and  $v = \partial^2 / \partial (\bar{s} \cdot \bar{s})^2 \ln \mathcal{N}$ .  $g_{ij}$  is the sum of a diagonal and a separable matrix. Its inverse has the same form and is easily found to be

$$(g^{-1})_{jk} = u^{-1} \left[ \delta_{jk} - \frac{v}{u + \bar{s} \cdot \bar{s} v} \bar{s}_j s_k \right] \quad (A.2)$$

Then according to (2.8) the fundamental brackets are

$$\{\bar{s}_j, s_k\} = (g^{-1})_{jk} = u^{-1} \left[ \delta_{jk} - \frac{v}{u + \bar{s} \cdot \bar{s} v} \bar{s}_j s_k \right]$$

$$\{s_j, s_k\} = \{\bar{s}_j, \bar{s}_k\} = 0 \quad (A.3)$$



These brackets enter in the definition of the symplectic structure (2.7). The factors  $u$  and  $v$  are evaluated explicitly from (A.1)

$$u = 1 + \frac{2 \exp - \frac{1}{2} \underline{s} \cdot \bar{\underline{s}}}{1 - \exp - \frac{1}{2} \underline{s} \cdot \bar{\underline{s}}}$$

$$v = \frac{\exp - 1/2 \underline{s} \cdot \bar{\underline{s}}}{(1 - \exp - 1/2 \underline{s} \cdot \bar{\underline{s}})^2}$$

Notice that, as  $\underline{s} \cdot \bar{\underline{s}} \rightarrow \infty$   $u \rightarrow 1$  and  $v \rightarrow 0$ .

c) Two body force:

The two body interaction between nucleons is taken to be a mixture of Rosenfeld + Serber force with a Gaussian radial dependence. The hamiltonian for the relative motion (with the CM energy subtracted) is

$$H = T_{\text{Rel}} + V + V_{\text{coul}}$$

$$T_{\text{Rel}} = \frac{1}{4Am} \sum_{i \neq j} (\underline{p}_i - \underline{p}_j)^2$$

$$V = 1/2 \sum_{i \neq j} v(r_{ij})$$

where

$$v(r) = V_0 (W + MP_x + BP + HP_x P_\sigma) \exp - \beta r^2 \quad (\text{A.4})$$

The parameters chosen are in complete agreement with ref.<sup>11)</sup> and are  $\beta = 0.46 \text{ fm}^{-2}$ ,  $V_0 = -72.98 \text{ MeV}$ ,  $W = 0.3742$ ,  $M = 0.4408$ ,  $B = 0.1159$ ,  $H = 0.0691$ . They correspond to a mixture of 94% Serber + 6% Rosenfeld force.

d) The hamiltonian kernel.

$$\mathcal{H}_{(\underline{s}, \bar{\underline{s}})} = \mathcal{Z} + \mathcal{V} + \mathcal{V}_{\text{Coul}}$$

$$\mathcal{Z}_{(\underline{s}, \bar{\underline{s}})} = \frac{\langle \underline{s} | T_{\text{Rel}} | \underline{s} \rangle}{\langle \underline{s} | \underline{s} \rangle} = \hbar \omega \left[ \frac{21}{4} - \frac{(\underline{s} - \bar{\underline{s}})^2}{4} + \frac{\underline{s} \cdot \bar{\underline{s}} E}{1 - E} \right] \quad (\text{A.5})$$

$$\mathcal{V}_{(\underline{s}, \bar{\underline{s}})} = \frac{\langle \underline{s} | V | \underline{s} \rangle}{\langle \underline{s} | \underline{s} \rangle} = \chi_d \mathcal{V}_d + \chi_e \mathcal{V}_e \quad (\text{A.6})$$

$$X_d = 8W + 4B - 4H - 2M$$

$$X_e = 8M + 4H - 4B - 2W$$

$$V_d = K_0 \left\{ 1 + I(s+\bar{s}) + \frac{[I(s-\bar{s})-I(s+\bar{s})]E^2 - 2[I(s)+I(\bar{s})-1-I(s+\bar{s})]E}{(1-E)^2} \right\}$$

$$V_e = K_0 + K_0 \left\{ 2 + I(s-\bar{s}) + I(s+\bar{s}) - 2[I(s) + I(\bar{s})] \right\} \frac{E}{(1-E)^2}$$

$$K_0 = 2 V_0 \left[ 1 + 2/(b^2 \beta) \right]^{-3/2}$$

$$I(x) = \exp \left[ - \frac{1}{4} \frac{\beta b^2}{1+2\beta b^2} x^2 \right]$$

$$E = \exp \left[ - 1/2 \underline{s} \cdot \underline{\bar{s}} \right]$$

In the final calculations the constant terms in  $V$  and  $\chi$ , yielding the ground state energy of the two separated alphas, were subtracted.

The Coulomb interaction was calculated neglecting antisymmetrization as

$$V_{\text{Coul}} \sim \int \rho(\underline{r}_1 - \underline{R}/2) \frac{e^2}{|\underline{r}_1 - \underline{r}_2|} \rho(\underline{r}_2 + \underline{R}/2) d\underline{r}_1 d\underline{r}_2 \quad (\text{A.7})$$

where  $\rho$  is the density distribution of each fragment. The normalization was determined by comparison to the point Coulomb interaction at large distance. The result is

$$V_{\text{Coul}}(R) = \frac{4e^2}{R} \operatorname{erf} \left( \frac{R}{\sqrt{2} b} \right) \quad (\text{A.8})$$

where  $\operatorname{erf}(x)$  is the error function<sup>20)</sup>

e) The folding limit.

In both  $\mathcal{V}$  and  $\mathcal{Z}$  a leading term can be isolated which dominates asymptotically for  $\underline{s}, \bar{s} \rightarrow \infty$ . Therefore in this limit the hamiltonian becomes

$$\mathcal{H} \rightarrow \mathcal{H}_f = -\frac{1}{4} \hbar \omega (\underline{s} - \bar{s})^2 + X_d K_0 I(\underline{s} + \bar{s})$$

which in terms of the real variables  $\underline{R}, \underline{P}$  takes the standard folding form

$$\mathcal{H}_f = \frac{1}{2m\mu} \underline{P}^2 + \mathcal{V}_f(\underline{R}) \quad (\text{A.9})$$

with

$$\mathcal{V}_f(\underline{R}) = X_d K_0 \exp - \frac{\beta R^2}{1+2b^2\beta}$$

Also from the overlap (A.1) we see that asymptotically the norm is  $N = \exp \underline{s} \cdot \bar{s}$  and therefore by the argument given at the end of section 2.1 the variables  $\underline{R}$  and  $\underline{P}$  form a canonical pair. This shows that in what we have called the folding region (outside of the dotted line in fig.2) the relative cluster coordinates are canonical and the hamiltonian is given by the local folding potential plus the kinetic energy corresponding to the relative mass. As the fragments come closer (in phase space) the energy (both the kinetic and potential parts) and the phase space are modified. Further in a forbidden region is reached which reflects the states of motion that violate the Pauli principle.

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SURPRISAL ANALYSIS OF ENERGY DISPOSAL IN  
HEAVY ION TRANSFER REACTIONS

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Quite a few nucleons can be transferred in a heavy ion collision at energies above the Coulomb barrier. There is also a significant dissipation of initial kinetic energy. One could have therefore expected that the final quantum state distribution in such processes, be statistical. That is, that all accessible quantum states be equally probable. The experimental result is very much to the contrary. While there is definitely dissipation of energy, there is a significant release of kinetic energy. For any exit channel the energy distribution has an asymmetric bell shape with an optimal  $Q$  value ( $Q$  is the difference between the final and initial kinetic energies in the center of mass system). In this lecture I shall outline an analysis of the deviation of the observed energy distribution from that expected in a statistical theory. Further details and examples can be found in two recent papers<sup>1,2)</sup>.

In the statistical limit all final quantum states (allowed by the conservation laws) are equally probable. Since we observe that this is not the case, it is necessary to find a way to assign non uniform weights to the quantum states. On intuitive (but also on theoretical<sup>3)</sup>) grounds we wish such weights to be as uniform as possible, since we do expect that such collisions are predominantly "statistical" in character. Of course, the weights cannot be completely uniform (i.e. equal) since that would be the purely statistical limit. To determine a distribution of quantum states which is "as random as possible" we use the procedure of maximal entropy<sup>4)</sup>.

Entropy is defined as

$$S = - \sum_i P_i \ln P_i \quad [1]$$

where  $P_i$  is the weight (the probability) of the  $i$ 'th quantum state. It should be stressed that equation [1] is correct only when the summation is over quantum states. Say how the only restriction is that the distribution over quantum states is normalized

$$\sum_i P_i = 1 \quad [2]$$

Then that distribution which is of maximal entropy subject to the constraint given by equation [2] can be found by the method of Lagrange multipliers,

$$P_i = \exp(-\lambda_0) \quad [3]$$

Here  $\lambda_0$  is the Lagrange multiplier. For the constraint [2] and its numerical value is to be determined by inserting the form [3] into [2].

Since  $\lambda_0$  is a constant, we have

$$\sum_i \exp(-\lambda_0) = \exp(-\lambda_0) \sum_i 1 = N \exp(-\lambda_0) = 1$$

or [4]

$$P_i = \exp(-\lambda_0) = \frac{1}{N}$$

where  $N$  is the number of quantum states.

It is therefore possible to recover the statistical limit by maximising entropy subject only to the constraint of normalization. Since the early work of Buttle and Goldfarb<sup>5)</sup> we expect however that heavy ion collisions be subjected to another constraint reflecting on "orbit matching". That is, since the ions are heavy and essentially classical in their motion the distance of closest approach for the incoming part of the trajectory should be equal to the distance of closest approach for the outgoing part of the trajectory. There should thus be an "optimal" or most probable final kinetic energy. Rather than most probable we shall use the mean value. Hence, from kinematic considerations we expect the mean  $Q$  value to be constrained. (Refinements of the considerations leading to an optimal

Q value can be found in ref.2). Since

$$E = E_i + Q_{gg} = E^* + E_f \quad [5]$$

where  $E$  is the total energy,  $E^*$  the excitation energy and  $E_f$  the final kinetic energy we can rephrase the constraint as a given mean value of  $E^*$ .

Let us then maximise the entropy subject to two constraints:

(i) normalization, given by equation [2] and (ii) given mean value of  $E^*$

$$\langle E^* \rangle = \sum_i E^*(i) p_i \quad [6]$$

where  $E^*(i)$  is the internal excitation energy of the quantum state  $i$ . There are now two Lagrange multipliers and the result is

$$p_i = \exp(-\lambda_0 - \lambda E^*(i)) \quad [7]$$

Among all the possible distributions which are (i) normalised and (ii) have a given mean excitation energy, distribution [7] has the highest entropy i.e. is as uniform as possible while still being subject to the two constraints. Note that using [2],  $\lambda_0$  can be expressed as a function of  $\lambda$ ,

$$\sum_i \exp(-\lambda_0 - \lambda E^*(i)) = 1$$

or

$$\exp(\lambda_0) = \sum_i \exp(-\lambda E^*(i)) \quad [8]$$

Hence there is only one parameter in equation [7], namely  $\lambda$ , and its value is to be determined from the implicit equation [6]. Using [8], the equation for  $\lambda$  is

$$\langle E^* \rangle = \frac{\sum_i E^*(i) \exp(-\lambda E^*(i))}{\sum_i \exp(-\lambda E^*(i))} \quad [9]$$

Experiments typically do not measure the distribution of quantum states particularly so at the energies of interest here where the quantum states are so dense that they are practically a continuum. Rather, what is measured is the distribution  $P(E^*)$  of energy such that  $P(E^*)\delta E^*$  is the probability of all states with excitation energy in the range  $E^*, E^* + \delta E^*$

$$P(E^*) \delta E^* = \sum_i p_i \quad [10]$$

all  $i$  such that

$$E^* \leq E^*(i) \leq E^* + \delta E^*$$

Figure 1:

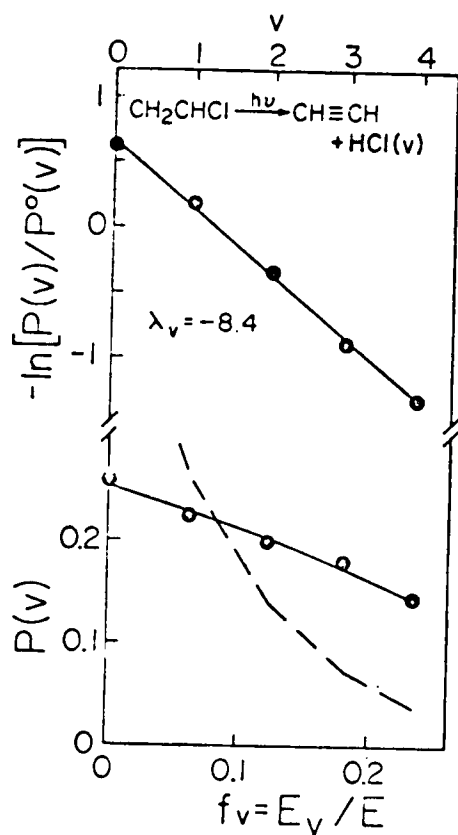
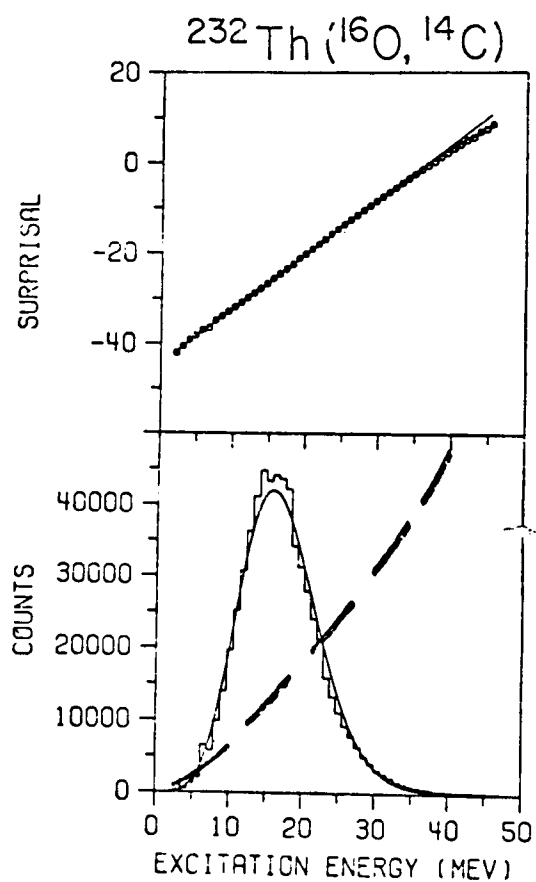


Figure 2



Using [7] for  $P_i$  and since for a small  $E^*, E^*(i)$  is constant,

$$P(E^*) \delta E^* = \exp(-\lambda_0 - \lambda E^*) \sum_i 1$$

all  $i$  such that

$$E^* \leq E^*(i) \leq E^* + \delta E$$

$$\text{or } P(E^*) = \rho(E^*) \exp(-\lambda_0 - \lambda E^*) \quad [11]$$

Here  $\rho(E^*)$  is the density of states

$$\rho(E^*) \delta E^* = N^*(E^* + \delta E^*) - N^*(E^*) \quad [12]$$

where  $N(E^*)$  is the number of states

$$N^*(E^*) = \sum_i 1 \quad [13]$$

all quantum states such that

$$E^*(i) \leq E^*$$

Typically, it is convenient to use a normalised density of states

$$P^0(E^*) = \rho(E^*) / \int_0^E \rho(E^*) dE^* \quad [14]$$

and hence examine the "surprisal" defined by

$$\text{"surprisal"} \equiv -\ln[P(E^*)/P^0(E^*)] \quad [15]$$

Figure 1 shows some actual results for the  $^{232}\text{Th}(^{16}\text{O}, ^{14}\text{C})$  channel at 105 MeV at the grazing angle. It is evident that equation [11] is very well satisfied. Many other examples are discussed in [2]. Other applications have been published, e.g. to fission processes [6]. Here we conclude with some general discussion.

First, if the approach is of general use it should also apply to other collisions where the final state distribution is expected to be statistical but is not. Figure 2 shows a molecular example, the photofission of  $\text{CH}_2\text{CHCl}^{7)}$ . The observed vibrational distribution of the HCl molecule (dots) is quite deviant from that distribution expected if all final quantum states are equally probable (dashed line) but the deviance,  $-[P(v)/P^0(v)]$  has a rather simple dependence when plotted vs. the vibrational energy  $E_v$ . ( $E$  is the total energy and  $P^0(v)$  is the normalised number of quantum states which correspond to the same vibrational quantum number of HCl).

Next, how about direct reactions. Figure 3<sup>8)</sup> shows the results for a very direct processes. The deviation from statistics is extreme, but it still has a simple structure. Reference 3 argues that this is not an accident but that our working hypothesis 'the distribution of final states is as random as it can be, subject to constraints' can be shown equivalent to quantum collision theory and that the constraints can be identified in terms of (i) the Hamiltonian of the system and (ii) the initial state. Further discussion and examples can also be found in the review<sup>9)</sup>.

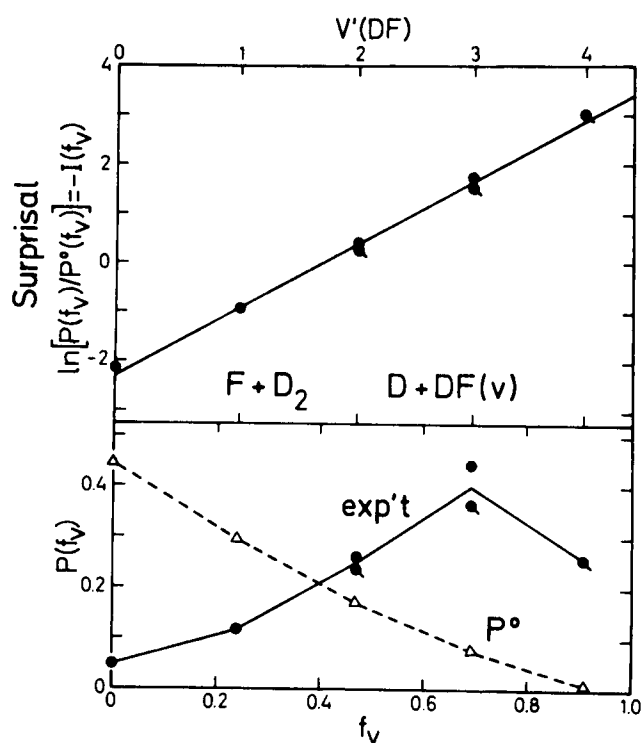


Figure 3

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Concluding Remarks

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Before leaving Buenos Aires I just collected the Proceedings of our first workshop in order to get inspiration in the concluding remarks there, which were delivered by a sensible fellow<sup>1)</sup>. Among these conclusions I found a diagram about the future time table of the Tandem Project which predicted the acceptance tests for the first half of 1982. We must remember that at that time not even the location of the project was decided. I would like to pay my respects to Emma and coworkers for the fact that, in spite of the many difficulties along the way, an important fraction of the machine is assembled inside the tank, the gas system is finished and the concrete building is almost ready. (It is also true that a small fraction of the concrete lies in the form of dusts inside the aluminum coins, which is not so good for the electrostatic properties of the machine, according to Dr. Ball).

The two first workshops were the result of a concentrated effort by the local community of physicists (assisted by selected friends from Brazil and Chile) in order to become acquainted with i) the problems that could be attacked with the tandem and ii) the details of the equipment (including manpower and costs) that was used at that time in other laboratories to investigate these problems. The main question that these two workshops tried to answer was what to do when the beam would be out. This problem has not been discussed again in the successive workshops. It is my opinion that this activity should be increased by an order of magnitude in our Department.

The third and fourth workshops had a different character. The main part of these workshops consisted of lectures delivered by three and two foreign visitors, respectively, during two weeks. An important by-products were the fact that computer codes brought by some visitors were implemented during the workshops and that a few of our younger colleagues found the possibility of excellent training opportunities in the visitor's laboratories.

There has been another change of character for the fifth workshop. It looks more as a small conference or symposium. In lieu of lectures, we have enjoyed the presentation of several review talks on different branches of many-body experimental and theoretical physics: the theory of high angular momentum states in nuclei (Prof. Mang); a summary of the first experiments performed in the similar machine at Oak Ridge (Dr. Ball); the possibilities associated with postacceleration techniques (Prof. Sala); the state of the art on phase transition in solids (Dr. Axe) and current developments on application of the theory of information to heavy ion reactions (Prof. Levine) and of methods of quantification the TDHF solutions (Dr. Saraceno).

A considerable amount of time was also devoted to the presentation of works performed in different institutions of Argentina, Brazil and Chile. They covered practically all of nuclear physics. They were presented in order of decreasing energies: fusion and fission, peripheral reactions, optical potentials, giant resonances, rotational and gamma-like structure of nuclei, life time measurements. There has also been a technical contribution on data acquisition.

Because of this wide range of interests, I do neither think it useful (nor I am able to do it) to comment on each individual presentation. I will rather try to make some general remarks:

1) The presentation of individual contributions in any large or small conference has the usual drawback that it doesn't necessarily lead to the cooperative endeavour implied by a workshop. This danger reminds me of the fact that several years ago the U.S. Post Office included the Physical Review in the category of "paid advertising mail". According to Prof. Lee Grodzins (who told me this anecdote) physicists were outraged until they realized that that was exactly what it was.

2) The amount of work presented constitutes solid evidence of the existence of an almost critical mass of nuclear physics in this region of the world. This appears to be more so since the presentation was far from complete: among the CNEA contributions only one dealt through only five minutes with experimental research. In view of my previous remark about self-advertising I am not sure whether to blame or praise my experimental colleagues by their neglect of the pressing requirements of our chairmen.

3) A promising fact is the appearance of a contribution signed by Chilean and Argentine coauthors. Let me remind you also about the existing Mexico - Sao Paulo - La Plata collaboration on the cluster structure of nuclear matter. Perhaps the most useful part of the workshop took place outside the lecture room. There have been many discussions on different projects between the participants. Should a sizeable fraction of these projects be accomplished, the workshop would have been an important success. The selection of a place outside Buenos Aires, in which the participants may spend together their leisure hours would also be completely justified.

Other most important aspect of the same subject will be dealt with in this afternoon's session on the policy of external users in the Tandem Project.

I like to express my regret because of the absence of our two British invited speakers. I owe my opportunity to work in physics to Niels Bohr's program of bringing together scientists from conflicting countries. Therefore I would like to transmit to Drs. Satchler and Gelletly the fact that we missed them as we missed Drs. Bayman and Bertrand. Although we understand their decision, I am relieved that the step to cancel their visit was not taken by us.

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## A DESCRIPTION OF THE TANDAR FACILITIES

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In December 1979 two significant events were celebrated at the Atomic Energy Commission: 25 years of continuous and fruitful operation of the 28 MeV Synchrocyclotron and the beginning of the construction of the building to host a new 20 MV Tandem Accelerator.

These two outstanding events occurring at the same time are a sign of the attention that the AEC has always paid to basic research. This attitude has proved to be particularly fruitful in developing our own capability in the fields of applied peaceful nuclear technology.

More than two years have already passed since that date and, as most of you have seen during your visit to the site last Monday, we are about to finish both the buildings and electromechanical works at least in those aspects that are relevant to the operation of the Accelerator. Provided God and NEC help us, we should have the facility operational some time late in this year or perhaps early next year.

Facing that possibility, our anxieties are being transferred from controlling our contractors, supervising technical drawings, and verifying cash-flows, into the way to secure the best possible utilization of the facility. This has been a point of real concern from the very beginning of the Project. This has been shown by the ambitious program for training young nuclear physicists (started in 1977) as well as by the series of meetings of the type of this one that we have held every year since 1978.

We are convinced that the best way of obtaining an optimum utilization of the facility is to make it available not only to other groups of CNEA but also to external users of our country and from abroad. That is why we want to take profit from this opportunity and the presence here of some of such potential users to express our intention regarding the character we wish to impose to the new Tandem laboratory.

Accordingly, the aim of my talk is to sketch the main features of the accelerator, its expected performance and the auxiliary equipment which will be available. Shortly, I shall recall those aspects which are interesting for the user. Anyhow, a more detailed description of the accelerator has been already presented elsewhere.

Regarding the machine I shall only mention that it is a 20 MV straight through Pelletron, built by NEC to fulfill CNEA specifications.

## 2. Ion source

Three interchangeable modules have been ordered for the ion source to provide most types of ions. One is a duoplasmatron ion source from which one can expect to get tenths of nAmps of  $H^-$ ,  $O^-$ ,  $Cl^-$ ,  $Br^-$  and  $I^-$ . It appears to be directly usable for all gases which form stable negative ions. This source is arranged to produce the high DC currents (50 to 100 nAmps) of  $H^-$  needed for the pulsed beam mode of operation.

The second is a negative Helium Ion Source of the charge exchange type.

Finally, the third is a sputter-cone type source which is very useful for producing beams of heavy metallic negative ions.

The three ion sources can quickly (several hours) be interchanged and made operational inside the injector system. The duoplasmatron ion source is now being successfully tested.

### 3. Pulsing system

Gas and foil stripping systems are contained in the interior of the terminal. The foil stripper consists of two modules each of which holds about 200 foils 9.5 mm in diameter. A second foil stripper with about 200 foils, 1.9 mm in diameter, is located in the first dead section of the high energy column.

### 5. The analyzer magnet

The analyzer magnet is a 90° double focusing unit with a mass-energy product of 500 supported on a rotatable base so as to be able to place the beam in the direction of any of the experimental lines. In addition to the 90° exit port, there are ports at 0° and a 15°. This latter port has been designed to mass analyze heavy molecular beams, which can not be bent through 90°.

### 6. Experimental area

We arrive now to the experimental area, whose design was considered of utmost importance. The experimental area occupies approximately a 270° sector consisting of two all purpose shielded rectangular rooms of about 22 x 23 meters, covering a 180° sector. There is also a heavy shielded area for high radiation experiments, which was planned to accommodate part of the on line facility now being used at the IALE Project. There are also two unshielded experimental rooms for off-line use. Finally, there is an experimental room in the basement directly below the analyzer magnet, as I mentioned before, to be devoted to studies with heavy molecular beams.

### 7. Experimental lines

For the time being, we are planning to have in operation 5 outgoing beam lines for experiments in nuclear physics and one assigned to other

fields of research or applications which will eventually require special equipment. For the six lines the standard equipment such as Faraday cups, steerings, vacuum systems, beam monitors, etc., is already available.

Let me now refer in particular to the different experimental lines for nuclear physics.

- Isotope Separator on line facility. A home-built isotope separator being used since 1970 to mass analyze fission products from thermal neutron induced fission of uranium will be upgraded and moved to the new laboratory. Recently the possibilities of the system have been broadened through the design and construction of a new non gaseous ion source. A further improvement will be possible at the TANDAR laboratory with the use of heavy ion projectiles.

- Gamma-ray spectroscopy. A general purpose gamma-line will be implemented, where the present studies of ( $\gamma$ -xn) reactions being performed at the Synchrocyclotron shall be continued through HI-xn reactions. A versatile set of reaction chambers for gamma-gamma coincidence measurements, for angular distributions and for timing using the pulsing system are going to be installed. Also a special chambers for on line measurement of conversion electrons will be built.

- Heavy Ion Scattering. The Tandem will be devoted to a large extent, to heavy ion studies. Various proposals for a heavy ion scattering chamber were considered and finally it was decided to build a reaction chamber based on the design used at the Weitzman Institute. It consists of a cylindrical body with a sliding steel foil to allow rotation of the chamber and two independent rotatable arms for angular distribution measurements. A number of interesting features have been incorporated to the original design such as the possibility of using the kinematical method or a combination of binary angular distribution with the times of flight of the fragments. Some developments on heavy ion detection are being also carried out.

- Light beam line. A general purpose high quality scattering chamber has been ordered from General Ionex to be used for light ion induced reactions. The chamber has already arrived and begins to be mounted and tested.

- Magnetic spectrometer. The fifth line will be equiped with a magnetic spectrometer. Considerable discussion on the type of spectrometer has preceded a final decision which favoured the split-pole type in stead of a Q-3D device. Of course considerations regarding the investment involved were not absent from this decision. Further discussions with Scanditronix showed the advantages in quality and cost of a new recent design by Enje known as "orange sector spectrometer".

#### 8. Data acquisition system

It has been planned so as to provide both a maximum of flexibility in the hardware implementation of the different required functions and a simplification of the software development.

For the conventional computing components we have purchased a DEC (VAX 11/780) as the main processor with a PDP 11/34 as front end processor. This will provide the link between the system and the display processor, a Versatex printer/plotter and the CAMAC subsystem which will handle the actual interaction with the experiments.

The CAMAC compatible experimental controller interface is presently under construction.

#### 9. Other facilities

Several support laboratories are being installed in the accelerator building, namely, for ion source maintainance and (future) developments, and for the production of detectors, targets and strippers.

A library and a well equiped workshop complement the support facilities.

## 10. Staff

Fourteen engineers, and thirty technicians integrate the Technical Assistance Division of the Physics Department, which is ready to take care of the accelerator operation and maintenance and of all the related technical jobs, including assistance to the experimental physicists in the developments for the different experimental lines.

## 11. Performance

The expected performance for the 20 MV Pelletron is summarized in the following table.

|                                                       |                                                                                                                   |
|-------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| Voltage rating                                        | 5.0 - 20.0 Megavolts                                                                                              |
| Voltage stability                                     | +2 kV                                                                                                             |
| Proton Energy Range                                   | 10.0 MeV - 40.0 MeV                                                                                               |
| DC Proton Current                                     |                                                                                                                   |
| - Design Capability                                   | 20 nAmp at 10.0 MeV to 40.0 MeV                                                                                   |
| - Acceptance Test Values                              | 3 nAmp at 10.0 MeV to 40.0 MeV, 75% transmission**.                                                               |
| DC Alpha Particle Current<br>(Acceptance Test Values) | 1.0 nAmps at 60.0 MeV                                                                                             |
| DC Sulphur Current*<br>(Acceptance Test Values)       | 150 nAmps at 10.0 MV terminal voltage, 15% transmission**<br>200 nAmps at MV terminal voltage, 15% transmission** |
| DC Iodine Current*<br>(Acceptance Test Values)        | 50 nAmps at 10.0 MV terminal voltage<br>100 nAmps at 20.0 MV terminal voltage                                     |
| DC Gold Current*<br>(Acceptance Test Values)          | 50 nAmps at 20.0 MV terminal voltage                                                                              |
| Pulsed Protons<br>(Acceptance Test Values)            |                                                                                                                   |
| 1.0 nanosecond FWHM                                   | . 3 milliamps 12 MeV                                                                                              |
| 4.0 MHZ frequency                                     | . 5 milliamps 26 MeV<br>. 8 milliamps 40 MeV                                                                      |
| Pulsed Iodine*<br>(Acceptance Test Values)            |                                                                                                                   |
| 3.0 nanosecond FWHM                                   | 1.0 nAmps at 6 MV terminal voltage                                                                                |
| 4.0 MHZ frequency                                     | 10.0 nAmps at 13 MV terminal voltage<br>10.0 nAmps at 20 MV terminal voltage                                      |

Charge Selector Efficiency

For the gold beam at 20 MV, the amount of beam in the most probable charge state plus one charge will be less than or equal to 0.8 pnAmps and the amount of beam in the most probable charge state minus one charge will be less than or equal to 1.0 pnAmps.

\* particle micro or nanoamperes of ions of the most probable charge state.

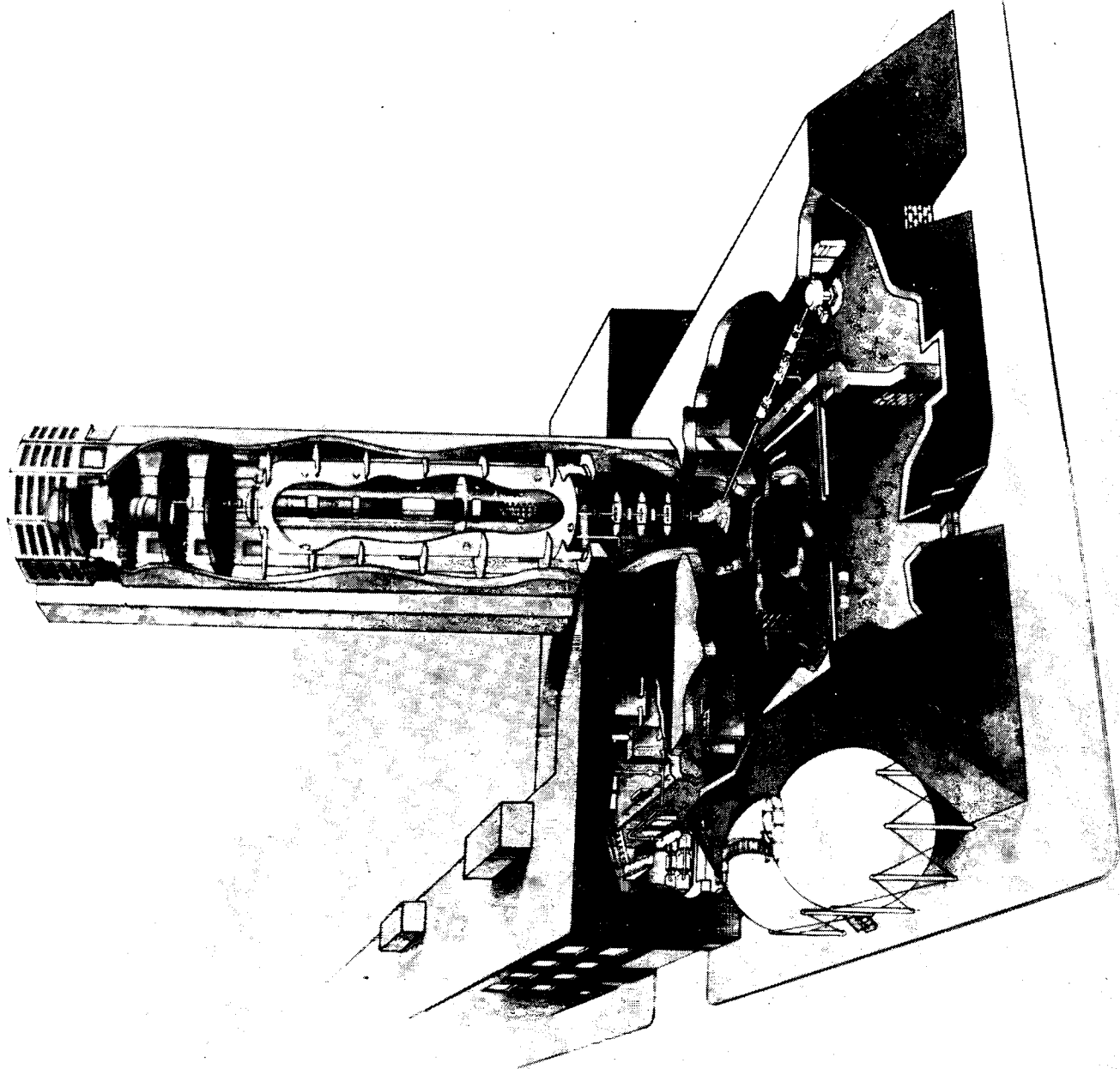
\*\* % transmission defined as particle current of most probable charge state measured after analyzer magnet image slit divided by particle current measured at input Faraday cup multiplied by 100.

12. Future upgrading

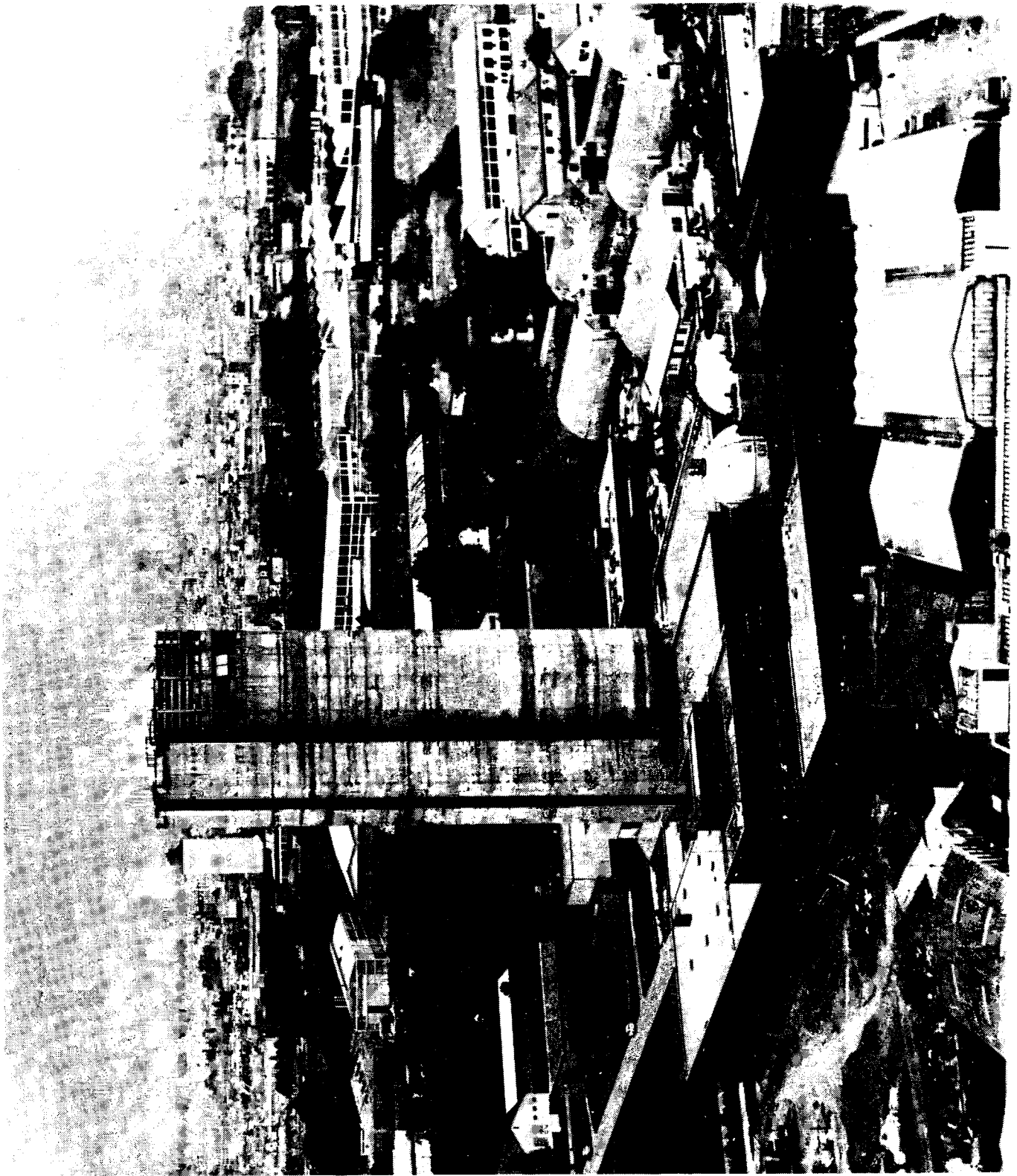
Some precautions were adopted in order to be ready for future improvements at a minimum cost. One, is the possibility of changing the NEC tubes into HVEC tubes by adequate dimensioning the inner diameter of castings. The second is to have the tank designed for higher SF<sub>6</sub> pressure than required, so as to be able to increase the terminal voltage in the case of future improvements of the tubes (this provided that they could admit higher voltage gradients). Finally we left plenty of room behind the experimental area in order to be able to adapt the building for the eventual installation of a post-accelerator facility.

When speaking about the future one should also mention that the building now in construction is what we call "Sector A" of a much larger building whose "Sector B" will locate all the activities of the Physics Department together with the Radiobiology Department. In a second stage of the TANDAR Project will be constructed within the same campus, the main building and special laboratories for all the basic research being done at the Atomic Energy Commission together with the main library, accomodation facilities, auditorium, etc.

Mariscotti will now tell you about our preliminary ideas about how all these facilities could be made available for external users.









SOME IDEAS ON EXTERNAL USERS

M.A.J.Mariscotti

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I would like to start with two preliminary remarks to help dispel the anxiety that you may experience as you prepare to listen to this talk. First, the sketch of the program suggests that my talk should go on and on until is time for the closing dinner. This is scheduled to take place at 21.00 hours, so that we have 5.00 hours from now. And not even a coffee break! Be your mind at rest. I'll do my best to be done with the job in the shortest time, probably no more than 20 minutes.

The second source of anxiety was pointed out to me by a friend while having dinner together the other night. After a few glasses of wine he asked me: "What type of external user do you have in mind?". I was startled. -He said: "I am referring to your announced talk. According to the tittle you are going to tell us some ideas on external users". I admit it took me a while, but then I realized that he was right. To all of you who may have been wondering but didn't dare to ask, I apologize. The tittle of this talk should be: "Some ideas on external users policy". It was not our intention to discuss the external users themselves such as preferred hair color, shoe size, etc.

The real purpose of this talk is to induce a discussion or rather, an exchange of ideas, on the subject. To do this I should like first to share with you the ideas we have been discussing ourselves. If I am not able to elicit any comments from you I will feel that I have failed. Even then my introduction would have served the purpose of informing you that we would like you to come and use the machine and also participate with comments, and suggestions at any time.

The basic philosophy is that the key to a productive scientific activity is exchange of ideas and exchange of information, hence the implementation of a policy that welcomes physicists from other laboratories will work to our benefit. Something about this we already said last year to some of you who attended the previous workshop.

There are several ways in which we can pursue this policy.

One of course is getting money to support a visitors program. This we shall try to do but will naturally be limited and hence will involve not many people and only specific projects.

Another is to arrange special bilateral agreements between institutions. These sometimes work, and sometimes get entangled in the bureaucracy. We have had some good experiences in this regard such as the cooperative program that is under way with Brookhaven National Laboratory supported by the CONICET and the National Science Foundation.

A third one may be achieved by means of the proposed International Center for Nuclear Physics which is going around and we have submitted to the International Atomic Energy Agency. The idea is to offer the TANDAR installation to set a Center tailored after the Center in Trieste. I think that there were a few copies of this proposal at hand here for you to see. I should say briefly that at present the response of the IAEA has been positive except for the funding which the Agency is reluctant to provide. In this regard, I should point out that we have been told unofficially by people from the Agency that to succeed one should secure as much support as possible from other countries. Therefore I take this opportunity to ask for your support and consideration as to how this could be done.

Finally, we have discussed, as I said, our external users program. In doing this we have taken into account the experience in other laboratories. Our ideas in this respect are the following:

First we considered at length if it was convenient to distinguish between internal and external users. We came to the conclusion that it was

best to do so, namely, to allocate beam time to internal and external users separately. Some laboratories do this, some others don't. An argument in favor of this is that it is to the benefit not only of the internal users but also of the external users, because in this way one makes sure that there will be people taking care of the equipment at each beam line, at least those set up locally.

The time allocation for external users would be distributed by an advisory committee to which proposals should be submitted. For practical reasons this committee would be formed mostly with local people. This committee would be assisted by a group of international advisers to whose the committee may refer to ask for their opinion in judging the academic value of the proposals. One should bear in mind that we expect to have proposals which belong to fields other than nuclear physics.

Consideration has also been given to the time distribution between internal and external users. It is actually too early for establishing a percentage of beam time for each group. Some of us have thought loosely of 50% each, some others 70% and 30%. It seems more reasonable to wait until the response from the potential external users makes it possible to obtain a fair estimate.

Periodically there would be a call for proposals (every three months? -perhaps more- experience should dictate the appropriate frequency to send out these calls). They would be sent to anybody who has expressed the wish to be included in the respective mailing list (for the moment and until the advisory committee is integrated anyone who so wishes can write to the Department).

Projects may involve internal and external users together. In such cases one can arrange to split the time between "internal" and "external" beam time. In general, it turns out to be convenient that an experiment be carried out in collaboration with local people but this of course will not be imposed on anybody.

External users may be encouraged to set their own committee to have a voice in recommending improvements in the facility to the Dept. Chairman.

External users will be entitled to use the instruments, electronics, data acquisition system and all conventional equipment in the laboratory.

In principle there would not be funds to pay for travel or per diem for external users.

This brief account gives you an idea of what we have in mind with respect to external users policy. I now would like to hear your comments.

Since this is the last talk of this Symposium I should now like to take a few minutes to acknowledge several people, if the chairman permits me.

In the first place I wish to thank, in behalf of all the members of the Physics Department, the presence here of our visitors who kindly accepted our invitation to come and who have contributed so much to this workshop by sharing their knowledge and experiences with us. Since we are a family of friends I do not need to be very formal and so I will say that they deserve a special thanks for having come in spite of the difficult circumstances which are taking place in our country at this time. We cannot dismiss the fact that more than one of you must have pondered seriously the convenience of staying home before coming. Thanks again for your friendly decision to come.

Now I would like to mention other group of people who have been mostly behind the "curtains", so to say, working very hard to make this workshop possible. I refer to the S.W.A.T Group for such is the name they have acquired after several heroic achievements. I don't really know what S.W.A.T means but most of you may be familiar with the popular TV series starring a group of very special guys who get where the action is always at the right time to save mankind from disaster. A member of this group is the fellow who has been driving people and goods for most of the time in the last two weeks: He is Horacio Soler.

Then there was a guy who was here over Easter and all this week making sure that we would get soap in our bathrooms, the plumber would be at hand for emergencies, took care of making this white board for us, made sure that the projectors would work and many other things (he had trouble to convince the man in charge that in order to have hot water at 7 a.m. it was necessary to switch on the furnace at an earlier time!): This is Jorge Vidallé.

Another member of S.W.A.T, provided us with newspapers, color TV, special tour guides (at one time I saw him directing the traffic in downtown Mar del Plata to make way for our buses). He is Libertario Rodríguez.

The head of S.W.A.T has been busy for several weeks now. He came to this place to arrange everything. I know he even had an encounter with the administrator here who said to him: "may be we won't be able to do this" -to what he responded firmly: "Sir, this shall be done" and so it was. This is Jorge Prieto.

Now I come to the girls which should never be absent in a S.W.A.T. type adventure as this surely one. One of them was the girl you saw coming forth and back with your plane tickets. You gave her your tickets (surely with some apprehension as to whether you would see them again) and there they came back with the right reservation (at least I hope it was so). She is Liliana González.

The other girl you know better. She was able to get many things for our comfort. Not to go into details, I will mention just one example. I was told that somehow she was able to convince a fellow from IBM that there wasn't any better deal for him than letting us have that big animal called "panther" that is sitting in the entrance hall (the computer that types) for nothing. I've been trying to imagine what could have seduced him so effectively into doing this. We are now thinking on how to best use her powers to get a Boeing 747 for the big Conference in 1985. She is María Teresa Carmuega.

Finally, we want to thank the two fellows who have been organizing this meeting all along and by that I mean many months. They are the ones who get all the blame and very little recognition. I know they are tired and will enjoy Sunday night going to sleep with the feeling that this is over, or is it? Well, we now have the opportunity to tell Roberto Perazzo and Olga Dragún that we thank them very much for having provided for a magnificent week of scientific and social exchange. Thank you all.



## Excitation of Giant Resonances Via Direct Reactions

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### I. Introduction

Giant resonances are manifestations of elementary modes of nuclear excitation. Since giant resonances are general properties of all nuclei, they lend themselves to description as general properties of nuclear matter. The term giant resonance has over the years come to be used nearly synonymously with the giant dipole resonance (GDR) discovered in the 1940's. However, as we know from theoretical considerations and now experimental observation, such a correspondence is largely a product of history.

In 1970 the first non-dipole giant resonance, the giant quadrupole resonance (GQR), was experimentally observed.<sup>1</sup> It is significant to note that the observation of this new giant resonance was not made by photo-nuclear reactions, the conventional means of exciting the GDR, but by direct reactions -- specifically, inelastic scattering of medium energy protons and electrons. Thus, new resonances and new techniques were simultaneously thrust upon the well established field of photonuclear physics. In the ten years since the discovery of the GQR the study of

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\*Operated by Union Carbide Corporation under contract W-7405-eng-26 with the U.S. Department of Energy.

so-called giant multipole resonances has grown into a bonafide subfield of nuclear physics, one that is pursued in most medium-energy nuclear physics facilities throughout the world.

In this talk I will review the current situation as regards observation of the new resonances. Since the bulk of the results have come from hadron measurements, I will concentrate on results from hadronic direct reactions. This selection is prompted by constraints of time and should not be interpreted as a negative opinion of results from electron scattering or particle capture, areas of study which have provided significant and important contributions to the new resonance field. More detailed information on this topic may be found in references 1-3.

Although this talk concentrates on non-dipole resonances, it is helpful to remind ourselves of the properties of what was for so many years the only giant resonance established experimentally. Figure 1 shows the spectrum of  $^{208}\text{Pb}$  as seen in the  $(\gamma, n)$  reaction.<sup>4</sup> The only structure observed in the spectrum is the peak from the GDR centered at about 13.5 MeV of excitation energy. The fact that photoabsorption proceeds overwhelmingly by dipole absorption leads to such beautiful GDR spectra which are uncomplicated by competing reactions. (Excitation of the GDR is 10-100 times stronger than E2 excitation via photoabsorption. This selectivity makes photoabsorption measurements of higher multipole resonances very difficult.) As shall be seen later such clean spectra are unfortunately not the case for excitation of giant resonances via direct reactions.

Consideration of the GDR systematics established over many years can provide a guide for the search for resonances related to other modes of

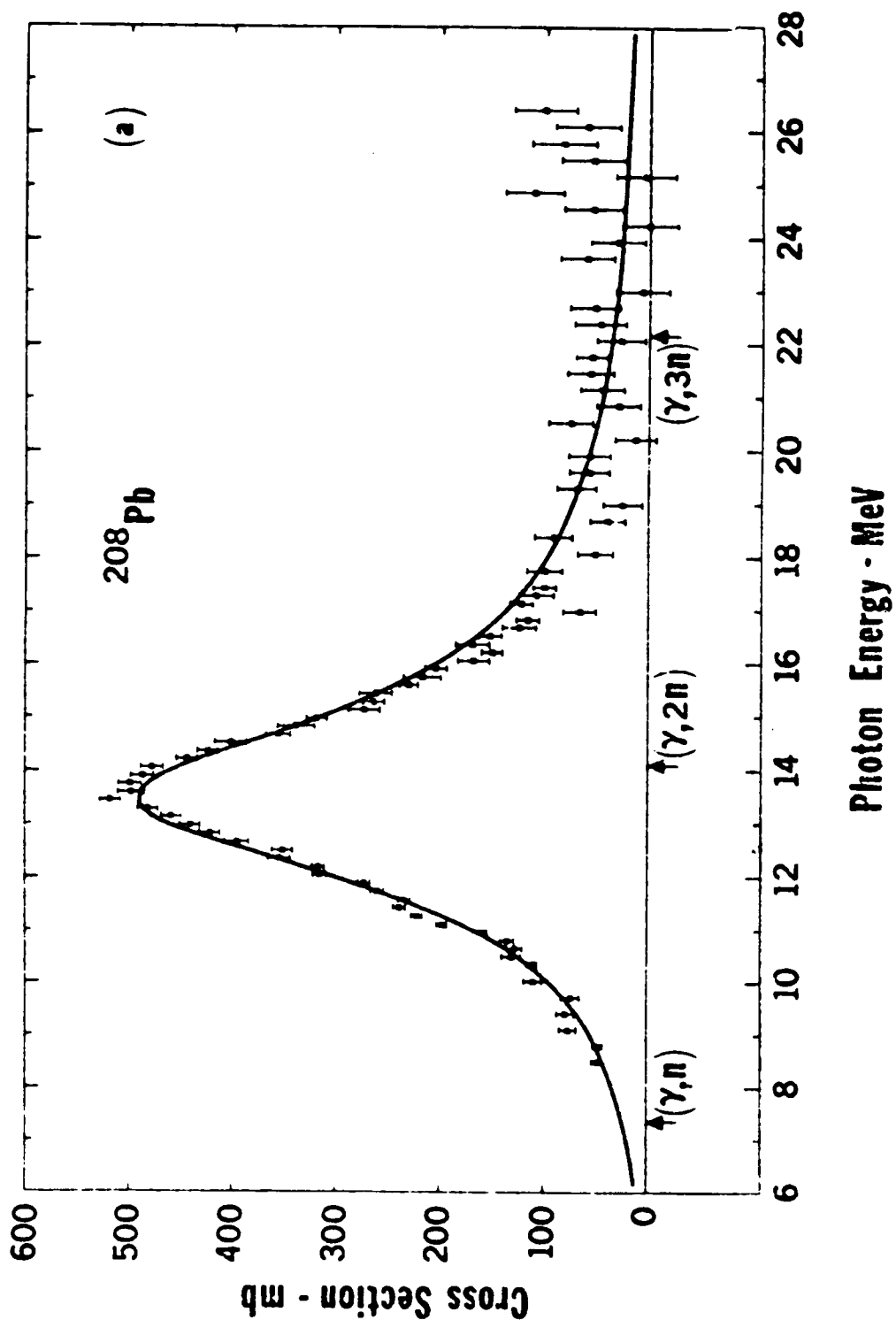


Fig. 1. Giant dipole resonance in  $^{208}\text{Pb}$  as observed in the  $(\gamma, n)$  reaction (ref. 4).

nuclear excitation. Shown on fig. 2 are values<sup>4</sup> of the excitation energy ( $E_X$ ) plotted as  $E_X A^{1/3}$  MeV, the width and sum rule depletion for the GDR for many nuclei spanning the periodic table. For nuclei with mass above  $\sim 130$ , the GDR is located at the systematic energy of  $\sim 78 A^{-1/3}$  MeV and most of the GDR sum rule is accounted for. For lighter nuclei the resonance energy falls steadily from the systematic value and less than 100% of the sum rule is accounted for (at least up to  $\sim 30$  MeV of excitation). The width of the GDR is narrowest near shell closures ( $A=90, 144, 208$ ) and widest for deformed nuclei (e.g., rare earth nuclei).

These systematics yield a few characteristics of giant resonances that may be useful for experimental searches for new resonances.

- 1) Giant resonances are general properties of nuclei.
- 2) The excitation energy of a giant resonance varies smoothly with nuclear mass (at least over most of the nuclear mass range).
- 3) Giant resonances exhaust an appreciable fraction of an appropriate sum rule.
- 4) Giant resonance strength is generally localized in excitation energy (more an experimental than theoretical necessity).

## II. Background

Giant resonances are often considered to be highly collective modes of nuclear excitation in which an appreciable fraction of the nucleons of a nucleus move together. Indeed the motion is so collective that it is appropriate to think of these modes of excitation in hydrodynamic terms like the oscillation of a liquid drop.

Figure 3 shows a representation of several of these modes of oscillation, where  $L$  represents the angular momentum quantum number of

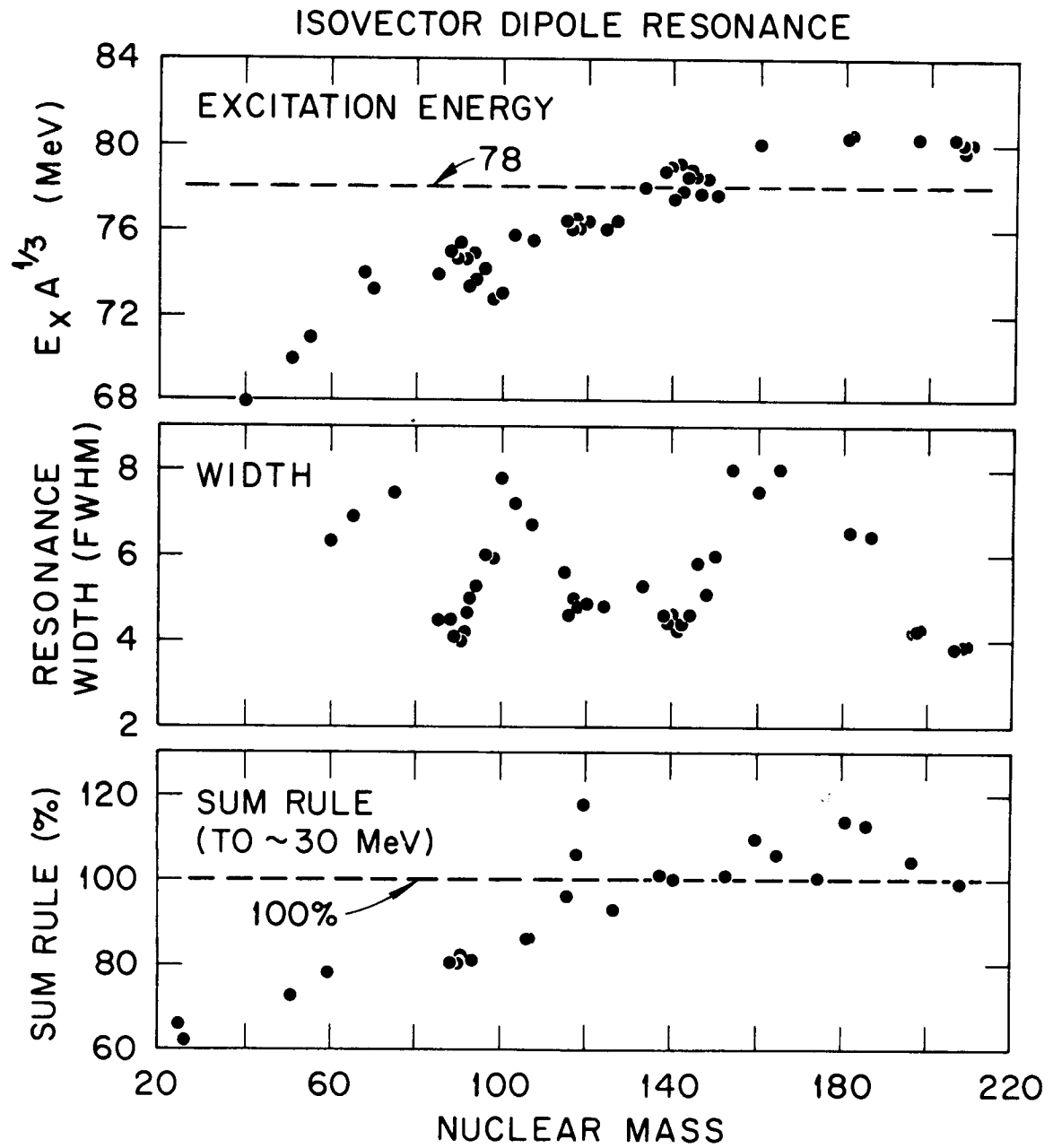


Fig. 2. Systematics of the isovector giant dipole resonance excitation energy, width and sum rule depletion (ref. 4).

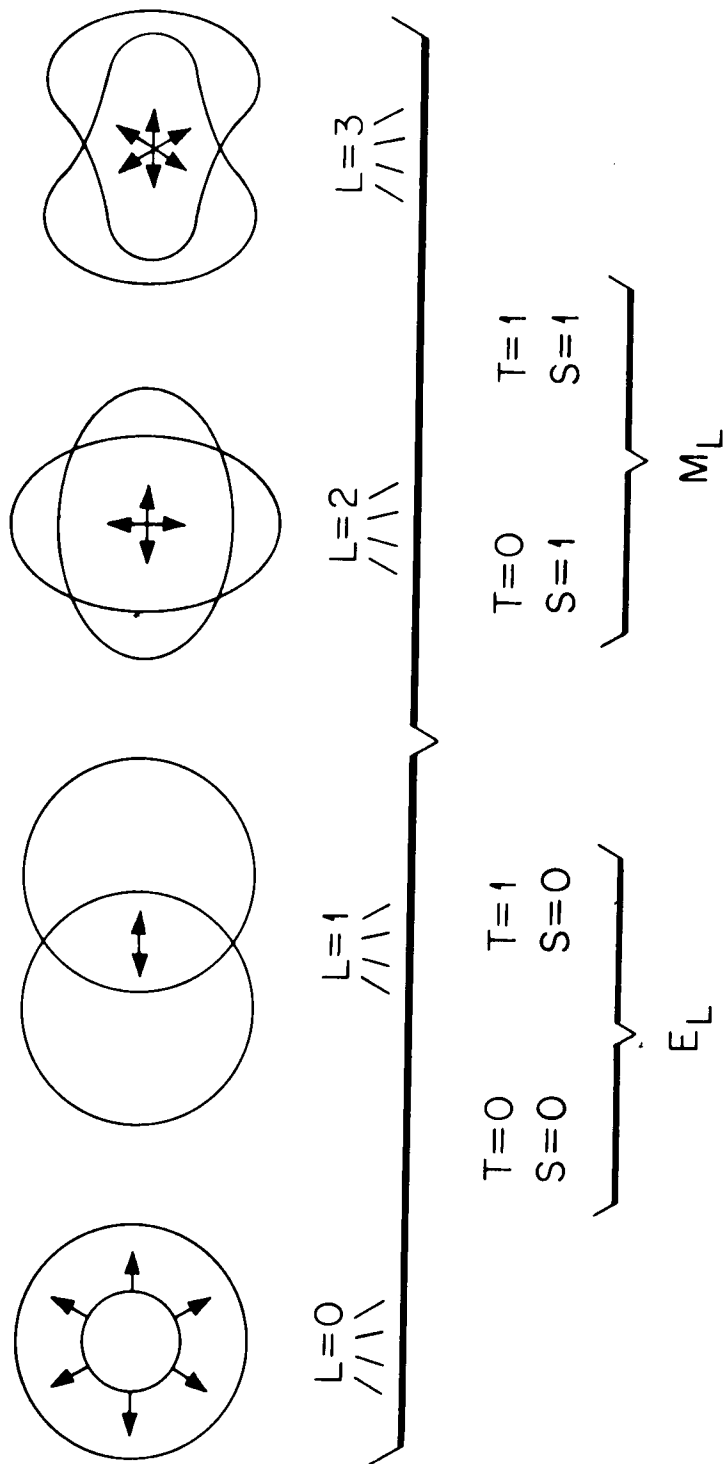


Fig. 3. Modes of oscillation of a nucleus.

the mode. The monopole ( $L=0$ ) mode is a spherically symmetric oscillation or compression of the nucleus; the dipole ( $L=1$ ) is pictured as a motion in which the neutrons and protons oscillate in bulk against each other while the quadrupole mode is an oscillation of the spherical nucleus to oblate shape then to prolate shape. Oscillations with  $L > 3$  are, of course, possible but are not shown. The nuclear fluid has neutron, proton, "spin-up" and "spin-down" components and hence, for each multipolarity ( $L$ ) there are four possible combinations of these components. Modes in which neutrons and protons oscillate in phase are characterized as isoscalar modes (denoted as  $T=0$  here) while those modes in which the neutrons and protons oscillate out of phase are called isovector ( $T=1$ ). Similarly, spin-up and spin-down nucleons oscillating in phase yield  $S=0$  modes while the so-called spin-flip modes ( $S=1$ ) are produced by spin-up and spin-down nucleons oscillating out of phase. The  $S=0$  oscillations are the electric modes while the  $S=1$  oscillations are the magnetic modes. The well known GDR is an example of an  $L=1$ ,  $T=1$ ,  $S=0$  mode of excitation, while the first  $2^+$  level of nuclei are examples of  $L=2$ ,  $T=0$ ,  $S=0$  modes.

The collectivity of these modes (i.e., of the nuclear states that are the observable manifestation of the various modes) can be deduced by studying the transition rates for their electromagnetic excitation (or de-excitation) or by measuring the cross sections for their excitation via direct reactions such as inelastic scattering. Useful "benchmarks" for comparison are the single particle transition rates and sum rules. While the former provide an estimate for a single nucleon promoted from

one shell-model level to another, the latter tell us how much total transition strength we can expect for a mode having particular (L,T,S) values. Throughout this talk giant resonance strength will be described in terms of the energy weighted sum rule (EWSR) which is a particularly useful sum rule because it is nearly model independent. Generally, the criterion for a transition to be considered collective is that its transition rate be many ( $\gtrsim 10$ ) times the single particle value, in which case the transition would exhaust an appreciable fraction of the EWSR for the mode in question. As we shall see giant resonances exhaust 20%-90% of their sum rules.

For direct reactions the transition rate and the angular momentum transfer are generally deduced by comparison of measured angular distributions with those calculated by use of the Distorted Wave Born Approximation (DWBA). The analysis procedure for giant resonance states is the same as has been established for direct excitation of low-lying states. In the case of electron scattering the transition rate,  $B(EL)$ , for the state in question can be directly obtained. In inelastic hadron scattering however, the situation is not nearly so direct. Comparison of the calculated cross sections with those measured, yields a quantity called the deformation parameter,  $\beta_L$ , as:

$$\beta_L^2 = \frac{d\sigma(L)}{d\Omega} \text{ measured} / \frac{d\sigma(L)}{d\Omega} \text{ calculated} \quad (1)$$

If  $\beta_L$  is assumed to be proportional to the mass multipole moment for a uniform distribution then

$$\beta_L^2 = B(EL) \left( \frac{4\pi}{3ZR^L} \right)^2 . \quad (2)$$



Whether or not the transition rate as directly determined in inelastic electron scattering or deduced in hadron scattering through use of equations (1) and (2) yield the same result has been an often debated topic. However, it is clear that hadron inelastic scattering is more model dependent than electron scattering. The application of the DWBA theory to giant resonance excitation by hadron inelastic scattering has been detailed by Satchler<sup>5</sup> and the experimental results have usually been analyzed following Satchler's formalisms. The EWSR is related to the transition rate as:

$$S_L = B(EL)(E_f - E_i) = \frac{L(2L+1)^2}{4\pi} \frac{\hbar^2}{2m} Ze^2 \langle r^{2L-2} \rangle, \quad (3)$$

where  $E_f$  and  $E_i$  denote the energies of the final and initial states in the transition and  $\langle r^L \rangle$  is the RMS charge radius of the ground state. For  $T=S=0$  transitions and  $\langle r^L \rangle = [3/(L+3)]R^L$  for a uniform mass distribution, the EWSR for a transition of multipole  $L$  can be written as:

$$S_L = \frac{3Ah^2}{8\pi m} LR^{2L-2}$$

where  $m$  is the nucleon mass and  $A$  is the nuclear mass number.

Using equation (2) relating  $\beta_L^2$  and  $B(EL)$  the following expression for  $\beta_L^2$  in terms of 100% EWSR depletion is obtained (for  $T=S=0$ ):

$$\beta_L^2 = \frac{2\pi\hbar^2}{3m} \frac{L(2L+1)}{AR^2} \frac{1}{E} \sim \frac{60L(2L+1)}{A^{5/3} E}, \quad (4)$$

if  $R = 1.2 A^{1/3}$  fm and  $E$  = excitation energy in MeV. Equation (4)

provides the limit of the EWSR for a transition of multipole  $L$  and energy  $E$ . Throughout this talk the measured giant resonance cross section will be presented as a fraction of the limit from equation (4). Sum rules for isoscalar monopole transition and isovector transitions are different and have been treated by Satchler.<sup>5</sup>

Figure 4 provides a representation of transitions that might comprise various electric modes. The figure schematically represents single-particle transitions between shell-model states of a hypothetical nucleus. Collective transitions result from coherent superpositions of many such single-particle transitions. Major shells are denoted as  $N$ ,  $N+1$ , etc. and are separated by  $\sim 1\hbar\omega$  or  $\sim 41 A^{-1/3}$  MeV. Giant resonances may be considered to result from transitions of nucleons from one major shell to another, under the influence of an interaction that orders these transitions into a coherent motion. The interaction for inelastic scattering can excite a nucleon by at most  $L\hbar\omega$ , or, to state it differently, the nucleon can be promoted by at most  $L$  major shells. The number of shells is either odd or even according to the parity.

Thus, the isovector giant dipole resonance (GDR) is built up of  $E1$  transitions spanning  $1\hbar\omega$ . The GDR might then be expected to be located at an excitation energy of  $\sim 41 A^{-1/3}$  MeV; however, it is located at  $\sim 77 A^{-1/3}$  MeV. This difference arises from the fact that the spin and isospin dependence of the nucleon-nucleon interaction ensures that the  $S=T=0$  collective states move down in energy, and that  $S=1$  or  $T=1$  states move up from the expected energy.

For  $E2$  excitations two different classes of transitions are allowed. The first of these, with lowest energy, is comprised of transitions

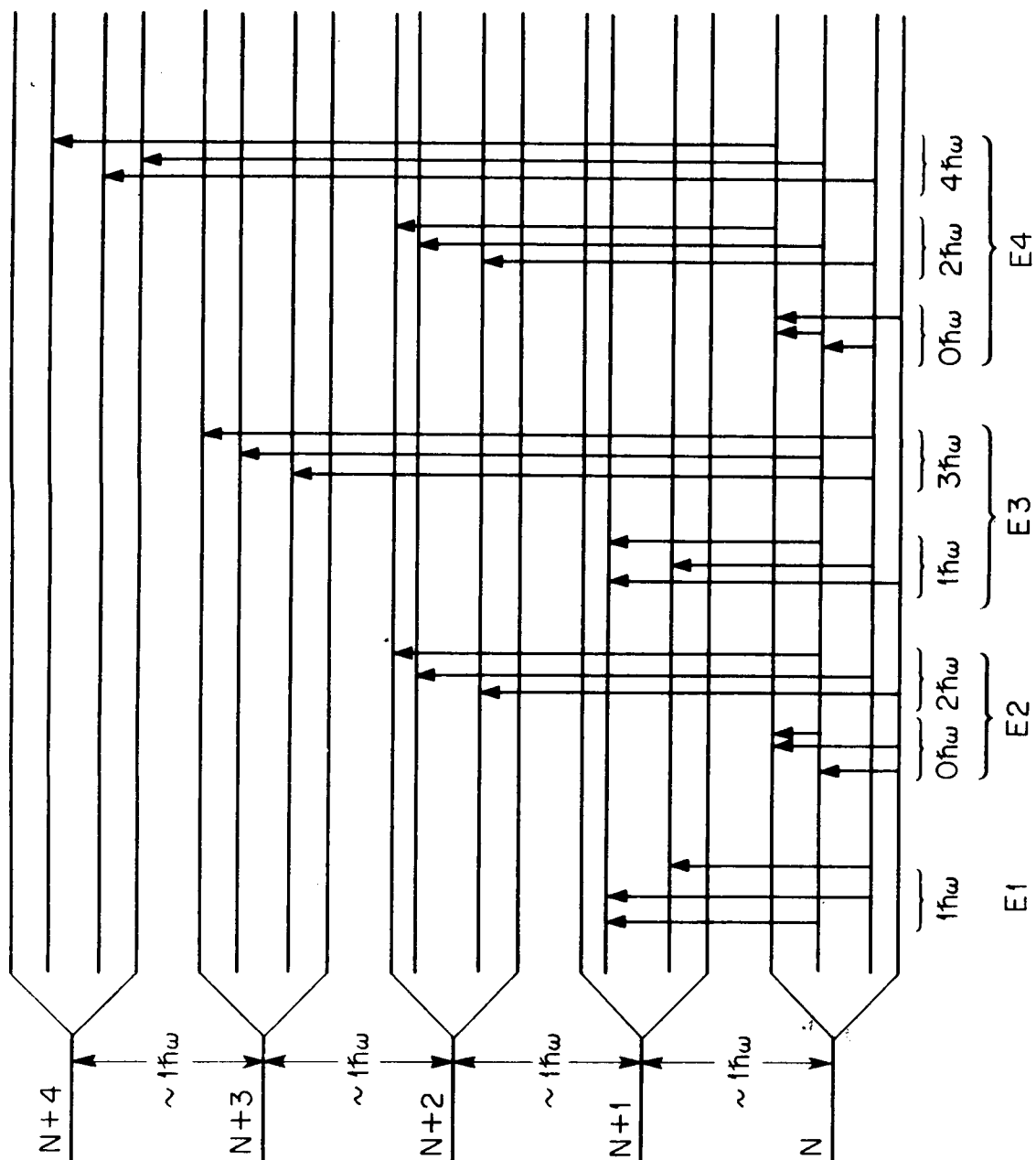


Fig. 4. Schematic representation of electric multipole transitions between shell-model states of a hypothetical nucleus. Major shells are denoted as  $N$ ,  $N+1$ ,  $N+2$ , etc. and lie  $\sim 1\hbar\omega$  or  $\sim 41 \times A^{-1/3}$  MeV apart.

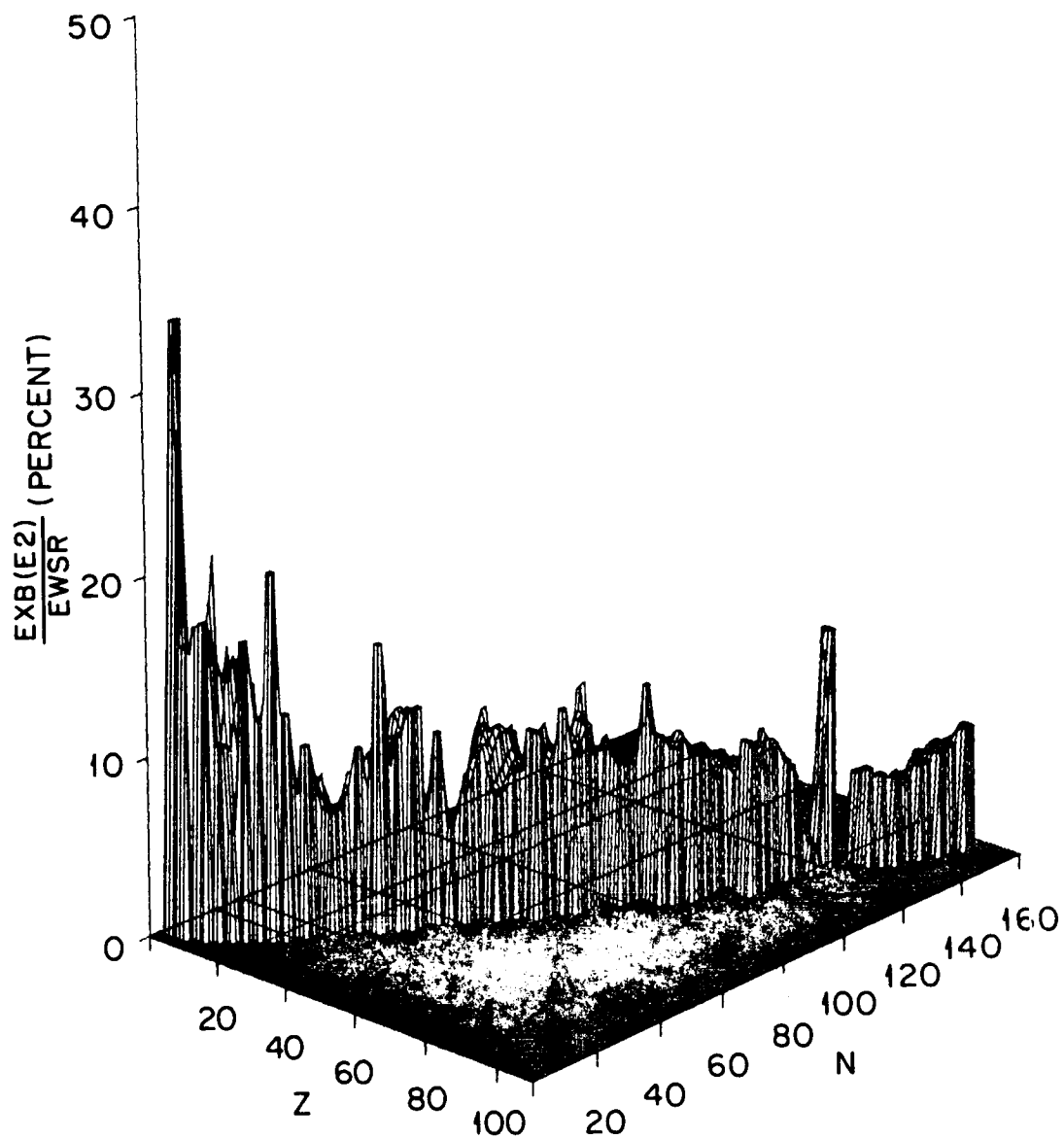


Fig. 5. Percent energy weighted sum rule depleted in the first excited  $2^+$  state plotted versus  $Z$  and  $N$ .

within a major shell, the so-called  $0\hbar\omega$  transitions. A second set is comprised of transitions between shells  $N$  and  $N+2$ , the  $2\hbar\omega$  transitions. These transitions would be pushed up or down in energy from  $2\hbar\omega$  for iso-vector or isoscalar modes respectively. While the  $0\hbar\omega$ , E2, excitations are identified with the familiar low-lying  $2^+$  levels, the  $2\hbar\omega$  class carry most of the EWSR and are associated with the GQR. By similar arguments E3 excitations of  $1\hbar\omega$  and  $3\hbar\omega$  and E4 excitations of  $0\hbar\omega$ ,  $2\hbar\omega$  and  $4\hbar\omega$  are expected.

For each class of transitions (E1, E2, etc.) the sum rule should be exhausted by the sum of the strength in all the transitions. For example, for E2 transitions the sum rule should be exhausted by the sum of the strength in the  $0\hbar\omega$  and  $2\hbar\omega$  transitions.

For the GDR we know 100% of the sum rule is accounted for (fig. 2). What about other multipoles? Figure 5 shows the percent of the EWSR depleted in the first  $2^+$  level of even-even nuclei. Except for the lightest nuclei, only  $\sim 10$ -15% of the EWSR is accounted for in the first  $2^+$  level. Table 1 shows a few cases (from inelastic scattering) where the entire bound state excitation region ( $\lesssim$  neutron separation energy) was studied. Only for  $^{24}\text{Mg}$  is any appreciable fraction of the  $L=2$  EWSR strength found in the bound-state region. Thus, there is good reason to expect considerable quadrupole strength to be in the  $\Delta N=2$  excitations. Indeed, less than one half of the possible sum-rule strength for any multipole is located in the bound states for the nuclei studied. In fact, there were predictions<sup>6</sup> that the  $\Delta N=2$  quadrupole strength would be appreciable and localized at an excitation energy of  $\sim 60 \times A^{-1/3}$  MeV.

TABLE 1. Percentage of isoscalar EWSR multipole strength depleted in bound states of  $^{24}\text{Mg}$ ,  $^{40}\text{Ca}$ , and  $^{208}\text{Pb}$ .

| Nucleus           | Multipole |   |    |    |    |    |   |     |   |
|-------------------|-----------|---|----|----|----|----|---|-----|---|
|                   | 0         | 1 | 2  | 3  | 4  | 5  | 6 | 7   | 8 |
| $^{24}\text{Mg}$  |           |   | 40 | 10 | 3  |    |   |     |   |
| $^{40}\text{Ca}$  | 0         | 0 | 14 | 38 | 7  | 11 | 1 | 0.2 | 0 |
| $^{208}\text{Pb}$ | 0         | 0 | 20 | 47 | 14 | 3  | 3 | 2   | 1 |

From these brief background comments one derives a complicated picture of giant multipole resonances. For each multipolarity there may be four independent modes and for each of these modes there may be more than one class of transitions, e.g. -- for  $L=4$ ,  $S=0$ ,  $T=0$  we could find a  $2\hbar\omega$  and  $4\hbar\omega$  giant resonance. How then do we sort out this complicated picture? The answer lies in the selectivity of the nuclear reactions we use to search for the resonances. We have already seen an outstanding example of such selectivity in the photonuclear excitation of the GDR.

Since we have said that photonuclear reactions are not especially well suited to study giant resonances other than dipole we must, of course, ask what is an appropriate technique? The answer has been through direct reactions, especially inelastic scattering, of medium energy projectiles. It has long been known that the inelastic scattering reaction provides strong excitation of low-lying collective  $T=S=0$  states. Thus, it seems reasonable to assume that high-lying collective states of

similar modes should also be excited. Inelastic electron scattering had been used for some time to study the GDR. However, while the electron scattering mechanism (electromagnetic) is well understood the selectivity of the reaction is rather low. The use of hadrons as projectiles provides much more variety for the reactions and thus, more selectivity. This can be seen by considering the effective interaction between a nucleon in the projectile and one in the target nucleus. The interaction is both spin and isospin dependent. For example, the central part of the interaction may be written as in Table 2.

TABLE 2

| $v_{TS}(i_{ij}) = v_{00}(r_{ij}) + v_{10}(r_{ij})\tau_i \cdot \tau_j + v_{01}(r_{ij})\sigma_i \cdot \sigma_j + v_{11}(r_{ij})\sigma_i \cdot \sigma_j \tau_i \cdot \tau_j$ |                             |                         |                                                               |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------|-------------------------|---------------------------------------------------------------|
| T=0<br>S=0                                                                                                                                                                | T=1<br>S=0                  | T=0<br>S=1              | T=1<br>S=1                                                    |
| GQR<br>1st $2^+, 3^-$                                                                                                                                                     | GDR                         | spin-flip<br>$2^-, 3^+$ | M1<br>Gamow-Teller                                            |
| IAS                                                                                                                                                                       |                             |                         |                                                               |
| (p,p')                                                                                                                                                                    | (p,p')                      | (p,p')                  | (p,p')                                                        |
| ( $\alpha, \alpha'$ )                                                                                                                                                     |                             |                         |                                                               |
| (d,d')                                                                                                                                                                    |                             |                         |                                                               |
|                                                                                                                                                                           | (p,n), ( $^3\text{He}, t$ ) |                         | (p,n), ( $^3\text{He}, t$ )<br>( $^6\text{Li}, ^6\text{He}$ ) |

The four modes for each multipolarity arise from a different term in the effective interaction. For example, the low-lying  $2^+$  and  $3^-$  nuclear

states are produced via the  $T=S=0$  term, the GDR and the isobaric analog resonance (IAS) are examples of excitations from the isospin term,  $T=1$ ,  $S=0$ . Isovector electric giant resonances arise from this term. The so-called "spin-flip" states,  $T=0$ ,  $S=1$  arise from the  $\sigma \cdot \sigma$  term while the spin-isospin term provides, for example, M1 states and the Gamow-Teller transitions. The various hadronic probes produce dramatically different strengths for the four components of the interaction. The  $(p,p')$  reaction can excite all four terms, but excitation of the  $T=S=0$  term would be  $\sim 9$  times (for cross section) greater than the other three terms. On the other hand alpha particles which have  $T=S=0$  will produce only  $T=S=0$  transitions while excluding excitation, to any observable extent, of states arising from the other three terms. Thus, inelastic scattering of medium energy hadrons will dominantly select the  $T=S=0$  states for excitation. However, in charge exchange reactions, such as  $(p,n)$ ,  $T=0$  does not contribute to the interaction and it is thus possible to study  $T=1$  excitations alone. The  $({}^6\text{Li}, {}^6\text{He})$  charge exchange reaction provides still further selectivity in that only  $S=1$ ,  $T=1$  modes are excited. Thus, use of a variety of hadronic probes and reactions may help to unravel the expected complicated giant resonance spectra. This discussion will concentrate on the results from inelastic scattering.

Since the inelastic scattering reaction has been utilized for low-lying level studies for so many years, why have such measurements not observed the giant multipole resonances earlier? At least part of the reason lies in what studies are fashionable and what are not at a given time. The advent of high resolution semi-conductor detectors and magnetic



spectrometers led to nearly a complete preoccupation by nuclear experimentalists with the first few MeV of nuclear excitation energy. The excitation region above  $\sim 10$  MeV was simply thought to be uninteresting.

Figure 6 shows a plot of a complete inelastic proton spectrum.<sup>7</sup> The spectrum is complete in that nearly all protons emitted at 27 degrees by the  $^{54}\text{Fe}$  target were observed (some protons are undetected for  $E_p \lesssim 1.5$  MeV). The region of this spectrum above an excitation energy of  $\approx 10$  MeV, the region which has been so often studied, contains only  $\approx 5\%$  of the integrated inelastic cross section. At the highest excitation energy end of the spectrum (lowest emitted proton energy) a large peak is observed which is produced by the nuclear evaporation process. The rather flat region between the evaporation peak and the neutron separation energy is called the nuclear continuum. This region is often described as arising from a pre-equilibrium particle emission process. The topic of this talk and for that matter of all the giant multipole resonance work during the past years is the small bump located at  $\sim 16$  MeV of excitation. Systematic observation of this peak in many nuclei showed that it was not produced by kinematical effects or solely by GDR excitation. Rather the peak has been shown to arise primarily from excitation of a giant quadrupole resonance (GQR).

Figure 7 shows spectra from a different inelastic scattering reaction in which the giant resonances are more strongly excited. These data<sup>8</sup> are from the  $^{120}\text{Sn}(\alpha, \alpha')$  reaction using 152 MeV alpha-particles at  $12^\circ$  and  $13^\circ$ . At both angles a large peak at  $\approx 12$  MeV rises from an otherwise flat continuum background. (A peak from hydrogen (H) contamination in the target is visible in the  $12^\circ$  spectrum.) Another broad peak is seen at

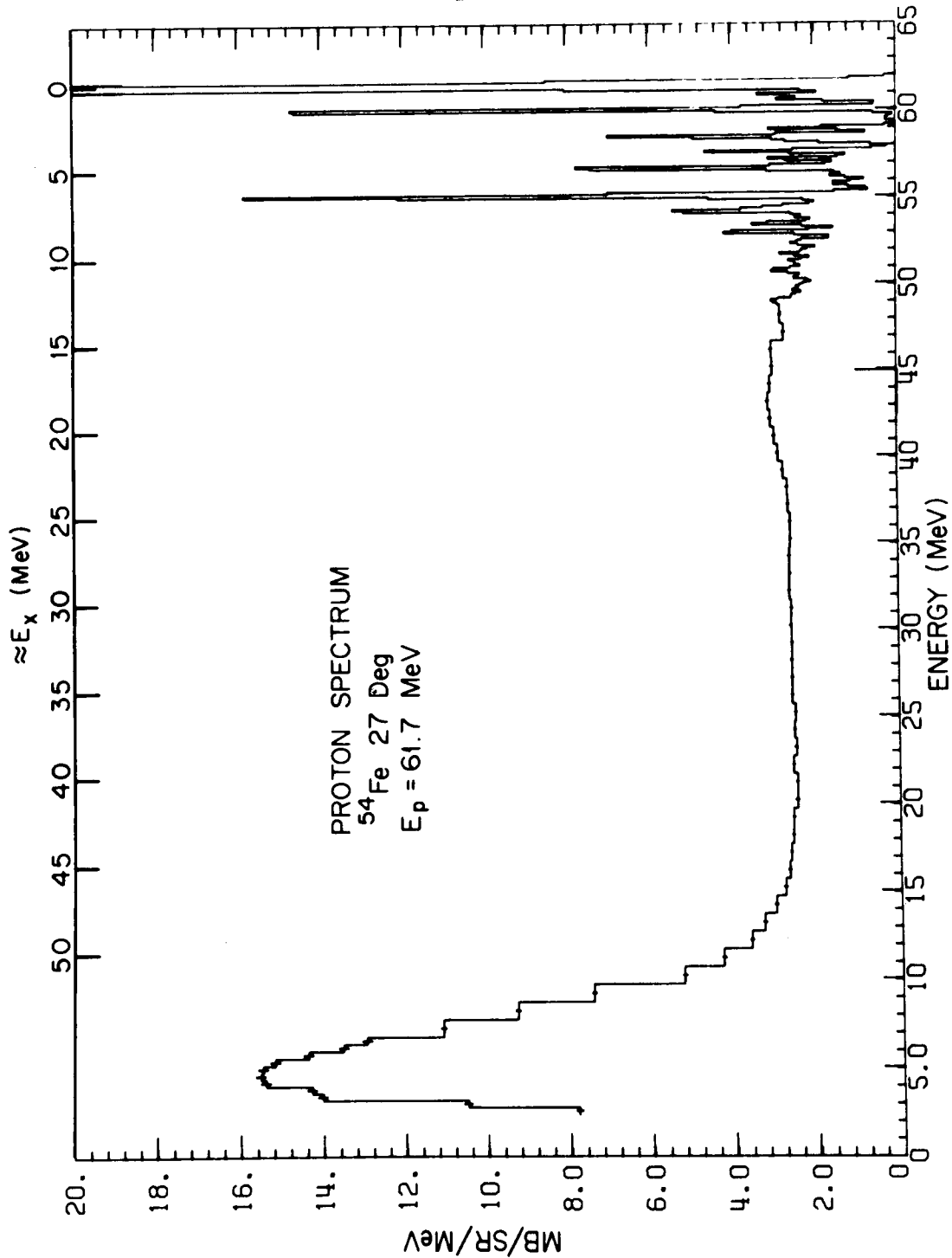
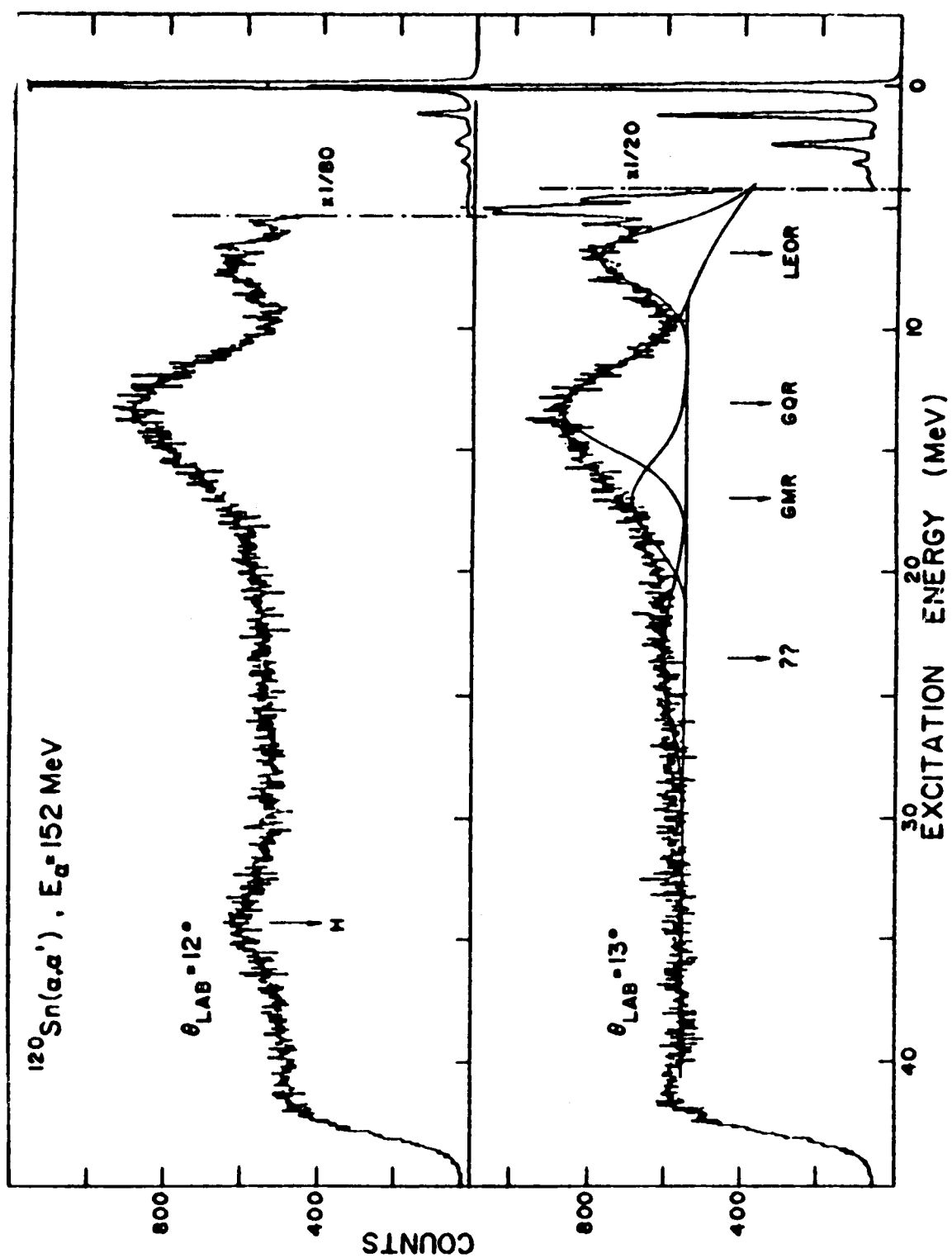


Fig. 6. Proton spectrum at  $27^\circ$  from 62 MeV protons in  $^{54}\text{Fe}$ . The energy of the outgoing proton is plotted at the bottom of the figure, while the approximate excitation energy is plotted at the top. Data have been plotted in  $\sim 1$  MeV-wide bins up to  $\sim 49$  MeV, then plotted in 50 keV-wide bins. Protons below  $\sim 1.5$  MeV were not detected in the experiment. The small, broad peak near  $E_x \sim 16$  MeV is identified as arising from excitation of the giant quadrupole and dipole resonances (ref. 7).



g. 7.  $^{120}\text{Sn}(\alpha, \alpha')$  spectra for  $E_\alpha = 152$  MeV (ref. 8). A decomposition of the spectrum into giant resonances (established and possible) and a continuum is shown on the 13 degree spectrum.

$\sim 7$ -MeV. The peaks from elastic scattering and inelastic scattering to low-lying levels are shown on a much reduced scale. It is immediately obvious that the experimental spectra we deal with in inelastic scattering measurements of multipole resonances are more complicated or, to state it differently, less clean than the photonuclear spectra used to deduce the parameters of the GDR. While the  $(\alpha, \alpha')$  reaction provides selectivity of  $T=S=0$  modes, all multipolarities may be, and indeed, generally are excited. We show on the 13 degree spectrum a decomposition of the spectrum into quadrupole (GQR), and monopole resonances (GMR) and a peak from the so called low-energy octupole resonance (LEOR). These resonances are discussed in some detail below. A possible additional resonance is shown centered at  $\sim 24$  MeV of excitation. This resonance is now known to arise from excitation of the giant octupole resonance (GOR) recently proposed from inelastic alpha, proton and helium-3 scattering.

Figures 6 and 7 serves to illustrate what is perhaps the most serious problem in inelastic scattering studies of giant resonances -- the giant resonance cross section is only a part (often a small part) of the total continuum cross section. Some assumption must be made about both the magnitude and shape of the continuum which lies under the resonance peak. It is assumed that the giant resonance cross section does not mix with the underlying continuum and the resonance peak is "stripped off" of the continuum by extrapolating the continuum magnitude and shape from higher excitation energies. I feel that cross sections for the giant resonances cannot be obtained from inelastic scattering with less than a 15-20% absolute uncertainty. We have not yet found a way to eliminate the continuum and leave just the resonance peaks.

The remainder of this discussion will be devoted to a summary of experimental results on the new giant resonances, especially the electric multipole resonances.

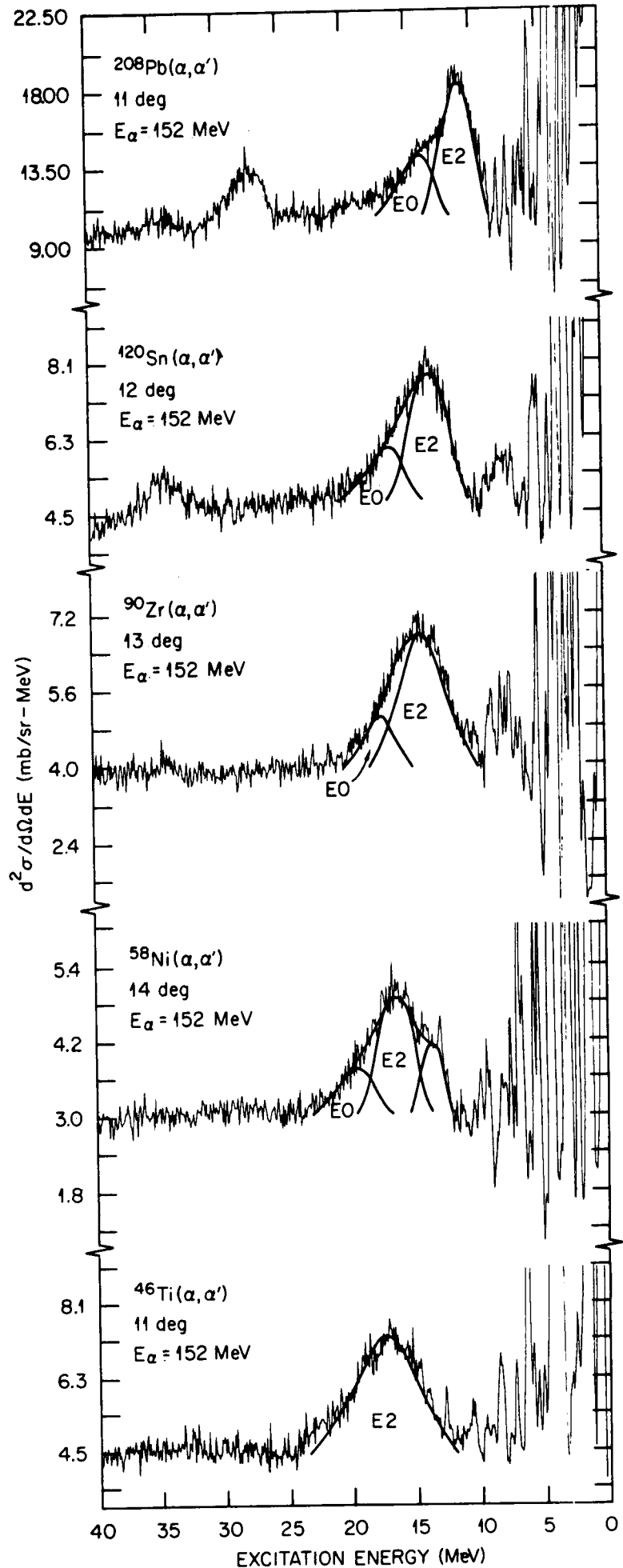
### III. Isoscalar Giant Quadrupole Resonance

In the notation of this presentation the isoscalar giant quadrupole resonance has quantum numbers  $(L,T,S) = (2, 0, 0)$ . The  $T=0$  GQR was the first of the new giant resonances to be discovered<sup>1</sup> and it remains the most well studied and carefully documented. In general there is excellent agreement between various types of hadronic measurements of the GQR and in most cases agreement with electron and particle capture reactions.

Figure 8 shows inelastic spectra<sup>8</sup> from five targets spanning a wide nuclear mass range, bombarded by 152-MeV alpha particles. In each spectrum a broad peak is observed at an excitation energy that varies smoothly with target mass. Prior to approximately three years ago the entire peak in each nucleus was attributed to excitation of the GQR. (For the  $(\alpha,\alpha')$  reaction the GQR is not excited to an observable extent.) However, as will be discussed below, we now know that (at least for nuclei heavier than about  $A=50$ ) the peak contains both monopole and quadrupole resonances. The identification of the resonance, i.e., the portion marked E2, as an  $L=2$  excitation is provided by comparison of the measured angular distributions with those calculated using the Distorted Wave Born Approximation (DWBA). The strength of the resonance in terms of percentage depletion of the EWSR is determined by the normalization of the calculated cross sections to those measured. Figure 9 shows such a comparison<sup>8</sup> for the five giant resonance spectra shown on fig. 8. The data

Figure 8

Spectra from inelastic scattering of 152-MeV alpha-particles on  $^{208}\text{Pb}$ ,  $^{120}\text{Sn}$ ,  $^{90}\text{Zr}$ ,  $^{58}\text{Ni}$  and  $^{46}\text{Ti}$ . The giant resonance structure located near the excitation energy  $63 \times A^{-1/3}$  MeV has been decomposed into contributions from the giant quadrupole and giant monopole resonances. The peak located at higher excitation energy in the  $^{208}\text{Pb}$  and  $^{120}\text{Sn}$  spectra are due to hydrogen contamination of the target (ref.8).



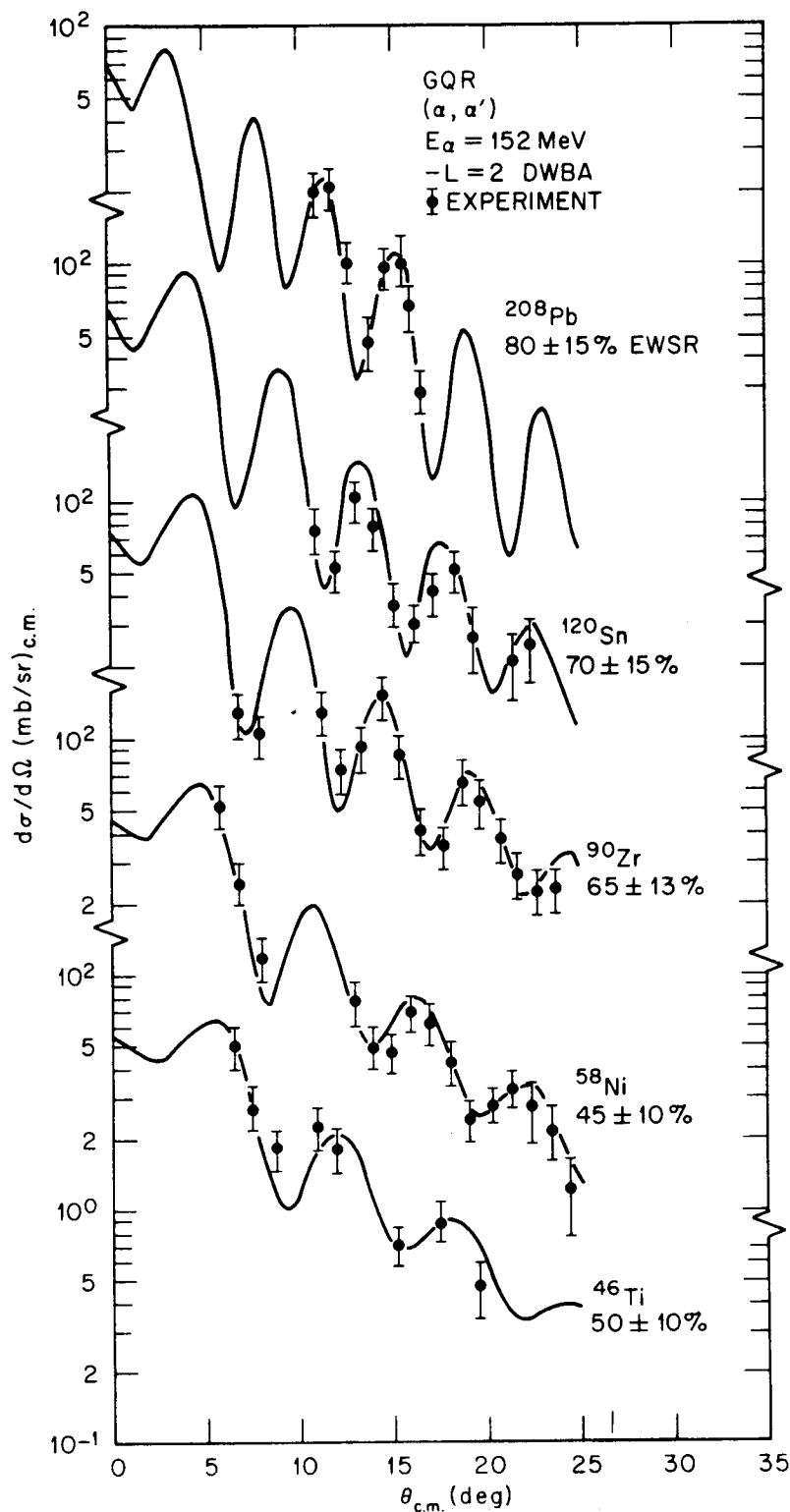


Fig. 9. Angular distributions of the E2 portion of the spectra from fig. 6. The data are compared to an L=2 DWBA calculation normalized to the indicated EWSR depletions (ref. 8).

are described very well by the DWBA calculation assuming the state to be quadrupole ( $L=2$ ). For all five nuclei most of the  $T=0$ ,  $L=2$ , EWSR is depleted in the resonance peak.

For nuclei having  $A < 40$  the character of the GQR changes dramatically as is seen in fig. 10. These data<sup>9</sup> were taken with a magnetic spectrograph which provided energy resolution considerably less than 100 keV (FWHM). For  $^{40}\text{Ca}$  one observes a broad peak located at  $\sim 63 A^{-1/3}$  MeV (position of arrow) similar to those in heavier nuclei. However, no such broad structure is found at  $63 A^{-1/3}$  MeV in the lighter nuclei. Rather, the excitation energy region between 12 and 20 MeV contains peaks arising from excitation of a large number of individual states. Most of these peaks are attributable to  $L=2$  excitations.<sup>10,11</sup> The sum of the  $L=2$ ,  $T=0$ , EWSR depleted in the individual quadrupole states is found to provide a significant fraction of the total EWSR.

Systematics for the energy, width and strength (sum rule) of the  $T=0$  GQR are presented in fig. 11. The data were mostly taken from ref. 1 and ref. 2. Where more than one measurement has been made on a given nucleus the results were averaged. In general the excitation energy for the GQR for  $A \gtrsim 100$  falls at the systematic energy of  $\sim 65 A^{-1/3}$  MeV. For nuclei lighter than mass 100 there is a clear tendency for the GQR peak to fall below the systematic energy. For nuclei lighter than  $A=40$  the energy of the strength centroid of the individual fragments has been plotted. The dashed line at  $64.7 A^{-1/3}$  MeV is taken from a recent calculation.<sup>12</sup>

The width of the GQR increases smoothly with decreasing nuclear mass. However, there are apparent shell effects in the data. The narrowest



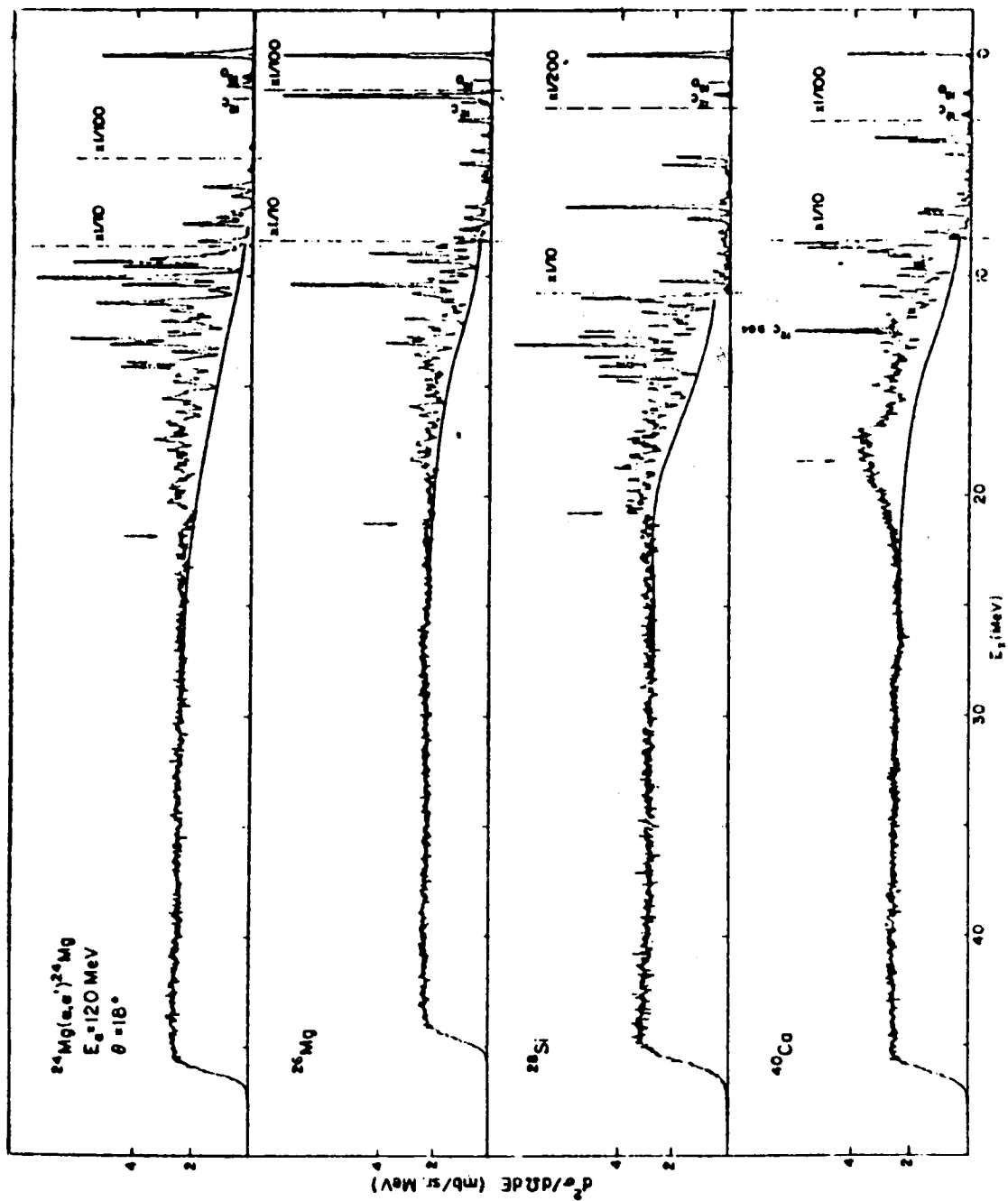


Fig. 10.  $(\alpha, \alpha')$  spectra for 120 MeV alphas on  $^{24}\text{Mg}$ ,  $^{26}\text{Mg}$ ,  $^{28}\text{Si}$  and  $^{40}\text{Ca}$  (ref. 9). The arrows are located at the excitation energy  $63 \times A^{-1/3} \text{ MeV}$ .

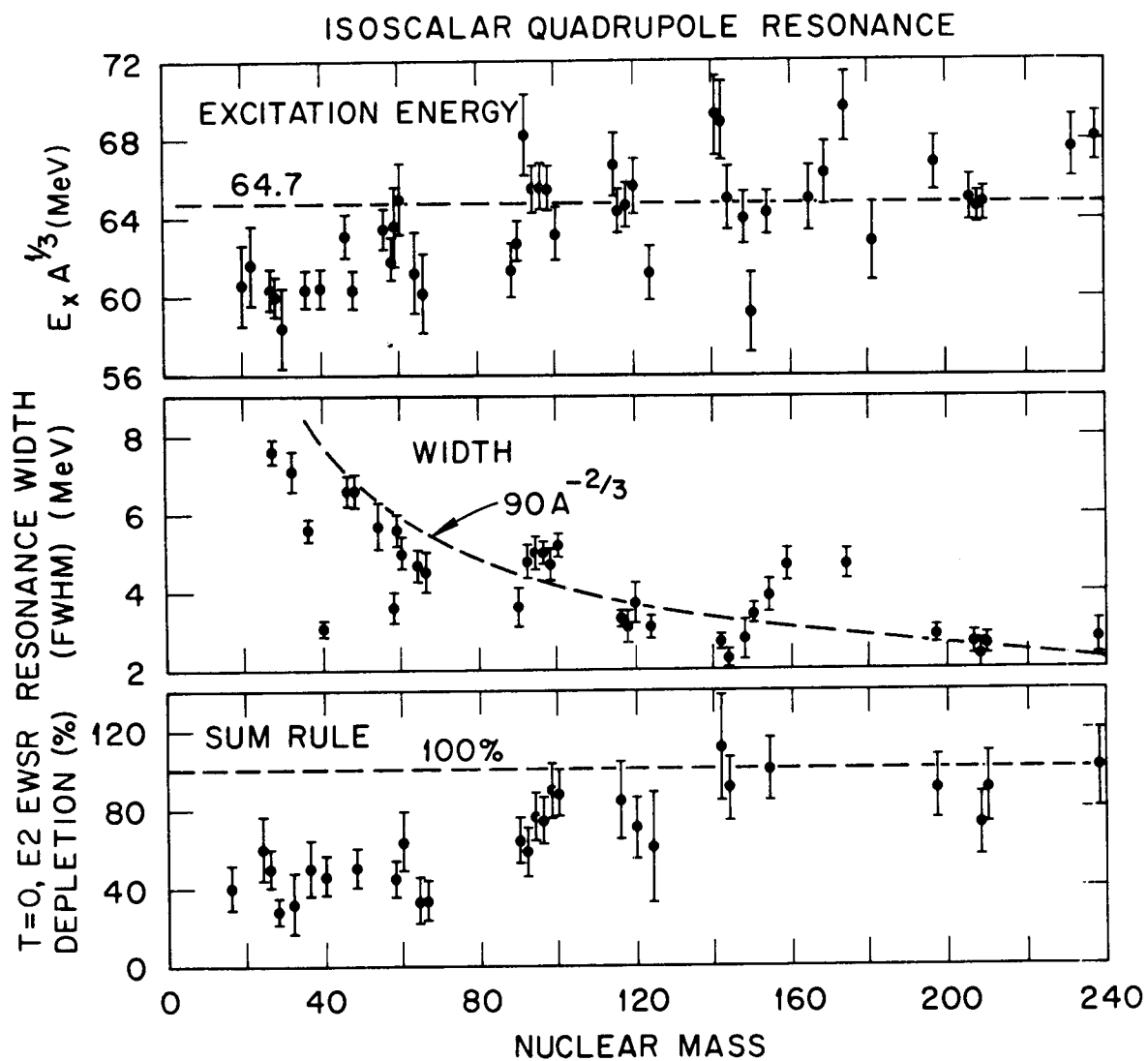


Fig. 11. Systematics for the excitation energy, width and sum rule depletion of the isoscalar giant quadrupole resonance. The data are mostly from refs. 1 and 2.

resonance widths occur for  $A=40, 90, 142$  and  $208$ . The resonance is widest in the region of the rare earth deformed nuclei. This behavior is similar to that observed for the GDR (fig. 2) although the increase in the GQR width in deformed nuclei is not nearly as great as for the GDR. The  $A^{-2/3}$  dependence of the width predicted in ref. 12 provides a good description of the data. The value of the multiplicative constant (90) has been increased from that suggested (58) in ref. 12 in order to fit the data.

The percentage of the  $T=0$ , E2 EWSR strength depleted in the GQR is shown at the bottom of fig. 11. It is to be noted that these values do not include contributions to the EWSR from low-lying ( $0^+_{\text{low}}$ ) quadrupole excitations. A trend to larger sum-rule depletion with increasing nuclear mass is clearly evident. For nuclei having  $A \gtrsim 100$  essentially 100% of the EWSR is found to be depleted in the GQR peak ( $2^+_{\text{low}}$  transitions), while for lighter nuclei 30-50% of the EWSR is typically found in the high-lying quadrupole states. However, considerably more EWSR strength is located in the low-lying  $2^+$  states of light nuclei than of heavy nuclei, so that the sum of the GQR ( $2^+_{\text{low}}$ ) and low-lying quadrupole ( $0^+_{\text{low}}$ ) strengths exhaust  $\sim 100\%$  of the  $T=0$ , E2 EWSR in light as well as heavy nuclei.

#### IV. Isovector Giant Quadrupole Resonance

As pointed out earlier, excitation of  $T=1$  (isovector) states via hadron inelastic scattering is much weaker than excitation of  $T=0$  states. Charge exchange reactions excite  $T=1$ ,  $S=0$  states and there is evidence<sup>13</sup> from  $(n,p)$  measurements that the GDR is excited. However,

charge exchange reactions have yet to clearly show excitation of the isovector giant quadrupole ( $L=2$ ,  $T=1$ ,  $S=0$ ). On the other hand, the electromagnetic ( $e,e'$ ) interaction provides equal strength both  $T=1$  and  $T=0$  excitations (all other things being equal, e.g. -- EWSR,  $L$ ,  $E_x$ ). For the lighter nuclei ( $A \lesssim 40$ ), particle capture reactions provide information on isovector strength, however most of the data on the  $T = 1$  GQR have come from inelastic electron scattering.

The left side of fig. 12 shows inelastic electron scattering spectra<sup>14</sup> from  $^{60}\text{Ni}$ . The broad peak observed at  $\sim 32$  MeV of excitation is identified as the  $T=1$ , GQR. This identification is supported by comparison of cross sections for the peak with calculated E2 form factors as shown on the right side of fig. 12 for  $^{58}\text{Ni}$  and  $^{60}\text{Ni}$ .

Plotted in fig. 13 are the systematics for the  $T=1$ , GQR. The data are all from inelastic electron scattering and are taken from ref. 15. Although much less data are available than for the  $T=0$  GQR the systematics are very similar. The excitation energy follows the systematic energy  $130 A^{-1/3}$  MeV for heavier nuclei while for lighter nuclei the energy drops off. The width of the resonance increases rather smoothly as the nuclear mass decreases. The available results show that  $\sim 100\%$  of the  $T=1$ , E2 EWSR is located in the resonance peak. The trend observed in the  $T=0$ , GQR for smaller depletion of the EWSR for nuclei having  $A \lesssim 100$  is not seen here, within the constraints of a much smaller available data base. The EWSR depletion shown on fig. 13 are based on use of the Goldhaber-Teller model. Use of the Myers-Swiatecki model for the ( $e,e'$ ) form factors results in a considerable reduction ( $\sim 30\%$  lower) in the EWSR depletion.

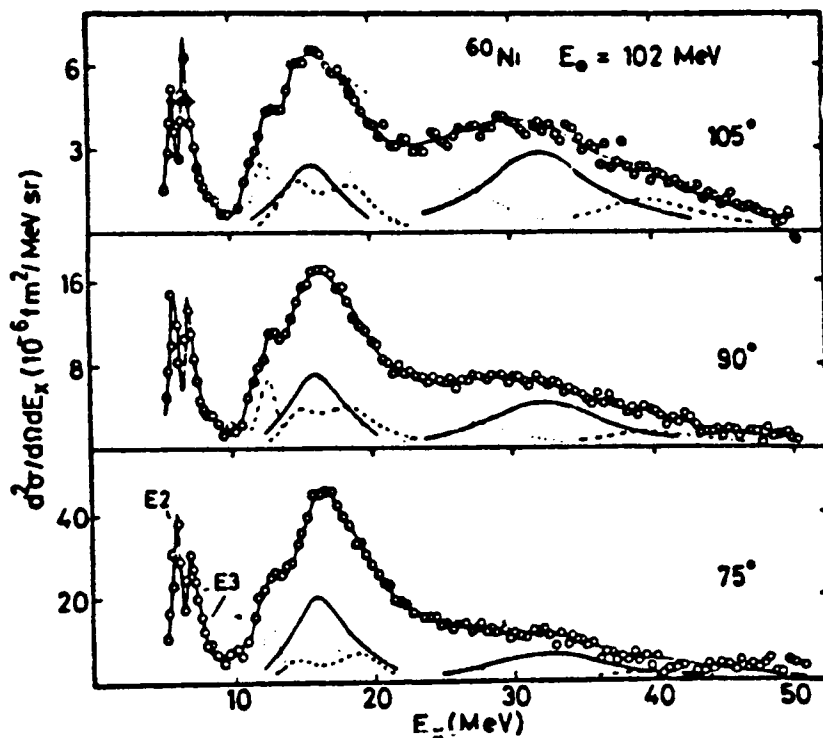
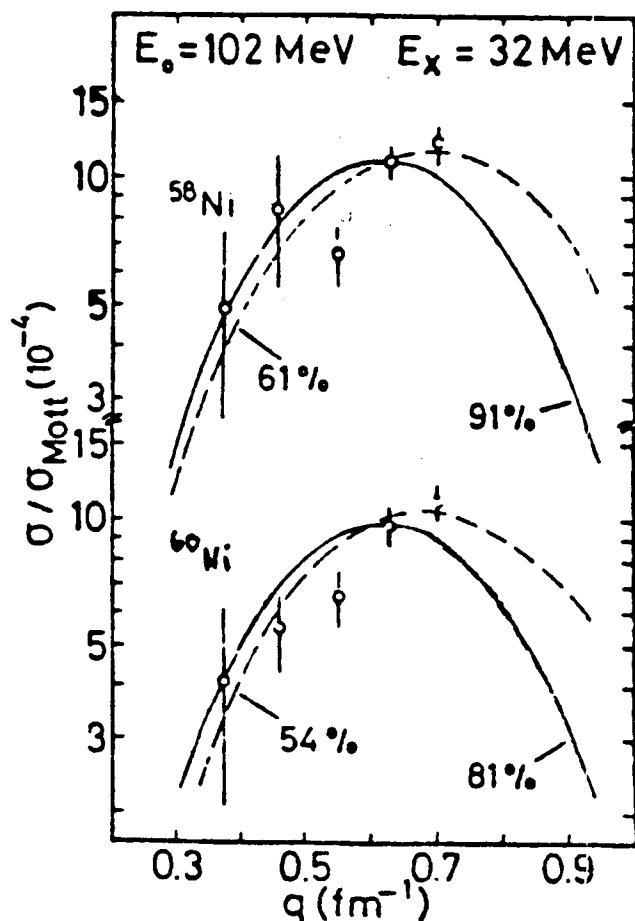


Figure 12

Left; Inelastic electron scattering from  $^{60}\text{Ni}$ . The broad peak at 32 MeV of excitation is assigned as isovector quadrupole resonance (ref. 14).

Figure 12

Right; Form factors for the isovector E2 states observed (ref. 14) in  $^{58}\text{Ni}$  and  $^{60}\text{Ni}$ . The solid curve is for the Goldhaber-Teller model while the dashed curve is from the Myers-Swiatecki prescription.



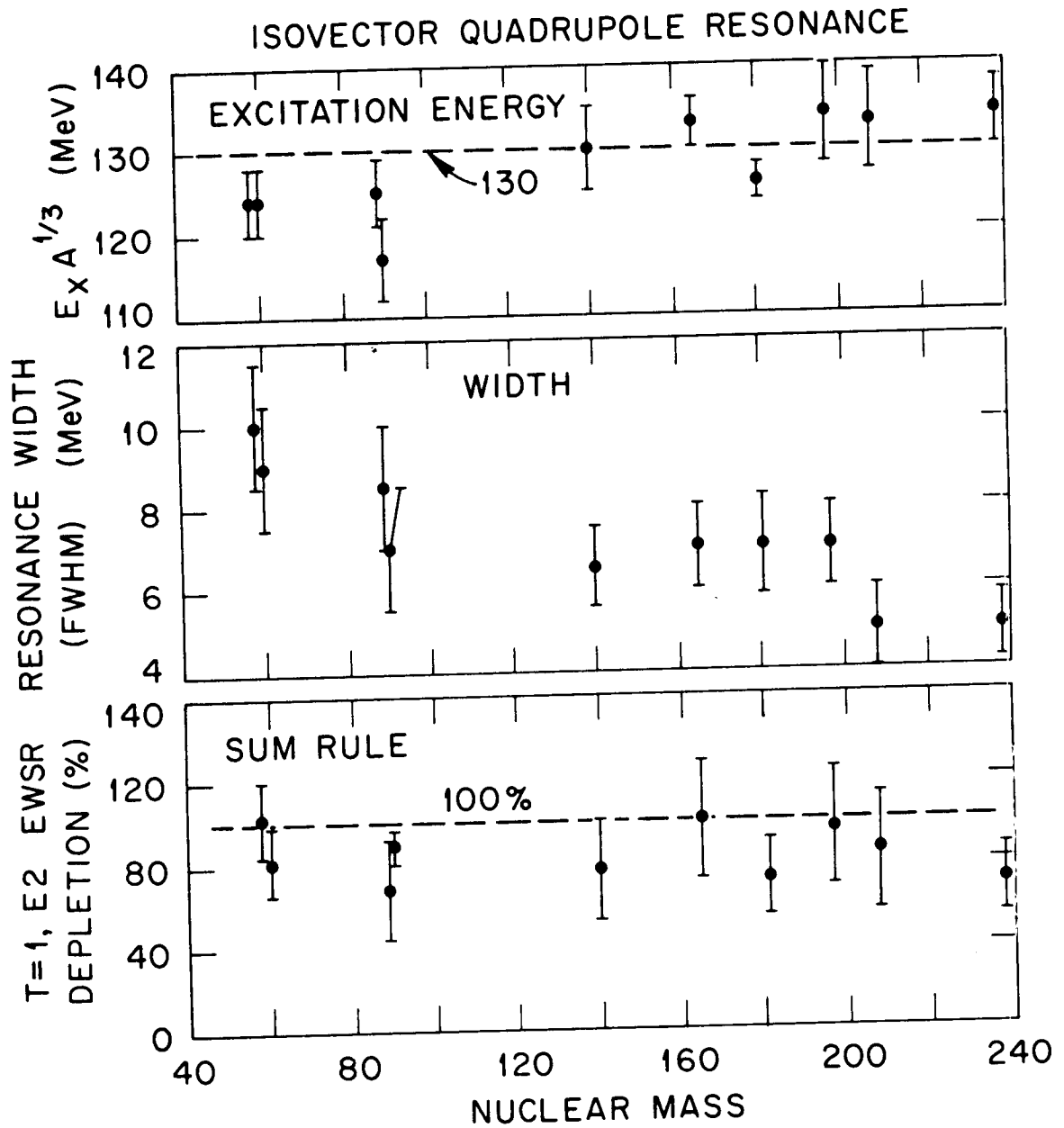


Fig. 13. Systematics of the excitation energy, width and sum rule depletion of the isovector giant quadrupole resonance. The data are from ref. 15.

## V. Isoscalar Giant Monopole Resonance

Although the most thoroughly studied of the new giant resonances is the GQR, the resonance which has generated the most interest is the monopole. We discuss here the isoscalar giant monopole resonance (GMR) ( $L=0, T=0, S=0$ ). Observation of the monopole, "breathing", or compressional mode of nuclear excitation is of special significance because knowledge of its excitation energy provides direct information on the nuclear compressibility. During the past few years several candidates for the E0 resonance have appeared but most have not withstood the test of further measurements. These early measurements are discussed in ref. 1. It is important to note that some early indirect evidence for an E0 resonance<sup>16,17</sup> placed the monopole excitation at an energy of  $\sim 80 \times A^{-1/3}$  MeV, a value in agreement with those from the recent more direct observations I will describe.

The first direct observation<sup>18</sup> of the resonance that was later confirmed to be the GMR was made using inelastic scattering of 120 MeV alpha particles from  $^{208}\text{Pb}$ . Those data show the presence of two broad peaks in the giant resonance region of the spectrum. A larger peak located at 11 MeV ( $65 \times A^{-1/3}$  MeV), the GQR, and a smaller peak located at 13.9 MeV or  $\sim 80 \times A^{-1/3}$  MeV. This latter peak is near the energy of the GDR in  $^{208}\text{Pb}$  (13.6 MeV), however the isovector GDR will not be excited by the  $(\alpha, \alpha')$  reaction with nearly enough cross section to account for this new peak. A similar peak was also found<sup>18</sup> in  $^{206}\text{Pb}$ ,  $^{209}\text{Bi}$ , and  $^{197}\text{Au}$ .

Thus it was established that a here-to-fore not directly observed resonance peak was located at  $\sim 14$  MeV for nuclei in the lead region. The obvious question was what is the nature of the peak? It is an unfortunate circumstance that the angular distributions for  $L=2$  and  $L=0$  excitation via the  $(\alpha, \alpha')$  reaction are identical in angular regions where the giant resonances are easily measured. This fact is demonstrated on fig. 14 which shows calculated monopole and quadrupole angular distributions for the  $^{208}\text{Pb}(\alpha, \alpha')$  reaction using 152 MeV alpha particles. The two curves are nearly identical for angles as small as  $\sim 5$  degrees with the  $L=0$  curve having somewhat larger peak to valley ratios. The 120-MeV  $(\alpha, \alpha')$  data of reference 18 were taken in an angle range where  $L=0$  and  $L=2$  angular distributions are not clearly distinguishable. Shown on fig. 14 are measured cross sections for the 13.9 and 10.9 MeV resonances obtained using 152 MeV alpha particle inelastic scattering. The measured cross sections<sup>8</sup> for the two resonances provide good agreement with the calculations, but not positive  $L=0$  identification.

However, as is seen in fig. 14, at very small angles the two angular distributions are out of phase and convincing identification of the  $L=0$  component could be made. Two groups have now provided measurements<sup>19,20</sup> on a number of nuclei utilizing detection systems especially designed to study very small angle (down to zero degrees) inelastic alpha and helium-3 scattering.

Spectra from inelastic scattering<sup>21</sup> of 129-MeV alpha-particles from  $^{144}\text{Sm}$  and  $^{154}\text{Sm}$  at 0, 4, and 6 degrees are shown on fig. 15. It is immediately obvious from this figure that use of small angle techniques does not eliminate the problem of excitation of the underlying nuclear



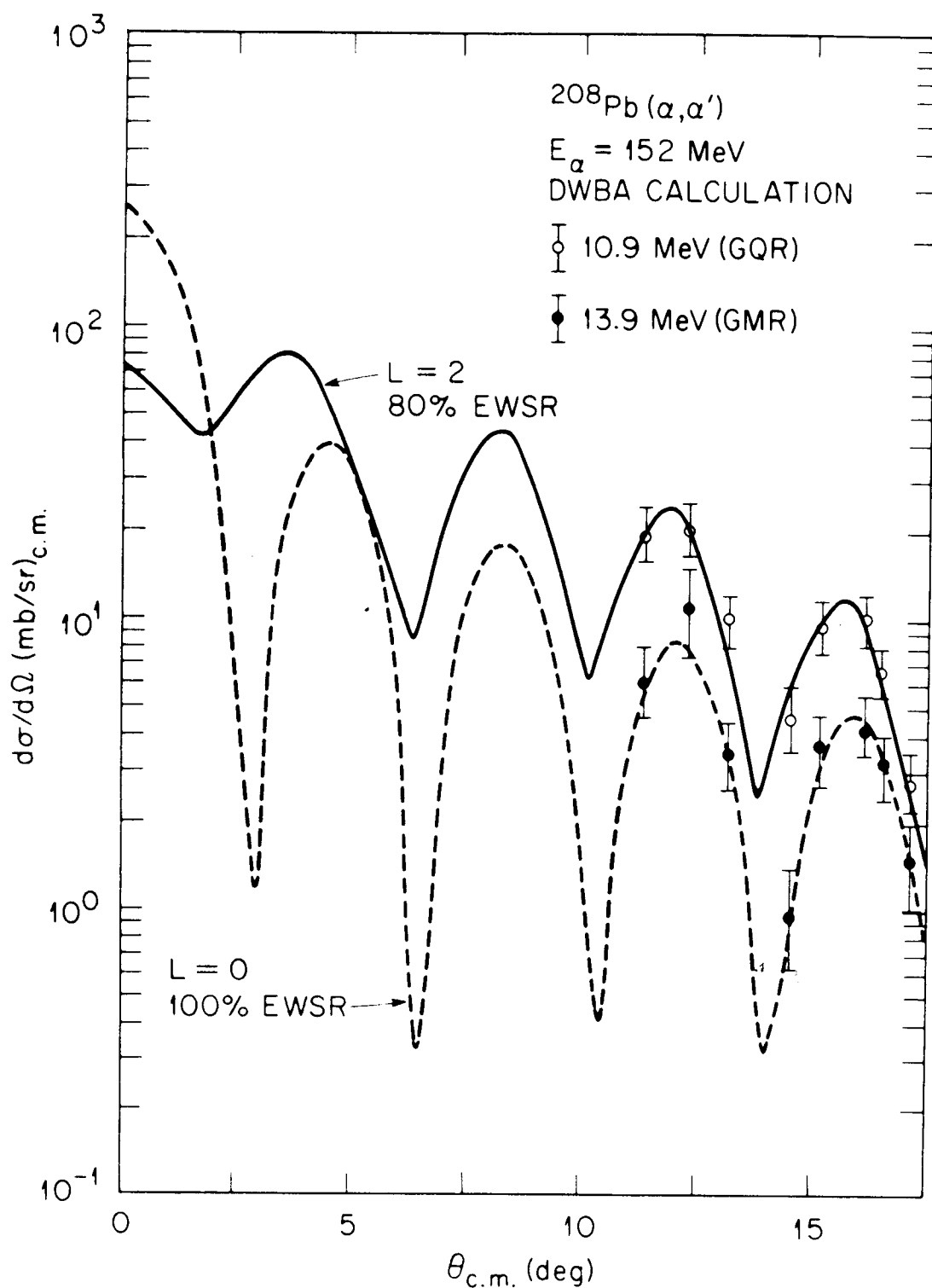


Fig. 14. Calculated inelastic scattering angular distributions for 152 MeV alpha particle excitation of the GQR and GMR in  $^{208}\text{Pb}$  compared with data from ref. 8.

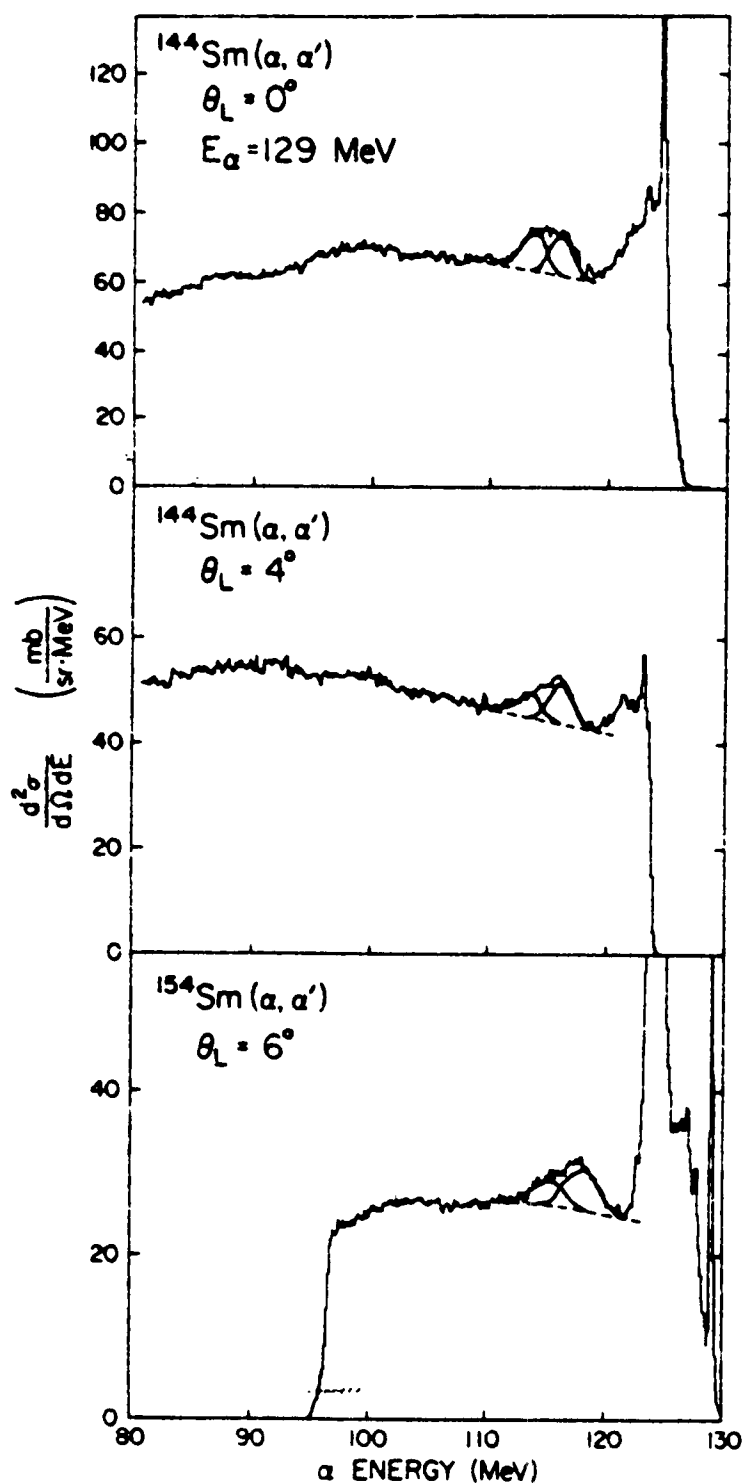


Fig. 15. Inelastic alpha particle spectra at very small angles for 129-MeV alpha-particles in  $^{144}\text{Sm}$  and  $^{154}\text{Sm}$  (ref. 21).

continuum. Indeed the resonance to continuum ratio may be poorer at very small angles than at larger angles. Some assumption about the shape and magnitude of the continuum underneath the resonances must be made in order to extract the resonance peaks from the spectrum. However, it is apparent, even in this figure, that the shape of the giant resonance peak changes considerably with angle.

Figure 16 shows the exciting advantages to be gained with small angle measurements.<sup>22</sup> The giant resonance peak, after subtraction of the continuum, is shown at several small angles for the reaction  $^{116}\text{Sn}(\alpha, \alpha')$  for  $E_\alpha = 129$  MeV. It is apparent that the centroid of the entire peak shifts and the width of the peak is not constant as the angle of observation varies, both effects being indicative of the existence of two different multipolarities within the single broad peak. The peak is shown decomposed into two components which clearly change relative magnitude rapidly with angle. The angular distributions for the two components of the resonance peak are plotted in the lower part of the figure. The lower excitation energy peak has an  $L=2$  angular distribution while the cross sections for the higher excitation peak follows the calculated  $L=0$  angular distribution. Such measurements have now clearly identified the peak seen earlier at larger angles as the isoscalar GMR resonance.

The small angle experiments and measurements using the  $(p, p')$  reaction<sup>23</sup> have now provided a significant body of information about the GMR in many nuclei.

Systematics of the monopole resonance are shown on fig. 17. The top plot shows the trend with nuclear mass number of the energy of the GMR.

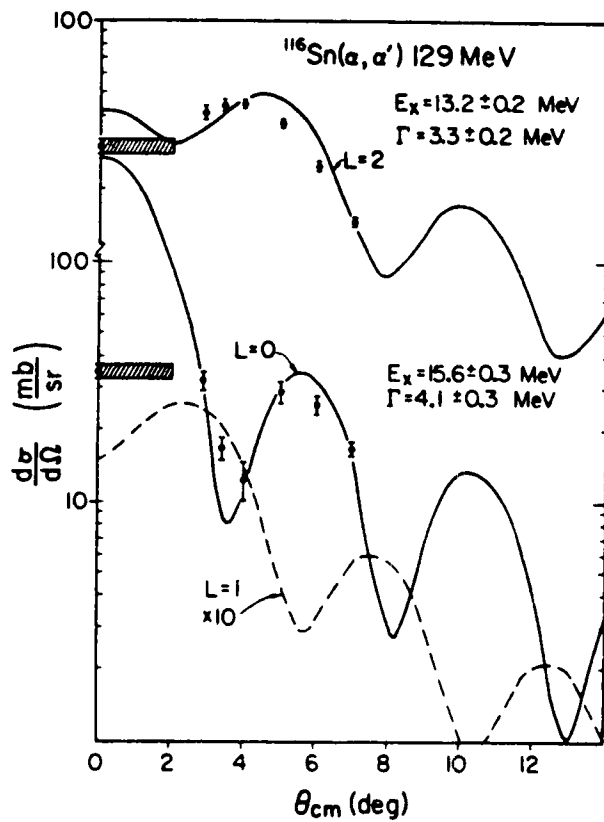
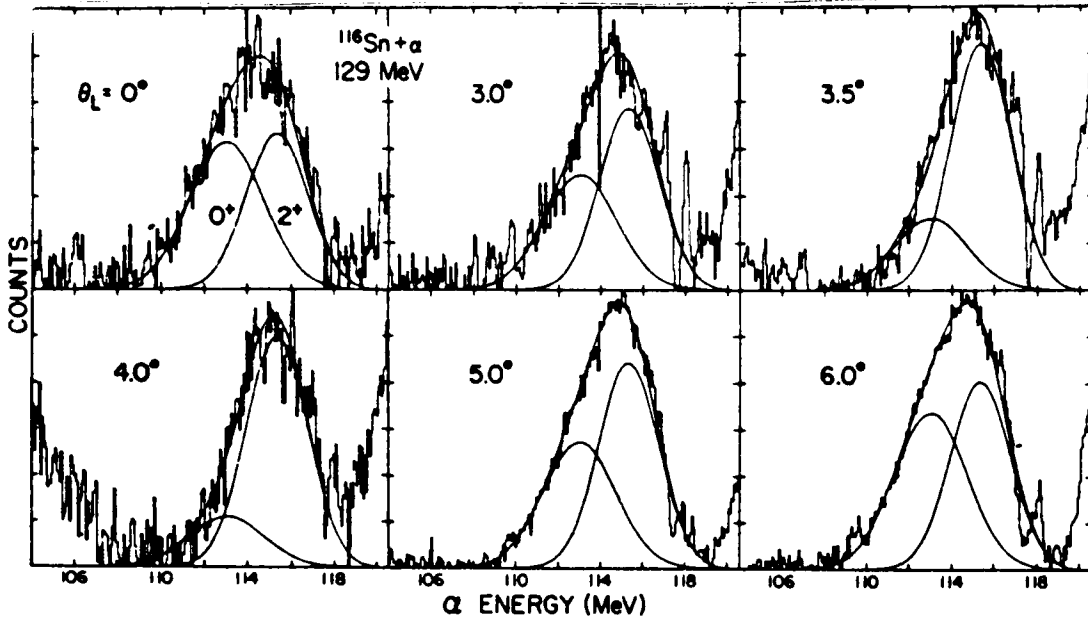


Fig. 16. Top; Giant resonance spectra from the reaction  $^{116}\text{Sn}(\alpha, \alpha')$  for  $E_\alpha = 129$  MeV. The nuclear continuum has been subtracted from the data. The peak is decomposed into monopole and quadrupole contributions (ref. 22).

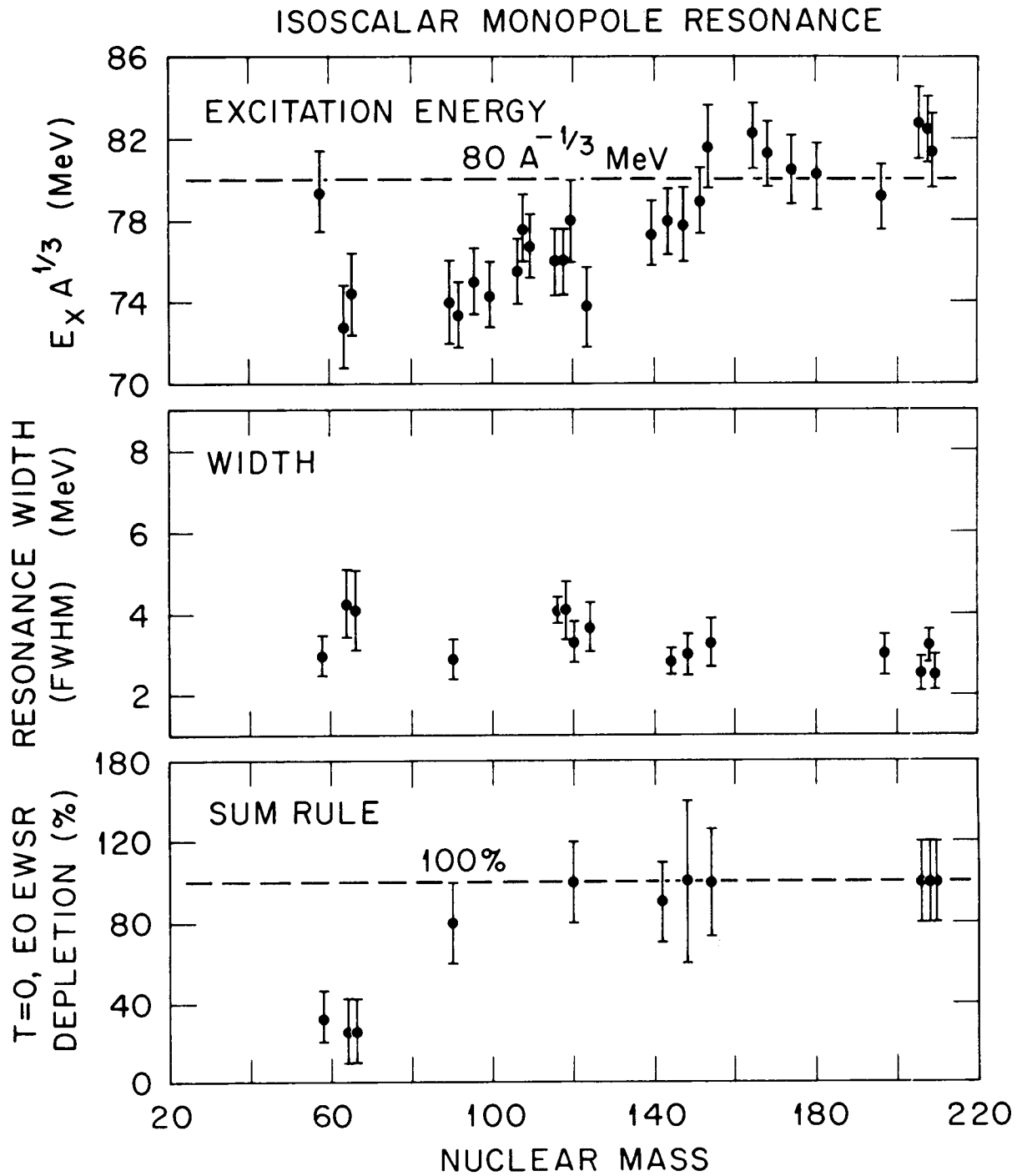


Fig. 17. Systematics for the excitation energy width and sum rule depletion for the isoscalar giant monopole resonance. Data are from refs. 19, 20, 21 and 23.

As has been the case for the resonances previously discussed, for heavier mass nuclei the resonance is located at a systematic energy, in this case  $\sim 80 \times A^{-1/3}$  MeV. For lighter nuclei the now familiar trend for the resonance to be found at lower excitation energy is observed for the GMR. The width of the monopole resonance shows a tendency to broaden as the nuclear mass decreases in agreement with the trend for other giant resonances. The sum rule depletion for the monopole resonance plotted at the bottom of fig. 17 shows that all of the  $T=0$ ,  $E0$ , EWSR is accounted for in nuclei as light as  $^{90}\text{Zr}$ . However, less than half of the sum rule strength is observed for the three lighter nuclei that have been studied.

Perhaps the most interesting aspect of the GMR to be derived from these studies is the lack of observation of the monopole resonance in nuclei having  $A \lesssim 50$ . There are no apparent theoretical reasons to suggest that the monopole should disappear in light nuclei. On the other hand, we know the strength for other giant resonances becomes very fragmented in light nuclei. If such is the case for the GMR and the resonance is spread over many MeV of excitation energy in light nuclei, then present measurement techniques may not be sensitive enough to locate the  $E0$  strength.

It now seems clear that the GMR has been identified and systematically observed in a wide mass range of nuclei. The question of the low sum rule depletion for mass 50-60 nuclei and the lack of observation of any  $E0$  strength in lighter nuclei is yet to be understood.

Observation of the giant monopole resonance is of special significance since the location of the GMR yields the value of the compressibility of nuclear matter. This comes about through the relationship from nuclear hydrodynamics:

$$k_{\infty} = \frac{3R_m^2}{h^2 \pi^2} [E_{(0^+)}]^2.$$

For an energy dependence of  $80 A^{-1/3}$  MeV as determined for the GMR (fig. 17) and  $R = 1.2 A^{1/3}$  fm, a value of  $k_{\infty} \sim 200$  MeV is deduced. The GMR observation provides the first measure of the compressibility of nuclear matter, a fundamental nuclear property. The knowledge of the nuclear compressibility is important to such diverse applications as the formation of shock waves in heavy-ion reactions and the creation of neutron stars in the supernova process.

#### VI. Higher Multipole Resonances

The basic experimental question pertaining to high-L ( $\geq 3$ ) giant resonances is whether the resonance strength is sufficiently localized in energy to be observable above the large nuclear continuum. Of course, similar questions were asked about the GQR before its observation. Nevertheless, there are good reasons, as discussed below, to expect that high-L resonances will be very difficult to observe.

- 1) For  $L \geq 4$  the sum rule strength should be divided between at least three distinct classes of transitions. As shown on fig. 4, E4 excitations should be found at  $0\hbar\omega$ ,  $2\hbar\omega$  and  $4\hbar\omega$ . Thus, 100% of the EWSR is in this case distributed among three transition classes rather than two as is the case for the GQR.
- 2) The excitation energy expected for the high-L resonances is much higher than that of, for example, the GQR. That is to say, the  $3\hbar\omega$   $L=3$ ,  $T=0$ , excitation should lie at a higher excitation energy than  $2\hbar\omega$  excitation. Since the EWSR value is inversely proportional to the resonance energy, and the inelastic cross section is directly proportional to the EWSR, the

cross sections for the same EWSR depletion are lower for higher excitation states.

- 3) It is expected that the width of the giant resonances will increase as the excitation energy of the resonance (and the multipolarities of the resonance) increases. For example, calculations<sup>12</sup> by Nix and Sierk indicate that the width of the  $L=3$  resonance should be  $\sim 3$  times wider than the GQR while a  $L=4$  resonance should be  $\sim 6$  times broader than the GQR.

From these simple considerations one sees that searches for high- $L$  giant resonances are faced with a decreased EWSR strength spread over a much wider excitation energy range than for low- $L$  resonances.

The difficulties described above to observation of high- $L$  giant resonances indicate the need for reactions which provide larger resonance cross sections and clear multipole identification through definitive angular distribution shapes. While no single reaction has appeared that will provide both of these desired features, the use of medium energy proton inelastic scattering provides excellent multipole selectivity and medium energy ( $E \gtrsim 25$  MeV/amu) heavy ion inelastic scattering provides very large resonance cross sections. The use of heavy ions in giant resonance research is just beginning and will be discussed later in this presentation. However, use of medium energy proton inelastic scattering has already made significant contributions to the giant resonance field.

The most important advantages to medium energy  $(p,p')$  studies of giant resonances are shown in fig. 18 which shows a plot of calculated angular distributions for the reaction  $^{208}\text{Pb}(p,p')$  at 200 MeV. Unlike



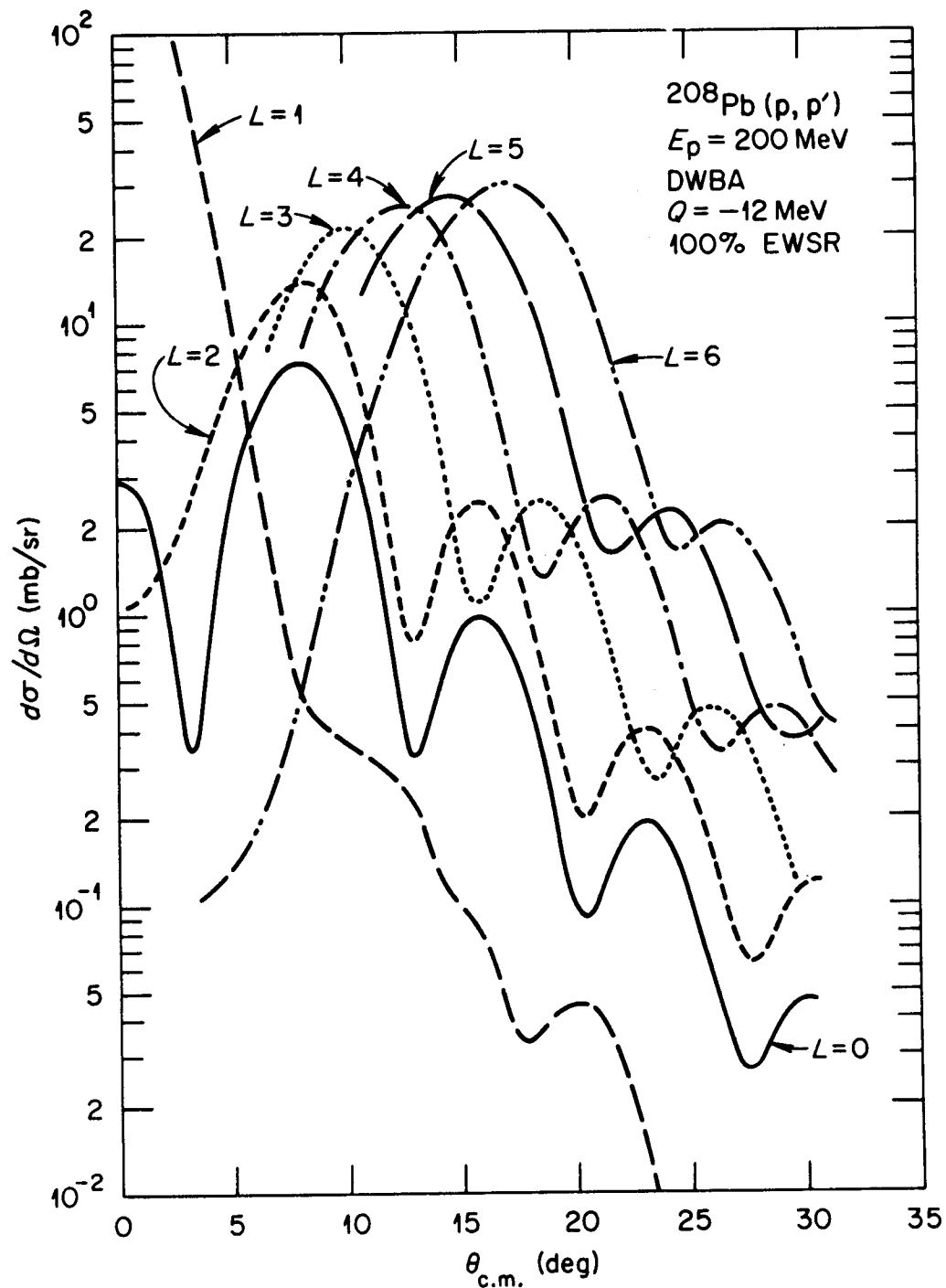


Fig. 18. DWBA calculations for excitation of various multipoles (all isoscalar except  $L=1$  is isovector) by the reaction  $^{208}\text{Pb}(p, p')$  for  $E_p = 200 \text{ MeV}$ .

the case for lower energy protons and comparable energy alphas, large differences exist between the angular distributions of neighboring L-transfer values. For example, the difference between the angular distribution for a pure L=2 giant resonance and a mixed L=2+4 even for only 15-20% L=4 (EWSR) is very large. In addition, the cross sections are considerably larger in the angular distribution maximum than for the same L-transfer using lower energy protons. An additional advantage lies in the fact that the L=1 (isovector GDR) excitation proceeds overwhelmingly by Coulomb excitation which decreases rapidly with increasing angle. For angles greater than  $\sim 5$  degrees the GDR cross section is predicted to be as much as thirty times smaller than the L=0 cross section which occurs at the same excitation energy.

#### A. Isoscalar Giant Octupole Resonances

As discussed earlier there are two classes of octupole transitions, the  $1\hbar\omega$  and  $3\hbar\omega$  classes. In addition, the low-lying collective  $3^-$  states generally account for very little of the T=0, E3 EWSR thus raising the possibility that considerable octupole strength may be found in the  $3\hbar\omega$  transitions.

Recently, what has been interpreted as a localization of  $1\hbar\omega$ ,  $3^-$  strength has been observed<sup>24</sup> by inelastic scattering of alpha particles from a large number of nuclei. Results from these measurements are shown on fig. 19. For most nuclei a broad peak is observed at the systematic energy of  $\sim 32 A^{-1/3}$  MeV. Although for  $^{197}\text{Au}$  the peak falls considerably below this energy. Angular distributions for the peak cross section above the dashed line indicates an L=3 assignment for the

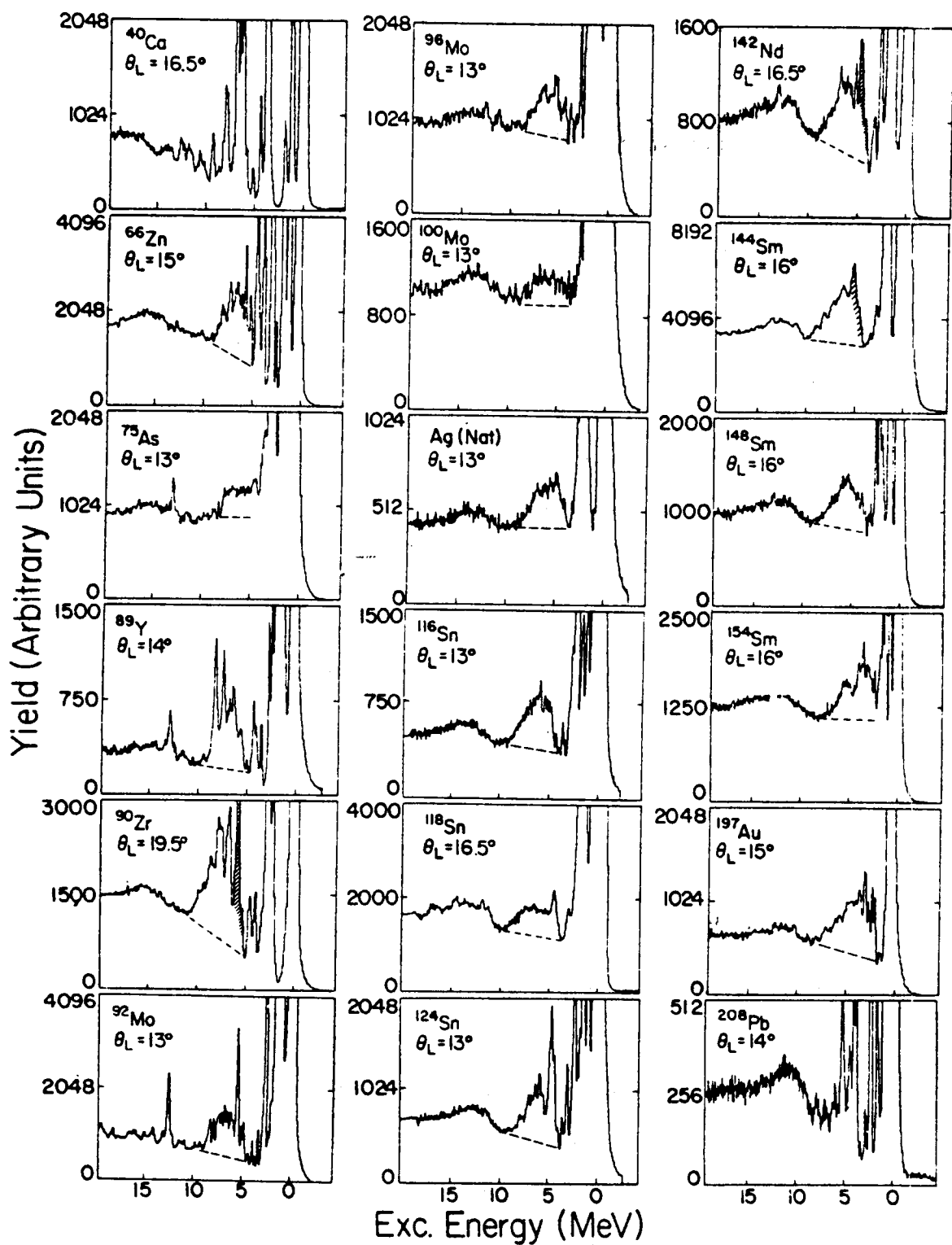


Fig. 19. Inelastic alpha-particle spectra from several nuclei bombarded by 96 and 115 MeV alpha-particles. The low-energy octupole resonance is located above the dashed line (ref. 24).

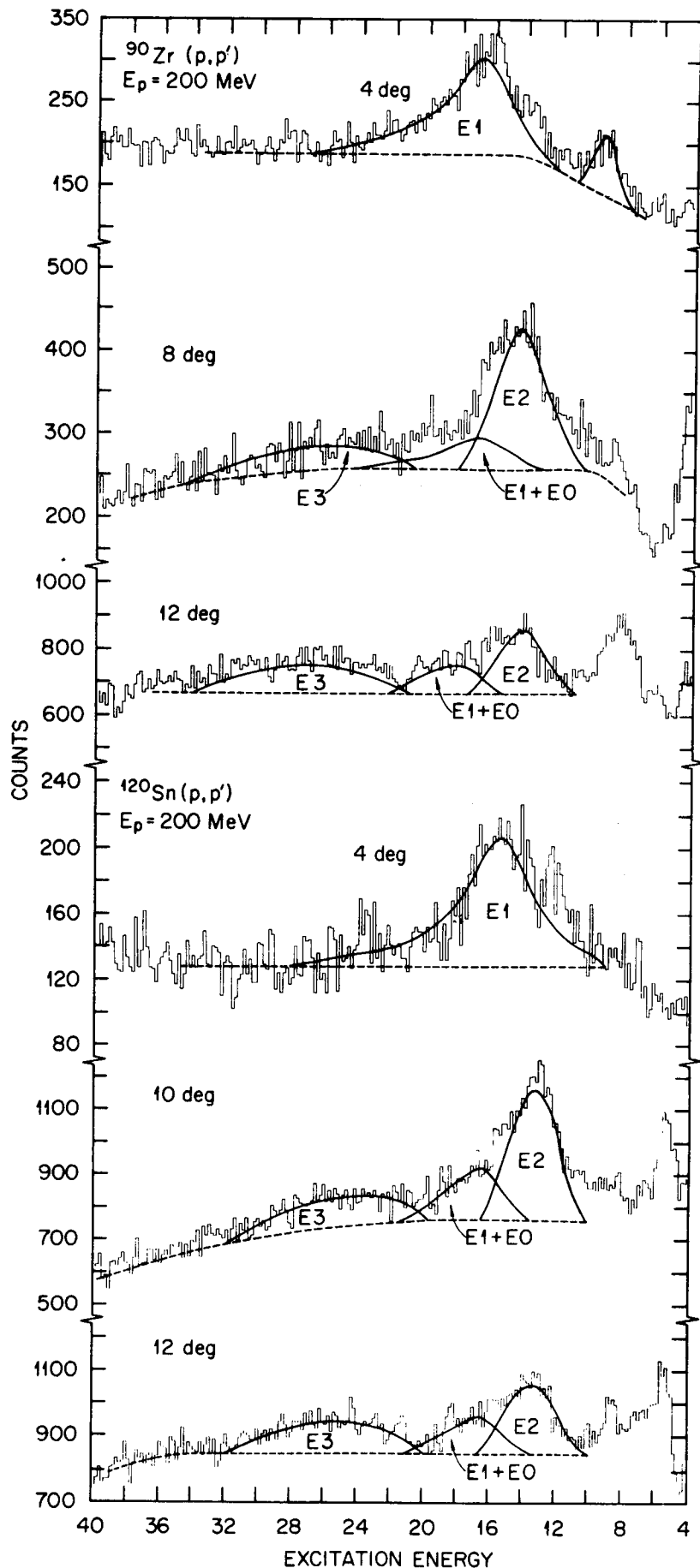
excitation. Suggestions of such strength were also made through the  $^{197}\text{Au}(p,p')$  reaction<sup>25</sup> and the  $^{116}\text{Sn}(e,e')$  reaction.<sup>26</sup> The fraction of the EWSR depleted in the so-called low-energy octupole resonance is 10-20% for most of the nuclei studied. No LEOR is observed in  $^{40}\text{Ca}$  or  $^{208}\text{Pb}$ .

Within the past year systematic observation of a  $3\hbar\omega$  giant octupole resonance (GOR) have been made through inelastic scattering of 800-MeV protons,<sup>27</sup> 172-MeV alpha-particles,<sup>28</sup> and 110-140 MeV helium-3 particles<sup>29</sup> and 200 MeV protons.<sup>30</sup> Figure 20 shows spectra<sup>30</sup> from the reactions  $^{90}\text{Zr}(p,p')$  and  $^{120}\text{Sn}(p,p')$  for 200-MeV protons. Data are shown at angles which should provide maximum cross sections for excitation of the GDR, GQR and GOR.

At  $4^\circ$ , a large peak is observed which is located at the excitation energy of the GDR. The solid curve shown on the  $4^\circ$  data is the GDR shape and energy from  $(\gamma,n)$  measurements,<sup>4</sup> with magnitude fitted to our data. The GDR accounts for virtually the entire observed peak at  $4^\circ$ . The GMR is located at nearly the same energy as the GDR but the DWBA calculations (e.g., fig. 18) indicate that the GMR cross section is at least an order of magnitude smaller than that of the GDR at  $4^\circ$ . Since the E1 cross section drops rapidly with increasing angle, the E0 cross section becomes increasingly more important at larger angles. The  $8^\circ$  and  $10^\circ$  spectra were obtained at a maximum for the L=2 angular distribution for  $^{90}\text{Zr}$  and  $^{120}\text{Sn}$ , respectively. The solid curve, taken from the shape and location of the GQR as determined from  $(\alpha,\alpha')$  measurements,<sup>8</sup> provides excellent agreement with the present data. The cross section

Figure 20

Spectra from inelastic scattering of 200-MeV protons from  $^{90}\text{Zr}$  and  $^{120}\text{Sn}$  (ref. 30). The multipolarities shown for the resonances are discussed in the text. The dashed line shows the shape and magnitude assumed for the nuclear continuum underlying the resonance peaks.



for the GDR at the larger angles is greatly reduced from that observed at  $4^\circ$ . It can be assumed that at these angles, the cross section in the excitation energy region where GDR is located is comprised of GDR and GMR components in unestablished proportions. In the  $12^\circ$  spectra broad peaks seen somewhat less clearly at  $8^\circ$  and  $10^\circ$  are observed at  $25 \pm 1$  and  $27 \pm 1$  MeV in  $^{120}\text{Sn}$  and  $^{90}\text{Zr}$ , respectively. Figure 21 shows angular distributions for these peaks. The data are seen to be very well described by the  $L=3$  DWBA calculation but not at all by the  $L=4$  calculation. Thus, we interpret these peaks as arising from excitation of the  $3\hbar\omega$ , GOR (E3).

Similar data were obtained<sup>27</sup> earlier from inelastic scattering of 800 MeV protons. Angular distributions from these data are shown in fig. 22. The data agree with the very characteristic angular distribution calculated for  $L=3$ . Figure 23 shows a spectrum from the  $^{208}\text{Pb}(\alpha, \alpha')$  reaction<sup>28</sup> using 172-MeV alpha-particles. In addition to the GQR at 10.9 MeV and the GMR at 13.8 MeV the data indicate another peak which the authors subdivide into a peak at 17.5 MeV and one at 21.3 MeV. The angular distribution for the 17.5 MeV peak agrees very well with an  $L=3$  DWBA calculation. (The authors assign the 21.3 MeV peak as an isoscalar dipole resonance [ $L=1, T=0, S=0$ ].) The inelastic helium-3 measurements<sup>29</sup> were made on a wider mass range of nuclei and the angular distributions of the peaks are again well described by  $L=3$ .

The results from these measurements are summarized on fig. 24. The resonance excitation energy follows the systematic trend of  $\sim 110 A^{-1/3}$  MeV. The dashed line gives the value of  $108.2 A^{-1/3}$  MeV calculated in

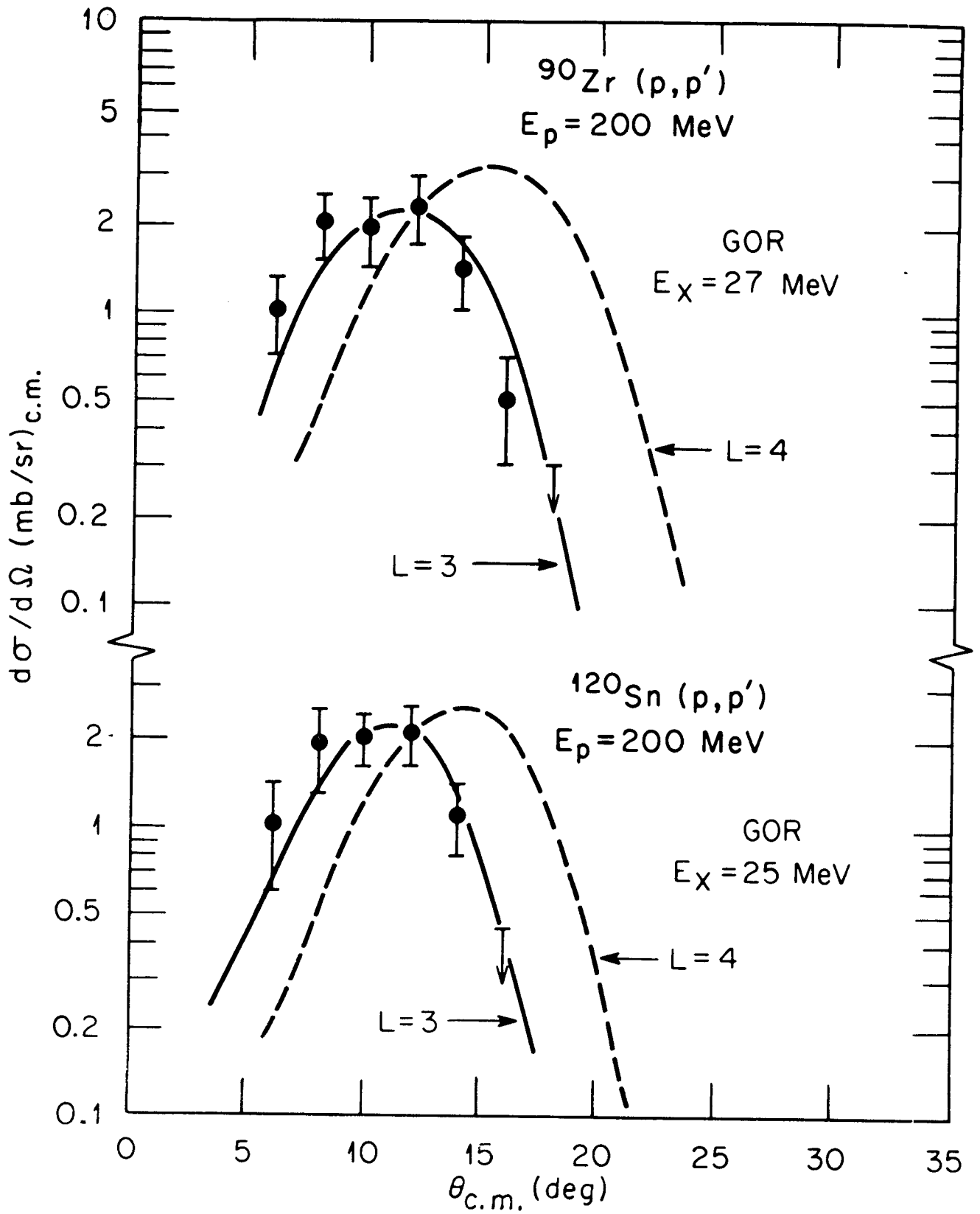


Fig. 21. Angular distributions for the GOR compared with DWBA calculations (solid and dashed curves) (ref. 30).

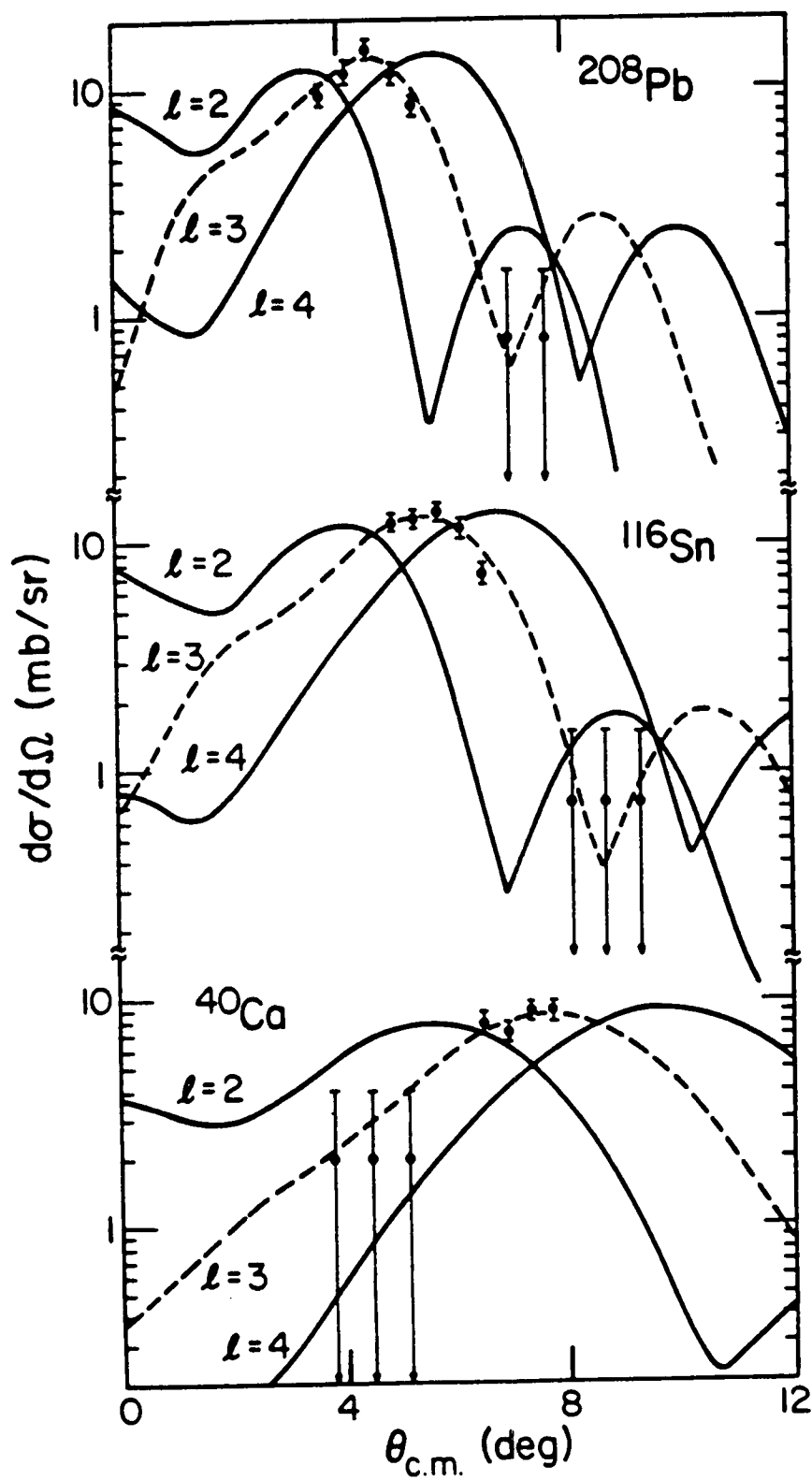


Fig. 22. Angular distributions for the giant octupole resonance as excited by 800-MeV protons (ref. 27).



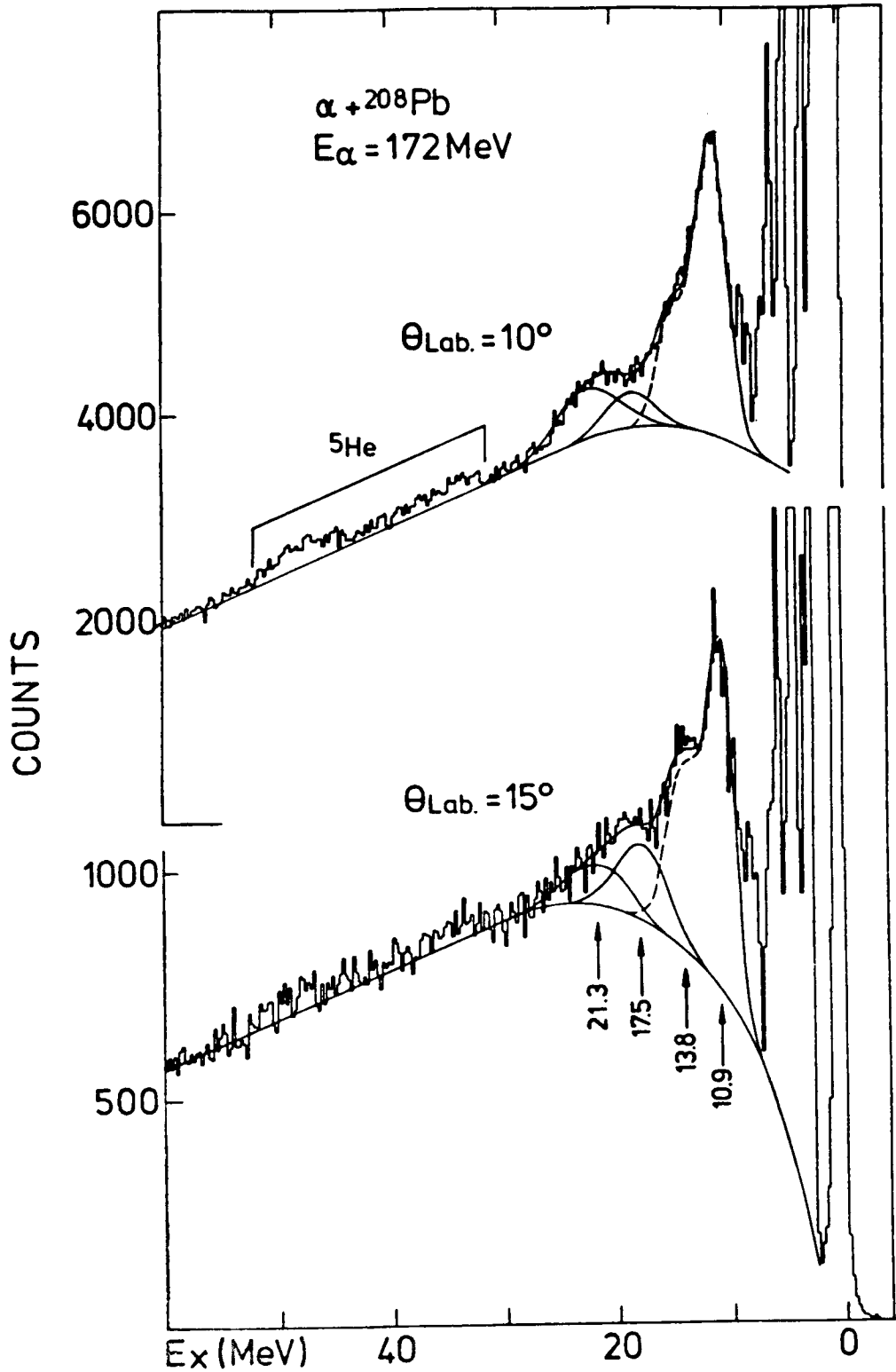


Fig. 25. Giant resonance spectra from inelastic scattering of 172-MeV alpha particles from  ${}^{208}\text{Pb}$ . The resonance structure is decomposed into a uQR (10.9 MeV), GMR (13.8 MeV), a proposed giant octupole resonance (17.5 MeV) and an isoscalar giant dipole resonance (21.3 MeV) (ref. 28).

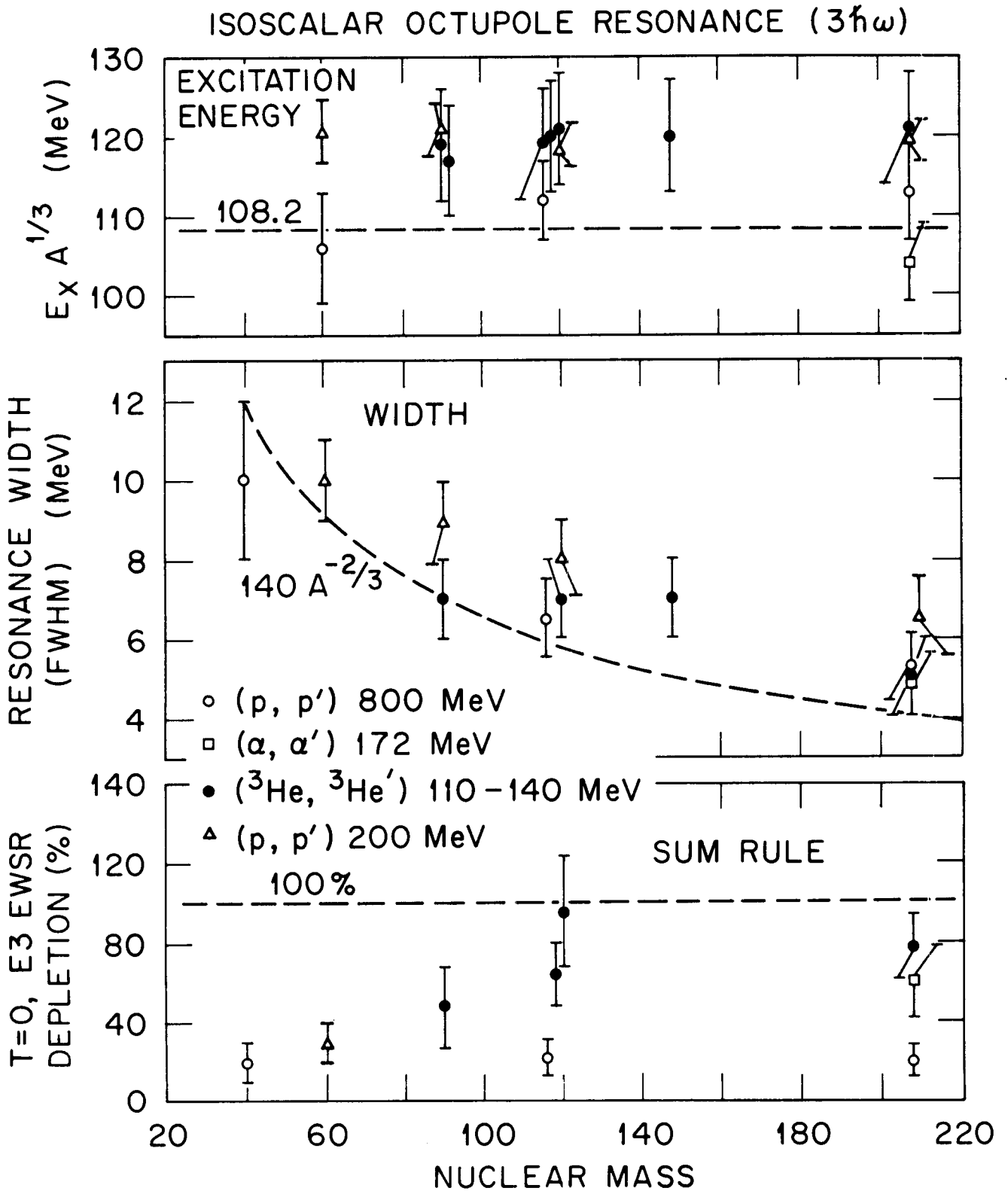


Fig. 24. Systematics of the excitation energy width and sum rule depletion for the isoscalar octupole resonance as proposed from the indicated experiments (refs. 27, 28, 29 and 30).

ref. 12. The resonance width again follows the familiar trend of increasing width with decreasing nuclear mass. The dashed curve for the value  $140 A^{-2/3}$  is from ref. 12 where the constant has been adjusted from the author's suggested value in order to fit the experimental results. There is clear discrepancy in the sum rule depletion as deduced from the  $(\alpha, \alpha')$  and  $(^3\text{He}, ^3\text{He}')$  results on the one hand and the  $(p, p')$  results on the other hand. The EWSR strength depleted in giant resonances is inherently difficult to extract from inelastic hadron measurements because uncertainties in both the continuum shape and the model dependent calculations must be contended with. If one accepts, for example, the EWSR value for the T=0 GOR of  $\sim 60\%$  for  $^{208}\text{Pb}$  from the  $(\alpha, \alpha')$  measurement then all of the T=0 E3 sum rule strength is accounted for since  $\sim 45\%$  of the strength lies in low lying states.<sup>31</sup>

These results suggest that the GOR ( $3\hbar\omega$ ) has been located. However, as was the case ten years ago for the GQR and three years ago for the GMR, more data are needed to firmly establish the identity and systematics of this resonance.

#### B. Isoscalar Hexadecapole Giant Resonance

There has been no direct, (i.e. - peak observed in a spectrum), experimental evidence for an E4 giant resonance (4,0,0). RPA calculations for  $^{208}\text{Pb}$  have shown<sup>32</sup> that the  $2\hbar\omega$  giant hexadecapole resonance (GHR) should occur at virtually the same excitation energy as the  $2\hbar\omega$  GQR. Other calculations<sup>33</sup> on  $^{208}\text{Pb}$  indicate that  $\sim 40\%$  of the L=4 EWSR should be located in the  $2\hbar\omega$  transitions and  $\sim 60\%$  in the  $4\hbar\omega$  transitions. It was shown<sup>34</sup> through microscopic DWBA calculations

that the resonance cross sections from 61 MeV (p,p') and 96 MeV ( $\alpha,\alpha'$ ) measurements on  $^{208}\text{Pb}$  were consistent with the existence of  $\sim 20\%$  of the  $T=0$  E4 EWSR within the (predominately) GQR peak. However, the inclusion of L=4 strength was based only on the total resonance cross section since the angular distributions could not definitively indicate a necessity for inclusion of GHR strength.

While the angular distributions for L=2 and L=4 excitations are not very different in low energy proton and alpha-particle inelastic scattering, as pointed out earlier, the differences are very large for higher energy inelastic proton scattering. The angular distribution differences between L=2 and L=4 should provide for rather easy detection of a 20-40% mixture of E4 strength in the GQR peak even with uncertainties on the data as large as 20-30%.

Figure 25 shows angular distributions<sup>30</sup> for the GQR peak from the reaction  $^{90}\text{Zr}(p,p')$  and  $^{120}\text{Sn}(p,p')$  (fig. 20). The comparisons of fig. 25 show that the GQR cross sections are well described by the L=2 DWBA calculation. The sensitivity of these results to higher multipole contributions in the GQR peak is shown by the dashed curve which results from a mixture of only 5% of the L=4, EWSR, with the L=2 angular distribution. Clearly, the GQR peak contains very little, if any, contribution from excitation of a  $2\hbar\omega$ , E4, giant resonance. This statement only indicates that there is no major L=4 strength within the GQR peaks shown in fig. 20. Strength from  $2\hbar\omega$ , E4, resonance excitation may be present but, may be many MeV wide and thus, escape detection in the present experiment. Similar results have been obtained for  $^{60}\text{Ni}$  and  $^{208}\text{Pb}$ .

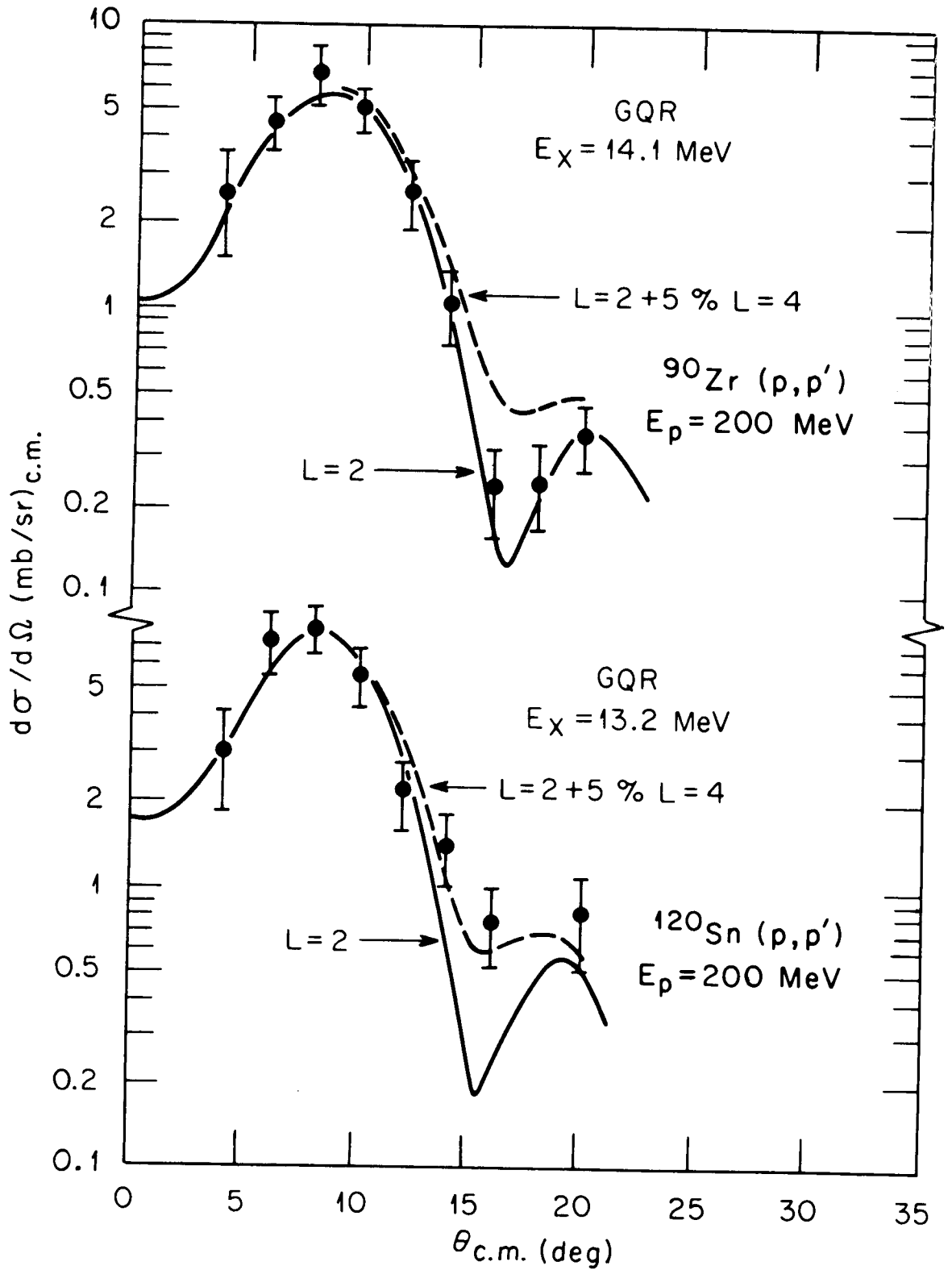


Fig. 25. Angular distributions for the GQR peak in  $^{90}\text{Zr}$  and  $^{120}\text{Sn}$  excited via 200 MeV ( $p, p'$ ) compared with DWBA calculations (solid and dashed curves) (ref. 30).

The location of the GHR is thus an unsolved question. If the strength is localized at all it will probably be situated at very high excitation energies. The calculations of ref. 12 place the  $L=4$  strength at  $150 A^{-1/3}$  MeV or  $\sim 25$  MeV in  $^{208}\text{Pb}$ . It is also very likely that the peak would be even broader than that reported for the GOR. Location of the GHR presents a considerable experimental challenge as does the location of even higher multipole resonances.

## VII. Future Directions

There are many uses of direct reactions for giant resonance studies that I have not described. Of these, the recent work in charge exchange studies of isospin/spin flip giant resonances is most notable. Such studies as  $(p,n)$ ,  $(n,p)$  and  $(\pi^{\pm}, \pi^0)$  require another lecture in order to do justice to the topic. I believe the recent results from these charge exchange measurements are only the beginning of a very exciting new era of measurement in giant resonances.

Since this discussion has dealt nearly exclusively with excitation of the so-called new giant resonances via inelastic scattering of medium energy hadrons, I would like to mention two inelastic reactions which show potential promise for giant resonance studies, but which have not yet been exploited. Some of the future direction in this field is certainly to be found in the use of heavy-ion inelastic scattering and pion inelastic scattering. I briefly discuss some very new results from the heavy-ion field.

The primary advantage to heavy-ion excitation of giant resonances is the large resonance cross section expected with higher energy incident beams. Figure 26 is a plot of maximum cross section (grazing

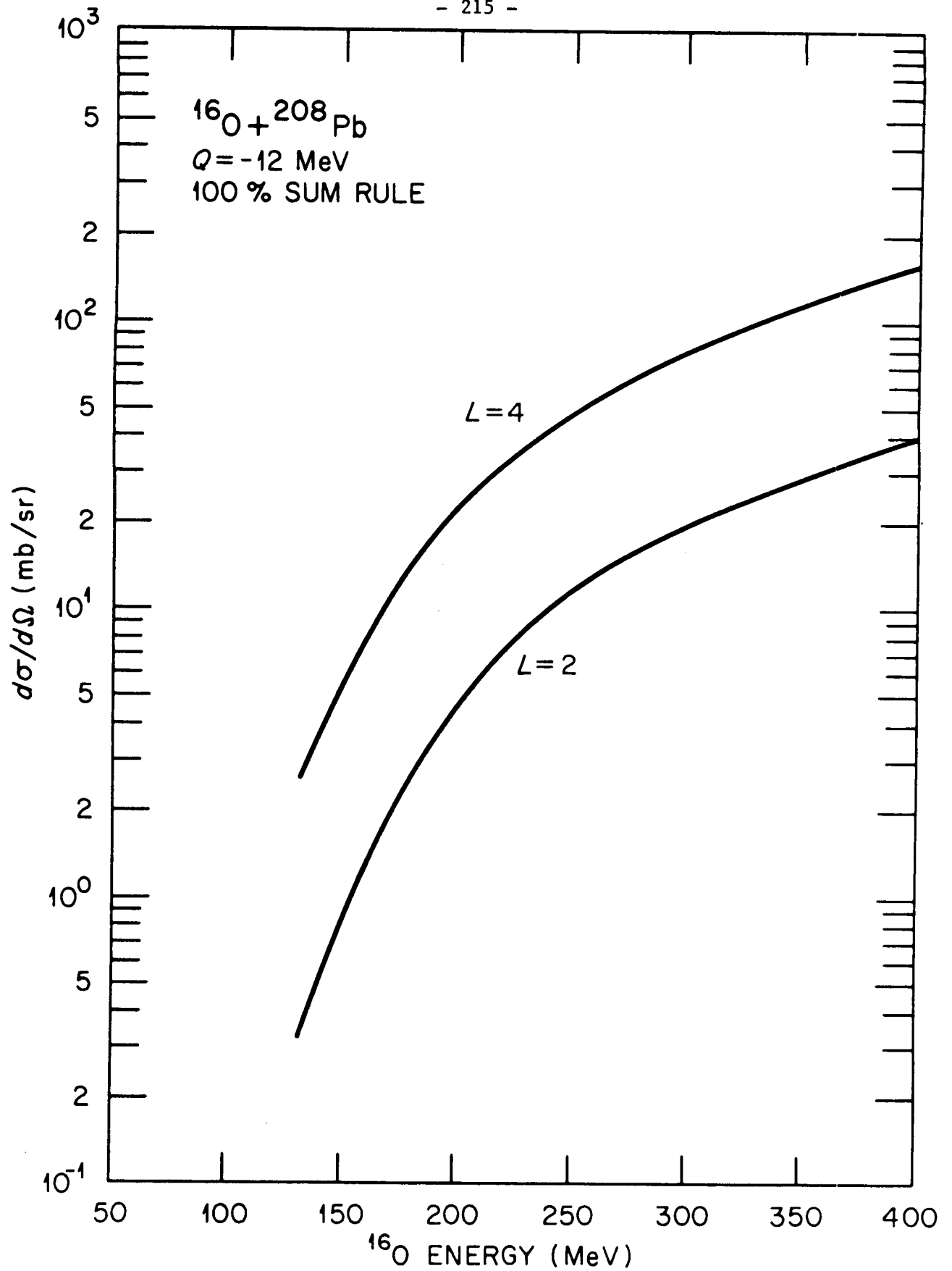


Fig. 26. Grazing angle cross sections calculated for  $L=2$  and  $L=4$  excitations by the reaction  $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O}')$  plotted as a function of incident  $^{16}\text{O}$  energy (100% EWSR).

angle) versus incident energy for  $L=2$  and  $L=4$  excitation ( $Q = -12$  MeV, 100% EWSR) in the reaction  $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O}')$ . For 400 MeV  $^{16}\text{O}$  beams the GQR cross section should be  $\sim 50$  mb/sr, nearly five times larger than for the 200 MeV  $(p,p')$  reaction and twice as large as achieved with 150 MeV alpha-particle inelastic scattering. Some measurements of giant resonance excitation using low-energy heavy-ions have been made.<sup>35</sup> However, the small cross sections realized in those measurements provides no advantage, in fact, provides a disadvantage over what can be achieved with other probes.

Recently, measurements<sup>36</sup> have been made of inelastic excitation of giant resonances using the 400 MeV  $^{16}\text{O}$  beam from the HHIRF accelerator at Oak Ridge National Laboratory. A partial spectrum of the giant resonances is shown on fig. 27. The GQR is clearly visible at 10.9 MeV as is a peak at 13.7 MeV which arises from excitation of the GMR and GQR. The peak at 17.6 MeV is not completely understood but may be the same as the peak reported<sup>28</sup> to be  $L=3$  in 172 MeV alpha-inelastic scattering (fig. 23). The peak at 24.8 MeV probably arises from kinematical nucleon pick-up and subsequent decay processes. The large peak (off scale) at  $\sim 7$  MeV of excitation arises from excitation of states in the  $^{16}\text{O}$  projectile which are Doppler broadened. The solid lines indicate fits to the peaks and the estimation of the shape and magnitude of the underlying continuum.

The most obvious giant resonance feature in the spectrum of fig. 27 is the very large ratio of the GQR peak height to the height of the underlying continuum. The ratio is  $\sim 2.2:1$ . Using 152-MeV alpha particles<sup>8</sup> the best ratio obtained for the GQR in  $^{208}\text{Pb}$  is  $\sim 1:1$ .



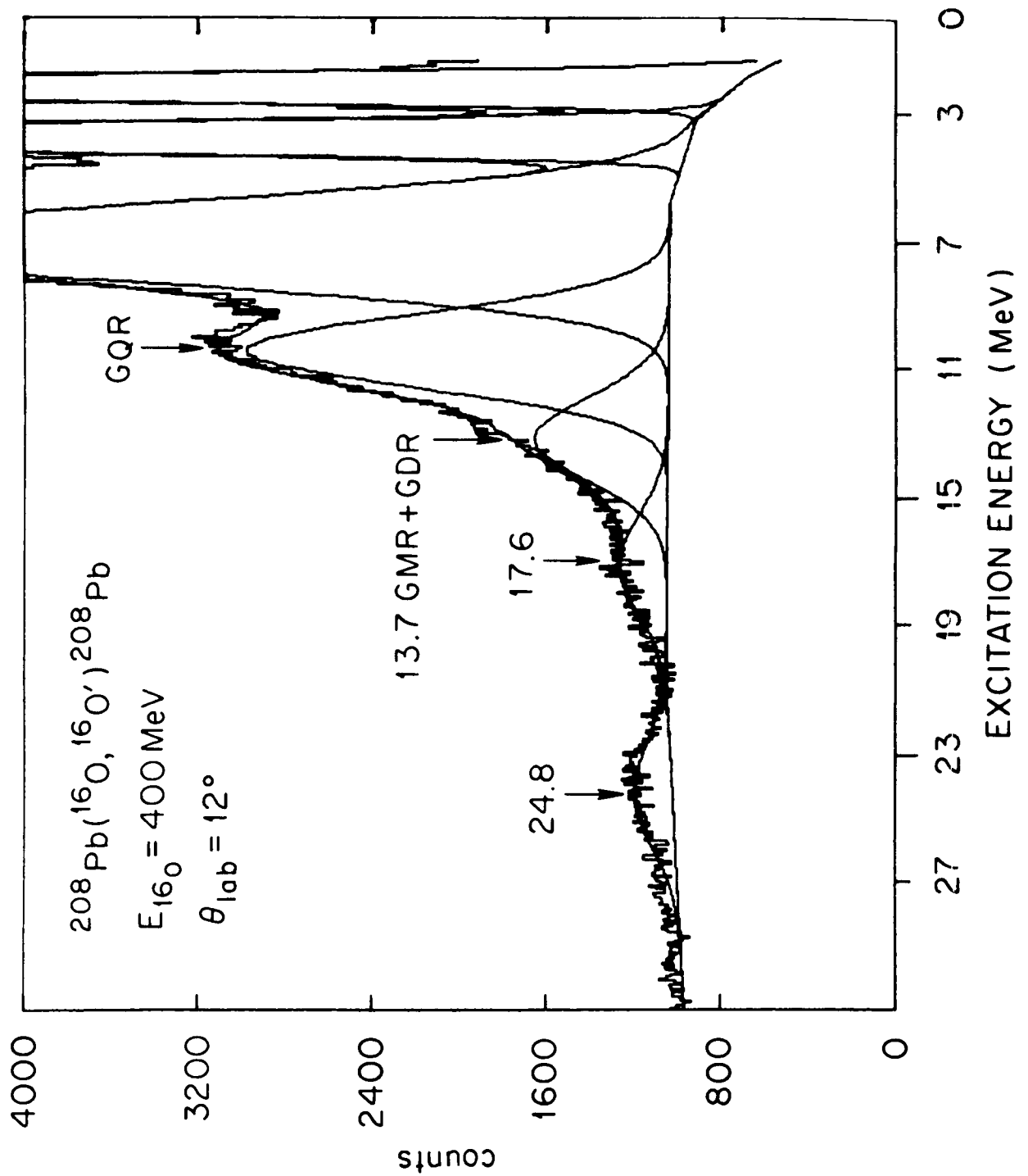


Fig. 27. Spectrum from the reaction  $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O}')^{208}\text{Pb}$  for  $E_{160} = 400 \text{ MeV}$ .

The solid curves are fits to the peaks and an estimate of the shape and magnitude of the underlying continuum (ref. 36).

The measured angular distributions for the GQR and GDR + GMR peaks are shown on fig. 28. Indeed the GQR cross section reaches a value of 50 mb/sr over twice that obtained using 150 MeV incident alphas. The calculations show that  $\sim 100\%$  of the GQR is accounted for, in excellent agreement with results using other probes. However, the calculations assuming 100% of the GMR cannot, by nearly a factor of three at small angles, account for the magnitude of the 13.7 MeV peak. Since the  $^{16}\text{O}$  projectile is isoscalar it is expected that nuclear excitation of the GDR would be very small. Coulomb excitation of the GDR is indeed possible and the calculated GDR cross section is shown on fig. 28. However, the sum of the GDR and GMR excitations do not quite agree with the data. It is to be noted that there is no low-lying GMR state in which to test the calculation.

Thus, use of heavy-ion inelastic scattering does indeed, provide a very large GQR cross section with the added benefit of a very large peak to continuum ratio. This combination makes the heavy-ion probe very useful for excitation of the GQR and measurement of the resonance subsequent decay modes, especially rare decay modes.

#### VIII. Summary

Through this rather rapid summarization of the experimental measurements of electric giant multipole resonances hopefully one sees that the field has progressed a great deal during the past ten years. The parameters of the giant quadrupole resonance are now firmly established by an extensive set of measurements. The GQR is providing a significant influence in other areas of nuclear physics. The monopole resonance has now been established

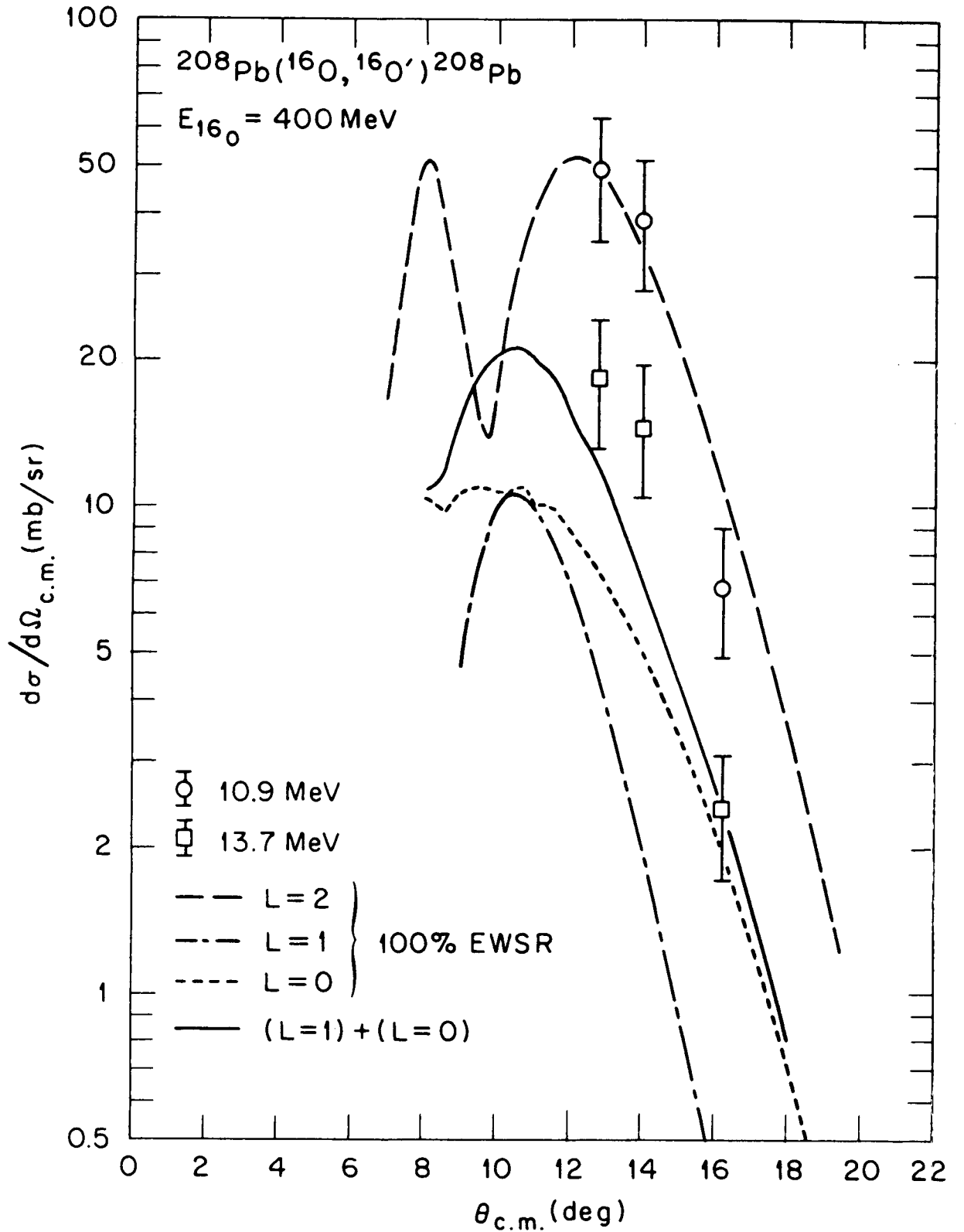


Fig. 28. Angular distributions for excitation of the GQR (10.9 MeV) and the GDR + GMR (13.7 MeV) peak. DWBA calculations for indicated L-transfers are shown as solid and dashed curves (ref. 36).

and its observation has provided the first direct measure of the nuclear compressibility. A strong case for the existence of a giant octupole resonance is now being made through a variety of hadron reactions.

However, we certainly have not exhausted the supply of giant multipole resonances. The newer techniques such as higher energy proton scattering, charge exchange reactions, heavy-ion scattering and pion reactions offer considerable hope for identifying new resonances during the next few years. We have come a long way in the past ten years but there is still a great deal to do in the giant multipole resonance field.

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The Experimental Tools to Exploit the Daresbury NSF

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Contents

- I. Introduction:- The Characteristics of the NSF
- II. Experimental Programme:- The Experimental Areas
  - II.I Gamma-ray experiments
  - II.II The QMG/2 spectrometer and 1 m scattering chamber
  - II.III The Recoil Separator:- A brief description of the properties of the Recoil Separator. An outline of the kind of experiment possible with this device
  - II.IV The Isotope Separator:- The design of the separator
- III. Initial Research Programme with the Separator:- Summary of possible uses
  - III.I Nuclear orientation:- The properties of the  $^3\text{He}$ : $^4\text{He}$  dilution refrigerator. Nuclear orientation experiments. First results. Limitations of these measurements
  - III.II Laser hyperfine studies:- What we can learn from laser hyperfine studies. Experiments at Birmingham. Proposed experiments at Daresbury. The collinear beam method.
- IV. The Poor Man's Alternative to a Recoil Separator and an Isotope Separator:-
  - In-beam  $\gamma$ -ray spectroscopy of nuclei far from stability with the use of neutron- and charged particle-detection. Studies of the light Sn nuclei and nuclei with  $N \sim Z \sim 40$ . Studies of radioactive decay products transported with a He-jet recoil transport system. Studies of the  $\beta$ -decays of the neutron-deficient Lu isotopes.

### INTRODUCTION

You are all familiar with the Segré chart shown in fig. 1 with all of the stable isotopes shown as black squares, and with the regions of deformation and the proton and neutron closed shells marked. Until the last few years our knowledge of the level structure of these nuclei has been limited in three main ways. Firstly it was limited to states of relatively low spin. Secondly it was confined to states at low excitation, and thirdly it was restricted to nuclei near the line of stability. In the 1970s with the advent of a new generation of accelerators and improved instrumentation this situation has changed rapidly. We have learned a great deal about high spin states<sup>(1)</sup>, and we now have 'complete' level schemes up to 2 MeV, within a certain range of spin, for a small number of nuclei<sup>(2,3,4)</sup> derived from  $(n,\gamma)$  studies. With heavy ion and high energy proton beams we have also pushed out far from stability with studies of both in-beam gamma-ray emission and radioactive decay. From these developments we hope not only to

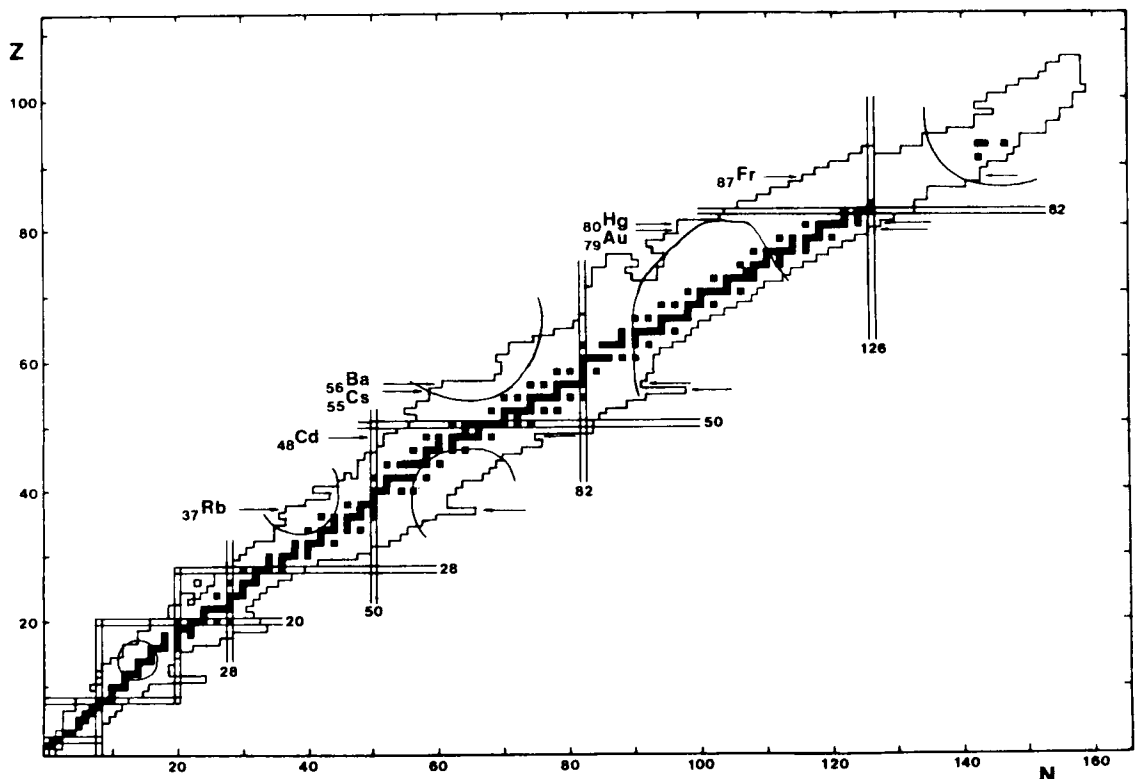


Fig. 1: A version of the Segré chart showing the stable nuclei as a function of  $Z$  and  $N$ . The magic numbers and known areas of deformation are shown.

observe new phenomena, but to gain greater insight into nucleon-nucleon forces from observations under a wider range of conditions, and to produce better models of nuclear structure. New and improved nuclear models have already appeared<sup>(5-7)</sup> over the last few years.

As you have recognised at C.N.E.A., to participate in these pioneering efforts near the frontiers of Nuclear Physics requires a suitable accelerator. Since 1974 a Tandem Van de Graaff with a planned terminal voltage of 30 MV has been under construction at Daresbury in Cheshire. This accelerator, known as the Daresbury Nuclear Structure Facility (NSF), is now complete and it is planned to have the first real beam from it during the week of this meeting. My brief in this lecture is to outline the experimental facilities available at or shortly after start-up, and to give you some feeling for the kind of Physics we plan to do with the facilities.

I will not say much about the accelerator itself other than to remind you of its main features and properties since they dictate the nature of the experimental programme to a large extent. The NSF is a conventional, vertical tandem, which should operate initially at 20 MV while we learn how to run it, and be capable of running at 30 MV terminal potential eventually. The design<sup>(8)</sup> incorporates a number of unconventional features aimed at achieving the required voltage and machine performance.

Figure 2 shows the expected energy per nucleon of the beam projectiles as a function of the projectile mass number  $A$ . Curves a) and b) are for terminal voltages 30 MV and 20 MV respectively. These results were calculated assuming foil stripping in the centre terminal and a second stripper foil placed one third of the way down the high-energy column. This figure also shows the Coulomb barrier for a Uranium ( $Z = 92$ ) target as a function of  $A_{\text{proj}}$ , and the Coulomb barrier for  $Z_{\text{proj}} = Z_{\text{target}}$ . At 20 MV we are over the barrier on U for ions up to  $A \sim 80$ , and at the design voltage of 30 MV ions with  $A \sim 160$  are over the barrier.

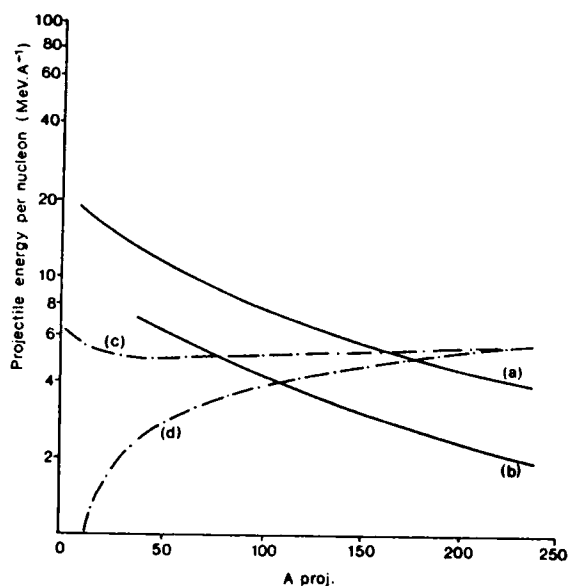


Fig. 2: This shows the predicted projectile energy per nucleon as a function of  $A_{proj}$  for the NSF at a) 30 MV, and b) 20 MV. Curve c) is the Coulomb barrier for  $Z_b = 92$ , and d) gives the Coulomb barrier for  $Z_{proj} = Z_b$ .

Some of the design figures for typical transmission efficiencies, charge states and beam currents for a variety of heavy ions are summarised in Table I<sup>(9,10)</sup>. The first and second strippers were assumed to be gas and foil strippers respectively. For some of these beams, tube loading and other factors will mean that the final beam currents will be well below the quoted values.

Table I. Transmission efficiencies, charge states and projected beam currents on target for some heavy ion beams. The slits of the 90° analysing magnet for the accelerator have been assumed to be wide open

| Ion type | 20 MV                |          |              |    |              |    |                    | 30 MV            |          |              |    |              |    |                    |                  |
|----------|----------------------|----------|--------------|----|--------------|----|--------------------|------------------|----------|--------------|----|--------------|----|--------------------|------------------|
|          | Initial beam (μA-ve) | % Trans. | 1st Stripper |    | 2nd Stripper |    | Final energy (MeV) | Final beam (pnA) | % Trans. | 1st Stripper |    | 2nd Stripper |    | Final energy (MeV) | Final beam (pnA) |
|          |                      |          | Charge state | %  | Charge state | %  |                    |                  |          | Charge state | %  | Charge state | %  |                    |                  |
| Cl       | 10                   | 14       | 8            | 35 | 13           | 39 | 246                | 1400             | 15       | 9            | 35 | 14           | 44 | 400                | 1500             |
| Fe       | 0.5                  | 8        | 8            | 29 | 17           | 27 | 300                | 40               | 8        | 9            | 29 | 19           | 29 | 500                | 40               |
| Br       | 10                   | 6        | 8            | 25 | 20           | 23 | 340                | 600              | 6        | 9            | 25 | 23           | 23 | 580                | 600              |
| I        | 10                   | 4        | 7            | 20 | 21           | 20 | 346                | 400              | 5.6      | 9            | 20 | 27           | 28 | 660                | 560              |
| Ta       | 1.3                  | 3.1      | 7            | 17 | 23           | 18 | 373                | 40               | 3.9      | 9            | 17 | 29           | 23 | 700                | 50               |
| Au       | 10                   | 3.1      | 7            | 17 | 23           | 18 | 373                | 300              | 3.7      | 9            | 17 | 30           | 22 | 720                | 360              |
| Pb       | 0.5                  | 3.1      | 7            | 17 | 23           | 18 | 373                | 15               | 3.6      | 9            | 17 | 30           | 21 | 720                | 18               |
| Bi       | 0.3                  | 3.1      | 7            | 17 | 23           | 18 | 373                | 9                | 3.6      | 9            | 17 | 30           | 21 | 720                | 10               |
| U        | 2                    | 2.9      | 7            | 16 | 23           | 18 | 373                | 58               | 3.2      | 9            | 16 | 30           | 20 | 720                | 64               |

The percentage transmissions quoted above are in terms of the number of particles in the incident negative ion beam. The charge states quoted for each stripper are the most probable states and the percentages quoted at each stripper are for the number of particles transmitted for further acceleration, taking into account the fraction in the charge state, the initial emittance of the incident particle beam and the multiple scattering introduced by the stripper. The estimated final beams are, as is usual, quoted in particle nanoamps. The maximum input beam has been limited to 10 μA for convenience.

The expected machine performance in terms of energy resolution is summarised in Table II. For light ions terminal stability is the limiting factor, but for heavier ions straggling in the second (foil) stripper eventually dominates. Even for the heaviest ion given here, namely Iodine, the resolution  $\delta E/E$  is expected to be less than or equal to  $3 \times 10^{-4}$ .

Table II. *Beam energy resolution. A two stripper system with a terminal potential of 30 MV was taken. The centre terminal stripper was assumed to be argon gas with an equivalent thickness of  $0.01 \text{ g m}^{-2}$  and the second stripper a  $0.1 \text{ g m}^{-2}$  carbon foil with a uniformity of 10%. The terminal voltage stability was taken as 1 in  $10^4$*

| Ion                                    | Lithium            | Carbon             | Sulphur              | Bromine              | Iodine             |
|----------------------------------------|--------------------|--------------------|----------------------|----------------------|--------------------|
| Energy loss in terminal stripper (keV) | 0.5                | 3                  | 15                   | 20                   | 20                 |
| Straggle from terminal stripper (keV)  | 0.1                | 0.6                | 3                    | 4                    | 4                  |
| $\bar{Q}_1$ terminal stripper          | 3                  | 6                  | 8                    | 8                    | 8                  |
| Energy loss in second stripper (keV)   | 4                  | 15                 | 150                  | 500                  | 730                |
| Straggle from second stripper (keV)    | 0.8                | 3                  | 30                   | 100                  | 150                |
| $\bar{Q}_2$ second stripper            | 3                  | 6                  | 12                   | 22                   | 25                 |
| Terminal stability (keV) energy spread | 12                 | 21                 | 35                   | 55                   | 61                 |
| Foil inhomogeneity (keV)               | 0.4                | 1.5                | 15                   | 50                   | 73                 |
| Final resolution (keV)                 | 12                 | 20                 | 50                   | 125                  | 180                |
| Final energy (MeV)                     | 120                | 210                | 350                  | 550                  | 610                |
| $\delta E/E$                           | $1 \times 10^{-4}$ | $1 \times 10^{-4}$ | $1.4 \times 10^{-4}$ | $2.3 \times 10^{-4}$ | $3 \times 10^{-4}$ |

The main advantages of a Tandem Van de Graaff are its energy precision, easy energy variability, d.c. beam, the wide range of beam species which can be accelerated, and the pulsing system we plan eventually. Thus the NSF should be a flexible and powerful tool for tackling problems in Nuclear Structure Physics.

Figure 3 shows the completed NSF Tower with the ion-source room on top and the beam rooms on the right.

Figure 4 shows a cut-away view of the NSF. The tower is 70 m high with the ion source and injector equipment housed at the top. Below the ion source room is the 45 m high pressure vessel. There is 13.7 m of active column above and below the centre terminal. This centre terminal contains quite a lot of equipment including gas and foil stripping, followed by an offset quadrupole triplet for charge state selection<sup>(11)</sup>. It also contains pumping, valves and diagnostic equipment. The centre terminal is surrounded

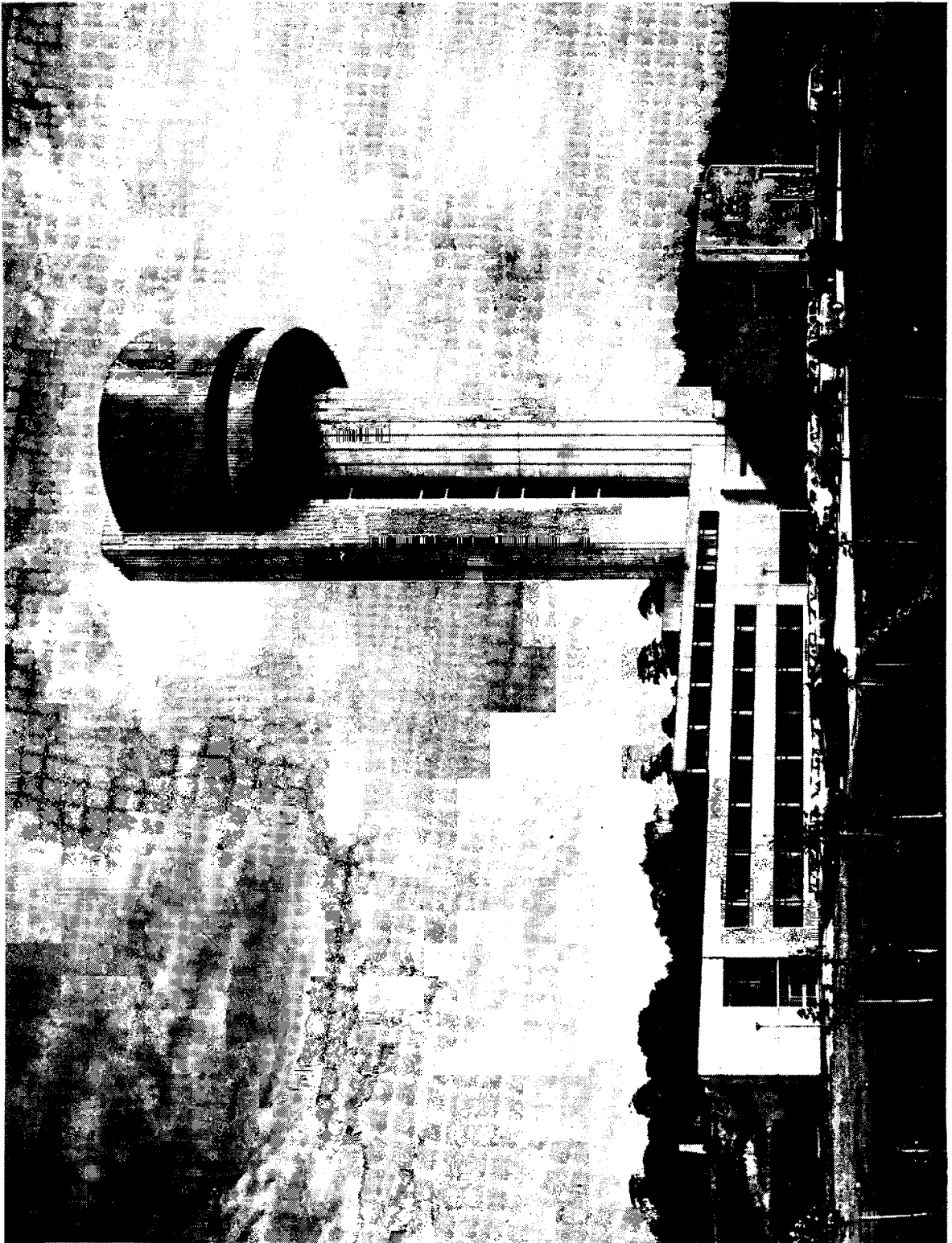


Fig. 3: A view of the NSF accelerator tower at Daresbury.

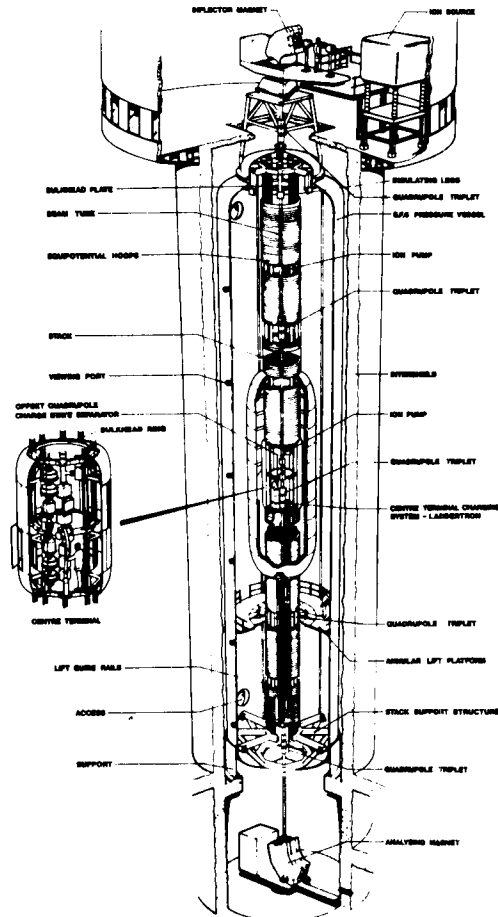


Fig. 4: A cut-away view of the NSF accelerator tower.

by an intershield which is required to reduce the Electric field at the terminal. To achieve 30 MV without an intershield one would have to increase the dimensions of the machine with considerable increase in cost.

While we waited for the machine to be ready we have been doing two things. We have been preparing equipment for use at the NSF, and at the same time we have been running experiments elsewhere. It is my task to give you some idea of both the experimental programme we have prepared and a few of the experiments we have been doing.

## II. EXPERIMENTAL PROGRAMME

Figure 5 shows the general layout of the beam rooms at the NSF and some of the equipment. We see the central accelerator tower in the middle



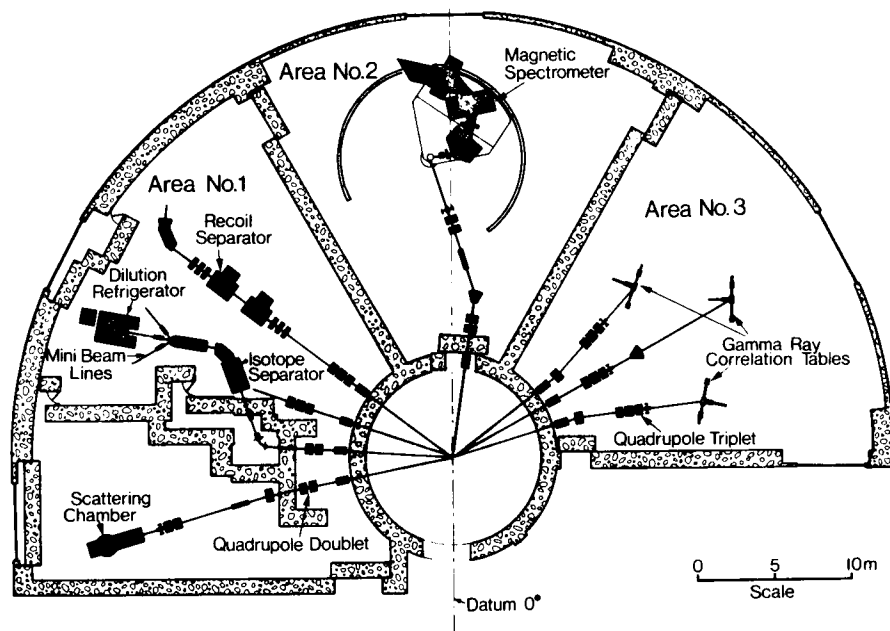


Fig. 5: This shows the layout of the three experimental areas at the NSF with the central accelerator tower in the middle of the diagram.

of the diagram. At the foot of the machine, below the pressure vessel, is a  $90^\circ$  analyzing magnet mounted on a hydrostatic bearing. This allows us to rotate the magnet to direct the beam into any beam line. The experimental area is divided into three rooms, which are separated by 1 m thick concrete walls.

### II.1 Gamma-Ray Experiments

These three areas are devoted to different experimental activities. Area 3 for example is mainly for  $\gamma$ -ray spectroscopy where we will have low radiation beams. There is a wide variety of  $\gamma$ -ray detection equipment available, which I will not describe in detail. (HI,xn $\gamma$ ) studies to obtain information on high spin states in rare-earth and actinide nuclei will be one of the main activities at the NSF, and one piece of apparatus developed for these studies is worth mentioning. A group at Liverpool University<sup>(12)</sup> has developed an array of Compton-suppressed Ge(Li) detectors which all

view a common target. In this system six, NaI Compton-suppression shields are mounted in a vertical plane through the target. The six Ge(Li) detectors within the shields view the target at  $\pm 30^\circ$ ,  $\pm 90^\circ$  and  $\pm 150^\circ$ , and the centre of each detector is 27 cm from the target. A NaI backscatter detector is mounted at the entrance of the suppression shields. A 'ball', consisting of 60 BGO crystals of hexagonal cross-section, surrounds the target and provides sum energy and gamma-ray multiplicity information. Analogue signals provide information about the sum energy and the number and location of detectors contributing to the sum. This system will provide Ge(Li) spectra with peak/total ratios of  $\sim 70\%$  at 1.33 MeV, and in  $\gamma$ - $\gamma$  coincidence experiments the proportion of peak-peak coincidences will approach 50% of the total. This is a powerful tool for gamma-ray studies, and is a development of an array already in use at the FN tandem of the N.B.I., Risø.

## II.II Magnetic Spectrometer

Area 2 is dominated by a large magnetic spectrometer which is shown in figure 6. This is a QMG/2 spectrometer<sup>(13)</sup>, one of the Q3D family. The basic geometry of the spectrometer is shown in figure 7. The spectrometer has a sliding seal target chamber which allows it to operate at any angle from  $-16.5^\circ$  to  $159^\circ$ . It has a large solid angle and high resolution so that it is well suited to a wide variety of different scattering and reaction studies with light and heavy ions. The essential characteristics of the spectrometer are listed in Table III. In the initial programme it will be used to study two proton transfer ( $^{16}\text{O}, ^{14}\text{C}$ ), ( $^{14}\text{C}, ^{16}\text{O}$ ) etc), exchange reactions such as ( $^7\text{Li}, ^7\text{Be}$ ), light nuclei far from stability with reactions such as  $^{24}\text{Mg}(^{18}\text{O}, ^{17}\text{F})$  or,  $^{26}\text{Mg}(^{18}\text{O}, ^{14}\text{O})$ . When the triton beam becomes available next year there are also users eager to study (t,p) reactions on the Actinide nuclei. A target chamber suitable for particle- $\gamma$  coincidence

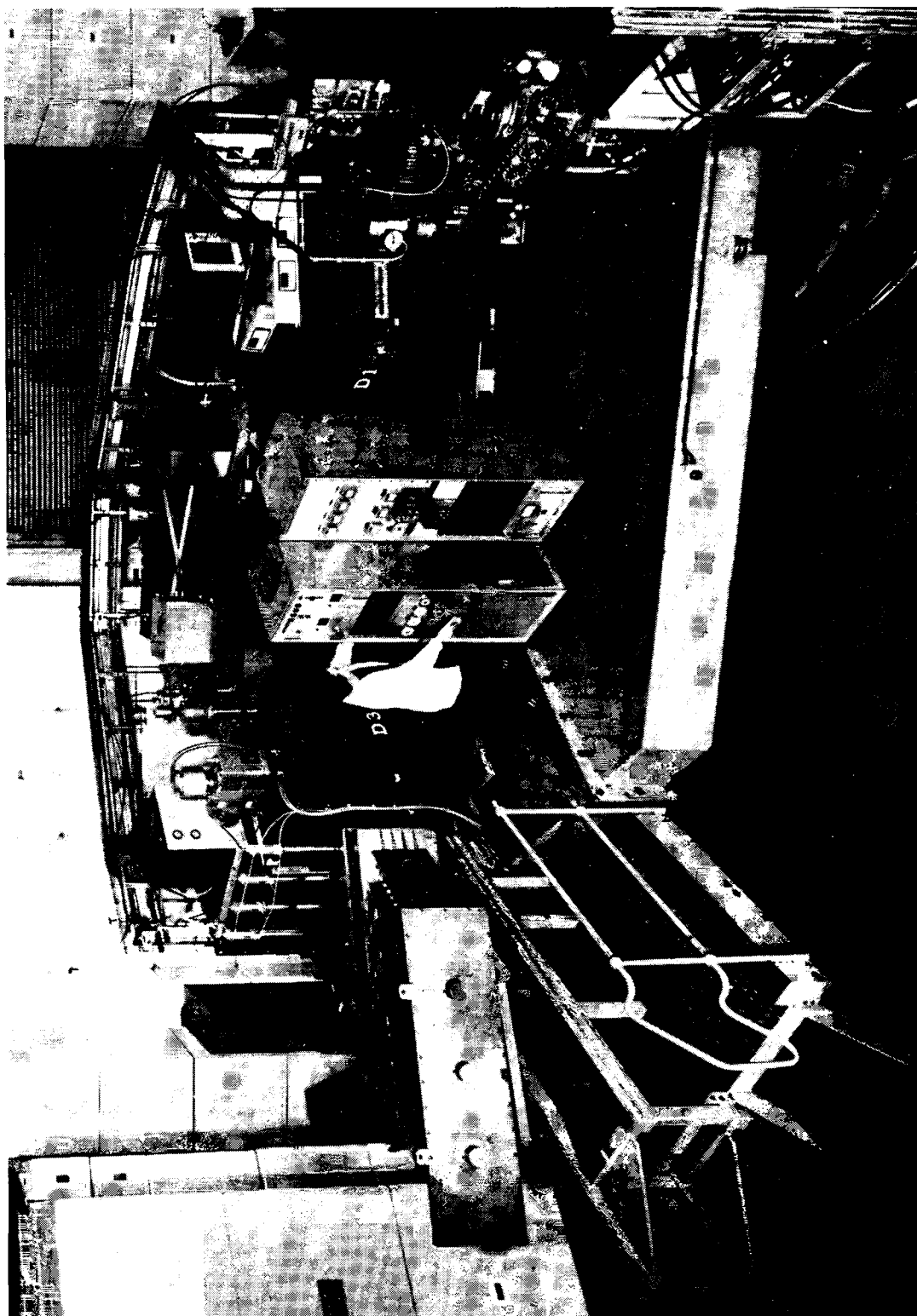


Fig. 6: A view of the QMG/2 spectrometer at the NSF.

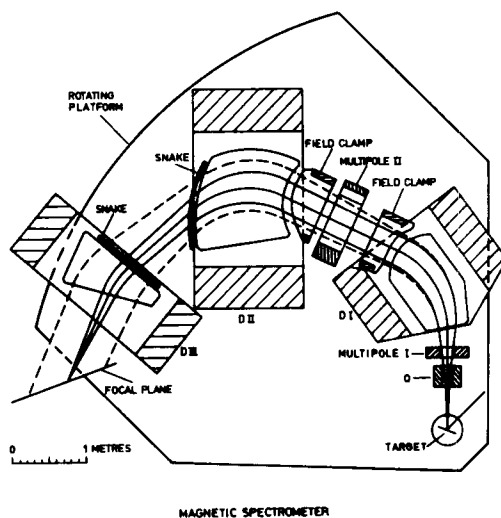


Fig. 7: The general layout of the QMG/2 spectrometer.

Table III

*Characteristics of the QMG/2 magnetic spectrometer*

|                                          |                         |          |
|------------------------------------------|-------------------------|----------|
| Mass energy product at $E_{\text{mean}}$ | (MeV. amu)              | 193      |
| First order resolving power              | $(E/\Delta E = D/2M_H)$ | 4300     |
| Dispersion along focal plane             | (cm/% mom.)             | 11.3     |
| Momentum range                           | (%)                     | 10%      |
| Solid angle                              | (msr)                   | 10       |
| Range of kinematic factors               |                         | 0.3      |
| Angular magnification                    |                         | 1.1      |
| Vertical magnification                   |                         | 4 → 7    |
| Focal plane geometry                     |                         | Straight |
| Focal plane length                       | (cm)                    | 113      |
| Focal plane angle                        | (deg)                   | 45°      |

studies will also be available, and experiments of this type will no doubt be undertaken.

The QMG/2 was built by AB Scanditronix of Sweden and was installed in 1980 and has been tested by Dr. N. Sanderson of Daresbury Laboratory. Measurements with  $\alpha$ -particle sources have shown that it performs to specification.

Further tests require heavy ions from the accelerator, and will begin as soon as some beam is available. The focal plane of the spectrometer is

1.2 m long, and the focal plane detector will consist of position-sensitive proportional counters combined with a gridded ionization chamber divided into several regions. It has a delay-line system to give the position, which must be determined with an accuracy of  $< 1$  mm if the counter is not to make a significant contribution to the overall resolution. A 0.5 m detector is already in operation<sup>(14)</sup> and measurements with a distributed parameter delay line indicate that a resolution of 0.64-0.75 mm can be obtained with a 600 mm line. These measurements show that the resolution deteriorates approximately as the square root of the length of the line. It is thought that a lumped parameter line will give better results.

In Area No. 1 there is a 1 m diameter scattering chamber built by Birmingham University<sup>(15)</sup>. It has two independently-movable arms which may be positioned to an accuracy of about one minute of arc. Detectors can be mounted singly or in arrays on each arm. There is a target ladder which allows targets to be introduced into the chamber via a vacuum look. This scattering chamber will be used for a variety of charged particle reaction studies. For example it is proposed<sup>(16)</sup> to look at the break-up of  ${}^7\text{Li}$  as a function of bombarding energy to observe how the reaction mechanism changes from break-up of the projectile in the Coulomb field at low energies to the rapid fragmentation at the nuclear surface observed at high energies. Other experiments will involve high resolution studies<sup>(17)</sup> of light particles in coincidence with projectile-like fragments and provide information about particle unstable states formed by inelastic excitation of the projectile, multinucleon pick-up and transfer leaving a highly excited ejectile.

### II.III The Recoil Separator

Area 3 will also contain a less common device, a Recoil Separator, designed jointly by Daresbury Laboratory and Liverpool University<sup>(18)</sup>. In

essence it is a device for separating fusion evaporation residues from beam particles. This will allow the measurement of coincidences between selected evaporation residues and gamma rays or other evaporated particles. It will also allow other experiments with selected residues.

In fusion evaporation reactions the reaction products emerge from the target with the same momentum as the beam particles but with different velocities ( $v$ ). One can then use the properties of the crossed-field analyzer to separate the reaction products from the beam. Figure 8 shows the device schematically. The beam particles and reaction products emerge from the target and enter a region with crossed electric ( $E$ ) and magnetic fields ( $B$ ). They are subject to a force  $F = q(E + v \times B)$ . If we adjust the magnitude and direction of these fields so that particles with the mean velocity of the fusion products pass directly through the system then the beam particles will be deflected into a beam dump. A quadrupole triplet prior to this crossed-field analyser is necessary to ensure that a large angular range is accepted by the device. The two crossed-field analysers produce a beam dispersed in velocity, and a second focussing device, a quadrupole triplet, brings it to a focus in a plane. Those ions in the same charge state, and with the same mass, but with different velocities, can be brought to a focus by a sector magnet with a uniform field. The beam is now dispersed in  $A/q$  rather than  $v$ .

The design is such that the first crossed-field analyser disposes of the beam, while the second operating independently controls the velocity dispersion in the intermediate focal plane. The magnetic

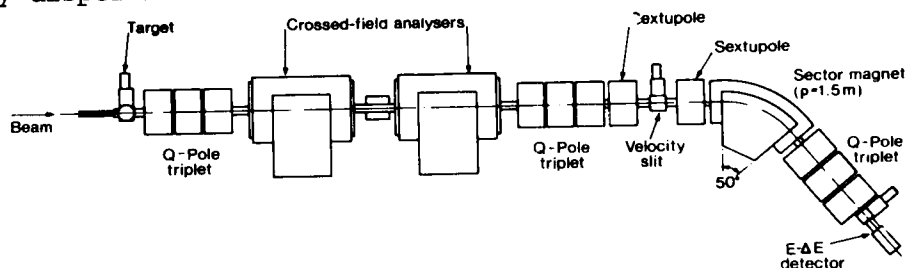


Fig. 8: The layout of the Recoil Separator.

focussing of the recoils has to be designed very carefully because of the spread in  $B\rho$  (magnetic rigidity) needed to accommodate the velocity range being accepted. Essentially this is because the focal length varies with velocity. The system chosen minimises this effect in the dispersion phase. The ions entering the sector magnet are selected first by velocity (typically  $\frac{\delta v}{v} = \pm 1\%$ ) and then by  $A_1/q$  to sufficient accuracy to resolve  $A_1$  to one unit if the correct value of  $q$  is known. This can be done if the detector measures the energy  $E_1$  of the ion to sufficient accuracy. A resolution  $\frac{\Delta E}{E} \sim 3\%$  would give  $A_1$  to 4% using the selected velocity and this is sufficient to determine the integer  $q$  to one unit using the selected  $A_1/q$ . This then gives  $A_1$  to the accuracy of the resolution of  $A_1/q$ . The main design parameters for the separator are summarised in Table IV.

Table IV

*Some parameters of the recoil separator*

|                                       |                                             |
|---------------------------------------|---------------------------------------------|
| Magnetic quadrupoles                  |                                             |
| Aperture                              | 155 mm                                      |
| Length                                | 250 mm                                      |
| Maximum field gradient                | 12 T m <sup>-1</sup>                        |
| Crossed field systems                 |                                             |
| Dipole Magnet                         |                                             |
| Aperture                              | 340 mm                                      |
| Length                                | 1000 mm                                     |
| Maximum field strength                | 0.6 T                                       |
| Electric field                        |                                             |
| Aperture                              | 100 mm                                      |
| Length                                | 1000 mm                                     |
| Plate voltage                         | $\pm 300$ kV                                |
| Dipole magnet                         |                                             |
| Aperture                              | 90 mm                                       |
| Radius                                | 1500 mm                                     |
| Bending angle                         | 50°                                         |
| Maximum field                         | 1.5 T                                       |
| Angular acceptance                    | $\Delta\theta = \Delta\theta = \pm 50$ mrad |
| Velocity acceptance                   | $\Delta V/V = \pm 1\%$                      |
| $A/q$ Image size                      | 2.3 mm horizontal<br>60 mm vertical         |
| $A/q$ dispersion                      | 4.8 mm/%                                    |
| Velocity dispersion at velocity slits | 11.5 mm/%                                   |
| Overall length                        | $\sim 12$ m                                 |

Figure 9 shows  $v/c$  versus  $A_1$  for  $A_1$  on Ca,  $A_1$  on  $A_1$  and Ca on  $A_1$ . Estimates based on our knowledge of the stopping powers of heavy ions suggest that one unit in  $A_1$  will be resolved above the line marked a and one unit in Z will be resolved above the line marked Z.

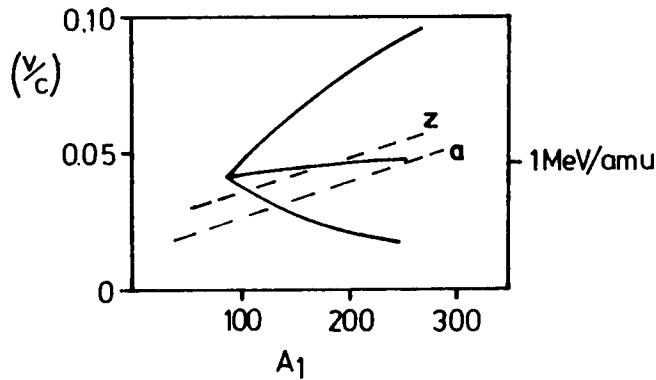


Fig. 9: For the Recoil separator one unit in  $A_1$  will be resolved in the region above the line a) by an E detector. One unit in Z will be resolved above the line (Z) by a  $\Delta E$  detector.

What form does the detector take? Figure 10 shows a schematic diagram of the detection arrangement. The focal plane is inclined at an angle of  $13^\circ$  to the beam and a thin C foil is placed at  $\approx 30^\circ$  to the beam. Grids on either side of the foil accelerate the electrons produced on to a multi-channel plate with an anode readout to give the position signal. There is no focussing between the foil and the multi-channel plate - the position signal is preserved by proximity. This should give  $\leq 1$  mm resolution over a 4 cm foil, and provides a fast time signal. The beam passes on through a thin window, which consists of  $\approx 45 \mu\text{gm}/\text{cm}^2$  of heat stretched polypropylene,

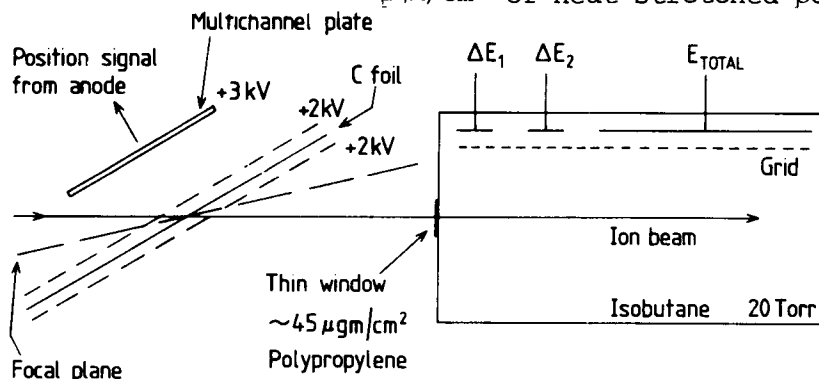


Fig. 10: A schematic of the detection system for the Recoil Separator.



into the Ionization chamber. This is filled with Isobutane at 20 Torr pressure. Three signals,  $\Delta E_1$ ,  $\Delta E_2$  and  $E_1$  are taken from the chamber.

The Recoil Separator was designed with a view to studying prompt  $\gamma$ -rays from fusion reaction products in order to study nuclei far from stability. In this mode  $\gamma$ -rays emitted from the recoiling nucleus in or near the target are detected in coincidence with ions which have passed through the separator. For an ion of energy  $\sim 1$  MeV/A the time-of-flight through the 12 m long separator is  $\sim 0.5$   $\mu$ secs. A time signal is derived from the multi-channel plate and the signals from the Ionization chamber determine  $A_1$  the projectile mass. Price<sup>(19)</sup> has described this mode of operation in terms of the specific case of  $^{114}\text{Xe}$ . The reaction chosen was  $^{58}\text{Ni} + ^{58}\text{Ni}$  at the Coulomb barrier of 180 MeV. A target thickness of  $\sim 300$   $\mu\text{gm}/\text{cm}^2$  would result in a velocity spread of  $\pm 5\%$  and angular spread of  $\pm 35$  mrad. With an acceptance of  $\frac{\delta v}{v} \sim \pm 1\%$  and  $\Delta\theta = \Delta\phi = \pm 50$  mrad the efficiency would be  $\sim 4\%$ . With the measured reaction cross-sections of Kirchner et al<sup>(20)</sup> one can calculate the expected A/q spectrum at the final image slits. Fig. 11 shows the calculated A/q spectrum. This distribution was calculated

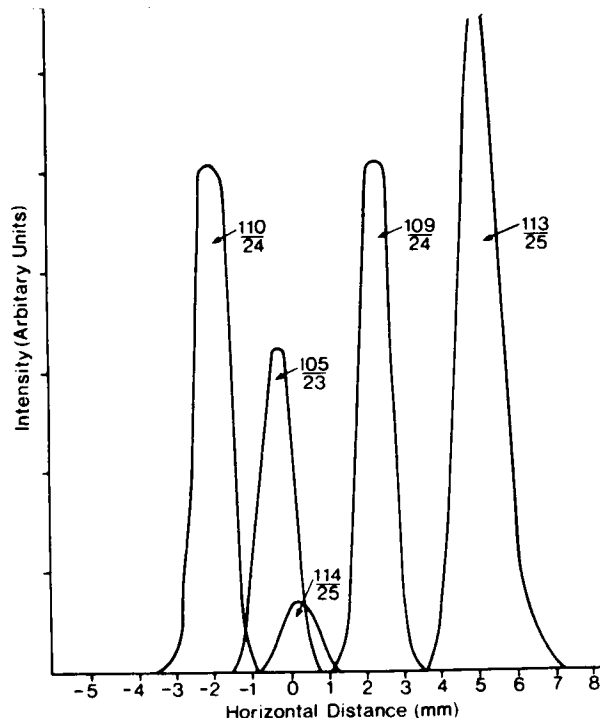


Fig. 11: The calculated A/q spectrum for the Recoil Separator in the case of  $^{114}\text{Xe}$  produced in the  $^{58}\text{Ni} + ^{58}\text{Ni}$  reaction (see text).

using the beam optics program TURTLE<sup>(21)</sup>, and the most probable charge states were calculated using the semi-empirical equations of Nikolaev and Dmitriev<sup>(22)</sup>. The A/q slits would be set to accept particles with A/q = 114/25, which would be unresolved from particles with A/q = 105/23. Since the separator has been set to accept a velocity spread of  $\pm 1\%$  about the centre-of-mass velocity the energy signal from the counter would differentiate between A = 114 and A = 105. In addition the  $\Delta E$  signals would allow Z identification. Table V summarises this calculation, and indicates that a  $\gamma$ -ray taking a large fraction of the  $^{114}\text{Xe}$  cross-section would lead to approximately 1000 counts in the photopeak in 45 mins. It should be noted, as pointed out by Price<sup>(19)</sup>, that a  $\gamma$ - $\gamma$  recoil experiment would be much slower, but having tagged the  $\gamma$ -rays in a  $\gamma$ -recoil coincidence experiment it would be possible to dispense with recoil detection for the  $\gamma$ - $\gamma$  coincidence measurements.

This instrument is in the course of construction and is expected to be operational by Summer, 1983.

Table V

Illustrative reaction  $^{58}\text{Ni}(^{58}\text{Ni}, 2p)^{114}\text{Xe}$  at 290 MeV

|                                                                                 |                                              |
|---------------------------------------------------------------------------------|----------------------------------------------|
| Total cross section for $\gamma$ -ray production                                | 10 b                                         |
| Cross section for production of $^{114}\text{Xe}$                               | 10 mb                                        |
| Target thickness                                                                | 300 $\mu\text{g cm}^{-2}$                    |
| Total rate in Ge(Li) detector                                                   |                                              |
| limited to                                                                      | $50 \times 10^3 \text{ s}^{-1}$              |
| Beam intensity                                                                  | 6 particle nA                                |
| Rate of $^{114}\text{Xe}$ in ionisation chamber                                 | $\sim 47 \text{ s}^{-1}$                     |
| Total rate in ionisation chamber                                                | $\sim 300 \text{ s}^{-1}$                    |
| Recoil transit time through system                                              | $\sim 800 \text{ ns}$                        |
| Coincidence timing resolution                                                   | $\sim 25 \text{ ns}$                         |
| Photo-peak rate of a major $^{114}\text{Xe}$ $\gamma$ -ray in a Ge(Li) detector | $\sim 0.35 \text{ s}^{-1}$ or 1000 counts in |
| in coincidence with the recoil separator                                        | the photo-peak in 45 mins.                   |

## II.IV The Isotope Separator

Another method of on-line separation, which allows us to look at the radioactive products of heavy ion reactions, is to stop reaction products inside an ion source and following ionization to extract them into an isotope separator. Area No. 1 also contains just such an isotope separator designed at Manchester University and Daresbury Laboratory for use both on- and off-line to separate by mass stable and short-lived nuclei. The design (23) of the separator was aimed at meeting the following design requirements:

- (i) Dispersion and resolution good enough to separate nuclei with mass number up to  $A \sim 250$ ,
- (ii) Simultaneous collection of a number of different isotopes in the focal plane, typically  $\pm 10\%$ ,
- (iii) Medium current ( $\sim 1\text{mA}$ ) operation for off-line use and low current operation for on-line use,
- (iv) Post-acceleration up to 100 keV to allow implantation, and
- (v) Coupled with (iii) flexible beam optics to allow the use of ion sources in different positions.

The overall layout of the isotope separator can be seen in figure 12. In essence it is a simple device which separates ions by momentum with the use of a dipole magnet. There are two ion source positions. In on-line operation the ion source/target is in the position farther away from the magnet. This position is well shielded, to the equivalent of  $\sim 1.5\text{ m}$  concrete, and allows high radiation levels and hence the use of intense, accelerator beams. In this position there is an accelerator gap following the extractor and an electrostatic quadrupole triplet, which allows us to re-form the beam at the off-line source position to act as the effective source at that point. The beam then passes through a cylindrical einzel lens and a planar einzel lens before entering the dipole magnet. The cylindrical einzel lens acts to increase the angular acceptance of the

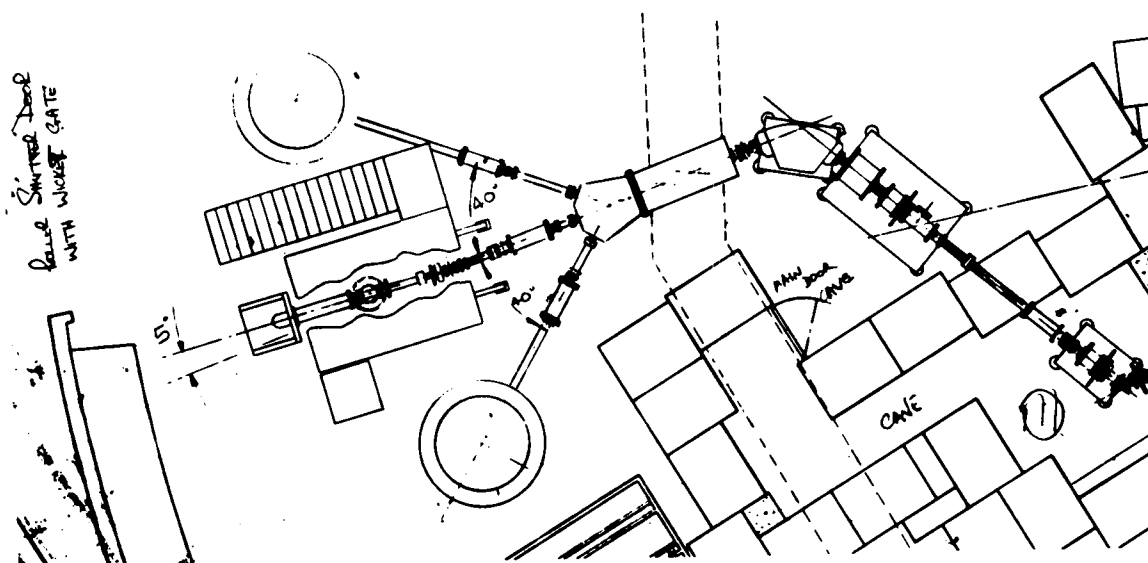


Fig. 12: The layout of the on-line isotope separator at the NSF - a detail from fig. 5.

separator. The planar lens allows us some vertical focussing. Figure 13 shows us schematically the optics of the separator. The electrostatic elements in the system are shown as vertical lines and the magnet has been straightened out to simplify the drawing. A scale is shown. The approximate

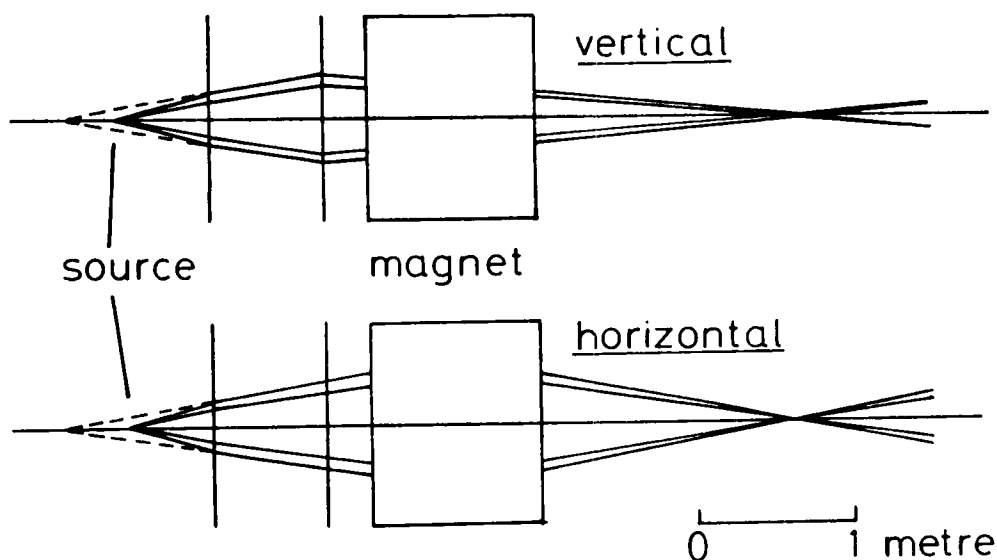


Fig. 13: The optics of the isotope separator showing the functions of the various elements. The lenses are shown as vertical lines and the magnet has been straightened out.

distance from the source to focal plane is 4 m, but the cylindrical einzel lens gives some control of the focal length. The figure shows the effect of the various elements in the vertical and horizontal directions. This optical scheme should give a focal plane inclined at  $\approx 25^\circ$  to the central beam direction in the horizontal plane, and at  $\approx 35^\circ$  in the vertical plane. The dispersion perpendicular to the central beam is approximately 10 mm for a 1% change in mass with the predicted resolution being better than 1 in  $10^3$ . In this mode of operation we anticipate beams which are of low intensity, with beam currents of the order of  $\mu\text{A}$ .

In off-line operation the ion source will be situated closer to the magnet. Beam currents of a few mA are anticipated in this case and the lenses will not be used since space charge is the main factor in determining resolution in this case. Here the focal plane lies closer to the magnet as we can see in the plan view of figure 12. Some of the basic characteristics of the separator are summarised in Table VI.

Table VI

Basic Characteristics of the Isotope Separator

|                               |                                        |
|-------------------------------|----------------------------------------|
| Source to magnet distance     | = 1.77 m                               |
| Magnet length                 | = 1 m                                  |
| Bending Radius                | = 1 m                                  |
| Bending Angle                 | = 1 radian                             |
| Magnetic Field                | = 0.15-1.0 T                           |
| Dispersion perpendicular beam | $\frac{1000}{A}$ mm for $\Delta A = 1$ |
| Pole gap                      | = 60 mm                                |
| Mass range m/M                | = $\pm 10\%$ .                         |

The optics of the separator are such that ions within  $\pm 10\%$  of the central mass may be collected in the focal plane box. Experiments may be

done with the separated beams at this point, either by collecting a particular mass on a tape which then transports the activity to a counting station or by direct counting of the beam intensities. Figure 13 shows an example of the results of a simple experiment of the latter type done at the McGill cyclotron<sup>(24)</sup>. Here the aim is to look at the independent yields of Rb and Cs isotopes following proton-induced fission at 80 and 100 MeV. The ion source used in this experiment was a surface ionization source and highly selective for the alkali elements. Figure 14 shows the measured independent cross-sections of the various Cs isotopes at the focal place of the McGill separator following proton induced fission of natural Uranium.

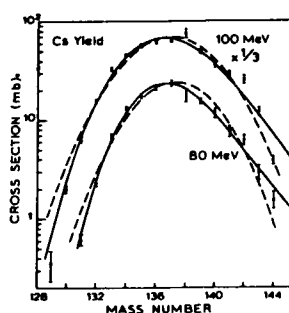


Fig. 14: Independent yields of Cs isotopes from p-induced fission of natural U.

Figure 15 shows the complementary independent yields of the Rb isotopes at the same bombarding energy. The dashed and solid lines represent second and third order polynomial fits to the measured points. These results allowed the conclusion that there does not seem to be any significant difference in the charge division for this mass split from the observed division in the thermal fission.

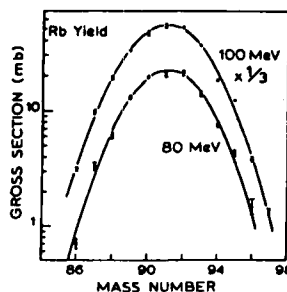


Fig. 15: Independent yields of Rb isotopes from p-induced fission of natural U.

Clearly one can apply this relatively simple technique to a wider range of elements and bombarding particles with the hope of obtaining a lot more information about the fission process.

This type of yield measurement enjoys a number of advantages over radiochemical methods:

- (i) Speed - a mass spectrum is obtained in a short period,
- (ii) Accuracy - the yields obtained do not depend on a prior knowledge of gamma ray intensities or other nuclear decay properties,
- (iii) Completeness - the yields of all of the isotopes with half-lives greater than a fraction of a second may be measured including those which are stable. As a result more significant tests of hypotheses about the mechanisms of charge division etc are possible,
- (iv) Corrections - the particularly troublesome problem of corrections for beta decaying precursors are much more readily made because of the short irradiation and decay times.

Figure 16 shows a picture of the separator as it was three weeks ago. It is mechanically complete up to the switchyard, it holds a vacuum of  $10^{-7}$  torr, and two ion sources are ready for use. A Freeman source has been tested on a separator we have built as a test rig at Manchester, and is functioning satisfactorily.

The target/ion source system is the key to successful operation of an on-line separator. We have chosen the FEBIAD source<sup>(25)</sup> developed at G.S.I. as our initial on-line source because of its flexibility and proven record of successful operation with a wide range of ions. We have built a modified version at Manchester and successfully run it on our test rig at Manchester. Figure 17 shows a diagram of the FEBIAD. Depending on whether the target is refractory or not it is housed just in front of or just inside the ion source cavity. Nuclear recoils from the target pass through a thin window on to a

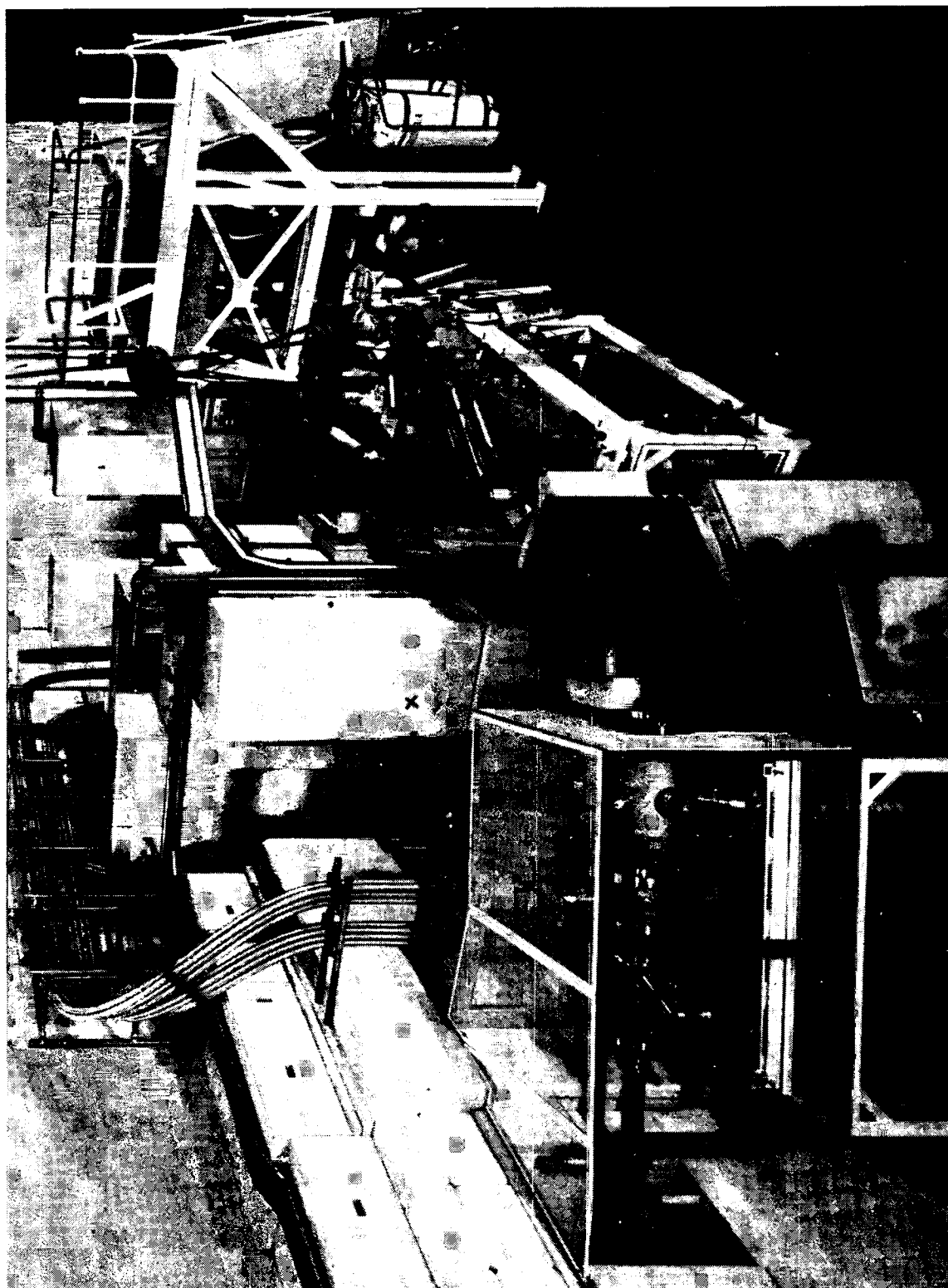


Fig. 16: A view of the on-line mass separator at the NSF.



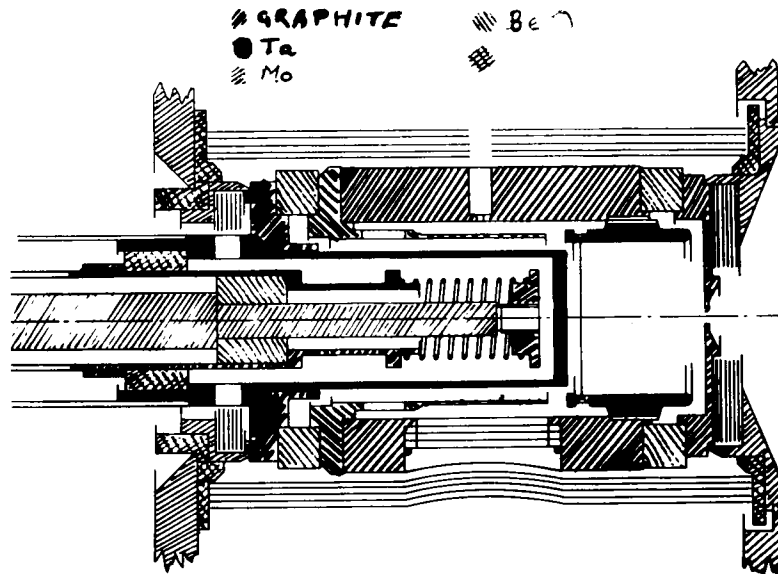


Fig. 17: A schematic diagram of the FEBIAD ion source.

hot surface, which is made of graphite felt or Ta. This is maintained at a high temperature, up to  $\sim 2300^{\circ}\text{C}$ , so that the nuclei stopped in the surface will diffuse out quickly into the small plasma region where an arc discharge is maintained by forced electron bombardment. The probability of ionization is high in this region and the ions are then extracted by the electric field of 30 kV on the extractor. They are then further accelerated up to 100 kV across the acceleration gap. This voltage was chosen because it is appropriate for ion implantation and we already have a number of applications in mind which require it.

### III. INITIAL RESEARCH PROGRAMME WITH THE SEPARATOR

What sort of programme do we plan with the separator? There will be a variety of off-line uses for the separator such as the production of thin targets for the magnetic spectrometer, the production of thin sources of heavy elements for conversion electron studies to provide a knowledge of heavy element decay schemes which is important for reactor design, and for the reprocessing of nuclear fuel, ion implantation for metallurgy and solid state studies etc.

In terms of on-line use we have four kinds of measurements in mind initially, which are related to some extent. There are some relatively straightforward  $\beta$ - and  $\gamma$ -ray spectroscopy, direct mass measurements, laser hyperfine studies and nuclear orientation. I will mention the first of these briefly later. We have three mini-beam lines coming out of the separator, which we can see in figure 12. On the left hand beam line (viewed from the separator) we are building an electrostatic sector designed to compensate for variations in the velocities of ions from the ion source. In the FEBIAD this depends on the anode-anticathode potential and corresponds to an energy spread of  $\approx 20$  eV. Comparison of masses with known masses will then allow us to determine mass differences directly.

### III.1 Nuclear Orientation

The central mini-beam line leads directly into a  $^3\text{He}$ : $^4\text{He}$  dilution refrigerator. A view of the refrigerator in use is seen in fig. 18, and a schematic diagram of the refrigerator is shown in fig. 19. This device is basically of a standard design, and was purchased from Oxford Instruments Ltd., U.K. This particular refrigerator can be loaded from the top without interrupting the refrigeration cycle, and can be used independently of both the separator and accelerator for nuclear orientation studies. It is fully operational.

The special feature of this refrigerator is the side access tube seen in fig. 19. This allows a beam of separated ions to be implanted into a sample at the end of the cold finger. For most nuclear orientation measurements the separated ion beam of interest will be implanted into a ferromagnetic host. The essence of the method of nuclear orientation is then simple. The probability of emission for any nuclear decay depends on the angle  $\theta$  between the initial nuclear spin  $I_i$  and the direction of emission for an ensemble of nuclei, when we average with equal weight over

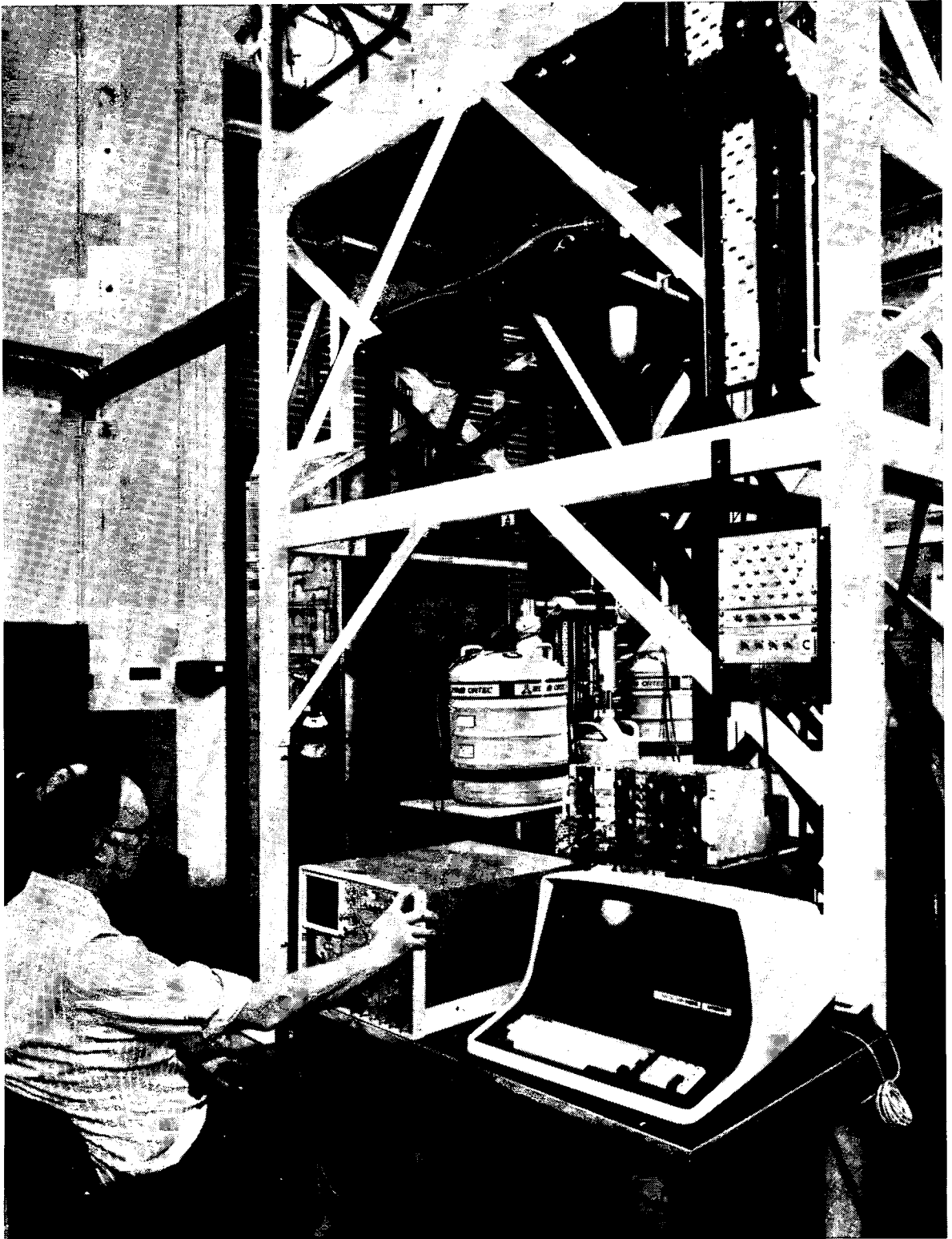
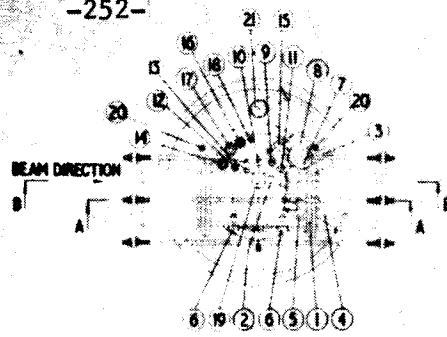
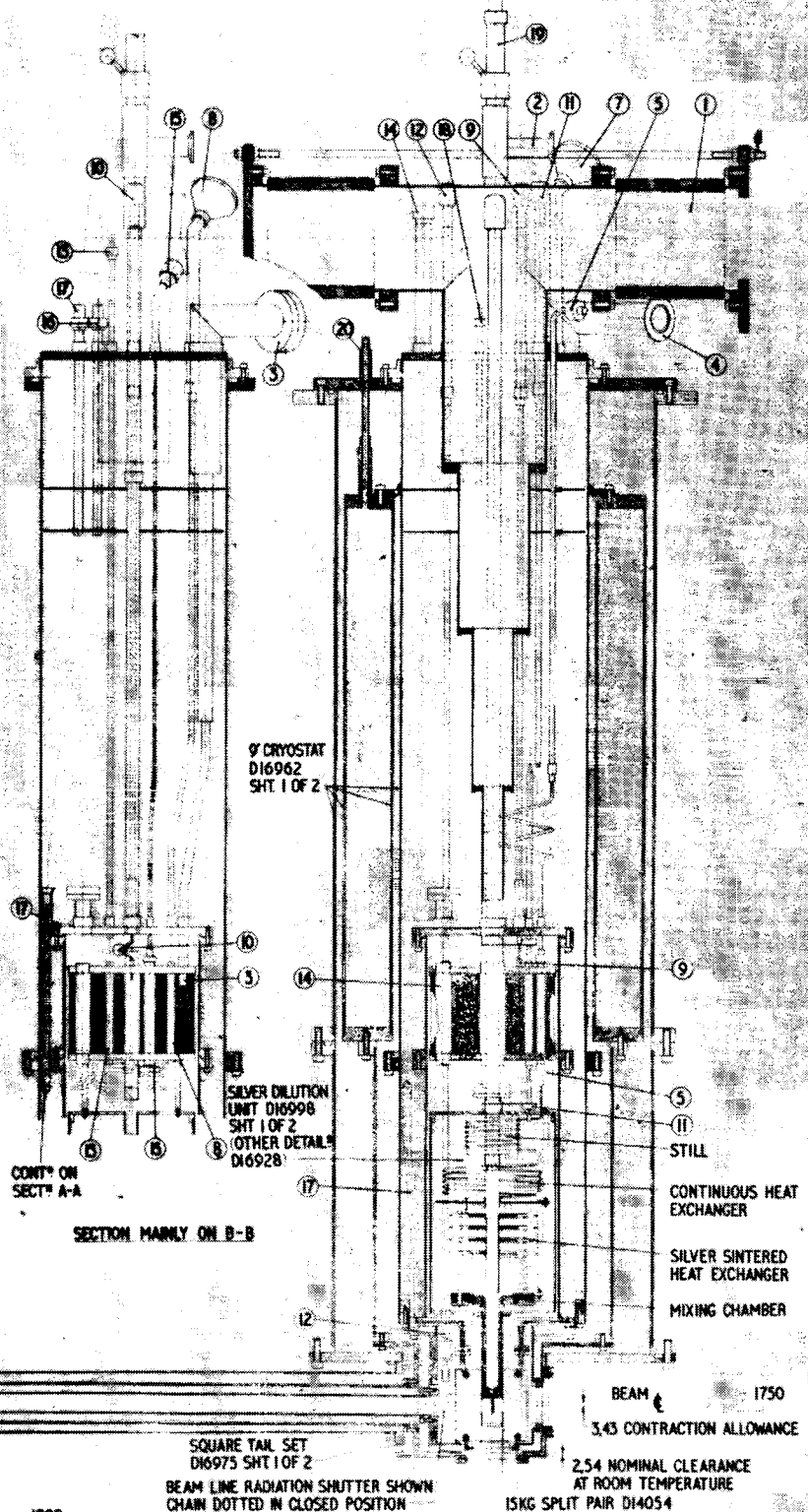


Fig. 18: A view of the  $^3\text{He} : ^4\text{He}$  dilution refrigerator in use with W.D. Hamilton (Sussex) viewing the results (see fig. 22).



|                                            |  |
|--------------------------------------------|--|
| STAINLESS STEEL                            |  |
| COPPER                                     |  |
| ALUMINIUM                                  |  |
| DURAL                                      |  |
| BRASS                                      |  |
| INNER VACUUM                               |  |
| OUTER VACUUM                               |  |
| LIQUID NITROGEN                            |  |
| $^4\text{He}$ AT 4.2 K                     |  |
| $^4\text{He}$ AT 1.2 K                     |  |
| $^4\text{He}-^3\text{He}$ (6%) BELOW 0.8 K |  |
| $^3\text{He}$ BELOW 0.8 K                  |  |



SECTION MAINLY ON A-A  
SHOWN AT ROOM TEMPERATURE  
SCALE  $\frac{1}{4}$  FULL SIZE

Fig. 19: Schematic diagram of the dilution refrigerator.

all directions we observe isotropic radiation. As we can see in figure 20, where the simple example of a  $1 \rightarrow 0$  gamma transition is illustrated, the various substates are separated in energy by an amount  $h\nu = g_N \mu_N B$  in a magnetic field  $B$ . If the temperature is reduced to the point where  $h\nu \sim kT$  then the Boltzmann factor  $\exp -h\nu/kT$  in the population ratio of adjacent substates is  $\sim 1$  so that there is appreciable ordering of the states. As a result the directional distribution of the emitted radiation is no longer isotropic.

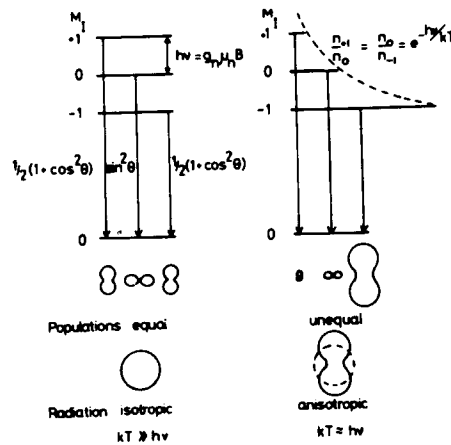


Figure 20: An example showing the effect of Nuclear Orientation

In general if we have a long-lived parent state of spin  $I_1$ , subject to a magnetic field  $H$  at temperature  $T$ , and it decays by unobserved transitions  $X_1$  to a state which decays with the emission of radiation  $X_2$  then we observe a normalized angular distribution

$$W(\theta) = 1 + \sum_{\lambda} B_{\lambda}(I_1, H, T) U_{\lambda}(X_1) A_{\lambda}(X_2) P_{\lambda}(\cos \theta)$$

where we are assuming an axis of cylindrical symmetry. The  $P_{\lambda}(\cos \theta)$  are Legendre polynomials,  $\lambda$  is integral and restricted to even values for parity-conserving decays. The termination of the series is determined by the level spins and multiplicities of the transitions involved. In most cases it is limited to  $\lambda \leq 4$ . The distribution is normalised to high

temperatures where all  $B_\lambda$  are zero and the distribution is isotropic.

This expression is convenient for analysis of the results in that the  $B_\lambda$  'orientation parameters' contain the temperature dependent part and the hyperfine interaction of the parent state, whereas the information about spins and multipolarities for the decay sequence is contained in the  $U_\lambda A_\lambda$  terms. It is obvious that if one decay sequence is known the  $B_\lambda$  terms can be found from the observed  $W(\theta)$  and the  $U_\lambda A_\lambda$  values for other sequences follow since the  $B_\lambda$  are the same for all.

How big are the effects? If we are to obtain a hyperfine splitting of 6 mK for a state with magnetic moment 1 nuclear magneton then a magnetic field  $B = 16T$  is required. Typically at 6 mK temperature the angular distribution of gamma rays under these conditions will be subject to changes of  $\approx 20\%$  compared to higher temperatures. A similar effect is obtained for an electric quadrupole moment of 1 barn in an axially symmetric electric field gradient of  $10^{18} \text{ V cm}^{-2}$ .

It is not easy to generate external magnetic fields of the required order-of-magnitude. The most convenient and common mechanism for producing the required polarization makes use of a dilute alloy of the element of interest in a ferromagnetic host. The effective magnetic fields experienced by impurity nuclei in an Fe lattice have been tabulated<sup>(26)</sup>. In many cases the field exceeds 20T and some are greater than 100T. Thus a polarised iron matrix produces magnetic fields at the nucleus of an impurity atom sufficient to give us the nuclear alignment we require for nuclear orientation measurements at 10 mK temperature.

The cooling power  $\dot{Q}$  varies with the temperature of the mixing chamber according to

$$\dot{Q} = Kn (T_{MC}^2 - T_{BASE}^2) W,$$

where  $n$  is the circulation rate of  $^3\text{He}$  in moles/sec,  $T_{MC}$  is the temperature

of the mixing chamber and  $T_{\text{BASE}}$  is the minimum temperature which can be attained. The  $T_{\text{MC}}$  vs  $\dot{Q}$  curve for the Daresbury refrigerator for two values of the circulation rate is shown in figure 21. Nuclear orientation measurements have been made with long-lived radioactive sources produced in  $(n, \gamma)$  reactions in the Universities Reactor at Risley, Cheshire. Figure 22 shows a portion of the  $\gamma$ -ray spectrum from the 1200y  $^{166}\text{Ho}$ - $^{166}\text{Er}$  decay taken with the refrigerator at 10 mK and 500 mK. The host in this experiment was Gd. One can clearly see differences in the relative peak intensities at the two temperatures.

Until now use of Nuclear orientation has been limited to radioactive sources of half-lives of several hours or greater. It should be possible to extend this type of measurement to nuclei with half-lives of a few seconds or lower. A very large number of nuclei fall into this category, many of them neutron-deficient nuclei readily produced in heavy ion reactions at the NSF.

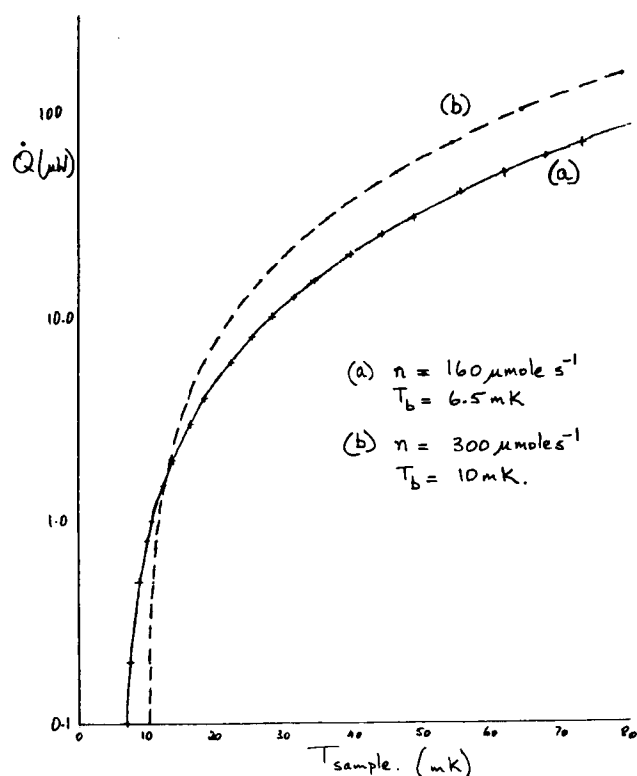


Figure 21: Cooling power versus the temperature of the mixing chamber at two different flow rates for the NSF refrigerator.

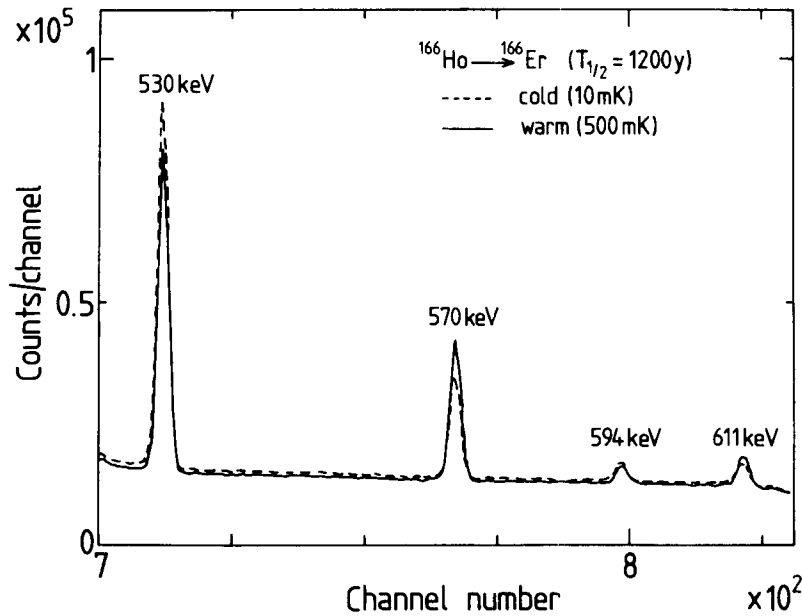


Figure 22: This shows a small part of the  $\gamma$ -ray spectrum from sources of  $^{166}\text{Ho}$  in a Gd host on the cold finger of the dilution refrigerator. The spectra were recorded at 10 mK and 500 mK. Changes in  $\gamma$ -ray relative intensities are clearly visible.

What are the anticipated limitations on these measurements at Daresbury? One limitation is the heat leak into the refrigerator during implantation which will limit the attainable temperature. A second limitation is the available counting rate and hence the time required to achieve good statistical accuracy. Thirdly there is a more fundamental limitation namely the spin-lattice relaxation mechanism.

The heat input when implanting directly into the refrigerator is due to both radiation and the kinetic energy of the implanted ions. Stone (27) has calculated that 1.6 nW will be deposited by  $10^5$  ions/sec with 100 keV energy. A five degree deflection in front of the entrance hole in the side access tube prevents neutrals reaching the cold finger. This is negligible compared with the heating by radiation. The side access tube is cooled to 4K except for a room temperature hole 1 cm in diameter a distance 1 m away. The direct radiation heating from this hole will be



$\sim 0.3 \mu\text{W}$ . The cooling curve shown in figure 21 shows that the refrigerator will maintain temperatures below 10 mK with this kind of radiation leak. We have tested it with this hole open and it does indeed maintain a temperature of about 7 mK.

Stone<sup>(27)</sup> also estimated the counting rates for a hypothetical experiment. He assumed a primary production rate of  $10^6$  isotopes/sec, a separation efficiency of 1%, and hence a maximum decay rate of  $\sim 10^4$ /sec. Allowing for Ge(Li) detector efficiencies at a distance of 7 cm we can expect counting rates of 1-100/sec in continuous, on-line operation. This should allow us to obtain  $W(\theta)$  to a statistical accuracy of  $\sim 0.1\%$  in a few hours.

Since the ions are implanted there will be a time delay before equilibrium is reached. The Korringa relaxation mechanism<sup>(28)</sup> via conduction electrons will dominate the spin-lattice relaxation time ( $T_1$ ). For this mechanism

$$T_1 T(\text{hyperfine splitting})^2 = \text{constant}$$

For dilute iron alloys

$$T_1 T(\gamma_n B_{\text{hf}})^2 \sim 5 \times 10^4,$$

where  $\gamma_n B_{\text{hf}}$  is the Zeeman splitting in MHz. This suggests  $T_1$  will lie in the range 1-100 seconds at 10 mK.

There may also be problems due to radiation damage in the host since it is being created at low temperature and will take a long time to heal because of the low mobilities.

Vanneste et al<sup>(29)</sup> have used this technique successfully to study the  $^{106}\text{In}$ - $^{109}\text{In}$  isotopes in the only on-line experiment of this type. They have

also discussed a variety of other possible orientation mechanisms.

Nuclear orientation should prove a powerful technique for measuring moments, level spins and multipolarities. It will also provide a useful link with solid state studies.

### III.II Laser Hyperfine Studies

On the remaining mini-beam line we will begin measurements<sup>(30)</sup> of moments and charge radii by laser hyperfine spectroscopy. This is an old technique revitalised by the use of tunable lasers. The hyperfine structure revealed in atomic spectra is due to the interaction of nuclear moments and the atomic electrons. A line such as the  $D_1$  transition (589.6 nm) in Na splits into a number of components due to the splitting of the levels into components characterised by the quantum number  $\underline{F} = \underline{I} + \underline{J}$ . This splitting is normally some 4-6 orders-of-magnitude smaller than the atomic fine structure. In addition if a sample contains a number of isotopes of the same element each component will split due to the so-called isotope shift. The isotope shift is the sum of two terms, the field effect and the mass shift due to change in reduced mass. The latter is particularly important in light elements. It can be calculated although there are difficulties due to the effect of collective motion of the electrons. The field shift is related to the change in the nuclear mean square charge radius in going from  $A_1$  to  $A_2$  ( $\delta \langle r^2 \rangle^{A_1 A_2}$ ), namely

$$\delta v^{A_1 A_2} \propto \Delta |\psi(0)|^2 \delta \langle r^2 \rangle^{A_1 A_2},$$

where  $\Delta |\psi(0)|^2$  is the change of electron density at the nucleus in the electronic transition. It is not possible at present to calculate  $\Delta |\psi(0)|^2$  but it is possible to calibrate the optical isotope shifts by comparison with sufficiently accurate measurements of  $K_\alpha$  X-rays or Muonic X-rays, or

measurements of electron scattering. This is the so-called King Plot procedure<sup>(31)</sup>. Uncertainties in this comparison dominate the final uncertainty.

In the initial experiments the technique used will be the crossed-beam technique developed in Birmingham<sup>(32,33)</sup>. A well-collimated atomic beam is formed (by stopping the separated beam in a Ta oven and forming a beam of the evaporated atoms by collimation), which is irradiated transversely by the beam from a tunable dye laser to reduce Doppler broadening of the scattered light. Resonance fluorescence may be observed with a sensitive, light detector. The isotope shifts and hyperfine splittings may be accurately measured by using a second dye laser locked to a transition in a reference atomic beam and detecting the heterodyne beat frequency between the two laser beams. An example of the kind of spectra obtained with this technique, which was developed at Birmingham<sup>(32,33)</sup>, is shown in figure 23. The figure shows the spectrum for the 586.8 nm line of Sm. In the upper half we see the frequency markers. In the lower half the resonance lines from the even isotopes of Sm are marked. The remaining lines are due to the odd-mass isotopes of Sm which are split by the hyperfine structure. While the shifts are very small ( $1$  in  $10^6$ ) Doppler free spectroscopy allows them to be measured to  $1$  part in  $10^4$ . If

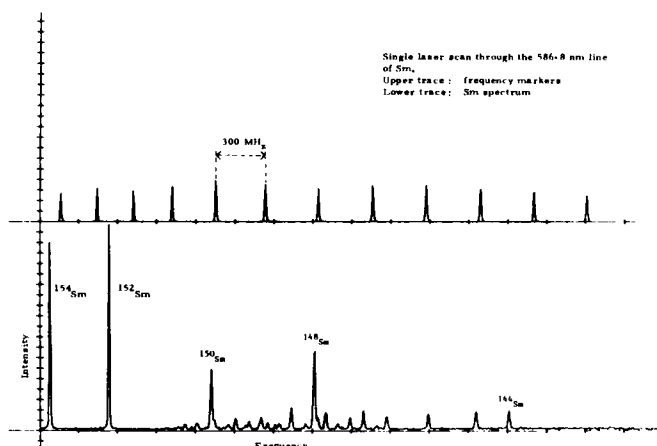


Figure 23: Results of a single laser scan through the 586.8 nm line of Sm with the crossed-beam technique developed by the Birmingham group<sup>(32,33)</sup>. The upper trace shows the frequency markers.

the even, Sm isotopes all had the same shape the spacings between the observed lines for these isotopes would be the same. One interesting feature readily observed in figure 23 is that there is a marked change in spacing as we go from  $^{150}\text{Sm}$  to  $^{152}\text{Sm}$  i.e. from  $N = 88-90$ . This is readily apparent in figure 24 where  $\delta\langle r^2 \rangle$  for the even Sm isotopes is plotted as a function of  $A$ . The results are normalised at  $A = 152$ . The values of  $\delta\langle r^2 \rangle$  for  $^{152}\text{Sm}$  and  $^{154}\text{Sm}$  follow the trend we would expect if the nuclear radius followed the liquid drop pattern,  $R = R_0 A^{1/3}$ . Instead we see a sudden change in going from  $A = 150-152$ . This transition has been known for a long time. One of the first measurements at Daresbury will be to use the separator to extend this sequence of measurements to much lighter Sm nuclei. This will take us beyond the closed neutron shell at  $N = 82$  to the predicted, abrupt transition<sup>(34)</sup> back to a deformed shape in the light Sm nuclei below  $^{139}\text{Sm}$ . Leander and Moller<sup>(34)</sup>, on the basis of deformed shell model calculations suggest, that there is a region of deformation in the light, rare-earth nuclei and that there is a promontary of deformed Pm and Sm nuclei reaching out towards stability. Measurements of isotope shifts should provide a good test of these predictions. It should be possible to make the Sm isotopes of interest in reactions such as

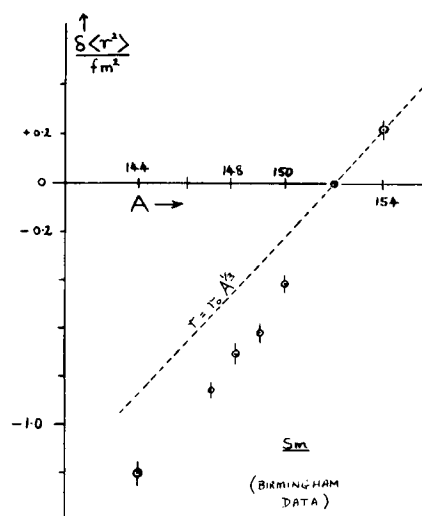


Figure 24: Values of  $\delta\langle r^2 \rangle$  for the even Sm nuclei normalised to  $^{152}\text{Sm}$ . These were derived from the data of figure 23. The variation expected from the liquid drop model is shown as a dotted line.

$^{138}\text{Ba} + ^{12}\text{C}$  or  $^{56}\text{Fe} + ^{95}\text{Nb}$ . There is considerable interest in this region, not only because of the well known  $N = 88-90$  transition, but because of the discovery of Ogawa et al<sup>(39)</sup> of the apparent shell closure at  $Z = 64$ . I will not develop this theme here but will return to it later. A range of Gd isotopes will also be studied.

It is likely that the collinear beam method<sup>(36)</sup> for laser hyperfine spectroscopy developed by the Mainz group will be adopted. The essence of the method is shown in figure 25. Here the separated ion beam from the separator is passed through a charge exchange cell where it is neutralized by alkali vapour and turned into an atomic beam. This eliminates any problems associated with ionic spectra. The laser beam is directed along the same path as the atomic beam. Resonantly scattered photons from the observation region are detected by a photomultiplier tube.

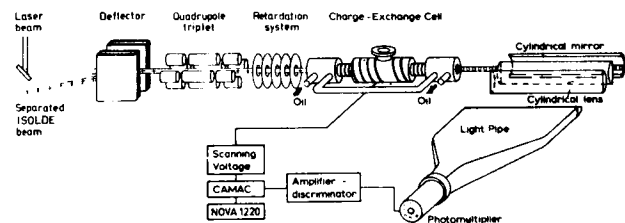


Figure 25: This shows schematically the collinear beam method<sup>(36)</sup> of laser hyperfine spectroscopy.

This method of collinear superposition of the ion and laser beams has two advantages. Firstly the separated ion beam is used directly as the sample so that losses associated with forming an atomic beam in an oven are eliminated. Secondly it reduces the Doppler width below the natural width. On leaving the ion source there is an energy spread

$$\delta E = \delta(mv^2/2) = mv\delta v,$$

which remains constant. If the beam is now accelerated through a potential

V then  $v = \sqrt{2eV/m}$ . In essence the Doppler width is reduced by the factor  $\sqrt{kT/eV}$  where  $kT$  is the thermal energy associated with the ion source ( $\sim 20$  eV for the FEBIAD<sup>(25)</sup>). The experiments can be carried out by tuning the laser frequency or by varying the acceleration voltage.

Figure 26 shows measurements of  $\delta \langle r^2 \rangle$  for the Cs, Ba, Xe isotopes accumulated using this method. In this case the results are normalised to the measurement for the  $N = 82$  closed shell nucleus. Again it is clear that the results do not follow the regular  $A^{1/3}$  dependence predicted by the liquid drop model. One feature of interest here is the pronounced, odd-even staggering. If pairing had no effect than we would expect no such effect. Instead adding an unpaired nucleon increases the size of the nucleus by more than half the amount due to adding a pair of nucleons.

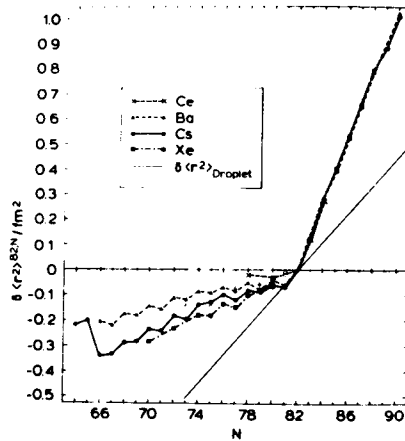


Figure 26: Measurements of  $\delta \langle r^2 \rangle$  for Cs, Ba and Xe isotopes normalized to  $N = 82$ .

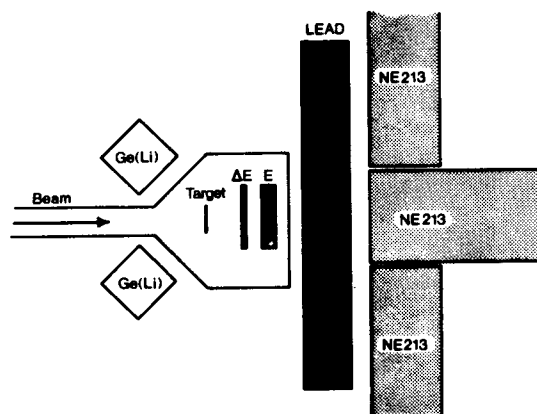
#### IV The Poor Man's Alternative to a Recoil Separator and an Isotope Separator

Up to this point I have tried to give you some idea of the experimental tools we have accumulated at Daresbury to try and exploit beam from the NSF. I spent some time on the Recoil Separator and Isotope Separator which are aimed at the study of in-beam spectroscopy of very neutron-deficient nuclei and the study of radioactive species far from stability respectively. Today

I thought I would talk about experiments aimed in the same direction but carried out with much cheaper and simpler apparatus. We will see that these simpler alternatives can be effective although they lack the sensitivity and flexibility of the Recoil and Mass separators.

If we want to study nuclei as far from stability as possible we are faced with a number of problems. In  $(HI, xnypz\alpha)$  reactions we find that the emission of protons and alphas is increasingly favoured as the proton and alpha binding energies fall relative to the neutron binding energy. Clearly to reach the most neutron-deficient nuclei we must be able to pick out channels with a small fraction of the total cross-section. Since the channels of interest involve neutron emission the logical way to study these nuclei is to observe the prompt  $\gamma$ -rays in coincidence with neutrons. This is the solution we have adopted and two sets of experiments are underway at Brookhaven National Laboratory using this technique - one aimed at nuclei with  $N = Z = 40$  and the other at the tin isotopes near  $^{100}\text{Sn}$ . They are both in the data-taking phase and at this stage I can only give you an idea of where they are heading.

The basic experimental set-up is shown in figure 27. The target is on a thick Pb backing,  $\approx 20 \text{ mgm/cm}^2$ , which stops the beam. Charged



Apparatus for studying very neutron deficient nuclei  
using  $(HI, xn, yp, z\alpha)$  reactions

Figure 27: Experimental arrangement for studying  $\gamma$ -rays in coincidence with neutrons and an E- $\Delta$ E charged particle telescope.

particles are detected in a  $\Delta E$ -E telescope directly behind the target and behind that is an array of neutron detectors filled with NE213 scintillator. Ge(Li) detectors at  $45^\circ$  view the target at backward angles. The data are recorded event-by-event with pulse shape discrimination being used to distinguish between neutrons and gamma rays in the NE213 scintillator. A Pb shield, of  $\frac{3}{4}$ " thickness, was also placed in front of the neutron detectors to cut down the number of gamma rays reaching them.

Figure 28 shows the E- $\Delta E$  spectrum in the telescope for the  $^{40}\text{Ca} + ^{70}\text{Ge}$  reaction studied by Varley et al<sup>(37)</sup>. The large peak is that due to protons, and we can see various peaks due to 2 protons, an  $\alpha$ -particle, etc. With software gates on these peaks and on the neutrons we can reconstruct

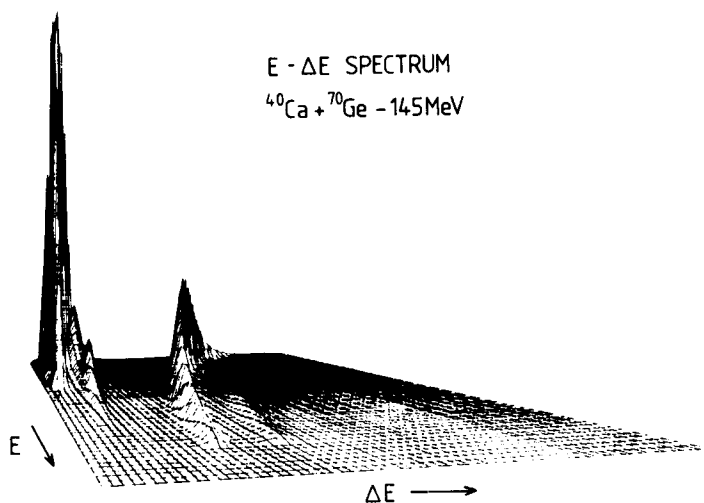


Figure 28: E- $\Delta E$  spectrum recorded by Varley et al<sup>(37)</sup> in studying the  $^{40}\text{Ca} + ^{70}\text{Ge}$  reaction.

the spectra. Figures 29, 30 and 31 show portions of a Ge(Li) detector spectrum in coincidence with a variety of different particle combinations. Gamma rays assigned to  $^{107}\text{Sn}$ , which involves the emission of two protons and a neutron are marked in keV at the bottom of the figure. It is a simple matter to assign a gamma ray to a particular reaction such as the



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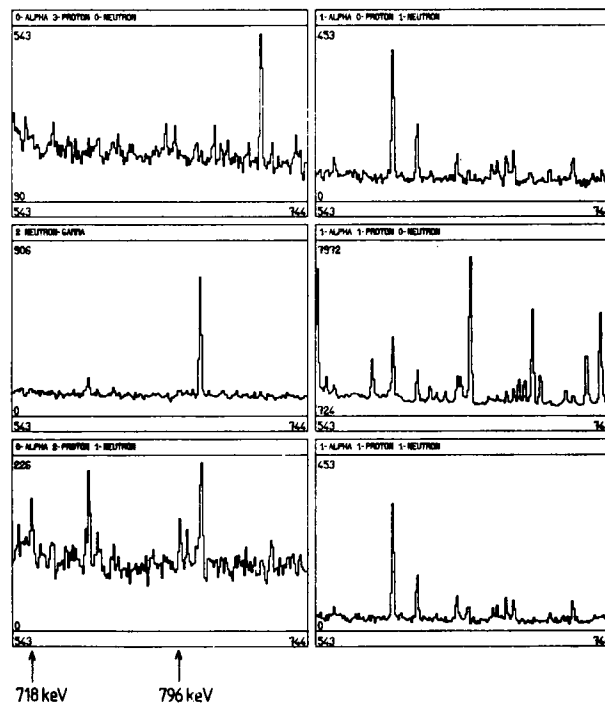


Figure 29: Ge(Li) spectra from  $^{40}\text{Ca} + ^{70}\text{Ge}$  gated by neutrons and charged particles in different combinations.

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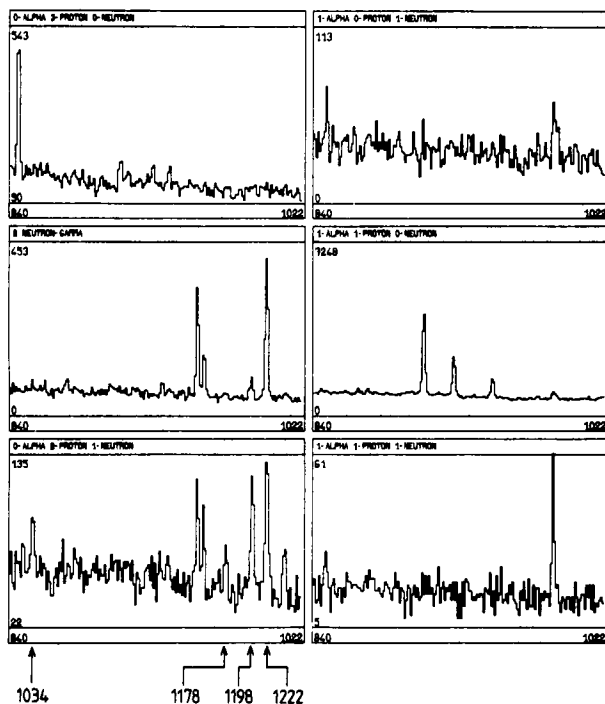


Figure 30: Ge(Li) spectra from  $^{40}\text{Ca} + ^{70}\text{Ge}$  gated by different combinations of neutrons and charged particles.

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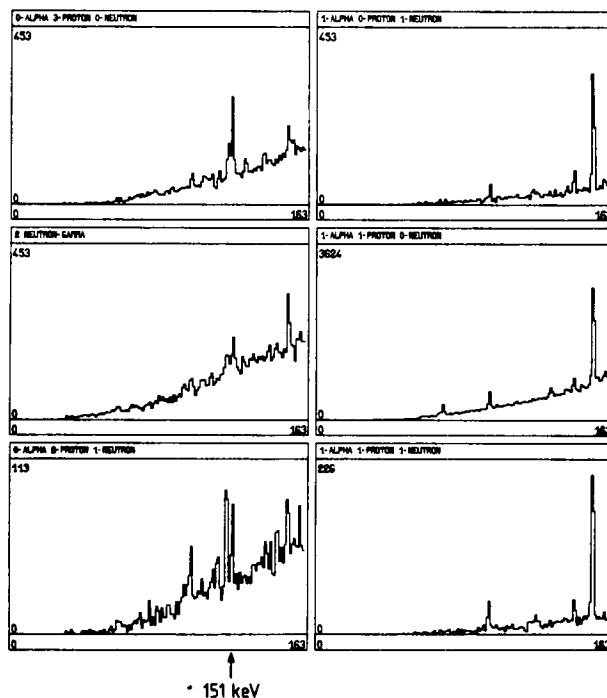


Figure 31: Ge(Li) spectrum from  $^{40}\text{Ca} + ^{70}\text{Ge}$  gated by different combinations of neutrons and charged particles. The line at 151 eV seen in the 2pn-gate is not the same as the line seen in the 3p-gate.

2pn reaction, since it must appear in any spectrum gated by a subgroup of these three particles, namely the pn- or p- or n-windows but not the  $\alpha$ n- or  $\alpha$ p-windows. One must still exercise care in interpreting the spectra. Firstly the 2pn is itself a subgroup of 2p2n or 3pn. Secondly one must look carefully at the gamma-ray energies. For example in figure 31 it appears that a gamma-ray at 151 keV appear in the 2pn- and 3p-windows. More careful inspection reveals that the line seen in the 2pn spectrum shifted a little to higher energy, and there are two gamma rays of almost the same energy.

This is confirmed by the results shown in figure 32 where we see part of a  $\gamma$ - $\gamma$  coincidence spectrum gated by the two parts of this gamma ray doublet. The lower spectrum due to the 152 keV gamma ray, now assigned to  $^{107}\text{Sn}$ , partly breaks through in the upper spectrum, but the spectrum

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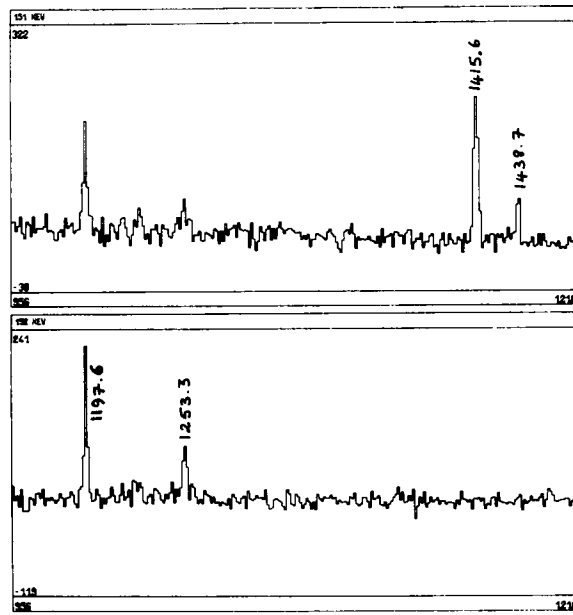


Figure 32: Gamma ray spectra gated with a single neutron and the two parts of the 151-152 keV doublet seen in figure 31 (see text).

associated with the lower energy gate does not break through in the lower spectrum.

The analysis of these data is still at a very preliminary stage. The partial level scheme for  $^{107}\text{Sn}$  extracted at this stage of analysis is shown in figure 33. It is too early to say much about this, but it is clear that one is not faced with the kind of simple band structure seen in deformed nuclei. From the systematics of the heavier Sn nuclei one would guess that the ground state and the first excited state are due to the  $g_{7/2}$  and  $d_{5/2}$  neutron orbits. Since all of the observed peaks are sharp we already know they have lifetimes long compared with the slowing down time. Measurements with a plunger and angular distributions will be carried out later this month to flesh out this level scheme.

With the same set-up Lister et al <sup>(38)</sup> studied the  $^{24}\text{Mg} + ^{58}\text{Ni}$  and  $^{28}\text{Si} + ^{58}\text{Ni}$  reactions to look at the nuclei with  $N \sim Z \sim 40$ . Figure 34 shows the gamma-ray spectrum from the  $^{24}\text{Mg} + ^{58}\text{Ni}$  reaction gated with 2

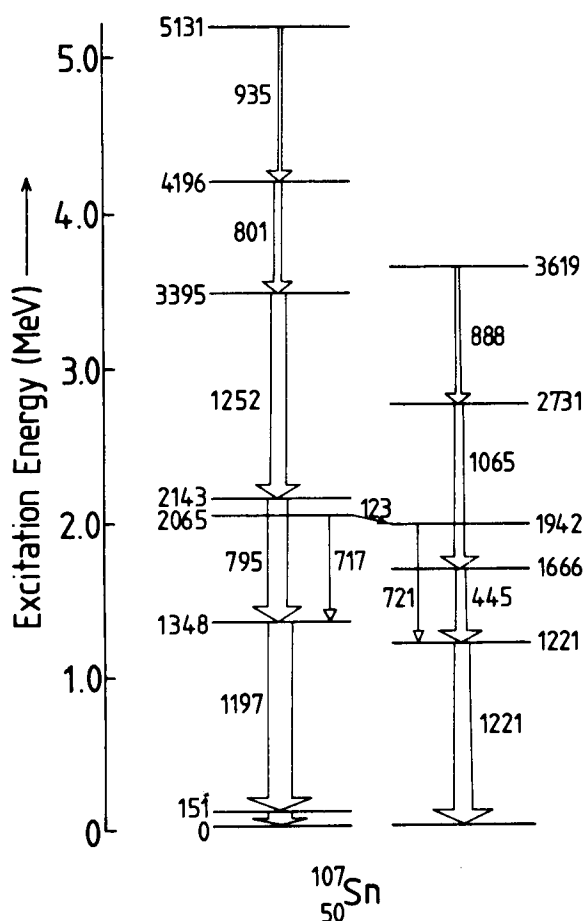


Figure 33: Preliminary<sup>(37)</sup> partial level scheme for  $^{107}\text{Sn}$ .

neutrons. Peaks from the p2n gate also appear. We see clearly the  $2^+ \rightarrow 0^+$ ,  $4^+ \rightarrow 2^+$  and  $6^+ \rightarrow 4^+$  lines in  $^{78}\text{Sr}$ . Similarly figure 35 shows the 2n-gated spectrum from the  $^{28}\text{Si} + ^{58}\text{Ni}$  reaction showing the  $2^+ \rightarrow 0^+$ ,  $4^+ \rightarrow 2^+$ , and  $6^+ \rightarrow 4^+$  lines in  $^{82}\text{Zr}$ .

In these experiments in-beam  $\gamma$ -rays from the nuclei  $^{77}\text{Sr}$ ,  $^{78}\text{Sr}$ ,  $^{80}\text{Sr}$ ,  $^{80}\text{Y}$ ,  $^{81}\text{Y}$ ,  $^{83}\text{Y}$  and  $^{85}\text{Y}$  have been observed as well as  $^{82-86}\text{Zr}$ . Figure 36 shows the systematic behaviour of the low-lying levels in the even Zr isotopes. We see a steady decrease in energy of the first  $2^+$  state for the neutron-deficient Zr nuclei. This and the increasing  $E_4/E_2$  ratio suggests that deformation is setting in. This is supported by the results of figure 37, where the energies of the first excited,  $2^+$  states in the even nuclei with  $N \sim Z \sim 40$  are plotted.

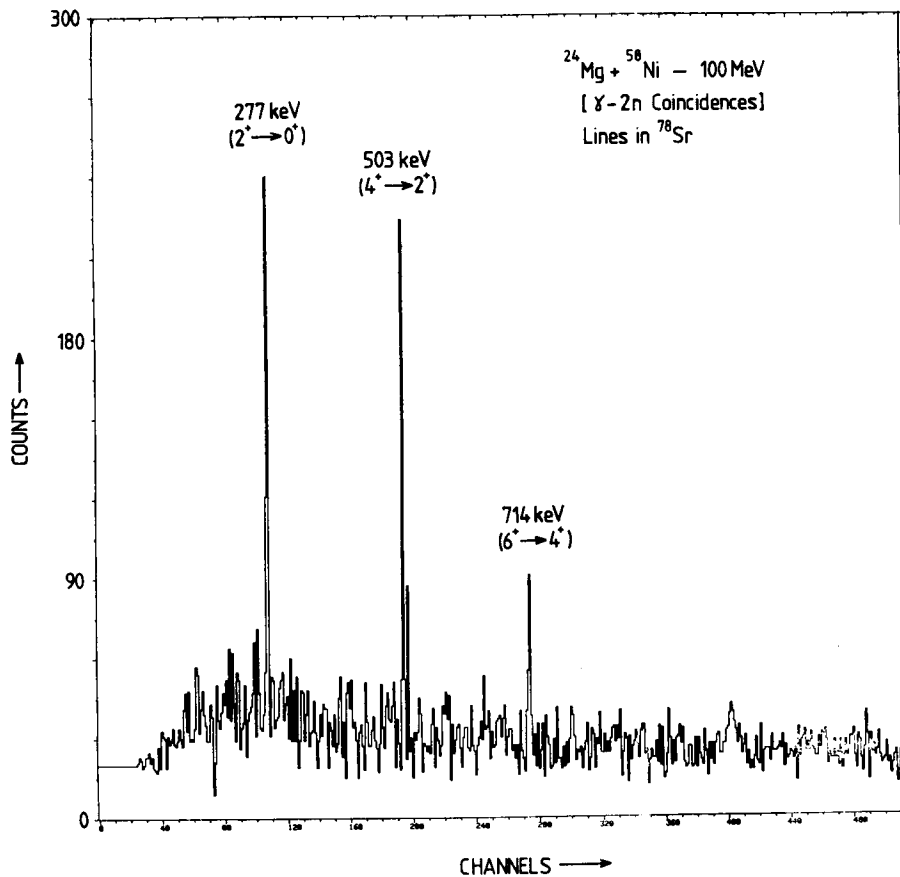


Figure 34: Ge(Li) spectrum from the  $^{24}\text{Mg} + ^{58}\text{Ni}$  reaction gated with two neutrons. Lines in  $^{78}\text{Sr}$  are marked.

These results are again preliminary but are interesting. In the spherical shell model the  $f_{5/2}$  subshell is complete at  $Z = 38$  and the  $fp$  shell is full at  $Z = 40$ . Thus in the Nilsson model there is a local minimum in the energy surface at zero deformation for both these nucleon numbers. For  $Z = 38$ , however, the Nilsson model also shows a minimum energy, when  $\delta \sim 0.28$ , associated with a large gap in the level spacing. The harmonic oscillator potential shows a shell closure at  $N = 40$  for a 2:1 deformation. These results clearly confirm the suggestion that the nuclei in this region are deformed. From this figure it seems likely that the Sr nuclei are more deformed than the Zr nuclei.

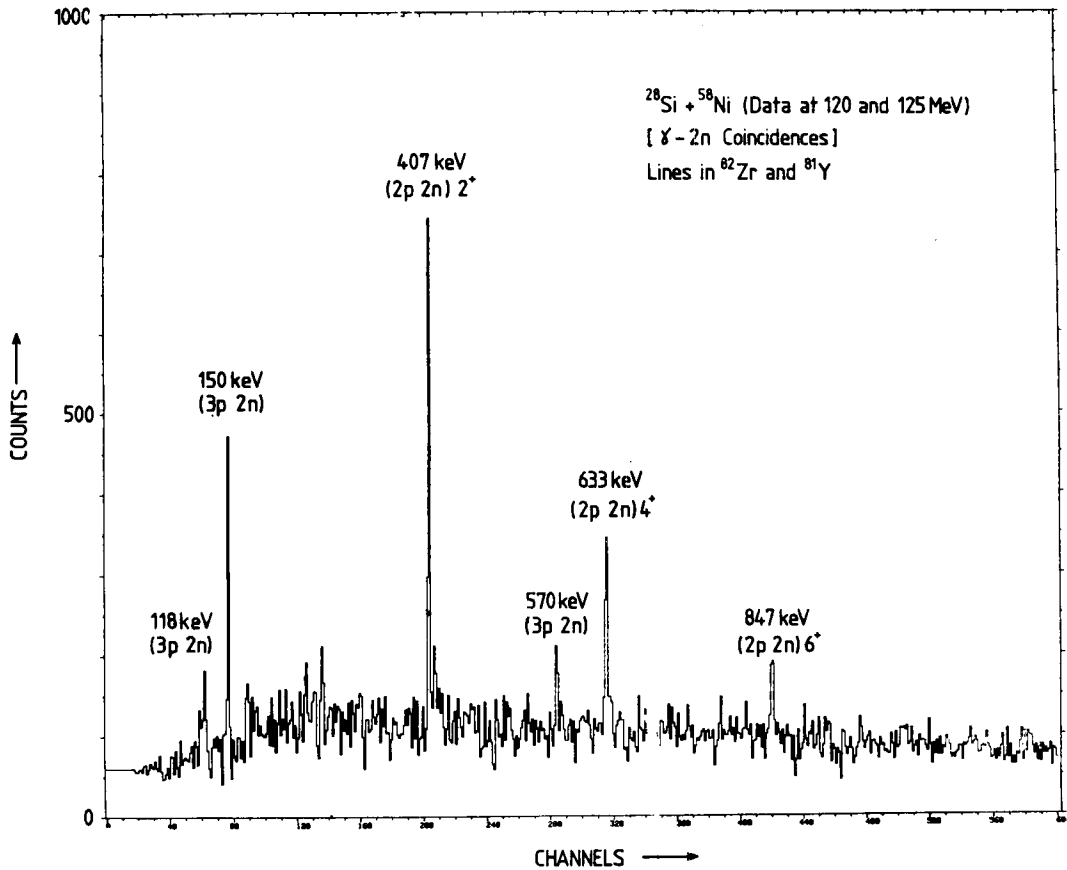


Figure 35: Ge(Li) spectrum from the  $^{28}\text{Si} + ^{58}\text{Ni}$  reaction gated with two neutrons. Lines in  $^{82}\text{Zr}$  are marked.

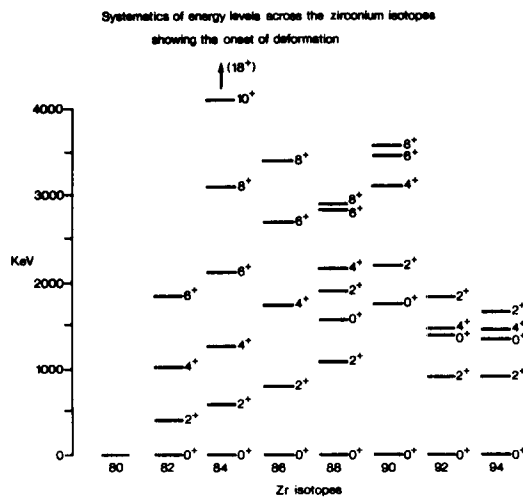


Figure 36: The systematic behaviour of low-lying levels in the even Zr isotopes.

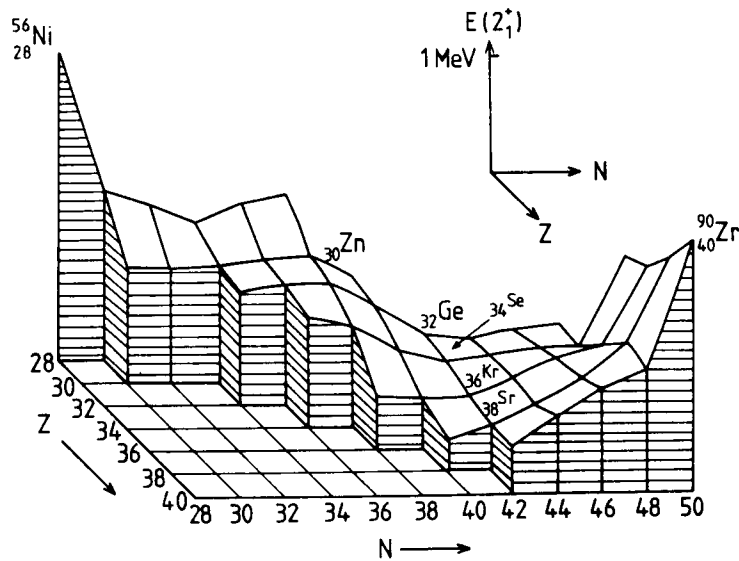


Figure 37: The energy of the  $2_1^+$  state as a function of  $Z$  and  $N$  for nuclei with  $28 \leq Z \leq 40$  and  $28 \leq N \leq 50$ .

The success of these experiments has encouraged further refinements of the techniques. At present the neutron detectors have an efficiency of  $\sim 10\%$  for 4 MeV neutrons. We have now designed a modular array of neutron detectors, again filled with NE213 scintillator with 12 detectors, each 8" x 4" x 24" in dimension, which we can stack in any way we want. This will increase our detection efficiency for 4 MeV neutrons to  $\sim 50\%$ , in these reactions where they all go forward.

It is also proposed<sup>(39)</sup> to concentrate on massive transfer to learn more about these nuclei. It has long been known that large numbers of high energy, light, charged particles are produced in heavy-ion reactions. It now seems that this is due to massive transfer, in which the projectile breaks up into heavy and light clusters at the nuclear surface with fusion of the target and heavy cluster and the light cluster continuing with a velocity close to that of the beam particles. In heavier nuclei it has recently been shown that this reaction mechanism leads to the selective excitation<sup>(40)</sup> of high spin states. For example, in the  $^{14}\text{N} + ^{144}\text{Sm}$  reaction there was no direct feeding of states with  $J \leq 26$ . One may

surmise that the use of heavier ion beams will allow the use of this mechanism to selectively excite high spin states, but this awaits experimental test. Even if the reaction does not have this degree of selectivity, however, it should still prove useful since the fast light cluster will carry away most of the excitation energy.

It is proposed to use a large, NaI sum spectrometer to measure the total  $\gamma$ -ray energy emitted in these reactions in coincidence with the various light particles. This allows the selection, for example, of those reactions in which  $\alpha$ -particles are emitted and lead to a residual state close to the yrast line. This state will then decay by a gamma-ray cascade. This can be selected with high efficiency and low background since the total cascade energy is known from the sum of the gamma ray and particle energies. The technique has the advantage of identifying the nucleus with certainty, allows the selection of the highest spin states and defines the entrypoint to the cascade with sufficient certainty to eliminate uncertainties due to feeding corrections. It is planned to study reactions such as  $^{40}\text{Ca}(^{40}\text{Ca},\alpha)^{76}\text{Sr}$ ,  $^{40}\text{Ca}(^{40}\text{Ca},2p)^{78}\text{Sr}$  and  $^{58}\text{Ni}(^{28}\text{Si},\alpha)^{82}\text{Zr}$  reactions.

Clearly these experiments are aimed at the same physics as the Recoil Separator. They have the considerable merit of being much cheaper and the technique has been shown to work. They have the disadvantage of not separating the residues from one another and hence do not allow one to do further experiments with the residues. They are also much less sensitive.

A similar situation exists for the isotope separator - the poor man's alternative in this case is the He jet recoil transport system (HeJRTS). Again this is a simple technique which does not separate radioactive species by either Z or A but allows us to transport the radioactivity away from the site of production, where the background is high, to a low back-



ground counting position. We can then use a variety of counting techniques to study individual decays. We have used a HeJRTS to study a number of radioactive<sup>(42-45)</sup> decays. Today I will describe some measurements we made on the neutron-deficient Yb isotopes populated in the  $\beta^+$ /EC decay of Lu isotopes, which were produced in heavy-ion reactions on the Manchester HILAC before its closure.

Fig. 38 shows the HeJRTS in schematic form. It was of the type described by Kosanke et al<sup>(46,47)</sup>. A thin Ti foil, of thickness  $\sim 1 \text{ mgm/cm}^2$ , was used to seal the vacuum of the accelerator and act as a window for the recoil chamber. The accelerator beam energy could be varied from the full energy of 9.6 MeV/nucleon with Al absorbers placed in front of the chamber. The recoil chamber could be varied in length and had an internal diameter of 0.75". A narrow, teflon capillary of internal diameter 1.2 mm protruded into the chamber just behind the target, which was close to the Ti window. In the experiments described here the recoil chamber was filled with He gas doped with controlled amounts of a monomer, usually benzene or diethyl sulphide. In general the chamber pressure was

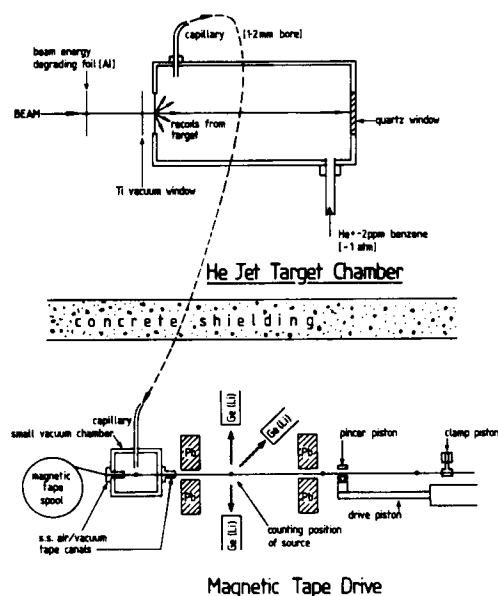


Figure 38: HeJRTS in schematic form (see text),

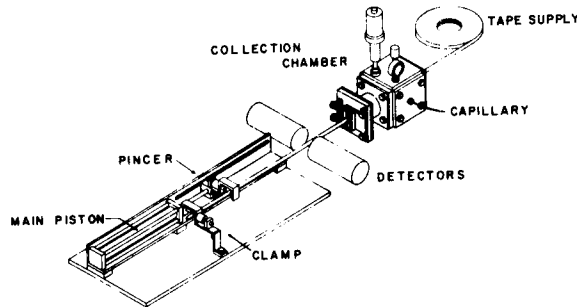


Figure 39: The tape transport system<sup>(48)</sup> in schematic form. The tape passes from the reservoir on the right through the evacuated collection chamber to a counting position viewed by the detectors. The arrangement of pneumatic pistons allows it to be moved a fixed distance at any pre-set interval.

vacancies in electron capture and internal conversion, this is an effective way of tagging gamma rays by element. Fig. 40 shows a Ge(Li) spectrum in coincidence with Yb  $K_{\alpha}$  X-rays. This particular spectrum was taken with

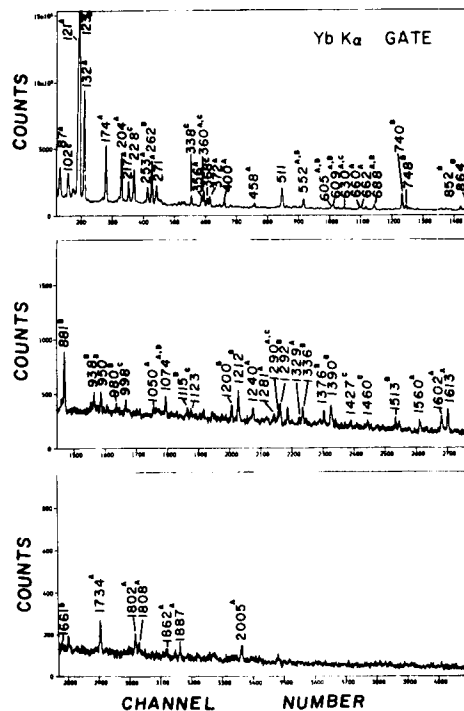


Figure 40: A Ge(Li) spectrum in coincidence with Yb  $K_{\alpha}$  X-rays. The sources were prepared in the  $^{155}\text{Gd} + ^{14}\text{N}$  reaction with beams of  $\sim 95$  MeV  $^{14}\text{N}$ . The decay of  $^{165}\text{Lu}$  is the predominant activity.

$\sim 2$  atmospheres. The capillary leads into a small chamber evacuated by a large rotary pump. The end of the capillary was placed about 1 cm from an IBM tape which passed through the chamber from a tape reel to the counting position. The tape could be moved a fixed distance at pre-set intervals to transport the activity to the shielded counting position.

Radioactive nuclei produced in the target recoil into the He-jet chamber and are thermalized by collisions in the He gas. They become attached to heavy, molecular clusters formed by the irradiation of the monomer. They are then swept into and through the capillary along with the clusters. At the output end the clusters strike the tape and stick to it. The tape system<sup>(48)</sup> is shown in figure 38. It is a cheap and inexpensive device and makes use of pneumatic pistons driven by compressed air to move the tape at any pre-set interval longer than a few seconds. The physical principles involved in the operation of a HeJRTS have been described by Wollnik<sup>(49)</sup>. In general it is better to use an aerosol generator to provide the polymer clusters in a controlled way rather than rely on polymer formation in the beam plasma, which varies<sup>(50)</sup> with beam intensity, the L.E.T. of the beam particles, etc.

In the experiments described here the Lu activities were produced in the  $^{14}\text{N} + ^{155}\text{Gd}$  and  $^{151}\text{Eu} + ^{16}\text{O}$  reactions. Initially excitation functions were measured with Ge(Li) detectors and low energy, photon spectrometers (LEPS), which are small, planar Ge detectors of high resolution. This allowed us to determine the optimum beam energies for producing the Lu isotopes of masses 158-165.

Since a variety of different elements are produced in these reactions, the observed gamma-rays were assigned to a particular element by measuring coincidences between Ge(Li) detectors and a LEPS with sufficient resolution to clearly resolve the  $K_{\alpha}$  X-rays of adjacent elements. Since K X-rays are emitted following the creation of inner shell

sources produced in the bombardment of  $^{155}\text{Gd}$  with  $\sim 95 \text{ MeV } ^{14}\text{N}$ . The dominant activity at this beam energy is  $^{165}\text{Lu}$ , although lines from  $^{164}\text{Lu}$  and  $^{166}\text{Lu}$  are also present in the spectrum.

A variety of measurements were made at the energies required to produce  $^{158}\text{Lu}$ , and  $^{160-165}\text{Lu}$ . In each case the pre-set source accumulation time was matched to the half-life of the isotope of interest to optimise the amount of activity from that isotope relative to others. Measurements were made of  $\gamma$ - $\gamma$ ,  $\gamma$ -X and X-X coincidences where X indicates measurements made with a LEPS spectrometer. The  $\gamma$ -decay half-lives associated with the  $\gamma$ -rays were measured by following these decays through sixteen, equal successive time intervals. The time intervals were chosen to cover several half-lives of the isotope concerned. Figures 41 and 42 show examples of these measurements. A pulser was also run into the spectrum and the recorded intensities in the pulser peaks were used to correct for dead-time effects. Decay energies were measured from the end-points of positron spectra recorded in coincidence with gamma rays. The positrons were detected in a hyperpure Ge detector. In the case of  $^{162}\text{Lu}$  and  $^{164}\text{Lu}$  decays the conversion electron spectrum was also studied with a Si(Li) detector and mini-orange spectrometer<sup>(51)</sup>. Some of the measurements are summarised in Table VII.

For  $^{160}\text{Lu}$ ,  $^{162}\text{Lu}$  and  $^{164}\text{Lu}$  decays gamma-gamma angular correlation studies were made in order to determine level spins and parities. The measurements were made with three Ge(Li) detectors. One was kept fixed at  $35^\circ$  to the tape carrying the source at a distance of 8 cms from the source. The other detectors were placed successively at angles of  $180^\circ$ ,  $10^\circ$ ,  $35^\circ$ ,  $45^\circ$ ,  $55^\circ$  and  $90^\circ$  to the fixed detector at the same distance of 8 cm from the source. The measurements were normalised by recording the singles spectrum from the fixed detector. Figure 43 shows part of the gamma-ray spectrum in coincidence with the 167 keV gamma ray emitted in  $^{162}\text{Lu}$  decay,

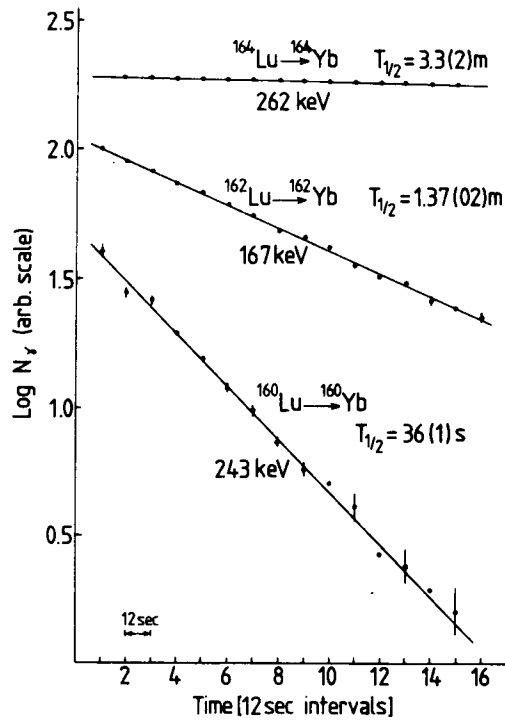


Figure 41: Measured decay curves for the 243, 167, and 262 keV  $\gamma$ -rays emitted in the decay of  $^{160}\text{Lu}$ ,  $^{162}\text{Lu}$  and  $^{164}\text{Lu}$  respectively. Each time interval is 12 seconds long.

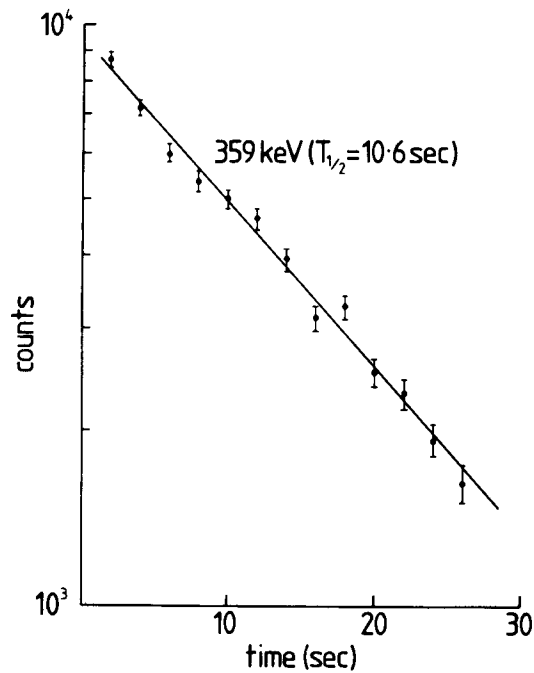


Figure 42: Decay curve of the 359 keV,  $2_1^+ - 0_1^+$ , gamma ray emitted in the decay of  $^{158}\text{Lu}$ .

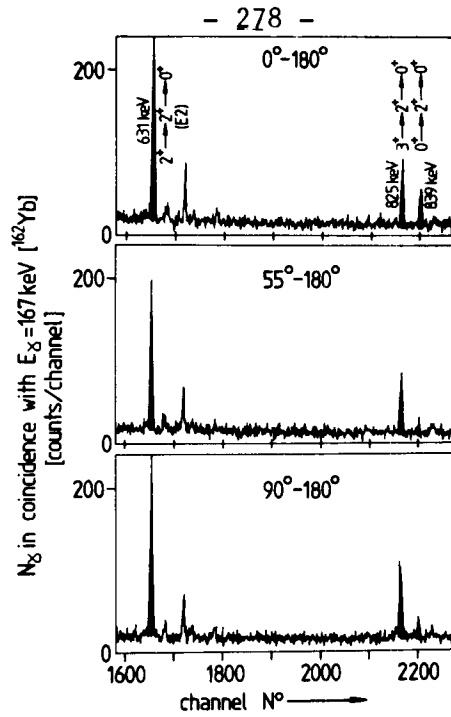


Figure 43: Part of a Ge(Li) spectrum in coincidence with the 167 keV,  $2^+ \rightarrow 0^+$  transition in  $^{162}\text{Yb}$  emitted in  $^{162}\text{Lu}$  decay observed at three different angles between the detectors.

the  $2^+ \rightarrow 0^+$  transition in  $^{162}\text{Yb}$ , at three different angles. The three spectra have been normalised and one can clearly see that the relative intensities of the 631, 823 and 839 keV peaks change from angle to angle. The resulting angular distributions of the 839-167 keV and 631-167 keV cascades are shown in figure 44. The various lines show the best fits to the usual angular correlation function

$$W(\theta) = 1 + Q_2 A_2 P_2(\cos \theta) + Q_4 A_4 P_4(\cos \theta),$$

for different initial spins in the cascade. Figure 45 shows  $\chi^2$  versus  $\arctan \delta$ , where  $\delta$  is the dipole-quadrupole mixing ratio in the upper transition, for the various possible spin sequences in the 632-167 keV cascade. It is clear that spin 2 with a pure E2, 632 keV transition is strongly favoured by the results.

These measurements have allowed us to construct decay schemes for the various Lu isotopes studied. Figures 46 and 47 summarise the decay

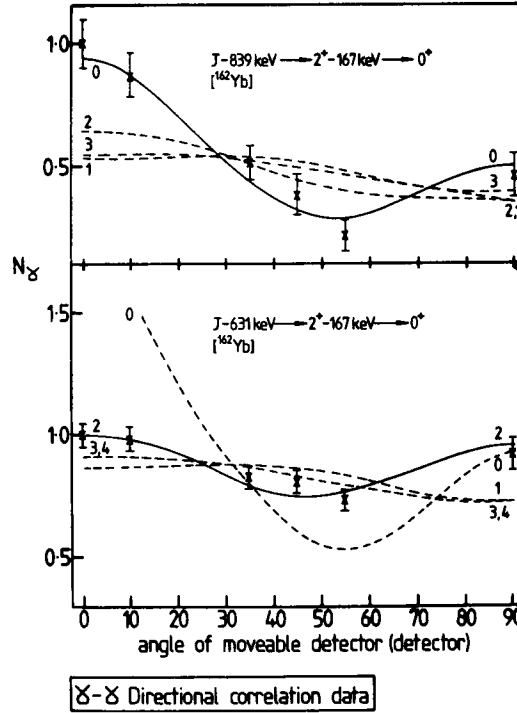


Figure 44: The measured angular distributions of the 839-167 keV and 631-167 keV gamma ray cascades emitted in  $^{162}\text{Lu} \rightarrow ^{162}\text{Yb}$  decay. The various lines show the best fits to the angular correlation for different values of the initial spin.

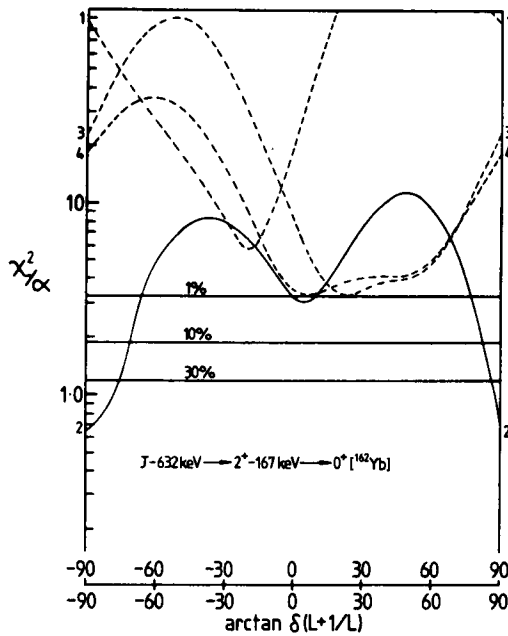


Figure 45: A plot of  $\chi^2$  vs.  $\arctan \delta$ , where  $\delta$  is the M1-E2 mixing ratio of the 631 keV transition, for the 631-167 keV gamma-ray cascade in  $^{162}\text{Lu}$  decay.

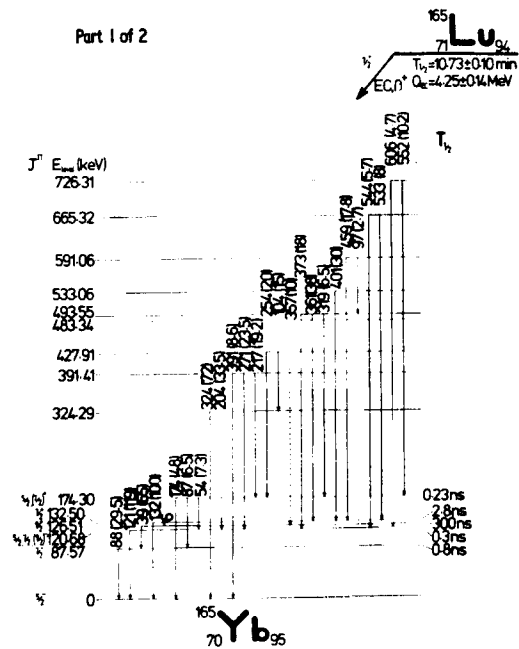


Figure 46: Part 1 of the  $^{165}\text{Lu}$  decay scheme (see fig. 47).

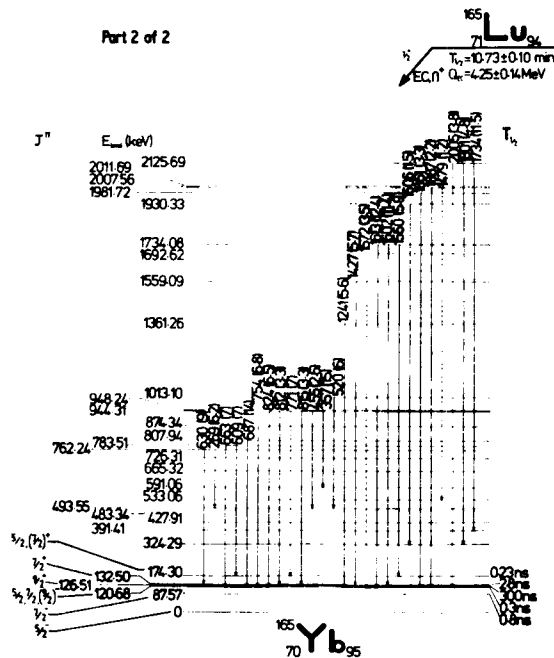


Figure 47: Part 2 of the  $^{165}\text{Lu}$  decay scheme (see fig. 46).



scheme<sup>(52)</sup> of  $^{165}\text{Lu}$  to levels in  $^{165}\text{Yb}$ . Figure 48 shows the decay scheme of  $^{163}\text{Lu}$ . Many of the transitions in these two cases are of low energy. Without the use of LEPS it would not have been possible to unravel these decays. I will not comment on these odd-mass decays except to say that our results suggest spin  $7/2$  for the  $^{165}\text{Lu}$  ground state in contrast to the assignment of spin  $1/2$  given by Ekstrom<sup>(53)</sup>. Our result would suggest that the odd-proton is in the  $7/2$  [404] orbit whereas his result is consistent with the  $1/2$  [411] proton orbital assignment. Our only explanation is that his sources were prepared in proton-induced reactions. It is possible that the two states lie very close together in energy and decay with effectively the same half-life, but it seems unlikely. One would have expected that Ekstrom would have observed the presence of a small fraction of a spin  $7/2$  resonance in his atomic beam magnetic resonance experiments.

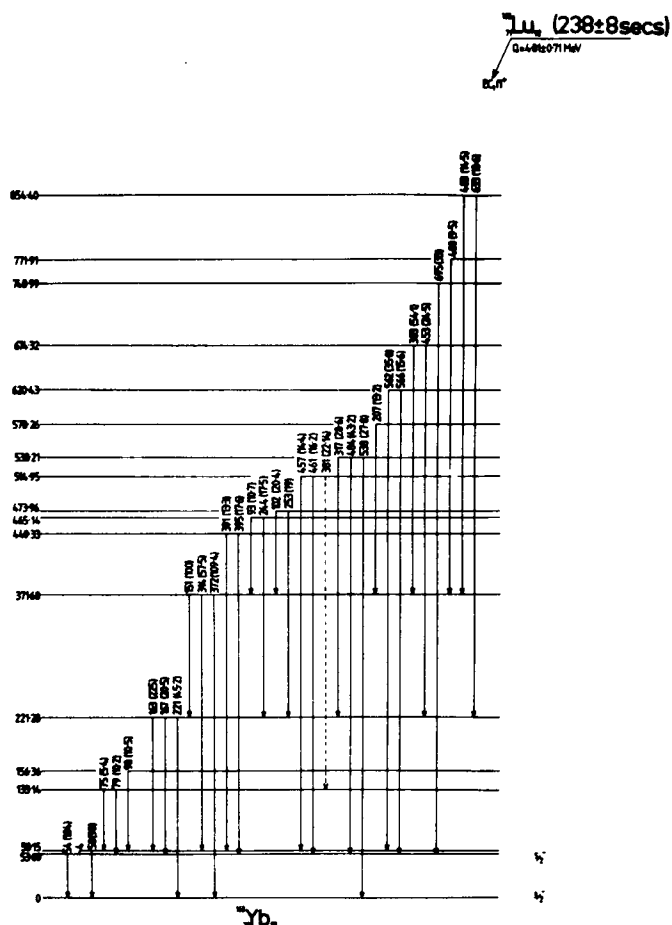


Figure 48: The decay scheme of  $^{163}\text{Lu}$ .

Our reason for studying the Lu decays was principally to obtain systematic information on the even Yb isotopes. The shape transition at  $N = 88-90$  in the Sm and Nd isotopes is well known and we wondered how it would be moderated by the addition of eight extra protons. Fig. 49 shows the low-lying levels in the even, Sm isotopes as a function of  $N$ . It is readily seen that the heavier isotopes are rotational and that there is a rapid change to a more vibrational spectrum as we go from  $N = 90$  to  $N = 88$ .

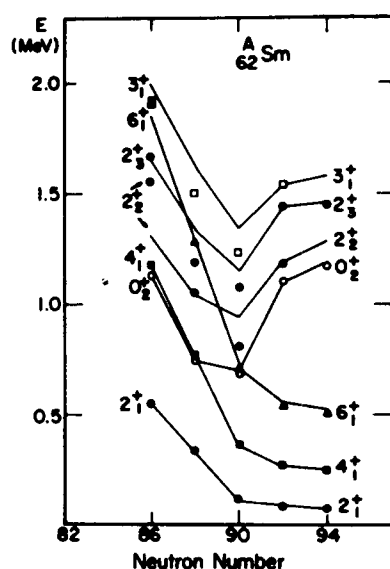


Figure 49: Systematic behaviour of the low-lying energy levels in the even Sm ( $Z = 62$ ) isotopes.

We can see the same feature in figs. 50 and 51, where the energy of the  $2_1$  state and  $E_4/E_2$  for nuclei in this region are plotted as a function of  $Z$  for fixed neutron numbers. The gaps in the middle of these figures emphasize again the rapid change in deformation in these nuclei. It is worth noting that the Gd nuclei ( $Z = 64$ ), which have a closed proton shell (35), make a jump from a near-spherical shape for  $N \leq 88$  to the most deformed of these nuclei for  $N \geq 90$ .

In fig. 52 the low-lying levels in the even, Yb isotopes are shown as a function of  $N$ . Here there is a smooth progression from rotational to vibrational nuclei. If we compare the results for Sm and Yb, as in

Table VII  
Measured Half-lives and Decay Energies of Lu Isotopes.

| <u>Isotope</u>    | <u>Half-life</u>     | <u>Decay Energy</u> |
|-------------------|----------------------|---------------------|
| $^{158}\text{Lu}$ | $10.6 \pm 0.3$ secs  | -                   |
| $^{160}\text{Lu}$ | $36 \pm 1$ secs      | $7.21 \pm 0.24$ MeV |
| $^{161}\text{Lu}$ | $78 \pm 2$ secs      | -                   |
| $^{162}\text{Lu}$ | $1.37 \pm 0.02$ min  | $6.74 \pm 0.27$ MeV |
| $^{163}\text{Lu}$ | $238 \pm 8$ secs     | $4.86 \pm 0.17$ MeV |
| $^{164}\text{Lu}$ | $3.15 \pm 0.08$ min  | $6.39 \pm 0.14$ MeV |
| $^{165}\text{Lu}$ | $10.74 \pm 0.10$ min | $4.25 \pm 0.14$ MeV |

Table VIII  
Comparison of Experimental and Theoretical Branching Ratios in  $^{160}\text{Yb}$

| <u>Transition</u> | <u>Energy (keV)</u> | <u><math>I_\gamma</math> (Rel.)</u> | <u><math>B(E2)</math> [Rel.]<sup>a</sup></u> | <u><math>IBA - 2^b</math></u> |
|-------------------|---------------------|-------------------------------------|----------------------------------------------|-------------------------------|
| $2_2 - 4_1$       | 181.7               | $\leq 1.2$                          | $\leq 388$                                   | 0.66                          |
| $2_2 - 2_1$       | 577.2               | 100                                 | 100                                          | 100                           |
| $2_2 - 0_1$       | 820.4               | 59                                  | 10                                           | 9.9                           |
| $0_2 - 2_2$       | 267.0               | $\leq 11.3$                         | $\leq 3476$                                  | 424                           |
| $0_2 - 2_1$       | 844.2               | 100                                 | 100                                          | 100                           |
| $3_1 - 4_1$       | 474.2               | 14.9                                | 33                                           | 22.6                          |
| $3_1 - 2_1$       | 869.2               | 100                                 | 10                                           | 8.3                           |
| $3_1 - 2_2$       | 292.2               | 4.5                                 | 100                                          | 100                           |
| $0_3 - 2_2$       | 401.3               | $\leq 14$                           | $\leq 1206$                                  | 1639                          |
| $0_3 - 2_1$       | 978.5               | 100                                 | 100                                          | 100                           |
| $4_2 - 3_1$       | 142.6               | $\leq 2.1$                          | $\leq 1325$                                  | 59.9                          |
| $4_2 - 2_2$       | 435.3               | 42                                  | 100                                          | 100                           |
| $4_2 - 4_1$       | 616.8               | 100                                 | 42                                           | 52.9                          |
| $4_2 - 2_1$       | 1012.4              | $\leq 1.2$                          | $\leq 0.04$                                  | 0.70                          |
| $2_3 - 0_1$       | 1293.5              | 100                                 | 12                                           | 18.3                          |
| $2_3 - 2_1$       | 1049.6              | 69                                  | 23                                           | 2.1                           |
| $2_3 - 4_1$       | 653.8               | 28                                  | 100                                          | 100                           |
| $2_4 - 0_1$       | 1358.5              | 11                                  | 3.6                                          | 5.0                           |
| $2_4 - 2_1$       | 1115.3              | 100                                 | 86                                           | 10.5                          |
| $2_4 - 4_1$       | 719.6               | 13                                  | 100                                          | 100                           |

- a) We have assumed that all of the observed transitions are pure E2 transitions (see text).  
b) See text for the basis of the calculations.

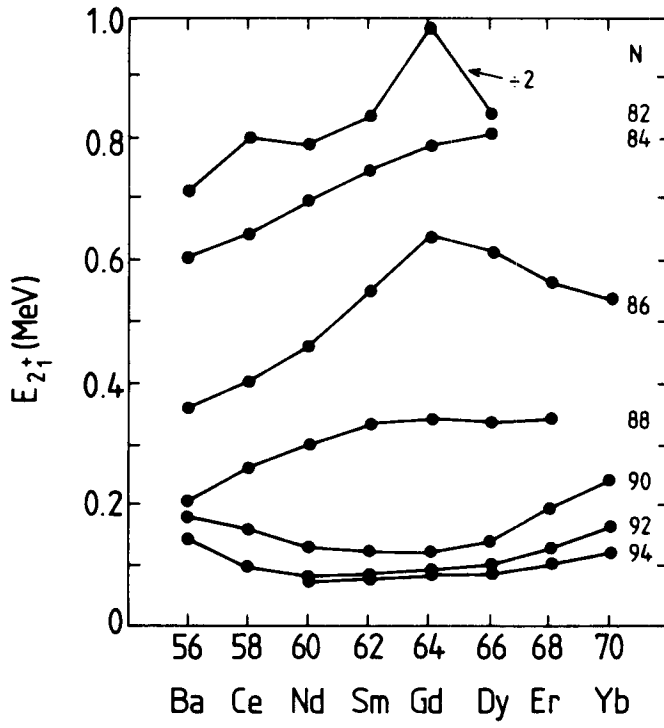


Figure 50: The energy of the  $2_1^+$  states as a function of Z for fixed N for the rare-earth nuclei.

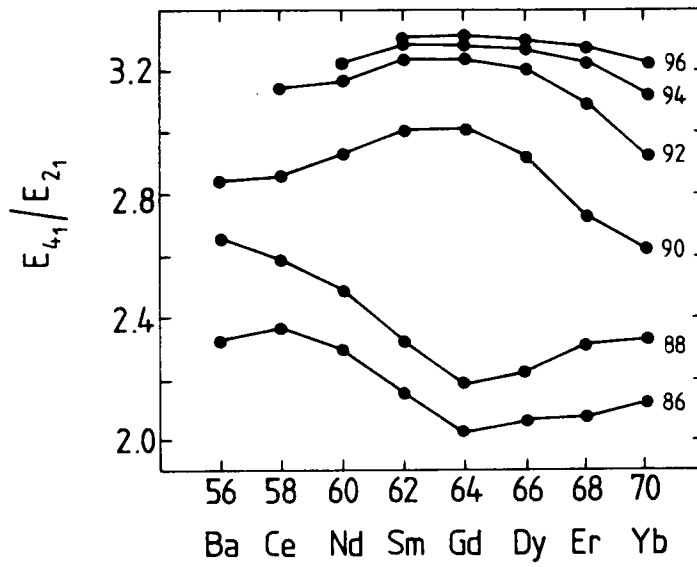


Figure 51: The ratio  $E_{4_1}/E_{2_1}$  as a function of Z for fixed N for the rare-earth nuclei.

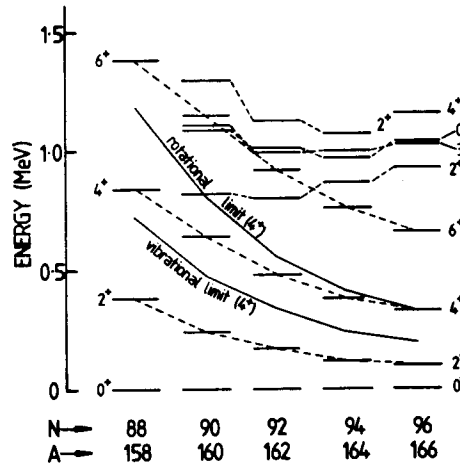


Figure 52: Systematic behaviour of the low-lying energy levels in the even Yb (Z = 70) isotopes.

fig. 53, we can see that the transition is much more abrupt in Sm than Yb. The extra protons seem to have smoothed out the transition. Casten et al<sup>(54)</sup> have explained the abruptness of the transition for Gd and Sm in terms of the idea put forward by Federman and Pittel<sup>(55)</sup> that it is due to the switching on of the strong n-p interaction for spin-orbit partner orbits. In the case of Gd, where the A = 146 nucleus is doubly-magic, the

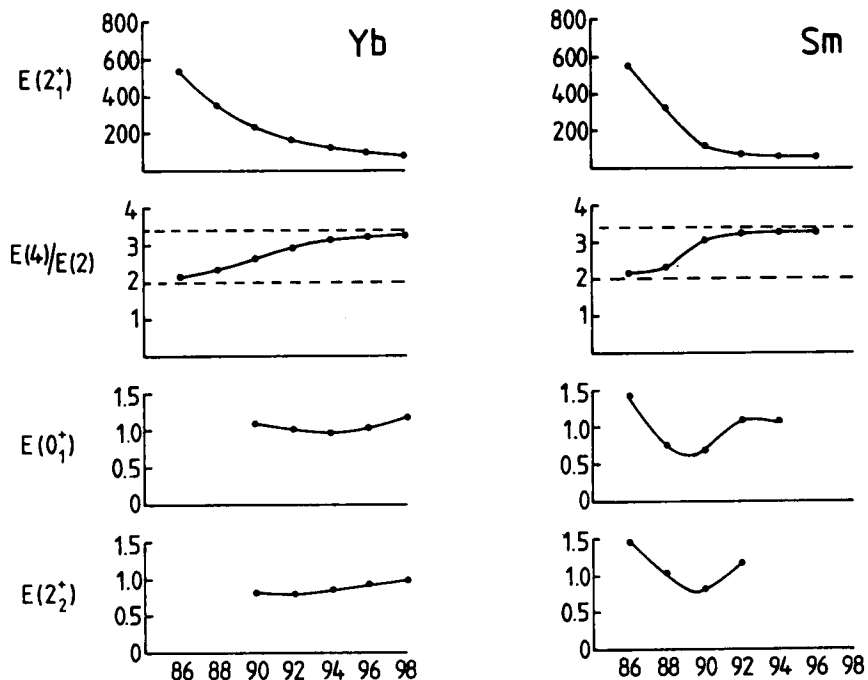


Figure 53: A comparison of  $E_4/E_2$ ,  $E(2_1)$ ,  $E(0_1)$ , and  $E(2_2)$  for the Yb and Sm isotopes as a function of N.

protons have filled the  $g_{7/2}$  and  $d_{5/2}$  orbits. Outside the  $N = 82$  shell the neutrons start to fill the  $f_{7/2}$  subshell but the spread in occupation amplitudes due to pairing means that they are already filling the  $h_{9/2}$  orbit near  $N = 88-90$ . The polarization of the proton shells causes the protons to begin to fill the  $h_{11/2}$  orbit. The n-p interaction causes a switch to a deformed structure with the consequent disappearance of the  $Z = 64$  gap. In Yb the  $h_{11/2}$  proton orbit is half-full. As a result when the neutrons start to occupy the  $h_{9/2}$  orbit the strong n-p interaction does not switch on suddenly but increases smoothly and we get the much more gradual transition observed. It should be noted that the transition occurs at a higher neutron number in Yb than Sm or Nd.

How else might we look at these nuclei? One way is in the framework of the Interacting Boson Approximation (IBA) of Arima and Iachello<sup>(6)</sup>. This model offers a simple description of collective states in even-even nuclei, with a Hamiltonian involving only a few parameters. In the version of the model<sup>(56)</sup>, IBA-2, which treats neutron and proton degrees of freedom separately, the parameters of the Hamiltonian can be related to the underlying shell structure and thus can be calculated microscopically. In the IBA an even-even nucleus with  $N_{\pi}(N_{\nu})$  pairs of protons (neutrons) is described as if it consisted of bosons, with total angular momentum  $L = 0$  or 2, each formed of two identical particles. The Hamiltonian may be approximated by

$$H = \epsilon(n_{d\pi} + n_{d\nu}) + K Q_{\pi} \cdot Q_{\nu} + V,$$

where

$$Q_{\pi(\nu)} = (d^{\dagger} s + s^{\dagger} d)_{\pi(\nu)} + \chi_{\pi(\nu)} (d^{\dagger} d)_{\pi(\nu)}.$$

The last term describes states of mixed neutron-proton symmetry, and can be neglected since they have not been identified in real nuclei and are expected to be at relatively high energies. If one makes the assumption

that  $\epsilon$  is the same for neutron and proton bosons then the level spectrum of an even-even nucleus is determined by four parameters, namely  $\epsilon$ ,  $K$ ,  $\chi_\pi$  and  $\chi_\nu$ . These parameters may depend on  $N_\pi$  and  $N_\nu$  but they are expected to vary smoothly with  $Z$  or  $N$ . Under these circumstances it should be possible by finding the parameters for an isotopic (isotonic) series to establish the parameters for any nucleus within or just outside the region bonded by the range of  $N_\pi$  and  $N_\nu$  values studied.

In IBA-1, where no distinction is made between protons and neutrons, one striking feature of the model is the appearance of three, limiting symmetries,  $SU(3)$ ,  $SU(5)$  and  $O(6)$ . These correspond to the familiar rotor, vibrator and  $\gamma$ -unstable oscillator of the collective model. Most nuclei lie somewhere between these limits and are naturally described in terms of the transition from one limit to another. In the case of the Yb nuclei whose level spectra are shown in fig. 52 they would appear to be described naturally in terms of the transition from  $SU(3)$  to  $SU(5)$  as the number of bosons decreases.

We may also look at how level properties change in this region with increasing proton number for a fixed number of neutrons. Scholten<sup>(57)</sup> has looked at the  $N = 90$  isotones, with  $Z = 60-68$ , within the framework of IBA-2. His results suggest that there is a transition from  $SU(3)$  to  $O(6)$  as  $Z$  increases.  $^{160}_{70}\text{Yb}_{90}$  is the next member of this isotonic sequence. It is possible to extrapolate the parameters used by Scholten to  $Z = 70$  and calculate the level spectrum of this nucleus. This has been done<sup>(58)</sup> with the programs NPBOS and NPBEM<sup>(59)</sup> using the parameters extrapolated from Scholten's calculations as shown in fig. 54. No attempt was made to adjust the parameters. The resulting level spectrum is compared with experiment in fig. 55. Strictly speaking the limiting symmetries of IBA-1 do not exist in IBA-2 but the  $O(6)$  limit corresponds approximately to  $\chi_\pi = -\chi_\nu$ . In  $^{160}\text{Yb}$  we have  $\chi_\nu = -1.2$  and  $\chi_\pi = 0.49$ . The calculation agrees quite

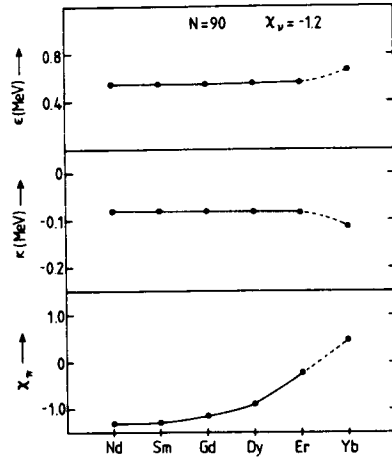


Figure 54: The parameters used by Scholten<sup>(57)</sup> in his IBA-2 calculations of properties of positive parity levels in the even Z, N = 90 isotones. The extrapolations to the values used<sup>(58)</sup> for  $^{160}\text{Yb}$  are shown as the dashed line.

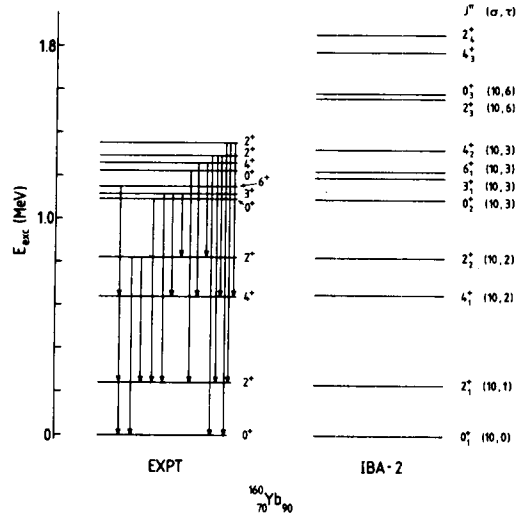


Figure 55: A comparison of the observed level spectrum of  $^{160}\text{Yb}$  with the IBA-2 predictions (see text) of ref. 58.

well with the observed level spectrum of  $^{160}\text{Yb}$  although the compression of the higher levels is not reproduced.

Table VIII compares the measured  $B(E2)$  ratios from the various levels with the predicted values. Unfortunately the  $E\gamma^5$  dependence of the  $B(E2)$  values means that the intensities of many of the low energy gamma rays are expected to be very low, and we could only extract limits. In general there is excellent agreement between theory and experiment, and since the details of the wavefunctions are insensitive to small changes in the



parameters this is unlikely to change in a calculation in which the parameters are varied to find a good fit to the level spectrum. It is worth noting that the  $B(E2)$  values have been derived on the assumption that the transitions are pure E2 transitions. Where the correlations were strong enough to be measured this assumption is justified as can be seen from figure 45. In the case of the higher lying states, where we have no reliable measure of the M1-E2 mixing ratio in the de-exciting transitions, the presence of an M1 admixture would improve the agreement between the measured and theoretical  $B(E2)$  ratios.

Fig. 56 shows how various properties change as a function of  $Z$  for the  $N = 90$  isotones. The experimental  $E_4/E_2$  ratio and the differences in energy between the quasi- $\gamma$  and quasi- $\beta$  bandheads and the  $4_1$  state are plotted on the left. It is seen that the  $E_4/E_2$  ratios change from a value of about 3.0, a bit below the rotational value of 3.33, towards the  $O(6)$  limit value of 2.5. At the same time the  $2_\gamma^+$  and  $O_\beta^+$  states cross-over with the  $2_\gamma$  level approaching the  $4_1^+$  state and the  $O_\beta$  level increasing in energy as we would expect in approaching the  $O(6)$  limit. On the right the

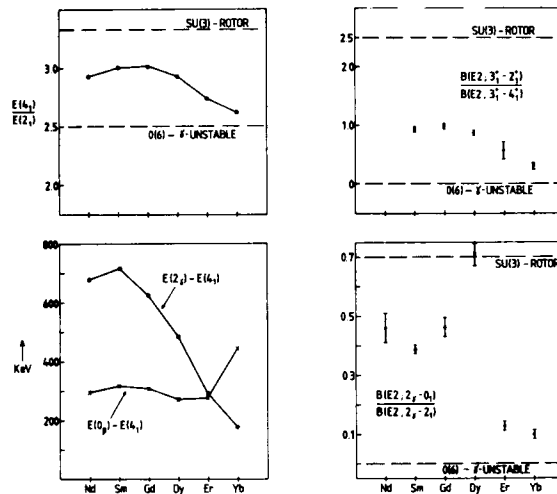


Figure 56: The upper left graph show the  $E_4/E_2$  ratio for the even  $Z$ ,  $N = 90$  isotones as a function of  $Z$ . The lower left shows  $\Delta(E(2_2) - E(4_1))$  and  $\Delta(E(2_\gamma) - E(4_1))$  as a function of  $Z$  for the  $N = 90$  isotones. The measured  $B(E2; 2_\gamma - O_1)/B(E2; 2_\gamma - 2_1)$  and  $B(E2; 3_1 - 2_1)/B(E2; 3_1 - 4_1)$  ratios are shown as a function of  $Z$  for the  $N = 90$  isotones on the right. The values of these ratios for the  $SU(3)$  and  $O(6)$  limits are indicated.

experimental  $B(E2; 2\gamma-0_1)/B(E2; 2\gamma-2_1)$  and  $B(E2; 3_1-2_1)/B(E2; 3_1-4_1)$  ratios are plotted as a function of  $N$ . Although they are not at the values associated with the  $SU(3)$  limit in the lighter nuclei they decrease towards the values of zero associated with the  $O(6)$  limit as  $Z$  increases.

In his calculations Scholten<sup>(57)</sup> adopted a value of 0.14 eb to match the experimental value of  $B(E2; 2_1^+-0_1^+)$  in  $^{152}\text{Sm}$ . The values of  $B(E2; 2_1^+-0_1^+)$  for all the  $N = 90$  isotones calculated with the same effective charge are compared with experiment in fig. 57. The theory predicts the trend very well with the initial increase being due to the increase in the number of bosons, and the subsequent fall being due partly to the decrease in the number of bosons beyond mid-shell and partly to the change towards the  $O(6)$  structure.

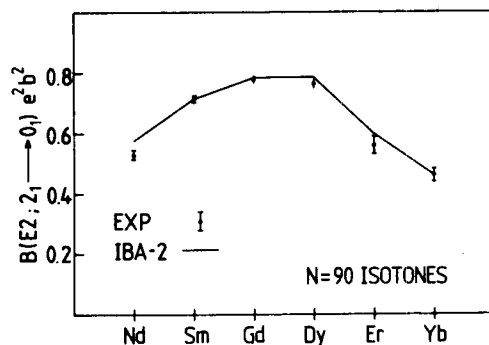


Figure 57: The measured  $B(E2; 2_1-0_1)$  values for the even  $Z$ ,  $N = 90$  isotones. The values predicted by IBA-2<sup>(58)</sup>, normalised to the value for  $^{152}\text{Sm}$  ( $Z = 62$ ), are indicated by the solid line.

Overall we conclude that the IBA-2 calculations describe the properties of the  $N = 90$  isotones very well. These nuclei show a transition from  $SU(3)$  to  $O(6)$  in the terminology of this model. Since  $^{160}\text{Yb}$  also lies in the middle of a transition from  $SU(3)$  to  $SU(5)$  in the series of Yb isotopes, it appears to be a nucleus very much at the cross-roads between all these limits. It will be very interesting to see the levels of  $^{162}\text{Hf}$  which should be closer to the  $O(6)$  limit.

Again it is clear that one can do many experiments with this simple device, the HeJRTS. One might ask why one should build an isotope separator

if this is so. As one might expect there are at least two disadvantages to the use of this technique. Firstly there are many experiments where one requires a single activity which has been physically separated from all others, e.g. for nuclear orientation studies. Secondly the sensitivity of these measurements is very limited. A more stringent test of the IBA-2 calculations would require the measurement of the intensities of the many weak, low energy transitions which should be present (see Table VIII). With a separated source these measurements should be possible. With the HeJRTS technique, where one relies on coincidence experiments, the sensitivity is limited and the measurements will become less sensitive as one looks at a smaller fraction of the total cross-section.

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RELATIONS BETWEEN THE SIMULTANEOUS AND SEQUENTIAL  
TRANSFER OF TWO NUCLEONS\*

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RELATIONS BETWEEN THE SIMULTANEOUS AND SEQUENTIAL  
TRANSFER OF TWO NUCLEONS<sup>\*,†</sup>

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1. Introduction

The transfer of two nucleons between projectile and target in a direct or peripheral reaction such as (p,t) or ( $^{16}\text{O},^{14}\text{C}$ ) may occur in one-step or two-steps. These we refer to as 'simultaneous' and 'sequential' transfers, respectively. In the former, the interaction acts once and both nucleons are transferred. In the latter, the interaction acts once to transfer one nucleon, the system then propagates in one or more intermediate states and is followed by a second action of the interaction to transfer the second nucleon. This process may be symbolized for the above examples as (p,d;d,t) and ( $^{16}\text{O},^{15}\text{N};^{15}\text{N},^{14}\text{C}$ ), implying the intermediate formation of a deuteron or the nucleus  $^{15}\text{N}$ . (Of course, the intermediate system may exist in more than one state of excitation.)

In terms of a perturbation theory expansion, such as the distorted-wave Born series, simultaneous transfer is possible in first order while sequential transfer requires second order. This is illustrated in Fig. 1. We are accustomed, perhaps, to thinking that a first-order process is more likely than a second-order one. However, a closer examination can prepare us for the possibility that this may not be so. The nuclear forces are predominantly two-body in

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character; hence, in first-order (Fig. 1a) only one of the two nucleons experiences an interaction. The possibility of finding that the other nucleon has also transferred arises only because its state within the projectile is not orthogonal to the state in the target into which it transfers. This process corresponds to a kind of quantum-mechanical tunneling from the projectile to the target.<sup>1</sup> However, in the two-step process (Fig. 1b) each nucleon is transferred under the direct influence of an interaction with the target; intuitively, this might seem more plausible. It requires an explicit calculation to determine which process is most likely in a given case, and such calculations are often beset with uncertainties. Nonetheless, it seems clear that the one-step and two-step amplitudes are frequently comparable in magnitude for light-ion reactions<sup>2</sup> while the two-step may dominate in reactions with heavy ions.<sup>3</sup> (The existence of strong Q-window effects, especially with heavy ions,<sup>4</sup> may enhance the sequential process when there is a large mismatch between the entrance and exit channels. The gap may be bridged more easily in two steps, with the interaction acting twice.) Consequently, it is not safe to ignore the existence of sequential transfer. However, one of the main reasons for studying two-nucleon transfers is to learn about the two-nucleon overlaps (existence and extent of pairing correlations, etc.). So we wish to know if this information is still available when the reaction proceeds by two, sequential, one-nucleon transfers. Our purpose here is to gain some insight into the relationship between the two amplitudes by using a simple approximate form of the theory. For simplicity, we shall discuss a light-ion reaction and, to be specific, we choose the (t,p) reaction (or the inverse (p,t) reaction). Similar considerations apply to other reactions.

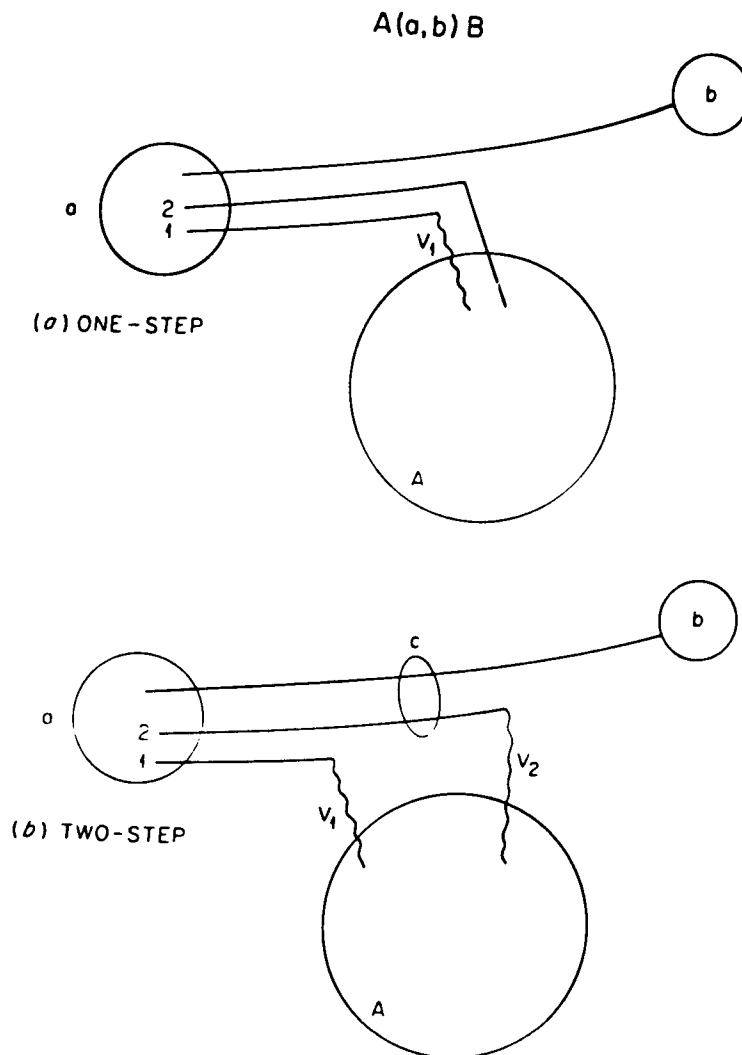


Figure 1

(a) Simultaneous transfer, (b) sequential transfer. The wavy line corresponds to one action of the projectile-target interaction.

## 2. The (t,p) or (p,t) Reaction

Apparently contradictory evidence exists for the relative importance of the two-step or sequential-transfer mechanism in (t,p) or (p,t) reactions. Indeed, the results of a confusing variety of sophisticated calculations are available, each of which treats properly some, but not all, aspects of these transitions. Among these aspects are: (i) the use of a realistic triton wavefunction and the associated realistic interaction;<sup>5-8</sup> (ii) an exact treatment of the finite-range of the interactions (as opposed to use of a zero-range approximation), in both the one-step<sup>5-9</sup> and the two-step<sup>8-11</sup> amplitudes; (iii) accounting for the non-orthogonality correction to the two-step term,<sup>2,9,11,12</sup> which tends to cancel the one-step amplitude;<sup>12-14</sup> (iv) allowing the intermediate n-p system of the two-step process to exist in continuum states, both spin-singlet and triplet, as well as in the bound, triplet deuteron ground state<sup>12,15</sup> (sometimes this is called the 'deuteron break-up effect'). In addition, there is a sensitivity of the results to the optical potentials employed, and a particular sensitivity of the cross section magnitudes to the nuclear wavefunctions used to construct the two-nucleon overlaps.<sup>6,7</sup>

In view of the complexity of the theoretical description of this transfer process, the agreement with measured cross sections obtained by calculations which only include some of these aspects must be viewed with some caution. In this situation it is valuable to have an understanding of any general, albeit approximate, features that the amplitudes may possess, independent of these details, which may illuminate the results of more detailed calculations.

The point of the present paper is to use a closure approximation to the second-order distorted-wave-Born expression for the two-step amplitude in order to exhibit the similarities and differences in comparison to the one-step

amplitude, especially with regard to the selection rules for spin-transfer.

This is by no means a new approach.<sup>3,16</sup> It has been used to explain why simultaneous and sequential processes tend to yield similar angular distributions and to depend upon the nuclear structure involved in the same way. Schaeffer and Bertsch<sup>16</sup> indicated schematically that, if the important intermediate states had similar energies, their Green functions could be replaced by an average one and closure applied to the nuclear states. This results in the nuclear overlaps for sequential transfer being the same as those for simultaneous transfer, with the same selection rules. A closer examination of the spin angular momenta involved modifies the conclusion about the selection rules a little. For example, in the two-nucleon transfer (t,p) reaction, if the n-p system or 'deuteron' associated with the intermediate states of the two-step amplitude appears in spin-singlet or spin-triplet states with equal weight, the selection rules for one- and two-step are identical. However, constraining it to be in a triplet state only (such as the common assumption that it is the physical deuteron ground state) acts as a spin-filter and determines that the two neutrons are transferred with a particular mixture of total spin  $S = 0$  or  $1$ . This mixture will differ from that for the one-step term. In this way, inclusion of singlet deuterons could completely change the results for vector analyzing powers and might modify conclusions that have been drawn from measurements of these quantities. It is sufficient, for our purpose, to consider the first and second-order distorted-wave Born amplitudes and to make various simplifying assumptions such as zero-range. While these may have large effects upon the magnitude of the amplitudes, they do not affect the underlying structure.

## 2.1. One-Step Amplitudes

The triton wavefunction can be decomposed into components in which the two neutrons have a definite total spin  $S$  (where  $S = 0$  or  $1$ ),

$$\psi_{\frac{1}{2} m_t} = \sum_S \psi_{\frac{1}{2} m_t}^S(\text{spins}) \psi_t^S(\text{space}). \quad (1)$$

Then, aside from uninteresting multiplicative constants, the first-order DWBA or one-step amplitude for the  $A(t,p)B$  reaction is a sum of components,

$$T_{BA}^{(1)}(S) = \int d\mathbf{R} d\mathbf{r} d\mathbf{e} \, x_p^{(-)*}(\mathbf{R}_p) \left( \frac{1}{2} m_p; B \middle| \left( \frac{1}{2}, S \right) \frac{1}{2} m_t; A \right) V(\mathbf{r}, \mathbf{e}) \psi_t^S(\mathbf{r}, \mathbf{e}) x_t^{(+)}(\mathbf{R}_t), \quad (2)$$

for each value of the total spin  $S$  of the two transferred neutrons. The spatial coordinates are defined in Fig. 2. The  $x_p$  and  $x_t$  are the usual distorted waves. The round brackets denote integration over the internal coordinates of nucleus  $A$  and the spin coordinates of the proton and two neutrons; this factor remains a function of the spatial coordinates  $\mathbf{r}_1$  and  $\mathbf{r}_2$  of the two neutrons. Also,  $\psi_t^S$  is the spatial part of the triton internal state as defined in Eq. (1). The transfer interaction  $V$  is taken to be the post form,  $V_{pB} - U_{pB} = V_{p,2n} + (V_{pA} - U_{pB})$ , with  $U_{pB}$  being the optical potential for the outgoing proton, and with the usual assumption that  $(V_{pA} - U_{pB})$  may be neglected. Then

$$V = V_{p,2n} = V_{pn_1} + V_{pn_2} = v(|\mathbf{e} - \frac{1}{2} \mathbf{r}|) + v(|\mathbf{e} + \frac{1}{2} \mathbf{r}|). \quad (3)$$

In general,  $V$  depends on the spins, but since it cannot change the spin  $S$  of the transferred neutrons,<sup>17</sup> it is sufficient for our purpose to treat it as spin-independent.

The spin matrix element gives a Clebsch-Gordan coefficient expressing the angular momentum coupling of the spins and Eq. (2) can be rewritten



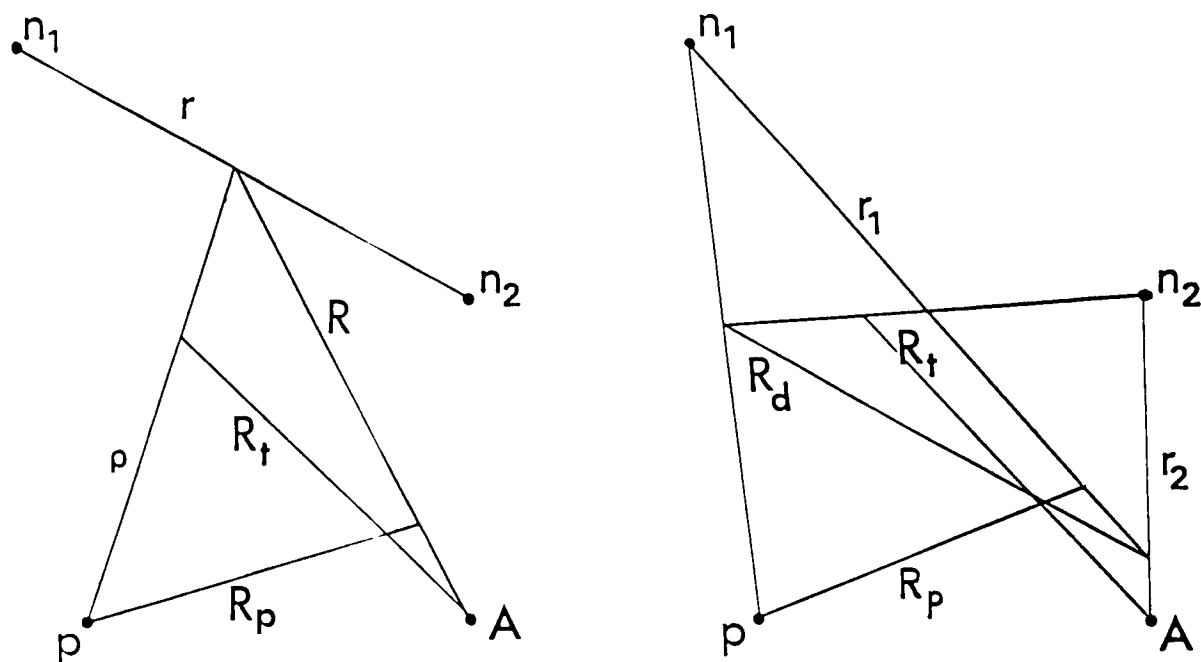


Figure 2

Coordinates used for (a) simultaneous, and (b) sequential transfer.

$$T_{BA}^{(1)}(S) = \langle S \frac{1}{2} M_S m_p | \frac{1}{2} m_t \rangle \int x_p^{(-)*}(\underline{R}_p) \phi_{SM_S}^{BA}(\underline{r}_1, \underline{r}_2) \times V(\underline{r}, \underline{\rho}) \psi_t^S(\underline{r}, \underline{\rho}) x_t^{(+)}(\underline{R}_t) d\underline{R} d\underline{r} d\underline{\rho}, \quad (4)$$

where the function  $\phi_S$  is the spatial part of the overlap of the two nuclei, i.e. the two-neutron form factor, corresponding to spin  $S$ ,

$$\phi_{SM_S}^{BA}(\underline{r}_1, \underline{r}_2) = (B | SM_S; A) \equiv \int \psi_B^*(\xi_A, \underline{r}_1 \sigma_1, \underline{r}_2 \sigma_2) \psi_A(\xi_A) \psi_{SM_S}(\sigma_1, \sigma_2). \quad (5)$$

It is this two-nucleon overlap which embodies the nuclear structure information about pairing, etc., that we wish to extract from measurements on two-nucleon transfer reactions.

The predominant part of the triton wavefunction is the symmetric  $S$ -state. Within it the neutrons have  $S = 0$ , and the space function  $\psi_t^0$  is a function of the single symmetric variable,

$$u^2 = r_{12}^2 + r_{23}^2 + r_{13}^2 = \frac{3}{2} r^2 + 2\rho^2. \quad (6)$$

Frequently, a zero-range approximation is used in which one sets

$$V\psi_t^0 \approx D(r)\delta(\underline{\rho}), \quad (7)$$

so that the proton is assumed to be at the centre of mass of the two neutrons and

$$T_{BA}^{(1)}(S=0) \approx \delta_{m_t m_p} \int x_p^{(-)}(\underline{R}') \phi_{00}^{BA}(\underline{r}_1, \underline{r}_2) D(r) x_t^{(+)}(\underline{R}) d\underline{R} d\underline{r}, \quad (8)$$

where  $\underline{R}' = \underline{R}A/(A+2)$ .

The importance of the mixed symmetry  $S'$ -state of the triton and of the  $D$ -state have been stressed by Nagarajan, et al.<sup>17</sup> We consider here the  $S'$ -state

only. It is a sum of two terms, one with  $S = 1$ , one with  $S = 0$ . We ignore the latter component, since it results in the same selection rules as the symmetric S-state. The term with  $S = 1$  has a spatial part of the form  $\underline{r} \cdot \underline{\rho} g(u)$ , so that there is a contribution to the amplitude

$$T_{BA}^{(1)}(S=1) = \langle 1 \frac{1}{2} M_S m_p | \frac{1}{2} m_t \rangle \int \chi_p^{(-)}(\underline{R}_p)^* \phi_{1M_S}^{BA}(\underline{r}_1, \underline{r}_2) V \underline{r} \cdot \underline{\rho} g(u) \chi_t^{(+)}(\underline{R}_t) d\underline{R} d\underline{r} d\underline{\rho}. \quad (9)$$

One cannot make the zero range ( $\rho = 0$ ) approximation on this term because of the factor  $\underline{r} \cdot \underline{\rho}$ . However, it is useful to consider the lowest-order non-vanishing terms in an expansion of the distorted waves in powers of  $\rho$ ;

$$\chi_t^{(+)}(\underline{R}_t) = \chi_t^{(+)}(\underline{R} + \frac{1}{3} \underline{\rho}) = \chi_t^{(+)}(\underline{R}) + \frac{1}{3} \underline{\rho} \cdot \underline{\nabla}_R \chi_t^{(+)}(\underline{R}) + \dots, \quad (10)$$

with a corresponding expansion for  $\chi_p^{(-)}(\underline{R} + \underline{\rho})$ . Since from Eqs. (2) and (4),  $V$  and  $g$  are even functions of  $\rho$ , we have

$$\int d\underline{\rho} g(u) V \underline{r} \cdot \underline{\rho} = 0, \quad (11)$$

so that the lowest-order term appearing in the expansion of  $T_{BA}^{(1)}(S=1)$  contains one factor of  $\underline{\rho} \cdot \underline{\nabla}_R \chi$ . If we define a function  $E$  according to

$$\int d\underline{\rho} g(u) V \underline{r} \cdot \underline{\rho} (\underline{\rho} \cdot \underline{\nabla}_R \chi) = E(\underline{r}) \underline{r} \cdot \underline{\nabla}_R \chi, \quad (12)$$

then we can write

$$\begin{aligned} T_{BA}^{(1)}(1) \approx \langle 1 \frac{1}{2} M_S m_p | \frac{1}{2} m_t \rangle \int d\underline{R} d\underline{r} E(\underline{r}) \phi_{1M_S}^{BA}(\underline{r}_1, \underline{r}_2) \\ \times \left\{ \frac{1}{3} \chi_p^{(-)}(\underline{R}^-)^* \underline{r} \cdot \underline{\nabla}_R \chi_t^{(+)}(\underline{R}) + \frac{A+2}{A} \underline{r} \cdot \underline{\nabla}_R \chi_p^{(-)*}(\underline{R}^-) \chi_t^{(+)}(\underline{R}) \right\}. \end{aligned} \quad (13)$$

Later we will show that the sequential transfer process generates amplitudes very similar to Eqs. (8) and (13).

## 2.2. Two-Step Amplitudes and the Closure Approximation

In second-order DWBA, the transition amplitude  $T^{(2)}$  for the two-step process  $A(t,d;d,p)B$ , with intermediate nuclear states  $C$ , has a part  $\hat{T}^{(2)}$  which is second order in the interactions and a non-orthogonality correction term  $T^{(NO)}$ . The first part has the form<sup>18</sup>

$$\hat{T}_{BA}^{(2)} = \sum_C \int \dots \int \chi_p^{(-)*}(\underline{R}_p) (p,B | V_{pn_1} | d,C) G_C^{(+)}(\underline{R}_d, \underline{R}'_d) (d,C | V_{dn_2} | t,A) \chi^{(+)}(\underline{R}_t) d\underline{R}_p d\underline{R}_d d\underline{R}'_d d\underline{R}_t, \quad (14)$$

where we ignore spins for the moment. Here  $G_C$  is the Green function for the propagation of the intermediate deuteron  $d$  (relative to the intermediate nucleus  $C$ ) under the influence of an optical potential. If this optical potential were neglected so that the propagation was in plane waves rather than distorted waves,  $G_C$  would reduce to

$$G_C^{(+)}(\underline{R}, \underline{R}') \sim \exp(ik_d s)/s, \quad s = |\underline{R} - \underline{R}'|. \quad (15)$$

(The wavenumber  $k_d$  could be reinterpreted as the local value  $k_d(\bar{R})$  at the position  $\bar{R} = \frac{1}{2}|\underline{R} + \underline{R}'|$ , thus introducing a 'local-energy' approximation<sup>3</sup> for the effects of the optical potential and the Coulomb field.) The quantities with round brackets are the usual nuclear overlap functions, or 'form factors', for one-nucleon transfer. In the post-post interaction form that we have adopted, the non-orthogonality term  $T^{(NO)}$  has a similar form except for having the opposite sign and for the omission of two scalar factors, the Green function  $G_C$  and the interaction  $V_{dn_2}$  for the  $(t,d)$  step. It tends to cancel the one-step amplitude.

The closure approximation consists of assuming that the intermediate states  $\psi_C$  that contribute importantly in the expression for  $\hat{T}^{(2)}$  are sufficiently close

together in excitation energy that the associated Green functions  $G_C$  can be replaced by an average  $\overline{G}_C$ . We then assume that we have a complete set of states  $\psi_C$  and use the completeness relation for the nuclear overlaps,

$$\sum_C (B|C)G_C(C|A) \approx \overline{G}_C \sum_C (B|C)(C|A) \quad (16a)$$

$$= \overline{G}_C(B|A), \quad (16b)$$

because of the closure relation

$$\sum_C |C\rangle\langle C| = 1. \quad (16c)$$

Despite the somewhat condensed notation, comparison with Eq. (5) shows that this procedure results in the appearance of the same two-neutron overlap functions or form factors  $\Phi^{BA}(\underline{r}_1, \underline{r}_2)$  as those which appear in the one-step amplitude (4).

Consequently, provided the important intermediate states are sufficiently close for the approximation (16a) to be valid, nuclear structure effects such as pairing correlations will manifest themselves in much the same way in one-step or two-step processes. In this way, one can understand how analyses of measured cross sections using the theory for a one-step transfer alone may yield useful information on the relative behavior of spectroscopic factors for different states and different nuclei,<sup>19</sup> even when the theory fails to explain the absolute magnitudes of the cross sections.

### 2.3. Further Comments on the Closure Approximation

One can easily construct model situations in which the procedure (16) becomes exact. For example, consider two-neutron transfer to a closed-shell target in which the intermediate states  $\psi_C$  are pure single-particle  $j$ -states and the residual state  $\psi_B$  has a pure  $j^2$  configuration. Only one intermediate state can contribute, namely the corresponding single-particle state in nucleus  $A+1$  with the same  $j$ , and Eq. (16b) becomes exact.

The expression (14) assumes a single intermediate state for the intermediate particle  $c = a-1$ , in this case a deuteron,  $c = d$ . More generally, both intermediate nuclei may be in more than one state and expressions like (14) need to be supplemented by an additional sum over the states of  $c$ . In a heavy-ion reaction, both  $c$  and  $C$  may be comparable and hence should be treated on the same footing. This was done by Feng, et al.<sup>3</sup> when they applied the closure approximation. The tendency has been to treat light-ion reactions differently, by assuming that the intermediate light particle remains in its ground state. This has been a matter of expediency; excited states of light ions are unbound and thus introduce a continuum of intermediate  $> 3$ -body states which greatly increases the complexity of the calculation. This has been done<sup>12,15</sup> for the intermediate 'deuteron' (i.e.,  $n$ - $p$  system) in  $(p,t)$  reactions.

The closure approximation (16) may be applied to the states of  $c$ , as well as those of  $C$ , if the important contributions still come from a sufficiently narrow band of energies (i.e. narrow enough that the Green function  $G_{CC}$  does not vary much). This results in the appearance of a second two-nucleon overlap or form factor  $(b|a)$ , which again is the same as that which occurs in the one-step amplitude  $T^{(1)}$ . Then the nature of the intermediate nucleus  $c$  plays no rôle and, for example, angular momentum selection rules are the same for one-step and two-step transfer. It is plausible that this is approximately valid for heavy-ion reactions<sup>3</sup> where the important intermediate states are likely to be bound states of moderate excitation energy. It is not so obvious for light-ion reactions. The explicit calculations<sup>12,15</sup> for  $^{48}\text{Ca}(p,t)$  at 20 and 40 MeV included the  $n$ - $p$  scattering states, both singlet and triplet. Important contributions came from the continuum states at 40 MeV, but the effect (on the cross section) was small at 20 MeV. The intermediate spin-singlet state is always associated

with continuum states of relative motion. Hence, the transition is dominated by triplet intermediate states if the contribution of the continuum is negligible. This constitutes a spin-filter through which the two-step process must pass and introduces differences in the spin selection rules compared to the one-step process. We return to this later. It is not clear from the results<sup>12,15</sup> for 40 MeV, where the continuum is important, whether this kind of spin selectivity is still active.

#### 2.4. Non-Orthogonality and the Second-Order Amplitude

If one introduces a truly complete set of intermediate states (for both intermediate nuclei), it is easy to show<sup>12,14</sup> that, in the post-post (or prior-prior) interaction form of the amplitude, the non-orthogonality correction term  $T^{(NO)}$  becomes equal and opposite to the one-step amplitude  $T^{(1)}$ . The two-step term  $\hat{T}^{(2)}$  is then left as the lowest-order contribution to the transition. In practice, however, it is not feasible to use a complete set. Formally, it implies non-convergence of the Born series because of the mathematical difficulties associated with the 3- and more-body states that appear when the intermediate states are unbound. One standard approach is to regard our calculations as being done within a truncated model space, limited to 2-body channels, and with effective interactions. Thus, a complete set is ruled out from the start. Nonetheless, the tendency remains for  $T^{(NO)}$  and  $T^{(1)}$  to interfere destructively, sometimes quite strongly (see Refs. 9 and 12-15 for examples), and this enhances the importance of the two-step process described by  $\hat{T}^{(2)}$ .

One final note: the separation of the second-order amplitude  $T^{(2)}$  into  $\hat{T}^{(2)}$  and  $T^{(NO)}$  is artificial. Despite the intuitive appeal of interpreting  $\hat{T}^{(2)}$  as "the two-step process", one cannot physically distinguish these two terms from each other (nor they from the first-order term  $T^{(1)}$ ). This is

emphasized by the lack of uniqueness in separating  $T^{(2)}$  into two parts; for example, if we had chosen the prior-post interaction form for  $T^{(2)}$ , there would be no  $T^{(NO)}$  term and hence no possibility of the cancellation of  $T^{(1)}$ . This lack of uniqueness also renders ambiguous even the conceptual division into one-step and two-step. Only the total amplitude has direct physical meaning; its breakdown into pieces is merely a consequence of the way we do calculations. Nonetheless, the discussion of this paper can be valuable as an aid to understanding the results of those calculations.

## 2.5. Approximate Two-Step Amplitude for (t,p) Reactions

We now consider the amplitude (14) in more detail. At first, we only allow ground-state (triplet) deuterons in the intermediate state. Further, for simplicity, we adopt the zero-range approximation for each of the single-nucleon transfers and only consider the dominant symmetric S-state for the triton.

Consequently, the spins of the two neutrons within the triton are coupled to  $S = 0$ . Then Eq. (14) becomes

$$\begin{aligned} \hat{T}_{BA}^{(2)} = & \sum_{C, m_d} D_0(t, d) D_0(d, p) \int d\mathbf{r}_1 x_p^{(-)*} \left( \frac{A+1}{A+2} \mathbf{r}_1 \right) \left( \frac{1}{2} m_p; B | s_d m_d; C \right) \\ & \times \int d\mathbf{r}_2 G_C^{(+)} \left( \mathbf{r}_1, \frac{A}{A+1} \mathbf{r}_2 \right) (s_d m_d; C | \frac{1}{2} m_t; A) x_t^{(+)}(\mathbf{r}_2), \end{aligned} \quad (17)$$

where  $D_0(a, b)$  is the usual zero-range normalization constant for one-nucleon transfer. The coordinates used are indicated in Fig. 2(b). Also,  $s_d = 1$  is the spin of the deuteron.

The spin overlaps are evaluated by first recoupling the spins in the triton in terms of the resultants  $\tilde{S}_d$  of the spins of the proton and the first neutron. Our assumption of a particular deuteron state (triplet,  $s_d = 1$ ) then selects the



$\tilde{s}_d = 1$  part of the triton. We then get

$$\begin{aligned} \hat{T}_{BA}^{(2)} = & \sum_{S=0}^1 (2s_d+1)^{\frac{1}{2}} (2S+1)^{\frac{1}{2}} W\left(\frac{1}{2} s_d S \frac{1}{2}; \frac{1}{2} \frac{1}{2}\right) \langle S \frac{1}{2} M_{S m_p} | \frac{1}{2} m_t \rangle D_0(t,d) D_0(d,p) \\ & \times \sum_C \int d\mathbf{r}_1 d\mathbf{r}_2 x_p^{(-)*} \left(\frac{A+1}{A+2} \mathbf{r}_1\right) (B|C) G_C^{(+)}(C|A) x_t^{(+)}(\mathbf{r}_2) \end{aligned} \quad (18)$$

where  $S$  is the angular momentum transferred due to the spins of the two neutrons. Although these spins were assumed to be originally coupled to zero within the triton, the 'spin-filter' introduced by interposing a deuteron with  $s_d = 1$  has resulted in the appearance of transfers with both  $S = 0$  and 1.

In order to simplify Eq. (18) further, we now make the closure approximation (16). Substituting the numerical values of the Racah coefficients in Eq. (18), we finally arrive at

$$\hat{T}_{BA}^{(2)} \approx \frac{\sqrt{3}}{2} \hat{T}_{BA}^{(2)}(S=0) - \frac{1}{2} \hat{T}_{BA}^{(2)}(S=1), \quad (19)$$

with

$$\hat{T}_{BA}^{(2)}(0) = \delta_{m_p m_t} D_0(t,d) D_0(d,p) \int d\mathbf{r}_1 d\mathbf{r}_2 x_p^{(-)}(\mathbf{r}_1)^* \phi_{00}^{BA}(\mathbf{r}_1, \mathbf{r}_2) \bar{G}_C x_t^{(+)}(\mathbf{r}_2), \quad (20a)$$

and

$$\hat{T}_{BA}^{(2)}(1) = \langle \frac{1}{2} M_{S m_p} | \frac{1}{2} m_t \rangle D_0(t,d) D_0(d,p) \int d\mathbf{r}_1 d\mathbf{r}_2 x_p^{(-)}(\mathbf{r}_1)^* \phi_{1M_S}^{BA}(\mathbf{r}_1, \mathbf{r}_2) \bar{G}_C x_t^{(+)}(\mathbf{r}_2), \quad (20b)$$

where

$$\mathbf{r}_1' = \frac{A+1}{A+2} \mathbf{r}_1.$$

Next, introduce the coordinates of the separation of the two neutrons, and of their CM relative to A,

$$\underline{r} = \underline{r}_1 - \underline{r}_2, \quad 2\underline{R} = \underline{r}_1 + \frac{A+2}{A+1} \underline{r}_2$$

or

$$\underline{r}_1 = \underline{R} + \frac{1}{2} \underline{r}, \quad \underline{r}_2 = \underline{R} - \frac{1}{2} \underline{r}.$$

Further, assume that  $\overline{G}$  has a short range in the variable  $r$ , so that we may approximate it by  $\overline{G}_0$ , the monopole part of  $\overline{G}$ , where

$$\overline{G} = \sum_{\lambda} \overline{G}_{\lambda}(r, R) P_{\lambda}(\cos \theta). \quad (21)$$

Here  $\theta$  is the angle between  $\underline{r}$  and  $\underline{R}$ . Then the lowest-order term arising from an expansion of the distorted waves in this range, like Eq. (10), should be a reasonable approximation. It is

$$\hat{T}_{BA}^{(2)}(0) \approx \delta_{m_p m_t} D_0(t, d) D_0(d, p) \int d\underline{R} d\underline{r} \chi_p^{(-)*}(\underline{R}') \phi_{00}^{BA}(\underline{r}_1, \underline{r}_2) \overline{G}_0(r, R) \chi_t^{(+)}(\underline{R}). \quad (22)$$

Equation (22) is identical in form to Eq. (8), with  $D(r)$  there replaced by  $\overline{G}_0(r, R)$ . (Note, however, that  $D(r)$  is usually assumed real, while  $\overline{G}_0$  is complex.)

Similar approximations can be made for  $\hat{T}_{BA}^{(2)}(1)$ . Because of antisymmetry, the overlap function  $\phi_{1M_S}$  must be an odd function of the variable  $\underline{r}$ ; therefore the non-vanishing term of lowest-order in an expansion of the  $\chi$ 's in powers of  $\underline{r}$  is

$$\begin{aligned} \hat{T}_{BA}^{(2)}(1) \approx & \langle 1 \frac{1}{2} M_S m_p \frac{1}{2} m_t \rangle D_0(t, d) D_0(d, p) \int d\underline{R} d\underline{r} \phi_{1M_S}^{BA}(\underline{r}_1, \underline{r}_2) \overline{G}_0(r, R) \\ & \times \left\{ -\frac{1}{2} \chi_p^{(-)}(\underline{R}')^* \underline{r} \cdot \underline{\nabla}_R \chi_t^{(+)}(\underline{R}) + \frac{A+2}{A} \underline{r} \cdot \underline{\nabla}_R \chi_p^{(-)}(\underline{R}')^* \chi_t^{(+)}(\underline{R}) \right\}. \end{aligned} \quad (23)$$

Equation (23), although not exactly the same as Eq. (13), is remarkably similar in form.

Clearly, a number of severe approximations have been made, so that the results obtained are qualitative and suggestive, not quantitative and rigorous. Nevertheless, they are useful aids in understanding the roles played by the simultaneous and sequential transfer amplitudes.

The remarkable similarity of Eqs. (8) and (22) for the  $S=0$  amplitudes confirms a result frequently observed in numerical calculations, that sequential transfer amplitudes tend to have the same dependence on angle as simultaneous transfer amplitudes. They also tend to depend in the same way on nuclear bound-state properties, so that relative spectroscopic factors obtained from analysis of data based on the one-step DWBA are expected to be quite reliable, even when sequential transfer amplitudes are significantly large. These points have also been made earlier.<sup>7,16</sup>

The similarity of Eqs. (13) and (23) for the  $S=1$  amplitudes is also interesting. These amplitudes have the same selection rules, the same dependence on bound-state properties and, although not of exactly the same mathematical form, appear to be very much alike. As a result, including sequential transfer amplitudes can be expected to have effects comparable to including the mixed-symmetry  $S'$  state in an exact finite-range treatment of the first-order amplitude. Both effects should influence the sensitive vector analyzing power. This injects some uncertainty into analyses based on the zero-range approximation, such as Ref. 20, although the calculations of the vector analyzing power presented there are quite suggestive of the importance of sequential transfer. It would be very interesting to compare the results of Ref. 20 with finite-range calculations made, for example, with the computer programs used in Refs. 7 or 9.

## 2.6. Importance of the Deuteron Singlet State

Only one author has reported<sup>12,15</sup> including the singlet state ( $s_d = 0$  in Eq.

(18)) of the deuteron in the calculations of sequential transfer. A proper treatment of the singlet-state is non-trivial since it lies in the continuum. If assumed to be quasibound, it could be included without difficulty in the same way as the bound triplet-state; however, it seems that this would be a poor approximation.<sup>15</sup> If the singlet continuum is treated properly, it is inconsistent to ignore the triplet continuum; indeed, the only reported calculations of the 'break-up' effect<sup>12,15</sup> do include both singlet and triplet contributions. Granted that the problem is complicated, it is still illuminating to discuss the consequences of the deuteron singlet states using a simplified approach.

Assume that the singlet and triplet intermediate n-p states contribute equally and that they propagate in the same way (i.e. that  $G_C$  is the same for both). For example, one might assume that the triplet deuteron ground-state and the zero-energy  $^1S$  resonance dominated, and neglect the differences between their energies and wavefunctions.

With this assumption, the total amplitude for sequential transfer is the same as Eq. (17) except that there should also be a summation over  $s_d = 0$  and 1. Since the Green functions for singlet and triplet deuteron propagation are assumed to be the same, summation of the projector  $|s_d M_d\rangle\langle s_d M_d|$  over  $s_d, M_d$  gives unity. As a result, the complete amplitude is equal to  $\hat{T}_{BA}^{(2)}(S=0)$  of Eq. (20a). The coefficient of  $\hat{T}_{BA}^{(2)}(S=1)$  is identically zero. Similar results are obtained for the non-orthogonality terms. The spin-transfer selection rules are then exactly the same for zero-range sequential transfer as for simultaneous transfer with a totally symmetric S-state triton. If this model were correct, including the sequential transfer amplitude could be expected to have little or no effect on the vector analyzing power. The model is oversimplified; for example, Hashimoto<sup>15</sup> finds the triplet states to be more important than the

singlet ones. Nonetheless, it serves as a warning that it may be impossible to completely decipher the reaction mechanism problem unless the deuteron continuum is properly included.

The following analogy is useful. The deuteron channel acts much like an optical polaroid filter. In the triton wave function the neutron spins are coupled to  $S = 0$ , which from recoupling we find is a linear combination of  $s_d = 1$  and  $0$ . The assumption of a triplet deuteron filters out the  $s_d = 0$  component, and the resulting triton function, after filtration, is no longer pure  $S = 0$ ; it also contains  $S = 1$  neutron components. If the singlet and triplet deuteron states are included with equal weights, the filter is removed and only  $S = 0$  neutron transfer is possible.

### 3. The $(\alpha, d)$ Reaction

A similar analysis has been made of the  $(\alpha, d)$  reaction.<sup>21</sup> Two sequential processes are available here,  $(\alpha, t; t, d)$  and  $(\alpha, h; h, d)$  (where  $h \equiv {}^3\text{He}$ ). Only  $s_d=1$  transfer is possible for the simultaneous transfer because of the zero spin of the alpha. If we assume an S-state of maximum symmetry for the alpha, only spin -  $\frac{1}{2}$  states are allowed for the triton and helion. Thus, although they act as spin -  $\frac{1}{2}$  'spin filters', they impose no further constraint on the spin transfer. The spin-transfer selection rules are the same for simultaneous and sequential transfer. There could be, however, an isospin filtering effect which would result from any difference in propagation of the  $t$  and  $h$  particles in unsymmetric, charged nuclear matter (i.e. if they had different Green functions).

By introducing similar zero-range and closure approximations, the one-step and two-step amplitudes are reduced to the same forms as Eqs. (8) and (22) for

the (t,p) reaction. Again, the two amplitudes are seen to depend upon nuclear structure in the same way. The same two-nucleon overlap, analogous to Eq. (5), appears in both. In particular, there does not appear any phase difference between them that depends upon the spin of the residual nuclear state.

The latter point is especially relevant because of some observations of the Pittsburgh group<sup>22</sup> on the  $^{208}\text{Pb}(\alpha, d)^{210}\text{Bi}$  reaction. Consider excitation of the  $(g_{9/2}h_{9/2})_J$  multiplet of states with  $J = 0, 1, \dots, 9$ . Simple angular momentum considerations, together with the tendency of  $(\alpha, d)$  reactions to favor large L-transfers, leads to the prediction that the one-step cross sections should show a regular stepped pattern when plotted against J, with the rise at each step increasing as J increases. DWBA calculations<sup>22</sup> confirm this (see Fig. 3 a). The discussion of the present paper suggests that a similar pattern should be seen for the two-step process by itself and again explicit calculations<sup>22</sup> confirm this (Fig. 3(a)).

The measured cross sections<sup>22</sup> show a rather different, saw-tooth pattern (Fig. 3); the predicted steady rise with L transfer is observed; however, the cross sections for the even-J members of the  $J=(L-1, L)$  pairs are larger than those for the odd-J members. Coupled-channels calculations<sup>22</sup> including direct and sequential transfers reproduced these trends and seemed to indicate that the saw-tooth pattern resulted from successively constructive and destructive interferences of direct and sequential amplitudes due to a J-dependence in their relative phase. No such J-dependent phase factor appears in the present treatment<sup>†</sup>, although it is able to account for the trends shown by the individual

<sup>†</sup>Although a coupled reaction channels code was used, the calculations of Refs. 22 were made in such a way as to correspond exactly to the second-order distorted-wave Born approximation discussed here, including the use of the zero-range approximation. Further, the non-orthogonality correction  $T(\text{NO})$  was not included. However, it is easy to see that this term, which tends to cancel the one-step amplitude, has exactly the same structure. Consequently, no J-dependent interference is expected from non-orthogonality corrections, either.

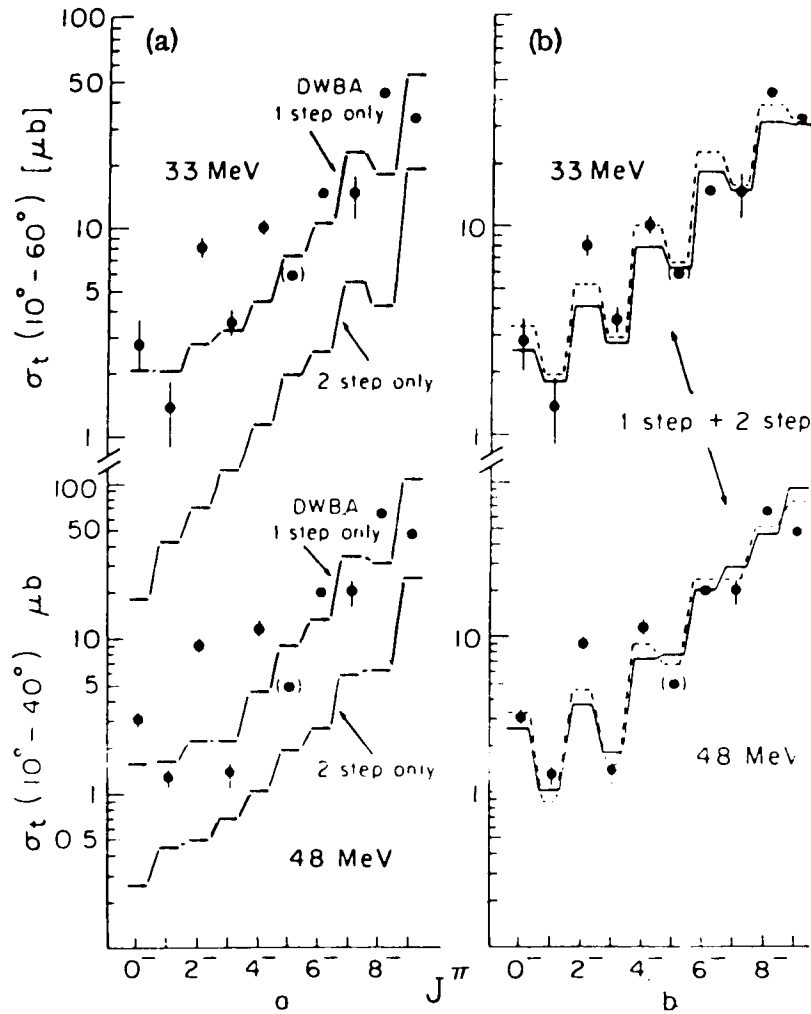


Figure 3

Integrated cross sections, versus  $J$ , for the  $^{208}\text{Pb}(\alpha, d)^{210}\text{Bi}$  reaction exciting the  $(h_{9/2}g_{9/2})_J$  multiplet. (a) Individual one-step and two-step cross section; (b) coherent sum of one-step and two-step; as reported in Ref. 22.

one-step and two-step amplitudes in terms of the reaction dynamics and the LS-jj transformation coefficients involved in the two-nucleon overlap factor.<sup>21</sup> Indeed, within our model, the integrand of the two-step amplitude differs from that for the one-step only by the replacement of the light-ion overlap function  $D(r)$  by  $\bar{G}_0(r,R)$ , the average of the triton and helion Green functions. The symmetric part of the two-nucleon overlap  $\phi(\underline{r}_2, \underline{r}_1)$ , from which any J-dependence must arise, is common to both amplitudes.

Our analysis makes it difficult for us to understand either the calculations presented in Refs. 22 or the experimental data, which seem to be nicely explained by the coupled-channels calculations. Cross sections for  $^{208}\text{Pb}(^3\text{He},p)^{210}\text{Bi}$  show a similar behavior.<sup>23</sup> The  $(^3\text{He},p)$  reaction differs from  $(\alpha,d)$  in that  $S=1$  and  $S=0$  transfer are both possible. However, nuclear structure factors make the  $S=0$  contribution much smaller than that for  $S=1$ , so that, except for kinematic effects, one should expect the two reactions to behave in a very similar way.

Our approach differs from that using coupled channels in that we do not consider intermediate states explicitly but make the closure approximation. However, with the assumption of a single  $(g_{9/2}h_{9/2})$  configuration for the final state in the  $^{208}\text{Pb}(\alpha,d)^{210}\text{Bi}$  case, the result of making the closure approximation yields the same result as restricting the intermediate states to be the  $^{209}\text{Pb}$  and  $^{209}\text{Bi}$  ground states, as was done in the coupled-channels calculations.<sup>22</sup>



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FOLDING MODELS FOR ELASTIC AND INELASTIC SCATTERING\*

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## FOLDING MODELS FOR ELASTIC AND INELASTIC SCATTERING\*

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### 1. Motivation

We are accustomed to the use of many simple models in nuclear physics. In the present context, the most widely used models are the optical model potential (OMP) for elastic scattering, and its generalization to non-spherical shapes, the deformed optical model potential (DOMP) for inelastic scattering. These models are simple and phenomenological; their parameters are adjusted so as to reproduce empirical data. Nonetheless, there are certain, not always well-defined, constraints to be imposed. The potential shapes and their parameter values must be "reasonable" and should vary in a smooth and systematic way with the masses of the colliding nuclei and their energy. Without these constraints, the potentials tell us very little, and they will be essentially useless for other purposes, such as in DWBA calculations.

One way of satisfying these constraints, without going back to a much more fundamental theory, is through the use of folding models. Remember that the basic justification for using potentials of the Woods-Saxon shape for nucleon-nucleus scattering, for example, is our knowledge that a nuclear density distribution is more-or-less constant in the nuclear interior with a diffuse surface. When this is folded with a short-range nucleon-nucleon interaction, the result is a similar shape with a more diffuse surface. Folding procedures allow us to

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incorporate many aspects of nuclear structure (although the nuclear 'size' is one of the most important), as well as theoretical ideas about the effective interaction of two nucleons within nuclear matter. It also provides us with a means of linking information obtained from nuclear (hadronic) interactions with that from other sources, as well as correlating that from the use of different hadronic probes. For example, inelastic electron scattering measurements may provide a transition density  $\rho_{tr}(r)$  for an excitation. Frequently this is represented by a simple collective (deformed density) model when the transition is strong, whose shape may be of the form

$$\rho_{tr}^C(r) = \beta_L^C R_C \frac{d\rho_C(r)}{dr} Y_L^M(\hat{r}), \quad (1)$$

where  $\rho_C(r)$  is the ground state charge density distribution,  $R_C$  is its radius, and  $\beta_L^C$  is an amplitude or 'charge deformation parameter' for the  $2^L$ -pole transition. The interaction for hadronic inelastic scattering is frequently represented in an analogous way (the DOMP) where the transition potential  $U_{tr}(r)$  is taken to be

$$U_{tr}(r) = \beta_L^N R_N \frac{dU(r)}{dr} Y_L^M(\hat{r}) \quad (2)$$

where  $U(r)$  is the optical potential for elastic scattering of that particular hadron,  $R_N$  is its radius, and  $\beta_L^N$  is the corresponding 'nuclear'  $2^L$ -pole deformation parameter.

The use of these models (1) and (2) (and their extensions) has led to much discussion as to how the  $\beta_L^N$  and  $\beta_L^C$  are related and, indeed, how the  $\beta_L^N$  obtained with different probes are to be related. (One popular prejudice<sup>1</sup> is that the 'deformation lengths' should be the same,  $\beta_L^C R_C = \beta_L^N R_N$ .) Folding provides some insight into this problem<sup>2-4</sup> and indeed may be used to construct the transition

potential  $U_{tr}(r)$  directly from the transition density by folding with a suitable interaction. As usual, such extensions of a model also open up new uncertainties. For example, the charge density (1) arises predominantly from proton excitations, while neutron and proton excitations contribute comparable amounts to the hadronic transition potential (2). However, we may turn this situation into an advantage and use it to give information about the neutrons.

Folding is still a model, and, at best, an approximation. It is a reasonable model for nucleon scattering. At low energies, it is closely related to the Brueckner-Hartree-Fock model for bound states. At high energies, it can be justified in terms of the impulse approximation. Extensions of the Brueckner-type theories provide support at intermediate energies. The situation is not so clear for composite projectiles, especially the scattering of two heavy ions. However, here the situation is helped because such systems usually exhibit strong absorption. Under such circumstances, the details of the interaction when the two nuclei overlap are largely irrelevant; all that matters is the interaction for peripheral collisions where again folding may be reasonable. Ultimately, the justification of the folding procedure must be two-fold: (i) it must be seen to work in practice (i.e. to explain experimental data), and (ii) it should be justified from a more fundamental theory. This is the common fate of physical models.

## 2. Further Theoretical Background

### 2.1. Coupled Equations and the Feshbach Projection Theory for the Optical Potential

We have spoken so far as though the inter-nucleus potential was defined uniquely. This is not so; there are various ways in which we may reduce the complicated  $(A_1 + A_2)$ -body scattering problem to an effective two-body problem.

Failure to remember this has sometimes led to confusion. A conventional optical model potential  $U(R)$  for two nuclei  $A_1, A_2$  is one which appears in a one-body Schrödinger equation

$$\left[ -\frac{\hbar^2}{2\mu_\alpha} \nabla^2 + U(R) \right] \chi(\underline{R}) = E \chi(\underline{R}), \quad (3)$$

where  $\mu_\alpha$  is the reduced mass of the pair,  $\underline{R}$  is the separation of their centers of mass, and  $E$  is the CMS energy of relative motion. The solution  $\chi(\underline{R})$  with the appropriate boundary conditions describes the elastic scattering of  $A_1 + A_2$ . Usually  $U(R)$  is assumed to be local (although its parameter values may vary with bombarding energy) and to have a smooth, simple functional form like that of the Woods-Saxon shape. Occasionally, some  $L$ -dependence is introduced, but only if one is forced to consider such a generalization by some intractable data. The primary consideration is simplicity.

How do we justify the use of an equation like Eq. (3) which makes no reference to the many internal degrees of freedom and the corresponding non-elastic channels except insofar as they lead to absorption from the elastic channel and require  $U$  to be complex? One standard way to make it plausible that such an equation may adequately describe elastic scattering is to use the projection operator formalism of Feshbach.<sup>5</sup>

Suppose that the complete Hamiltonian for the colliding pair is given by

$$H = H_1 + H_2 + T + V \quad (4)$$

where  $H_1$  and  $H_2$  denote the internal Hamiltonians for the isolated nuclei,  $T$  is the kinetic energy of relative motion in the center-of-mass system, and  $V$  is the coupling interaction between the ions. At this point, we ignore any explicit reference to antisymmetrization between nucleons in different ions, although we assume that the wavefunctions for the internal states of the individual nuclei

are antisymmetric. We denote a complete set of states for the internal Hamiltonians of nuclei  $A_1$  and  $A_2$  by  $\psi_i$  and  $\phi_j$  respectively, where

$$(H_1 - \epsilon_{1i}) \psi_i = 0, \quad (H_2 - \epsilon_{2j}) \phi_j = 0. \quad (5)$$

If the complete wavefunction is denoted by  $\Psi^{(+)}$ , we note that measurements of elastic or inelastic scattering provide us with information about the projection of the complete wavefunction onto the channel subspace defined by the particular measurement. We denote these  $\underline{R}$ -dependent projections (amplitudes) by

$$x_{ij}(\underline{R}) \equiv \langle \psi_i \phi_j | \Psi^{(+)} \rangle, \quad (6)$$

which means that we can express the total wavefunction as

$$\Psi^{(+)} = \sum_{i,j} x_{ij}(\underline{R}) \psi_i \phi_j. \quad (7)$$

A set of coupled equations governing the  $x_{ij}(\underline{R})$  can be found by inserting the above expansion into the complete Schrödinger equation

$$(H-E) \Psi^{(+)} = 0. \quad (8)$$

This yields

$$[T + (\psi_i \phi_j | V | \psi_i \phi_j) - E_{ij}] x_{ij}(\underline{R}) = - \sum_{[k, \ell \neq i, j]} (\psi_i \phi_j | V | \psi_k \phi_\ell) x_{k\ell}(\underline{R}). \quad (9)$$

where  $E_{ij} = E - \epsilon_{1i} - \epsilon_{2j}$  and  $(i, j = 0 \dots \infty)$ .

The rounded brackets denote integration over the internal coordinates of the two nuclei and serve to remind us that the matrix elements remain a function of  $\underline{R}$ .

Now Eqs. (9) represent an infinite set of coupled equations which, although complete, are not very useful. Practical calculations require a severe



truncation of these to a small set of equations coupling a few states of particular interest; this is what is involved in the so-called 'coupled-channels' method. The use of an equation like Eq. (3) in fact involves truncation to just that one channel associated with the ground states,  $i = j = 0$ . The price we pay for any such truncation is that the interaction  $V$  should be replaced by an effective interaction. This becomes clear in Feshbach's formalism<sup>5</sup> in which all explicit couplings to channels other than the one of interest are transformed away and their effects incorporated into an effective interaction. For that channel in which both nuclei remain in their ground states this procedure results in an equivalent one-body Schrödinger equation for the  $x_{00}(R)$ ,

$$[T + U_{op} - E] x_{00} = 0 \quad (10)$$

with the effective interaction or optical potential operator  $U_{op}$  given by

$$\begin{aligned} U_{op} &= (\psi_0 \phi_0 | V | \psi_0 \phi_0) + (\psi_0 \phi_0 | V Q \frac{1}{E - H_{QQ} + i\epsilon} Q V | \psi_0 \phi_0) \\ &= U_F + \Delta U. \end{aligned} \quad (11)$$

$U_F$  is defined as the first term in Eq. (11)

$$U_F = (\psi_0 \phi_0 | V | \psi_0 \phi_0), \quad (12)$$

and is called the folded potential for elastic scattering. (Note that this same folded potential and its generalization to excited states and off-diagonal couplings also appears in Eqs. (9).)  $Q$  is the operator which projects off the ground states of the two nuclei, so that only excited states appear as intermediate states in  $\Delta U$ . By its construction, the exact elastic scattering amplitude may be obtained from  $x_{00}$  as  $R \rightarrow \infty$ .

A similar result is obtained when the truncation is to a model space which includes one or a few excited states as well as the ground states.  $U_{op}$  becomes a matrix in the model space and provides the effective interactions to be used

in the corresponding truncated coupled-channels problem. In particular, it yields (complex) corrections  $\Delta U$  to the interchannel couplings in addition to the folded terms that already appear in the Eqs. (9).

The folded potential  $U_F$  is real (provided that  $V$  is real). The remaining term  $\Delta U$ , which we may refer to as a 'dynamic polarization potential', arises from coupling to all the other states, is much more difficult to calculate even approximately, and, in general, is complex, non-local, energy- and angular-momentum dependent. (In practice, because of the strong short-range repulsion in the bare nucleon-nucleon interaction, the  $V$  itself will be an effective interaction or  $G$ -matrix which itself includes some 'polarization' corrections, primarily those associated with the short-range correlation between the interacting nucleons and hence high excitation energies. In principle, this leaves open the possibility of some double counting.)

In phenomenological approaches it is this object  $U_{op}$  which is approximated by a local, complex model potential  $U(R)$  in Eq. (3). However, although this formalism provides us with the form of the optical model, it does not guarantee that any given simple representation of  $U_{op}$  will be adequate. This question may be explored both empirically (is a given model successful in fitting data?) and by studying the structure of  $\Delta U$ .

## 2.2. Other Approaches to the Nucleus-Nucleus Interaction Energy

A particular point to be stressed is that with the potential  $U_{op}$  (and by implication an equivalent model potential  $U$ ) defined in this way, the solution  $\chi(R)$  of Eq. (3) represents  $\chi_{00}(R)$  of Eqs. (9) and (10) and describes the relative motion of the two nuclei while they both remain in their respective ground states. This may be a very small component of the total wavefunction in that region of space where the two nuclei overlap appreciably; in that case, the

strong absorption into other channels manifests itself through  $\chi_{00}(\underline{R})$  becoming very small for  $R \lesssim R_1 + R_2$ .

This is to be contrasted with most of the potentials calculated microscopically for heavy-ion collisions (see Refs. 6-10, for example). These calculations may use the energy-density approach, the Thomas-Fermi approximation, the liquid-drop model, the proximity theorem, etc., but they all attempt to follow to a greater or lesser degree the readjustments that the two nuclei must make as they begin to interact and overlap; distortion of the nuclear shapes, reaction to the Pauli principle, effects of the saturating nature of the nuclear forces, etc. Such an interaction energy function does not determine just the  $\chi_{00}$  component of the wavefunction (7), but is to be used in a description of the motion of a wave packet which includes a wide range of excited states of the separated systems. It is not to be identified with  $U_{op}$  of Eq. (11), and, in principle, it should not be used in an equation like Eq. (3).

It is possible that the equations for elastic scattering from these other approaches may be formally recast into the same form as the optical model Eq. (3) (see Pal, et al.<sup>11</sup> for a recent discussion of this). The corresponding effective optical potential  $\bar{U}$  appearing in such an equation will be different from  $U_{op}$ . It will contain not only the interaction energy function just referred to but also correction terms. For example, in one model situation<sup>12</sup> there are corrections corresponding to changes in the kinetic energy due to changes in the inertial properties as the system begins to interact. The corresponding scattering solutions  $\tilde{\chi}(\underline{R})$  from such equations will differ from  $\chi_{00}(\underline{R})$ . Before approximations are made, we must have  $\tilde{\chi}(\underline{R}) \rightarrow \chi_{00}(\underline{R})$  asymptotically because the various theories describe the same elastic scattering, but these wavefunctions will differ in the interaction region at small  $R$ .

The actual physical processes that are to be described in the two types of approaches are, of course, the same. In the Feshbach theory they manifest themselves in the dynamic polarization potential  $\Delta U$ ; in some ways this formalism is deceptively simple, being most convenient to use when the effects of a few non-elastic channels are to be studied explicitly. The other approaches provide vehicles for including, perhaps in intuitively more obvious ways, various bulk physical effects. Nonetheless, the 'potential' functions they generate should not simply be identified as optical potentials without bearing in mind the considerations already mentioned.

In practice, of course, approximations are made in both kinds of approach. Frequently an adiabatic approximation is used when calculating an ion-ion interaction energy. This implies that the collision is slow enough for the two nuclei to readjust, both as they begin to interact and overlap and as they separate, so that they return to the elastic channel; there is no 'absorption' into non-elastic channels and the calculated interaction energy is real. The absorption into other channels that actually occurs must then arise from the correction terms mentioned above. In practice, these are usually replaced by a phenomenological absorptive potential. On the other hand, the folding model evaluates only the  $U_F$  term of Eq. (11), identifying it with the real part of the potential and implicitly assuming that the polarization potential  $\Delta U$  is predominantly imaginary and can be represented by a phenomenological imaginary potential. This corresponds to a sudden approximation in which excitations ('polarization') of the system tend not to de-excite back to the elastic channel, but result in absorption.

### 2.3. Folding Model Potentials

The idea of a folded potential has a long history. The electrostatic potential  $U_C(r)$ , due to a charge distribution  $\rho_0(r)$ , is given by a single folding,

$$U_C(\underline{r}) = \int \frac{\rho_C(\underline{r}')}{|\underline{r}-\underline{r}'|} d\underline{r}', \quad (13)$$

while the electrostatic interaction between two charge distributions is given by a double folding,

$$U_C(R) = \iiint \frac{\rho_{C1}(r_1)\rho_{C2}(r_2)}{r_{12}} d\underline{r}_1 d\underline{r}_2 \quad (14)$$

where  $\underline{r}_{12} = \underline{R} + \underline{r}_2 - \underline{r}_1$  (see Fig. 1). In the nuclear case, the Coulomb interaction  $1/r_{12}$  is replaced by a nucleon-nucleon (effective) interaction  $v_{12}$ . The first term  $U_F(R)$  in Eq. (11) was identified as a folded potential,

$$U_F(R) = \iiint \psi_0^*(\xi_1)\phi_0^*(\xi_2) \sum_{ij} v_{ij} \psi_0(\xi_1)\phi_0(\xi_2) d\xi_1 d\xi_2 \quad (15)$$

where nucleon  $i$  is in nucleus 1, nucleon  $j$  is in nucleus 2, and  $\xi_1$  and  $\xi_2$  refer to the internal coordinates of these two nuclei, respectively. If  $v_{ij}$  is a local interaction,  $v_{ij} = v(\underline{r}_{ij})$ , then the potential (15) may be reduced to the analogue of Eq. (14):

$$U_F(R) = \iiint \rho_1(\underline{r}_1)\rho_2(\underline{r}_2)v(\underline{r}_{12})d\underline{r}_1 d\underline{r}_2, \quad (16)$$

where  $\rho_i(\underline{r}_i)$  is the one-body density for the  $i$ th nucleus, usually assumed to be spherically symmetric. This we call a double-folded potential. The properties and applications of this expression have been discussed in detail elsewhere (see Refs. 14-16), where many other references are also given). We just mention here that many results are more easily understood in momentum space by using Fourier transforms. If we denote the transform of a function  $f(\underline{r})$  by

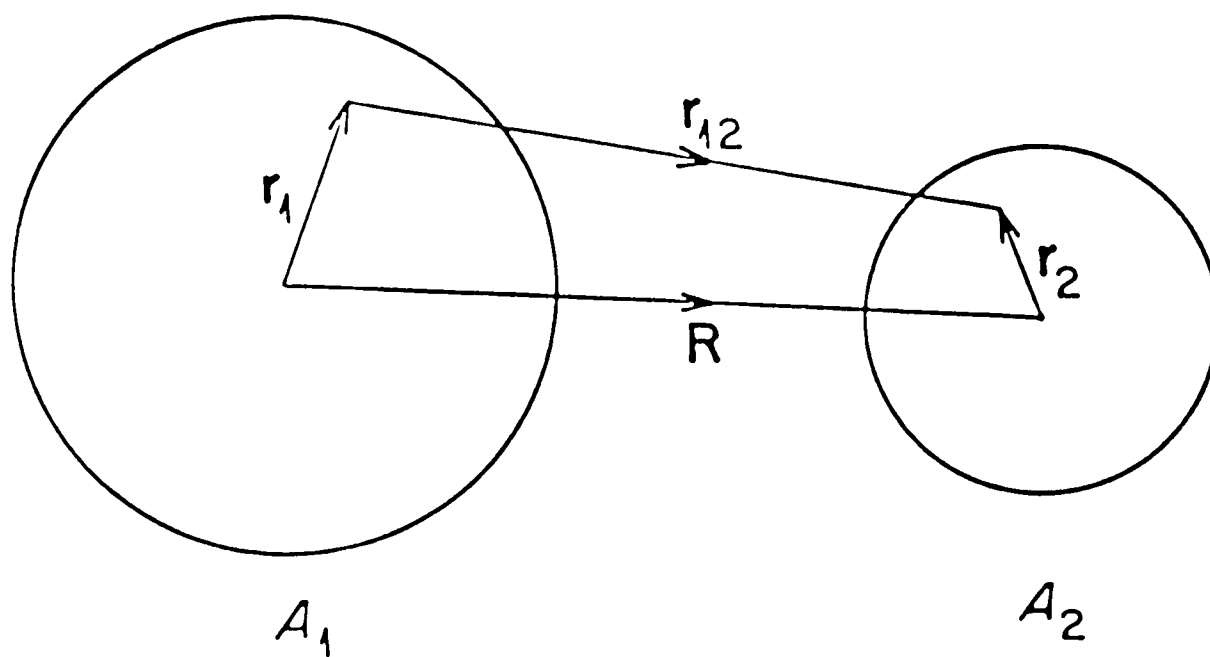


Figure 1

Coordinates used in folding calculations.

$$\tilde{f}(\underline{q}) = \int d\underline{r} \exp(i\underline{q} \cdot \underline{r}) f(\underline{r}), \quad (17)$$

then in momentum space Eq. (16) becomes

$$\tilde{U}_F(\underline{q}) = \tilde{v}(\underline{q}) \tilde{\rho}_1(\underline{q}) \tilde{\rho}_2(-\underline{q}). \quad (18)$$

Then, for example, if the densities  $\rho_1$  and  $\rho_2$  are spherically symmetric,

$$U_F(R) = \frac{1}{2\pi^2} \int q^2 dq j_0(qR) \tilde{v}(q) \tilde{\rho}_1(q) \tilde{\rho}_2(q). \quad (19)$$

When only small momentum transfers are involved in the scattering, it is helpful to expand the various factors in powers of  $q$  in order to see which are their important characteristics. For example, if  $f(r)$  is scalar in Eq. (17),

$$\tilde{f}(q) = J_0 \left[ 1 - \frac{1}{6} \langle r^2 \rangle q^2 + \frac{1}{120} \langle r^4 \rangle q^4 + \dots \right], \quad (20)$$

where  $J_0$  is the volume integral of  $f(r)$  and  $\langle r^n \rangle$  is its  $n^{\text{th}}$  radial moment.

A simple generalization of the formula (16) is applicable to inelastic scattering.<sup>14-16</sup> The transition potential for exciting one of the nuclei is obtained by replacing the corresponding ground-state density by the appropriate transition density (for example, like the simple collective model one of Eq. (1)). The resulting folded transition potential is non-spherical (if  $L \neq 0$ ). There is no reason to believe that its radial shape will be exactly like the radial derivative of the diagonal potential  $U_F(R)$  of Eq. (16), as is assumed in the simple DOMP model of Eq. (2). Indeed, it is known<sup>15,16</sup> that the folded shape depends upon the multipolarity, whereas the DOMP prescription (2) gives a shape that is independent of the multipole order.

We may break the double folding into two steps and rewrite Eq. (16):

$$U_F(R) = \int d\underline{r}_1 U_{2N}(\underline{r}_{12}) \rho_1(\underline{r}_1), \quad (21)$$

where

$$U_{2N}(\bar{r}_{12}) = \int d\mathbf{r}_2 \rho_2(\mathbf{r}_2) v(\mathbf{r}_{12}), \quad (22)$$

and  $\bar{r}_{12} = \mathbf{r}_{12} - \mathbf{r}_2 = \mathbf{R} - \mathbf{r}_1$ . Then  $U_{2N}$  is a folded potential for the interaction of nucleon 1 with nucleus 2 (analogous to Eq. (13)). Clearly Eq. (13) is the appropriate folding model for nucleon scattering from a nucleus.<sup>13</sup>

### 3. Single-Folded Potentials

#### 3.1. Introduction

In practice, the single-folded form (21) has often been invoked for the scattering of composite nuclei, but with  $U_{2N}$  replaced by an empirical, phenomenological nucleon-nucleus  $A_2$  optical potential. Usually this is what is meant by a single-folding model. Clearly the procedure is symmetrical in the two nuclei  $A_1$  and  $A_2$ ; one may equally well take an empirical nucleon-nucleus  $A_1$  optical potential  $U_{1N}$  and fold it with the density distribution for nucleus  $A_2$ . Since the potentials  $U_{1N}$  and  $U_{2N}$  are empirical, there is no á priori guarantee that the two results will be identical.

The phenomenological nucleon-nucleus optical potential  $U_{2N}$  will be complex if it is obtained from analysis of nucleon-nucleus scattering. However, it should not be concluded that single-folding with the imaginary part will yield the correct imaginary potential for nucleus-nucleus scattering, even if use of the real part is successful. The imaginary part of  $U_{2N}$  accounts for absorption due to the excitation of one nucleus by a free nucleon; it does not account for excitation of the other nucleus in which the interacting nucleon resides, or effects due to the presence of the other nucleons in that nucleus. In the simplest case of the scattering of the loosely-bound deuteron, the absorption due



to break-up of the deuteron is not included; the single-folding model gives an absorptive potential which is much too weak.<sup>17</sup> (Such break-up can also modify the effective real potential.<sup>18,19</sup>)

Of course, if the potential  $U_{2N}$  (or its imaginary part) is not taken from empirical nucleon scattering data, but is adjusted to fit nucleus-nucleus scattering data, as in the alpha scattering case discussed below, then it is reasonable (though not necessarily correct!) to use the same adjusted interaction for, say, application to inelastic scattering.

The other ingredient of the single folding model is the density distribution for the nucleus into which the interaction is being folded (or transition density, if an inelastic excitation is being considered). The tail of the folded potential, important for the scattering of strongly absorbed particles, is very sensitive to the extent of this density distribution, raising the possibility of learning something about the matter distribution provided some interaction potential can be shown to be valid. This is important because our other source of data on nuclear densities is electron scattering which primarily tells us about the proton distributions. Hadrons, on the other hand, interact as strongly with the neutrons. Only for nuclei with  $N = Z$  is it reasonable to assume that neutron and proton distributions are the same (and even then there can be small differences because of the Coulomb forces). It is sometimes assumed that the distributions are in the ratio  $N/Z$  when  $N \neq Z$ , or  $\rho_n(r) = (N/Z)\rho_p(r)$ . Although reasonable, this assumption must be used with caution; it is believed, for example, that neutron distributions in the ground state extend to slightly larger radii than do the protons, and this can have quite large effects.

The mean square radius (MSR) is a useful characteristic of the ground-state densities. Although the folded potential does depend to some extent upon the detailed surface shape of the density, it is largely determined by the MSR.<sup>15</sup> In the case of  $2^L$ -pole excitations, this rôle is taken by the  $r^{L+2}$  moment of the transition density. These results can be easily understood<sup>14-16</sup> in momentum space, using the expansion (20).

A variety of representations of the ground-state density have been used. There is the standard Fermi (= Woods-Saxon!) shape, or the convoluted form of Helm (itself a folding model) which has some analytic advantages.<sup>20</sup> Independent-particle models have also been used, either from Hartree-Fock calculations or more phenomenological shell-model calculations.<sup>14,15</sup> These provide a way of incorporating our prejudices about shell effects and about the behavior of the neutrons, for example.

Transition densities may be constructed in similar ways. One popular form is like that given by Eq. (1), except for the appearance of the matter, rather than the charge, distribution. Another is the Tassie or hydrodynamical model whose radial part differs from Eq. (1) by an additional factor of  $r^{L-1}$ ; for  $L > 2$  and isoscalar excitations,

$$\rho_{tr}(r) = \alpha_L r^{L-1} \frac{d\rho(r)}{dr} Y_L^M(\hat{r}). \quad (23)$$

This has some appeal if the transition is strong (such as for a giant resonance), for it has been shown<sup>21</sup> to be correct for a single transition that exhausts the classical energy-weighted sum rule (EWSR).

### 3.2. Alpha Scattering

Single folding has been applied with some success to alpha-nucleus elastic and inelastic scattering using either model  $N$ - $\alpha$  potentials or ones derived from

data on nucleon-alpha scattering.<sup>14</sup> When we are concerned only with data for forward scattering angles (or small momentum transfers), the most relevant properties of the effective interaction are its strength (volume integral) and mean square radius (see Eq. (20)). In these circumstances, a simple form like a Gaussian may be used. A recent example<sup>22</sup> adjusted the strength  $-(V+iW)$  and range  $\alpha$  of a Gaussian N- $\alpha$  interaction to fit the forward angle ( $\theta \lesssim 30^\circ$ ) elastic scattering of 140-MeV alphas. As had been found in earlier studies,<sup>14</sup> there is a remarkable consistency in the results (Fig. 2) with  $V \approx 36$  MeV,  $W \approx 23$  MeV, and  $\alpha \approx 1.94$  fm.

This empirical interaction was then applied to inelastic data for the excitation of both low-excited states and giant-resonance states. The Tassie form (23) was adopted for the transition densities and the amplitudes  $\alpha_L$  deduced by comparison with the data. Figure 3 shows an example for the lowest  $3^-$  state in  $^{208}\text{Pb}$ ; the amplitude  $\alpha_3$  required corresponds to 18% of the EWSR, which is exactly what one would deduce from the measured  $B(E3)$  value for this transition if it is assumed that the proton transition density is just  $(Z/A)$  times the matter density. This illustrates how folding allows one to directly correlate information from different sources in a relatively unambiguous way. (The excellent agreement, of course, has to be somewhat fortuitous. Equation (23) is still a model; the proton and neutron contributions do not have exactly the same shape, and the proton part is not in exact agreement with the measured charge transition density. On the other hand, when a transition density of the form of Eq. (1) was used, the  $B(E3)$  value deduced from the  $(\alpha, \alpha')$  data was 50% larger than the measured one. We can conclude that the Tassie model (23) is more appropriate.)

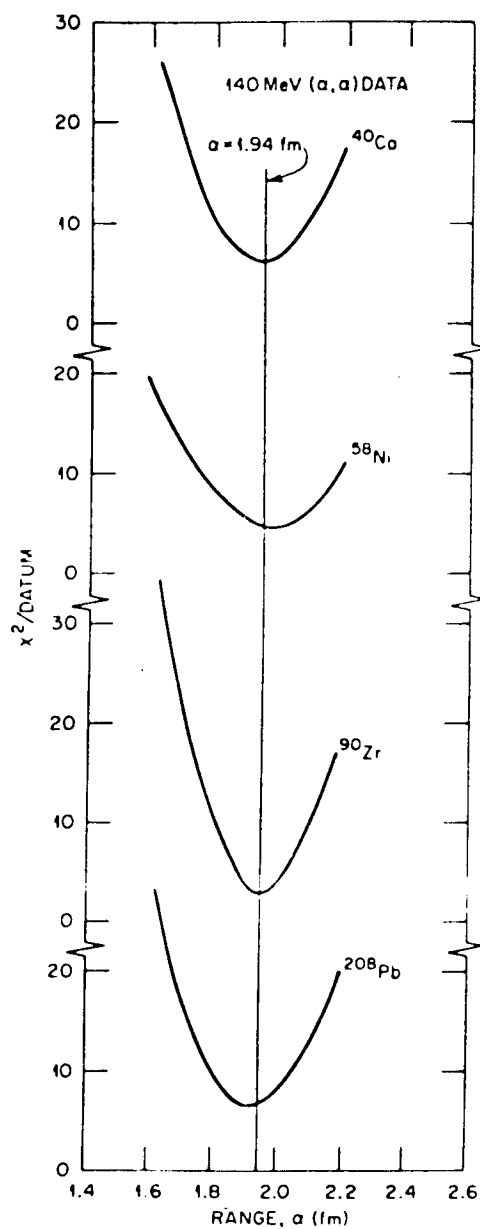


Figure 2

Variation of  $\chi^2$  with the range of the Gaussian nucleon-alpha interaction used in a single-folding model when real and imaginary strengths are optimized to fit elastic data at forward angles.<sup>22</sup>

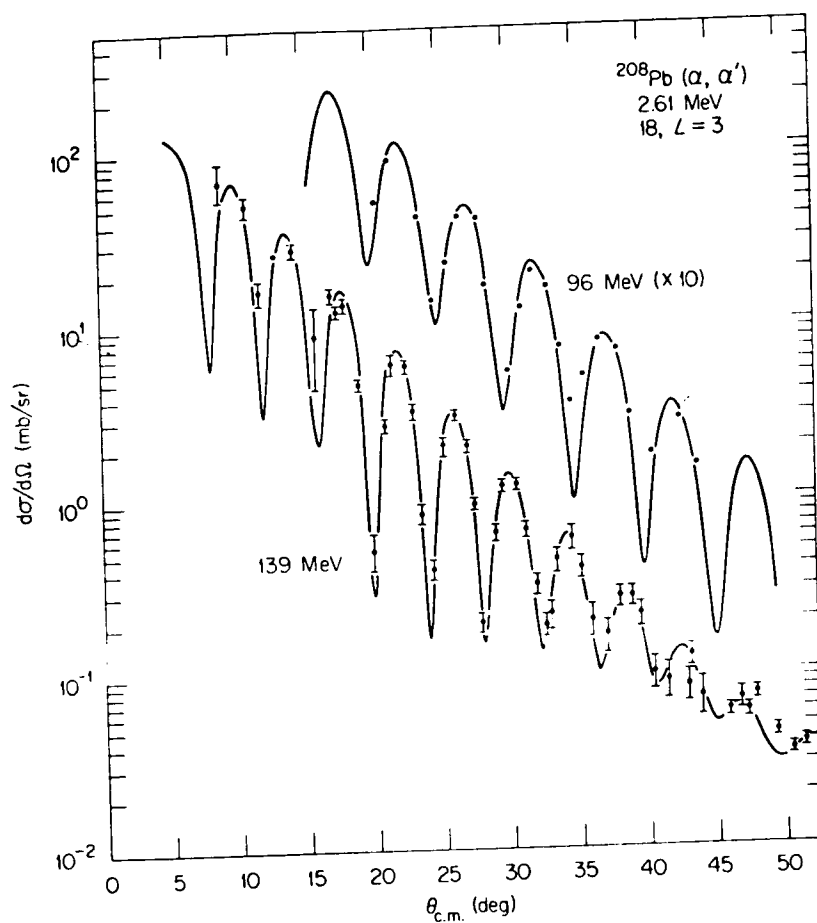


Figure 3

Comparison of single-folding model predictions,<sup>22</sup> using 18% of the energy-weighted sum rule with measured cross sections for the  $3^-$  state in  $^{208}\text{Pb}$ .

### 3.3. Heavier Ions

The single-folding approach to the optical potentials for heavy ions starts with some standard phenomenological nucleon optical potential obtained from nucleon-nucleus scattering. It invariably overestimates the strength of the real potential (near the strong absorption radius, where it matters most) by a factor of about two.<sup>23</sup> Several reasons have been offered in explanation of this,<sup>24</sup> including the improper treatment of any density-dependence present in the underlying nucleon-nucleon effective interaction.

It has been suggested<sup>25</sup> that single folding of this kind may be adequate for estimating both the real and imaginary contributions to the optical potential from one or a few loosely-bound 'valence nucleons', even if it does fail for the bulk of the more tightly bound nucleons. In this approach, the nucleus is divided into 'core' and 'valence' parts. The core contribution is taken to be the same as the phenomenological potential which describes scattering from the corresponding core nucleus, while the valence nucleons are treated 'microscopically' within the single-folding model. Successful applications have been made<sup>25</sup> to the elastic and inelastic scattering of  $^{17}\text{O}$  and  $^{18}\text{O}$  where, of course, the core nucleus is taken to be  $^{16}\text{O}$ . The relatively weak binding of the valence nucleons results in their contribution to the interaction being of longer range than that due to the core nucleus. Thus, they may play a disproportionately large rôle in generating the potential at the large distances that are important for strongly absorbed scattering. In particular, they may have important effects on the Coulomb-nuclear interference pattern when the nucleus is excited.

A recent example of the use of this model is shown in Fig. 4. The curves represent a preliminary analysis by Rhoades-Brown of Oak Ridge data<sup>26</sup> for the excitation of 120 MeV  $^{18}\text{O}$  to its first  $2^+$  state by scattering from  $^{208}\text{Pb}$ . The

calculations are almost free of adjustable parameters. The complex core potential is taken from analysis of 130-MeV  $^{16}\text{O} + ^{208}\text{Pb}$  elastic data. The interaction of the valence neutrons of  $^{18}\text{O}$  with the target is an empirical (complex) optical potential taken from analysis of  $n + ^{208}\text{Pb}$  scattering. The valence neutron wavefunctions result from shell-model calculations.<sup>25</sup> These alone are not sufficient to account for the  $2^+$  excitation; for example, being neutrons they would result in a vanishing  $B(E2)$  value unless they are assigned an effective charge. This implies polarization of the core protons. This core polarization is included<sup>25</sup> by using the standard prescription (e.g., Eq. (21)) of deforming the core potential by an amount required to explain the observed  $B(E2)$  for this transition. The coupling is sufficiently strong that a first-order DWBA is not sufficient for a description of this excitation. Particularly important is the re-orientation of the  $2^+$  state after its initial excitation (i.e. transitions between the various magnetic substates). The quadrupole component of this reorientation interaction is constructed in the same way,<sup>25</sup> normalized to the observed electric quadrupole moment of the  $2^+$  state. In addition, it was found advantageous to include an hexadecapole interaction as well. Although there is no independent information on the electric hexadecapole moment of this  $2^+$  state, the values chosen are consistent with our expectations for nuclei in this region. The main effect of the additional reorientation due to this term is to decrease the cross section at the larger angles (see Fig. 4). (It is a common feature of this and other analyses of these data that such a term seems to be required. Consequently, we may obtain a measure of this hexadecapole interaction and, through folding, of the underlying mass moment.)

The agreement with the data is remarkable considering that no parameters were adjusted. The standard type of DOMP analysis has great difficulty in

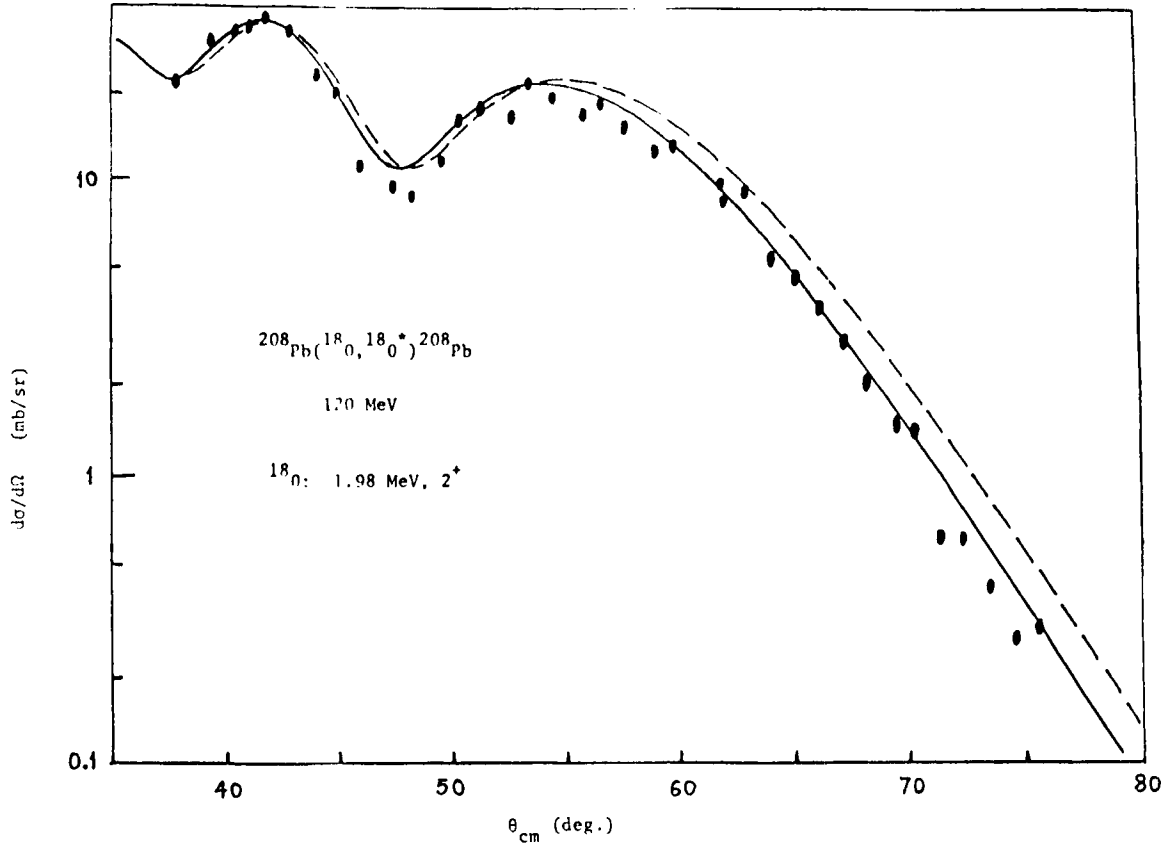


Figure 4

Coupled-channels predictions using the hybrid folding model of Landowne, et al.<sup>25</sup> for exciting the first  $2^+$  state of  $^{18}\text{O}$  when scattering at 120 MeV from  $^{208}\text{Pb}$ , compared to measured cross sections.<sup>26</sup> The solid curve includes reorientation due to a hexadecapole moment of the  $2^+$  state, the dashed curve does not. (Both include reorientation due to the quadrupole moment.)



reproducing accurately the Coulomb-nuclear interference minimum near  $50^\circ$  without destroying the fit at other angles. Even the agreement shown in Fig. 4 probably could be improved by small parameter adjustments; this would be valid because all of the empirical quantities used to determine the input have appreciable experimental errors associated with them.

#### 4. The Double-Folding Model

##### 4.1. The Ingredients

The double-folding model has been reviewed in detail several times in the last few years (for example, see Refs. 15, 27, and 28). We only mention a few points here.

The discussion in Sec. 3.1 on the nuclear densities or transition densities applied equally here. The other ingredient in the formula (16) is the effective nucleon-nucleon interaction  $v$ . At one time (see Ref. 13 for example), it was popular to use for  $v$  a simple potential, such as a Gaussian, which fits low-energy nucleon-nucleon scattering. However, this has been shown<sup>23</sup> to overestimate the heavy-ion optical potential by roughly a factor of two. (It has also been shown<sup>29</sup> to predict  $(p,p')$  cross sections which are too large.)

It is desirable to use an effective interaction  $v$  which is based upon a realistic nucleon-nucleon force, since one goal is to obtain a unified description of nucleon-nucleon, nucleon-nucleus and nucleus-nucleus scattering. As a consequence, our effective interaction is some kind of  $G$ -matrix. There is no guarantee of the existence of such a single, simple, effective interaction of this kind which may be used in a variety of physical circumstances. However, it is certainly worth seeing how far we can go with such a simple concept and to find what kind of corrections we may have to make in particular cases.

A 'high-energy' approach to this problem was used by Dover and Vary.<sup>16,28</sup> In the high-energy limit (or impulse approximation),  $v$  would become the (complex)  $t$ -matrix for free space nucleon-nucleon scattering. However, for the low-energy ( $\lesssim 20$  MeV per nucleon) collisions that are encountered particularly in heavy-ion scattering, there are very large corrections to be made for the effects of the nuclear medium in which the two interacting nucleons are embedded (Pauli principle, off-shell propagation and Fermi motion of the nucleons). One 'low-energy' approach<sup>30</sup> assumes that the effective interaction  $v$  is similar to the  $G$ -matrix for two nucleons bound near the Fermi surface. One consequence of this assumption is that  $v$  is real, so that the imaginary interaction has to be treated phenomenologically. An example of this real interaction which has been widely (and successfully) used<sup>15,30</sup> has become known as the M3Y interaction. The part that is independent of spin and isospin has the simple form

$$v_{M3Y}(r) = \left[ 7999 \frac{e^{-4r}}{4r} - 2134 \frac{e^{-2.5r}}{2.5r} \right] - 262 \delta(r), \quad (24)$$

where the ranges are in fm and the strengths in MeV. The other spin, isospin-dependent terms have similar forms. The last, zero-range term in Eq. (24) is a pseudo-potential that represents the effects of knock-on exchange between the incident and the struck nucleon. (This is believed to be the leading correction when the formula (16) is properly antisymmetrized with respect to the interchange of nucleons between the two nuclei  $A_1$  and  $A_2$ . The individual nuclear wavefunctions  $\psi$  and  $\phi$  are themselves assumed to be antisymmetric.)

Other work on nucleon-nucleus scattering<sup>27,31</sup> has taken into account that the projectile nucleon is in the continuum above the Fermi sea. This yields a complex  $G$ -matrix; however, it seems probable that at least the absorptive processes in the scattering of composite nuclei, especially heavy ions, are very

different so that it may still be preferable to treat the imaginary potential phenomenologically. More recent work<sup>32</sup> has recognized that, in the collision of two composite nuclei, both of the two interacting nucleons are embedded within two pieces of nuclear matter that are moving with respect to each other. This work provides both real and imaginary components of the interaction. The imaginary part provides the absorptive potential due to the bulk properties of the overlapping nuclei; it is supplemented by additional surface terms, calculated according to the  $\Delta U$  term of Eq. (11), which arise from the finite extent of the two pieces of nuclear matter and correspond to the excitation of collective surface vibrations.<sup>33</sup>

It is worth remarking that replacing the bare nucleon-nucleon potential by an effective or G-matrix interaction is a process analogous to the derivation of the effective interaction  $U_{op}$  in Sec. 2.1. The G-matrix transformation itself incorporates some polarization effects, namely the short-range correlations between pairs of nucleons and, in principle, allows us to use simpler nuclear wavefunctions such as those of the shell model. Then a full derivation of the operator  $U_{op}$  of Sec. 2.1 would go explicitly through this two-stage projection from the original many-body problem (see, for example, Ref. 34).

From its origin as an effective interaction or G-matrix, it is clear that, in general, this  $v$  should be both energy-dependent and density-dependent.<sup>†</sup> For example, the volume integral of the effective interaction (including, implicitly, the knock-on exchange<sup>35</sup>) for a nucleon scattering from symmetric nuclear matter was found by Jeukenne, et al.<sup>31</sup> to be represented approximately by

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<sup>†</sup>The MBY interaction (24) does not depend upon density or energy (except for a weak energy-dependence of the strength of the zero-range pseudo-potential). From its construction<sup>30</sup> within a truncated space of oscillator functions, one can see that it represents a certain average over a range of kinetic energies and densities.

$$J_0(\rho, E) \approx F(E)(1 - d\rho^{2/3}), \quad (25a)$$

with

$$d = 2.03 \text{ fm}^2, \quad F(E) = (903 - 7.67E + 0.22E^2) \text{ MeV fm}^3 \quad (25b)$$

and the density  $\rho$  in  $\text{fm}^{-3}$ . ('Normal' nuclear matter has  $\rho \approx 0.17 \text{ fm}^{-3}$ ; then  $1 - d\rho^{2/3} \approx 0.38$ , a large reduction from the value in free space.)

The usual way of handling the density dependence is to use the local density approximation. In this we assume that the effective interaction between two nucleons embedded in a nucleus at some position where the density has a certain value is given by the G-matrix for infinite nuclear matter with the same density value. This ignores the effects on G of density gradients. Also, because the force has a finite range, two positions,  $\underline{r}_1$  and  $\underline{r}_2$ , are involved; frequently one uses the density at the center of gravity,  $\frac{1}{2}(\underline{r}_1 + \underline{r}_2)$ . This kind of uncertainty is further aggravated in heavy ion double folding because there are two densities,  $\rho_1(\underline{r}_1)$  and  $\rho_2(\underline{r}_2)$ . The simplest assumption is a sudden (or 'frozen density') approximation in which the two undisturbed densities are simply superposed. Then one uses  $\rho = \rho_1 + \rho_2$  in Eq. (25). This may be adequate at high energies (relative energy per nucleon high compared to the Fermi energy) where the Pauli principle has little effect and there is little time for the two nuclei to readjust as they collide. It seems less appropriate at lower energies; unfortunately, attempts to do better are quite complicated.<sup>32,33</sup> Fortunately, the region of strong overlap is not experienced in most heavy-ion collisions because of the strong absorption, and the frozen density approximation is more plausible for the low-density surface region. For a recent application to heavy-ion scattering, see Ref. 36. In the example of alpha scattering at 140 and 172 MeV discussed below, one might argue that the energy per nucleon is

moderately high and that the tight binding of the alpha discourages its polarization.

#### 4.2. Applications to Heavy-Ion Scattering

The M3Y interaction (24) has been used extensively to construct the real parts of optical potentials for the elastic scattering of heavy ions (see Refs. 15 and other references quoted there). For example, analyses of some 156 sets of data for some 67 systems at energies mostly between 5 and 15 MeV per nucleon were reported in Refs. 15. Except for  ${}^6,{}^7\text{Li}$  and  ${}^9\text{Be}$ , these data were fitted by renormalizing the real folded potential by a factor  $N$  which was found to be close to unity,  $\bar{N} = 1.06 \pm 0.11$ . Although the imaginary potentials had to be chosen phenomenologically, good data do allow one to determine the strength required for the real part with little ambiguity.

Figure 5 shows another example of the use of the M3Y interaction. The experimental deformed potential contours were deduced<sup>37</sup> from measurements on excitations of  ${}^{160}\text{Gd}$  up to its  $12^+$  state by the inelastic scattering of  ${}^{40}\text{Ar}$ . The folded (non-spherical) potential was generated by using a non-spherical, quadrupole-deformed density for  ${}^{160}\text{Gd}$ .

The exceptional cases of  ${}^6,{}^7\text{Li}$  and  ${}^9\text{Be}$  scattering require an optical potential whose real part is only about half the strength predicted by the M3Y interaction. One feature that all three ions share is a dissociation energy much smaller than for other stable nuclei, so one is tempted to attribute the failure to the effects of break-up. This seems to have been confirmed, at least for  ${}^6\text{Li}$ , by some recent calculations.<sup>19</sup> A similar overestimate of the real potential when the M3Y interaction is used has been observed<sup>38</sup> for the scattering of the light ions  $d, t$ , and  ${}^3\text{He}$ , which are also weakly bound.

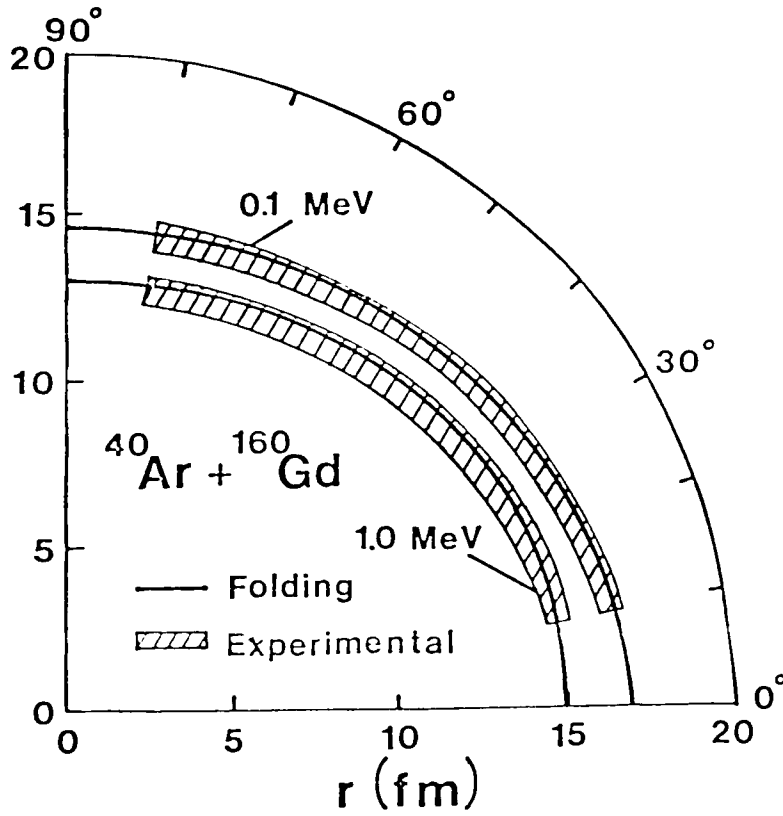


Figure 5

Non-spherical optical potential contours<sup>37</sup> for  $^{40}\text{Ar} + ^{160}\text{Gd}$ , referred to the body-fixed axes for  $^{160}\text{Gd}$ , compared to folding-model calculations using the M3Y interaction.

#### 4.3. Density Dependence and Alpha Scattering

It was noted in Sec. 3.2 that a simple nucleon-alpha potential of Gaussian form was adequate, within the single-folding model, to reproduce the observed alpha scattering at forward angles. Use of the M3Y interaction (24) in the double-folding model (plus a phenomenological imaginary potential) also provides agreement with data at 140 and 172 MeV in the diffraction region at small angles,<sup>39</sup> with only a few percent renormalization required for an optimum fit. This indicates that the model is providing the correct real potential for peripheral collisions.

It has long been known<sup>40</sup> that observation of alpha scattering at energies high enough ( $\gtrsim 100$  MeV) and angles large enough to exhibit the 'nuclear rainbow' refractive effects can resolve the discrete ambiguities associated with the interior depth of the optical potential. Such data indicate that a real depth of  $\sim 140$  MeV is required. However, the folded potential is  $\sim 230$  MeV deep in the interior; consequently it will generate incorrect scattering at large angles. This is indicated in Fig. 6. Reducing the strength of the interaction (by  $N \approx 0.5$ ) to give the correct large-angle scattering completely destroys the agreement at smaller angles because the potential in the surface is then too weak.

A natural way to rectify this situation is to introduce the expected density dependence, like that of Eq. (25), which reduces the strength in the interior relative to that in the low-density surface region. Explicit calculations<sup>39</sup> confirmed that this was so, as shown by the example in Fig. 6. For these illustrative calculations, the M3Y interaction (24) was simply multiplied by a density-dependent factor. This factorization is not strictly correct but makes the calculations easier. A factorized dependence on the densities, different from (25a), was adopted for computational convenience<sup>41,42</sup>

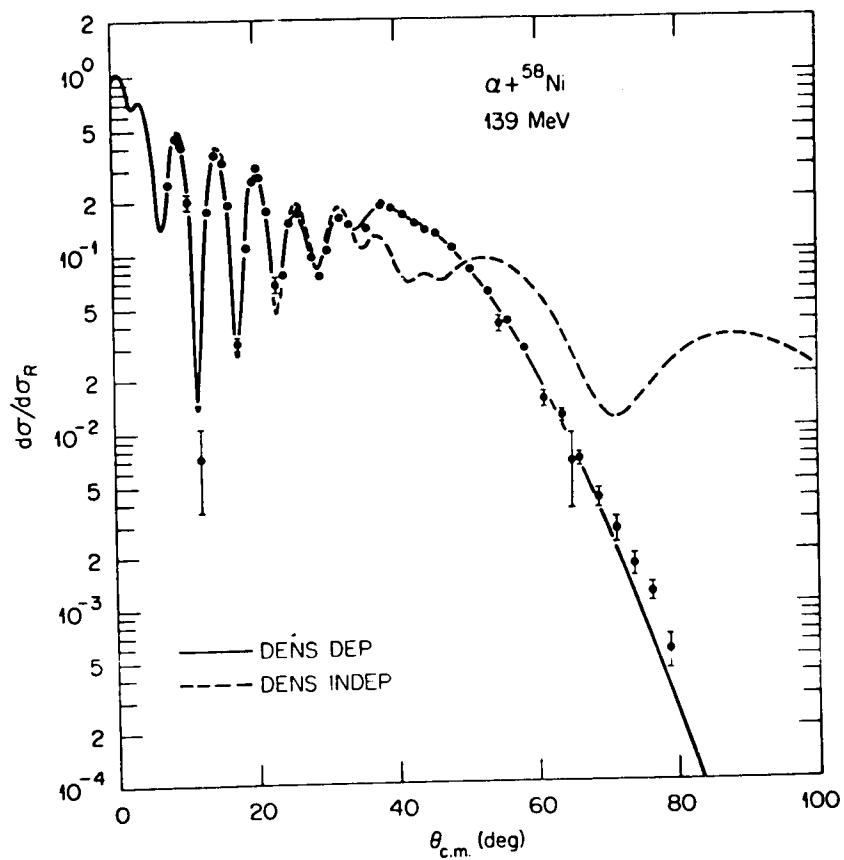


Figure 6

Comparison of measured elastic scattering of  $\alpha + {}^{58}\text{Ni}$  with double-folding calculations using either the M3Y interaction (dashed curve) or the same interaction modified by a density-dependent strength (full curve).



$$v(r, \rho) = f(\rho) v_{M3Y}(r), \quad (26a)$$

with

$$f(\rho) = C(1 + \alpha e^{-\beta\rho}), \quad \rho = \rho_1(r_1) + \rho_2(r_2). \quad (26b)$$

The (energy-dependent) constants in  $f(\rho)$  were chosen to reproduce the magnitude and density dependence of the volume integral of the G-matrix of Jeukenne, et al.<sup>31</sup> at the appropriate energy per nucleon. An overall renormalization of the real folded potential by a factor of  $N$  was allowed in fitting the 11 sets of alpha-scattering data. Quite consistently the optimum value was  $N \approx 1.3$ . With this, the factor  $N f(\rho)$  is unity for a density of  $\rho \approx 0.15 \text{ fm}^{-3}$ , nearly equal to the 'normal nuclear matter' density of  $\rho_0 \approx 0.17 \text{ fm}^{-3}$ . This is somewhat larger than had been expected,<sup>15,41,42</sup> but it is difficult to interpret this result precisely, in the light of the hybrid nature of the simple model interaction (26). We also note that the G-matrix of Ref. 31 is only given for  $\rho \lesssim \rho_0$ , whereas larger values of  $\rho = \rho_1 + \rho_2$  are encountered when the alpha is within the target nucleus. Further, the use of  $\rho = \rho_1 + \rho_2$  may introduce too strong a dependence on density. The two densities are actually moving with respect to each other, thus weakening the repulsive effect of the Pauli exclusion principle compared to that for the static superposition  $\rho_1 + \rho_2$ . Further studies with a more sophisticated interaction would be of interest.

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## CONTRIBUTED SEMINARS



# A SEARCH FOR THE LOW-ANGULAR MOMENTUM CUTOFF IN THE $^{12}\text{C}+^{16}\text{O}$ FUSION

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The Time-Dependent-Hartree-Fock (TDHF)<sup>1)</sup> theory prediction of the inhibition of low-impact-parameter fusion in nucleus-nucleus collisions at high energies has been investigated. The shapes of the angular distributions in the  $^{12}\text{C}(^{16}\text{O}, \alpha_0)^{24}\text{Mg}$  reaction from  $E(^{16}\text{O})=28.5$  MeV to 100 MeV were studied and analysed within the statistical model framework.

Angular momentum matching conditions in the statistical compound nucleus decay to a discrete state force a localization of the contributing partial cross sections  $\sigma_J$ , in angular momentum space. The distribution of  $\sigma_J$ , is strongly peaked with a center of gravity  $\langle J \rangle$ , the mean compound nucleus contributing to the excitation of the final state. If a low-angular momentum cutoff occurs, the center of gravity of the distribution is shifted to higher values, decreasing the period of the oscillations of the angular distributions<sup>2)</sup>. The position of the first minimum (and maximum) of the oscillatory angular distributions to the  $J^\pi=0^+ ^{24}\text{Mg}$  ground state is expected to be sensitive to the presence of a low-angular momentum cutoff. This effects has been investigated by following the energy dependence of the angles of the first minimum ( $\phi$ ) and first maximum ( $\bar{\phi}$ ) of the  $\alpha_0$  angular distribution.

The statistical model gives approximately  $\phi=3\pi\langle J \rangle$  and  $\bar{\phi}=5\pi/4\langle J \rangle$  where  $\langle J \rangle$  is approximately the grazing angular momentum  $L_g^\alpha$  of the outgoing

particle. Thus in the absence of the low- $L$  cutoff one expects that  $\phi L \sim 3\pi/4$  and  $\frac{1}{2}L \sim 5\pi/4$  are constant and independent of the bombarding energy.

Measurements were made on the University of Sao Paulo 8UD Pelletron for  $E(^{16}\text{O}) < 50$  MeV using a solid state Position Sensitive Detector and the University of Rochester MP tandem accelerator using a split pole spectrograph (40 MeV  $E(^{16}\text{O}) < 100$  MeV).

Results are shown in figures 1 and 2. No evidence for a low- $L$ -cutoff was found even at energies well beyond the predicted threshold. The excitation functions as well as the angular distributions are well described by the statistical model calculations if no low- $L$ -cutoff is used<sup>3)</sup>. If the predicted values of  $l_{\min}$  are used, the calculations display discrepancies with experimental data (see figs. 1,2).

Due to the localization of the  $\sigma_J$  distribution, our method is not sensitive to very low  $l$  values cutoff. ( $l_{\min} < 5\hbar$  for  $E(^{16}\text{O}) > 100$  MeV) therefore measurements using the neutron channel and out of plane particle-particle correlation will be performed.

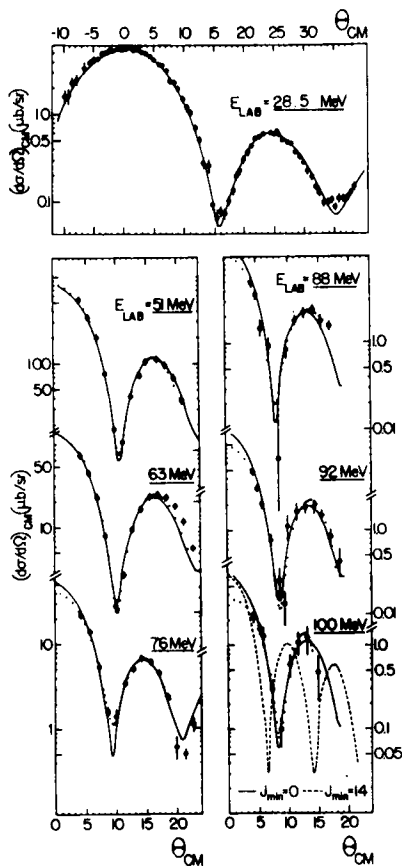


Figure 1:

Typical angular distributions. The small points correspond to least-square fits by sums of Legendre polynomials. The full lines show the result of statistical-model calculations assuming  $J_{\min} = 0$ . The dashed line at  $E_{\text{lab}} = 100$  MeV is the result of the same calculation, in which a cutoff at  $J_{\min} = 14\hbar$  is applied.



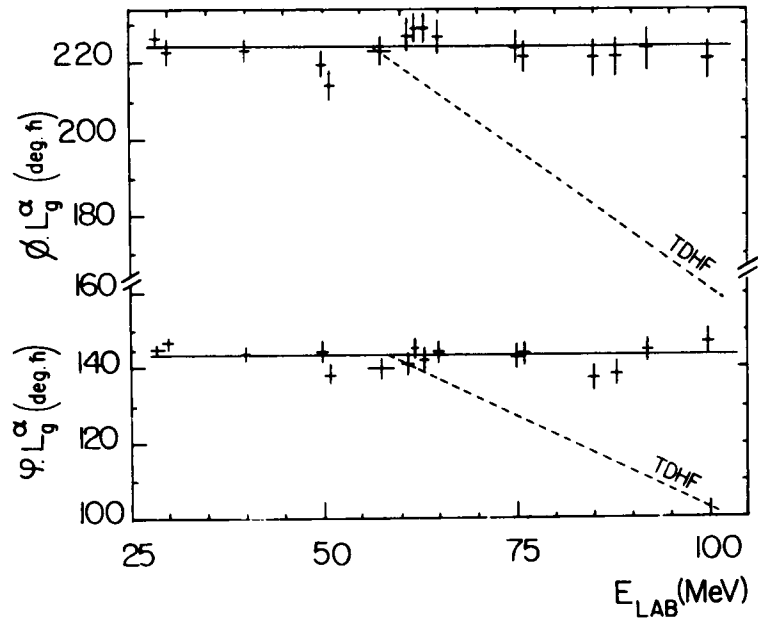


Figure 2:

Experimental values obtained for  $\phi_L^{\alpha}(E_{lab})$  and  $\phi_L^{\beta}(E_{lab})$ . The full line describes the Hauser-Feshbach theoretical trajectory if  $J_{min}=0$  is assumed. If the TDHF-predicted  $J_{min}$  values are considered in the calculations, the dashed trajectory is obtained.

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# SOME CONFLICTING ASPECTS OF COLLISIONAL NUCLEAR DYNAMICS

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Collisional nuclear dynamics has become fashionable in the scope of the search of improvements for mean-field theories of nuclear motion<sup>1-5</sup>. Being the latter independent-particle approaches to collective dynamics, their lack of particle-particle correlations is responsible of some deficiencies in the description of two-body observables, typically dispersions in mass or charge yields in low energy, peripheral damped reactions between heavy ions. Incidentally, nucleon emission in the course of the reaction could be regarded as an indication of some amount of nucleon residual interaction, manifesting in the broadening of the single-particle (s.p.) levels of the mean field. Such an effect lies certainly beyond the s.p. model and claims for the complementation of the latter with a proper treatment of the coupling to the remaining nuclear degrees of freedom.

In the independent-particle frame, the time-dependent Hartree-Fock (TDHF) view refers the problem of collective motion to that of the evolution of a Slater determinant of s.p. wave functions in a mean field. It has been proven useful for describing the time-evolution of density profiles, energy and mass exchange during an encounter between two large nuclei, excitation of collective modes, and the dynamics of one-body operators. Its limitations show up when two-body operators and fluctuations

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come into the picture. Attempts to correct the TDHF version of many body dynamics could be roughly classified into two categories, according to the purpose. Should one be interested in the construction of a scheme that brings the evolution closer to the exact one, the research ought to proceed through either of the following alternatives:

- a) modify the trial wave function, giving up the determinantal way of thinking, or
- b) propose other evolution law for the uncorrelated many body wave function.

On the other hand, were the intention just to observe the time-evolution on a macroscopic or asymptotic time-scale, irreversibility would manifest through the broadening and subsequent decay of the s.p. levels in the mean field. Thus instead of an evolution law that reads, for the one-body density operator,

$$i\dot{\rho} = \mathcal{L}_{\text{HF}}(\rho)\rho = [\mathcal{H}_{\text{HF}}(\rho), \rho] \quad [1]$$

in the usual TDHF appearance, one would expect a non-hermitian generator of the time-evolution, i.e.,

$$i\dot{\rho} = [\mathcal{L}_{\text{HF}}(\rho) - i\mathcal{K}(\rho), \rho] \quad [2]$$

Here the real kernel  $\mathcal{K}(\rho)$  contains the average effect of those relevant two-particle correlations that are usually disregarded in the mean field approach to the dynamics. In addition,  $\mathcal{K}(\rho)$  may include any other coupling that affects the s.p. of interest, typically the interaction with the collective modes of the nuclear fluid.

Equation [2] possesses the form of a "linearized kinetic equation", well known in nonequilibrium statistical mechanics. Under suitable assumptions regarding,

- i) the macroscopic conditions of the many body system under consideration, such as its temperature and degree of dilution, and
- ii) the microscopic conditions, such as the strength and range of the two-body interaction;

and provided that the time and length parameters defining the microscopic scales are much shorter than their macroscopic partners, the form of the kernel  $\mathcal{K}(\rho)$  can be explicitly displayed<sup>5)</sup> in its most general version.

Computational limitations definitively rule out any collisional calculation involving realistic, i.e., two- or three-dimensional systems that could describe a low energy encounter between two heavy nuclei, either in momentum or coordinate space. Nevertheless, the interest deposited upon either "nuclear half-matter" or slab collisions in the frame of the TDHF studies has proven successful, in view of the amount of dynamical information extracted from these one-dimensional approaches<sup>6)</sup>. However, should a collisional regime be established, a straight forward calculation of the collisional kernel  $\mathcal{K}(\rho)$  in eq.[2] would lead to an unavoidable divergence<sup>7)</sup>. This is justified combining two facts:

- a) the interaction responsible of the nonhermitian kernel is the weak, residual one;
- b) in a one-dimensional system, the unperturbed s.p. level density  $dn/d\epsilon$  behaves like  $\epsilon^{1/2} \sim |\vec{k}|^{-1}$

These conditions create a relaxation time for the slab system, shorter than the duration of a single nucleon-nucleon collision. This unphysical situation can be formulated as an impossibility statement, namely, about the non existence of a frame for a collisional slab dynamics. Thus, irreversibility in nuclear systems must be examined in a realistic approach, where numerical restrictions play an important role. Like in other fields of physics, the way out of these limitations might be the consideration of a low-dimensional set of observables, whose mean values in the collisional field could be assigned an approximate law of motion.

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"One and two step processes contribution to the cross section in the preequilibrium regime"

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We have calculated the contribution of the one and two step processes to the total cross section of (p,d) inclusive reactions up to very large energy loss. The analysis was applied to several targets and incident energies. We have adopted the basic ideas of the multistep direct reaction (MSDR) theory developed by Tamura et al.<sup>1)</sup>. In transitions to the continuum regime a large number of complicated states is involved. It is quite natural to assume that when summing over a great number of actual nuclear states the interference terms between different steps processes average out in a large extent. Thus the calculation reduces to the incoherent sum of cross sections for each step.

The one step is calculated in terms of the well known DWBA theory. We assume that any nuclear state is well described by a pure single particle shell model. Thus, the summed cross section leading to actual residual states related with the pick up of a neutron from an (nlj) shell is

$$\sum_{n_f} \frac{d\sigma^{I_i \rightarrow n_f}}{d\Omega} = \frac{3}{2} \frac{D_0^2}{10^4} \frac{d\sigma_{nlj}^{DW}}{d\Omega}(E_{nlj}) \sum_{n_f} S_{nlj}^{I_i \rightarrow n_f} \quad [1]$$

where  $\sum_{n_f} S_{nlj}^{I_i \rightarrow n_f} = \bar{n}[(nlj)]$  is the number of neutrons in the (nlj) shell, and  $E_{nlj}$  is the average excitation energy of these residual states. Thus, the absolute values of the one step differential cross section are obtained adding up [1] for all occupied (nlj) neutron shell.

For the two step process we considered the two main reaction paths: an inelastic scattering of the incident proton followed by the pick up of a neutron (p,p',d) and the transfer reaction followed by an inelastic interaction of the exit deuteron (p.d,d'). In describing this two step reaction process we adopted the coupled reaction channel formalism. The two step differential cross section for a given final state  $\alpha I_F$  is written

$$\frac{d\sigma}{d\Omega}^{\alpha I_F}(E) = \sum_{l_{tr}, j_{tr}} \int \rho_{l_{tr}}(E'') S^{j_{tr}}(E'') \frac{d\sigma}{d\Omega}^{\alpha I_F}(E'', E) dE'' \quad [2]$$

Here  $L_{tr}$  and  $I_{tr}$ , (that are to be coupled to  $I_F$ ) are the angular momenta transferred in the inelastic and neutron pick up steps, respectively, while  $E(E')$  is the projectile (ejectile) energy, and  $E''$  is the intermediate proton or deuteron energy.  $d\sigma^{nI}/d\Omega(E'',E)$  is a purely dynamical factor that can be obtained from a coupled reaction channels code like CHUCK2.  $\rho_{L_{tr}}(E'')$  plays the role of an spectroscopic density for the inelastic process and  $S^{J_{tr}}(E'')$  is the spectroscopic factor for the transfer step. When summing over all final states the spectroscopic factor  $S^{J_{tr}}(E'')$  is obtained through the pick up sum rule for (p,d) reactions. Furthermore, in actual calculations, the integral in [2] is replaced by a sum over appropriately chosen intermediate energy intervals  $\Delta E''$ . Then

$$\rho_{L_{tr}}(E'') \Delta E'' = \sum_{E' = E'' - \Delta E''/2}^{E'' + \Delta E''/2} \beta_{L_{tr}}^2(E'') \quad [3]$$

where  $\beta_{L_{tr}}^2$  is the strength due to particle-hole excitations and is obtained through an RPA formalism.

Numerical calculations were made for all the ejectile energy spectra in the  $^{54}\text{Fe}(p,d)$  reaction at  $E_p=62$  MeV. Due to the great number of representative states in the  $^{209}\text{Bi}(p,d)$  reaction at  $E_p=62$  MeV only two points of the spectra were calculated.

The following conclusions were obtained: the one step process contribution exhausts the experimental yield over the first 5-8 MeV of excitation energy in agreement with ref<sup>2)</sup> for the (p,p') reaction but in contradiction with results obtained by Tamura et al<sup>1)</sup>. For the two step contribution the (p,d,d') path is appreciable greater than the (p,p',d) route due to dynamical effects. Despite the use of a mass dependent model, results for  $^{54}\text{Fe}$  and  $^{209}\text{Bi}$  are very similar, according with experimental facts. As a concluding remark for excitation energies  $E_x \gtrsim 25$  MeV is necessary to include more than two step processes.

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# EXCITATION OF THE OCTUPOLE VIBRATION IN $^{154}\text{Sm}$

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In reference [1 , 2] the multiple excitation of rotational states in deformed nuclei, due to the nuclear and Coulomb interaction, was considered using a novel approach. Letting the wave functions (and therefore the phase shifts) depend on the angle  $\theta'$  between the symmetry axis of the deformed nucleus and the vector  $\vec{r}$  which joins the centers of the target and the projectile, it is possible to calculate the differential cross section to different final states, avoiding a coupled channel calculation. A numerical study for  $^{16}\text{O}$  colliding with several Dy isotopes showed that with this method one obtains approximately the same kind of fit to the experimental data as the coupled channel calculations [3] .

In reference [4] the method was generalized to scattering processes in which, together with the excitation of rotational states, one has another process present which can be considered as a perturbation.

In the present communication, we will illustrate the method with the excitation of the  $v=0$  octupole vibration-rotational band of  $^{154}\text{Sm}$  in a collision with  $E_{\text{lab}}=50$  MeV  $\alpha$ -particles. In order to get the differential cross sections, the DWBA expression for the T-matrix is used, but considering only the potential responsible for the excitation of the octupole vibration. All processes related to the rotational excitation being included in the distorted waves.

Let  $V_{\text{tot}}(r, \theta')$  be the total interaction potential between the target and the projectile. The distorted waves, which depend parametrically on  $\theta'$ , are obtained from an optical model code for spherical nuclei using the potential  $V_{\text{tot}}(r, \theta')$ . Figu-



re 1 shows that these wavefunctions (from which angle dependent phase shifts are obtained) indeed reproduce the cross sections for the rotational states. Figure 2 shows the differential cross section for the excitation of the  $1^-$  and  $3^-$  states of  $^{154}\text{Sm}$ .

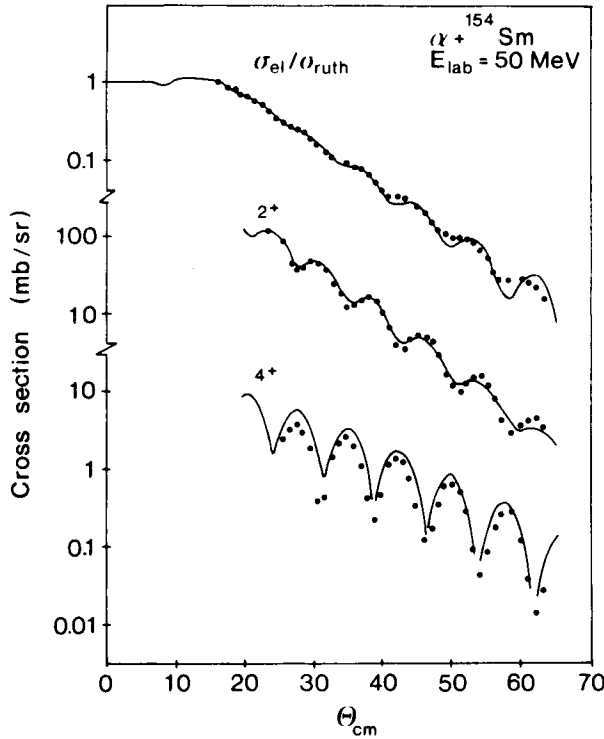
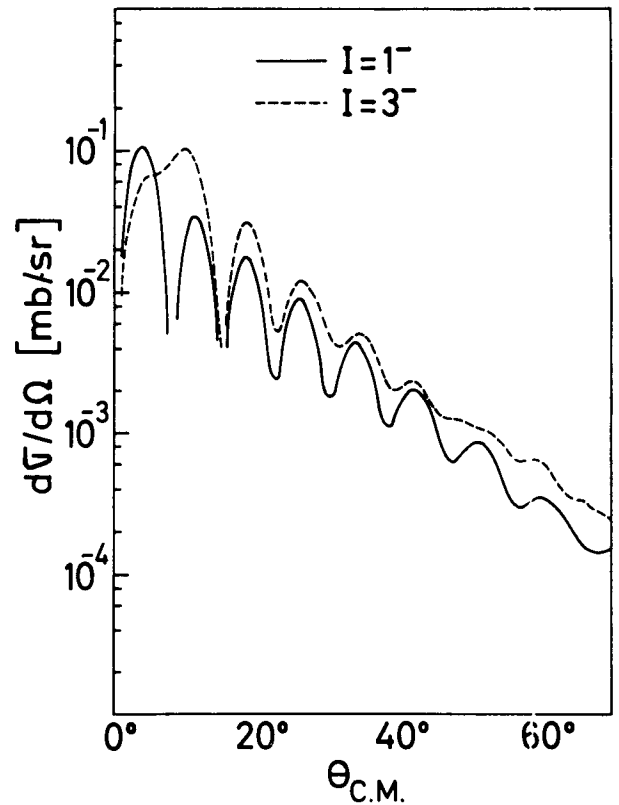


Figure 2. Differential cross section for the excitation of the vibration-rotational states  $1^-$  and  $3^-$  of  $\text{Sm } 154$ .

Figure 1. Differential cross section for the excitation of the rotational states of  $\text{Sm } 154$ . The parameters for the deformed Saxon-Wood potential were taken from reference [5].



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## Wigner versus Heisenberg processes in pair-exchange reactions

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In the 1981 Workshop we suggested the possibility of using pair-exchange reactions such as  $C^{14}, O^{14}$  in order to populate states which are both i) interesting from the point of view of nuclear structure and ii) difficult to obtain otherwise. These reactions may specifically populate certain collective (n-particle)-(n-hole) states with  $n=2,3,4$  (in addition to the well known g.s. and particle-hole states). On the previous occasion we made a very rough but physically simple prediction on the cross sections of the reaction  $Po^{208}(C^{14}, O^{14})Pb^{208}$  based on experimental results of two-neutron transfer reactions.

In the present contribution we analyze the mechanisms through which direct pair-exchange reactions may proceed and we perform a proper DWBA calculation for the reactions  $Po^{208}(C^{14}, O^{14})Pb^{208}$  and  $Ca^{40}(C^{14}, O^{14})Ar^{40}$ . The first reaction has a large interest because of nuclear structure considerations, although it is difficult because of the target half-life. The second reaction has been performed experimentally<sup>1)</sup>.

We may think of a Wigner and of a Heisenberg type of process. In the first type there is a simultaneous diproton pick-up and a dineutron stripping. In the second type, the effective interaction transforms a diproton into a dineutron in the target, while simultaneously a dineutron becomes a diproton in the projectile. In the following, each type of process is separately treated, although in principle they should add coherently. This is due to the fact that we do not have an indication of the relative size (or sign) of the Wigner and Heisenberg terms in the effective four-body force which is responsible of the reaction.

Both Wigner and Heisenberg predictions confirm the results of the simplified calculation concerning the relatively large population of states belonging to the pairing family and the very small cross sections to states with two particle-hole phonons. Therefore, the "new" states that may be located through pair-exchange reactions are the proton pairing vibrations at 5.3 Mev ( $I=0$ ) and at 6.4 Mev ( $I=2$ ), and the alpha-vibration at  $\approx 10$  Mev.

The relative population of the  $I=2$  state in  $Ar^{40}$  is not well reproduced by either mechanism.

| $E_x$              | J               | CONFIGURATION                | Previous Estimate | Wigner processes | Heisenberg processes |
|--------------------|-----------------|------------------------------|-------------------|------------------|----------------------|
| 0 <sup>a)</sup>    | 0 <sup>+</sup>  | g.s.                         | 1                 | 1                | 1                    |
| 2.61 <sup>a)</sup> | 3 <sup>-</sup>  | (p-h,3)                      | 0.66              | 0.29             | 0.16                 |
| 3.20 <sup>a)</sup> | 5 <sup>-</sup>  | (p-h,5)                      | 1.5               | 1.1              | 0.03                 |
| 4.86 <sup>a)</sup> | 0 <sup>+</sup>  | (r,n,0)(a,n,0)               | 0.44              | 2.4              | 0.60                 |
| 5.22               | 6 <sup>+</sup>  | (p-h,3) <sup>2</sup>         | 0.01              | 0.002            | 0.00001              |
| 5.30               | 0 <sup>+</sup>  | (r,p,0)(a,p,0)               | 2.3               | 0.15             | 0.32                 |
| 5.65 <sup>b)</sup> | 2 <sup>+</sup>  | (r,n,0)(a,n,2)               | 1.1               | 0.13             | 0.04                 |
| 5.81               | 8 <sup>+</sup>  | (p-h,3)(p-h,5)               | 0.05              | 0.005            | 0.0002               |
| 6.37               | 2 <sup>+</sup>  | (r,p,2)(a,p,0)               | 1.8               | 0.13             | 0.01                 |
| 6.40               | 10 <sup>+</sup> | (p-h,5) <sup>2</sup>         | 0.04              | 0.004            | 0.003                |
| 10.16              | 0 <sup>+</sup>  | (r,n,0)(a,n,0)(r,p,0)(a,p,0) | 1.0               | 0.22             | 0.03                 |

a) state experimentally known

b) state experimentally known to be splitted at 5.55 MeV and 5.80 MeV.

Estimated total cross section (relative to g.s.) for the reaction  $Po^{208}(C^{14},O^{14})Pb^{208}$ . The excitation energies listed in the first column are in MeV and the incident energy is 260 MeV. In the case of states with two or four phonons, the listed energy is the sum of the energies corresponding to the single-phonon states. The third column displays the phonon configurations of each final state. The fourth column displays the rough estimate given in the previous workshop using two-neutron stripping and pick-up experiments with light ions.

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# CALCULATION OF TRANSITION OPERATORS FROM REALISTIC INTERNUCLEON FORCES

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In this work we discuss two methods for calculating the on and off-shell scattering amplitude for nucleon-nucleon scattering. The main advantage of these two methods is that any of the presently available realistic internucleon forces (<sup>1-3</sup>) can be easily handled, whether they have a hard core or not. Thus we have a unique and systematic scheme to compare the different internucleon interactions and to study how they differ among themselves.

It is well known that all the phenomenological nucleon-nucleon forces are defined by fitting the elastic scattering amplitudes for nucleon-nucleon scattering and the deuteron binding energy. However they give different results in many-body calculations due to their different off-shell properties. These off-shell properties can be studied by calculating the transition operator matrix elements for the two nucleon system (<sup>4</sup>). The transition operators play an important role in many-body calculations since they may be used to eliminate the reference to the, generally singular, internucleon force.

The transition operator (t-matrix) is defined by the Lippmann-Schwinger integral equation (<sup>4</sup>)

$$t(z) = V + V \frac{1}{z - K} t(z) , \quad (1)$$

where  $z$  is a complex energy parameter,  $V$  is the internucleon force and  $K$  is the kinetic energy operator. In order to calculate the t-matrix elements  $\langle \vec{k}' | t(z) | \vec{k} \rangle$ , with  $|\vec{k}\rangle$  a plane wave state of momentum  $\vec{k}$ , we should like to discuss two alternative methods:

I) Transform eq. (1) into an inhomogeneous Schrödinger equation (<sup>5</sup>). In fact, if we introduce a correlated wave function  $|\Psi_{\vec{k},z}\rangle$  for the nucleon pair defined by

$$t(z) |\vec{k}\rangle = V |\Psi_{\vec{k},z}\rangle , \quad (2)$$

the correlated state  $|\Psi_{\vec{k},z}\rangle$  is solution of the following equation

$$(K + V - z) |\Psi_{\vec{k},z}\rangle = (k^2 - z) |\vec{k}\rangle . \quad (3)$$

We take  $\hbar = M = 1$ , with  $M$  the nucleon mass. Eq. (3), which is a Schrödinger equation with a source term, seems particularly suitable to handle hard core interactions as we can easily impose the correct boundary conditions in the core region. The momentum space t-matrix elements are given by

$$\langle \vec{k}' | t(z) | \vec{k} \rangle = (2\pi)^{-3/2} \int e^{-i\vec{k}' \cdot \vec{r}} V(r) \Psi_{\vec{k},z}(\vec{r}) d\vec{r} . \quad (4)$$

II) Solve directly the integral equation, eq. (1). In this method it seems convenient to calculate the t-matrix elements in the mixed coordinate-momentum representation,

$$\langle \vec{r} | t(z) | \vec{k} \rangle = V(r) \left| \langle \vec{r} | \vec{k} \rangle + \langle \vec{r} | \frac{1}{z - k} t(z) | \vec{k} \rangle \right| \quad (5)$$

and

$$\langle \vec{k}' | t(z) | \vec{k} \rangle = (2\pi)^{-3/2} \int e^{-i\vec{k}' \cdot \vec{r}} \langle \vec{r} | t(z) | \vec{k} \rangle d\vec{r} . \quad (6)$$

The main reason for eq. (5) is that we can treat the hard-core potentials as easily as the soft-core ones. Although the potential may have a hard core in eq. (5), from a technical (and practical) point of view, we can divide by  $V(r)$  provided that it does not vanish when we discretize the integral equation in the  $\vec{r}$  coordinate. Then the hard core problems disappear.

We have developed computer programmes to evaluate  $\langle \vec{k}' | t(z) | \vec{k} \rangle$  using both methods. All the components of the nuclear interaction have been considered. Therefore we can calculate the t-matrix elements in all the possible quantum states for the interacting nucleons. As an example we show in fig. 1 the results for the real and imaginary parts of the t-matrix for the singlet even state with total angular momentum  $J = 0$  at  $z = 30$  MeV. The input potentials are the Reid soft and hard core force (2). We observe in this state that the t-matrix elements are very similar at low momentum ( $k, k' < 2 \text{ fm}^{-1}$ ) but the presence of different cores makes a considerable difference for the high momentum components. It is worth mentioning that the same calculation with the Hamada-Johnston potential (1) produced extremely close results to those obtained with the Reid hard core force.

As a summary we may say that we have at present two very accurate methods to evaluate the on and off-shell t-matrix elements. These matrix elements can be directly used as an input

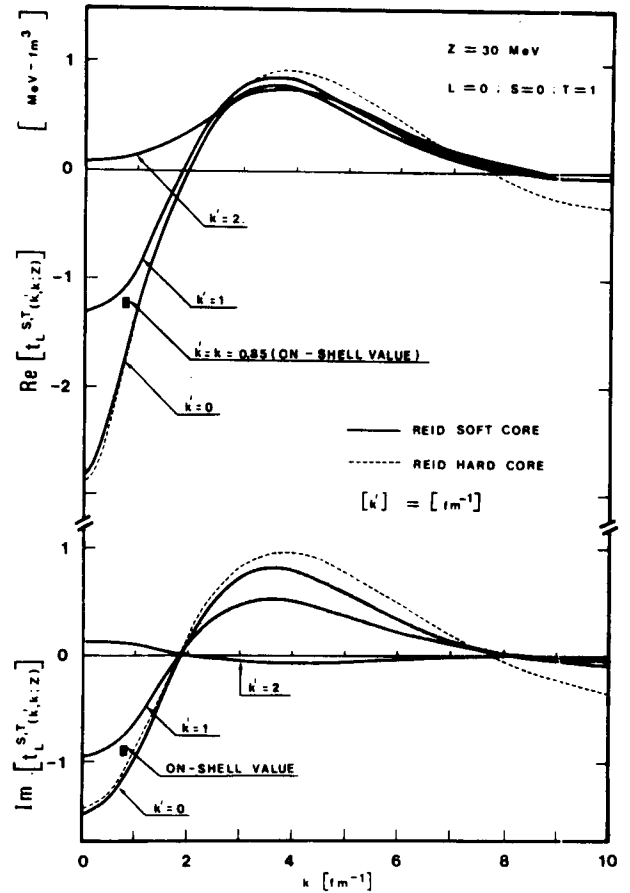


Fig. 1. Off-shell behaviour of the transition operator in a singlet even state.

for many-body calculations. They also provide a tool to evaluate effective forces.

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# ELASTIC SCATTERING FOR HEAVY ION SYSTEMS WITH ALPHA STRUCTURE

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It is a well known fact that the elastic scattering angular distributions of s-d shell nuclei with alpha structure present strong oscillations in the backward angles and a rise in the cross-section for  $\theta_{cm} = 180^\circ$ .

In order to interpret these oscillations as an interference between the elastic amplitude and a double alpha transfer amplitudes, the angular distributions of the elastic scattering  $^{24}\text{Mg}(^{12}\text{C}, ^{12}\text{C})^{24}\text{Mg}$  and of the alpha-transfer reaction  $^{24}\text{Mg}(^{12}\text{C}, ^{16}\text{O})^{20}\text{Ne}_{gs}$  and 1st. excited state, have been measured at 40 MeV incident energy in the  $20^\circ \leq \theta_{CM} \leq 110^\circ$  and  $20^\circ \leq \theta_{CM} \leq 70^\circ$  angular regions respectively.

The elastic scattering and the alpha transfer reaction angular distributions have been fitted by Frahn's closed formalism<sup>1,2)</sup> using the Ericson parametrization for the nuclear S-matrix. This permitted to obtain all the pertinent paramters:  $\Lambda$ ,  $\Delta$ ,  $\alpha$  for the elastic scattering  $\Lambda_T$ ,  $\Delta_T$ ,  $\alpha_T$  for the transfer and a normalization factor A which contains the alpha transfer spectroscopic factor and a dynamic factor.

The coupling between the elastic channel and the double alpha transfer channel was calculated using the closed formalism expressions derived by Frahn and Hussein<sup>3)</sup>, and the parameters determined by the elastic and transfer reactions. The results are presented on figure 1. This calculation shows that the coupling between the alpha transfer and elastic channel can account for the intermediate and backward angle oscillations observed in the elastic scattering angular distributions.

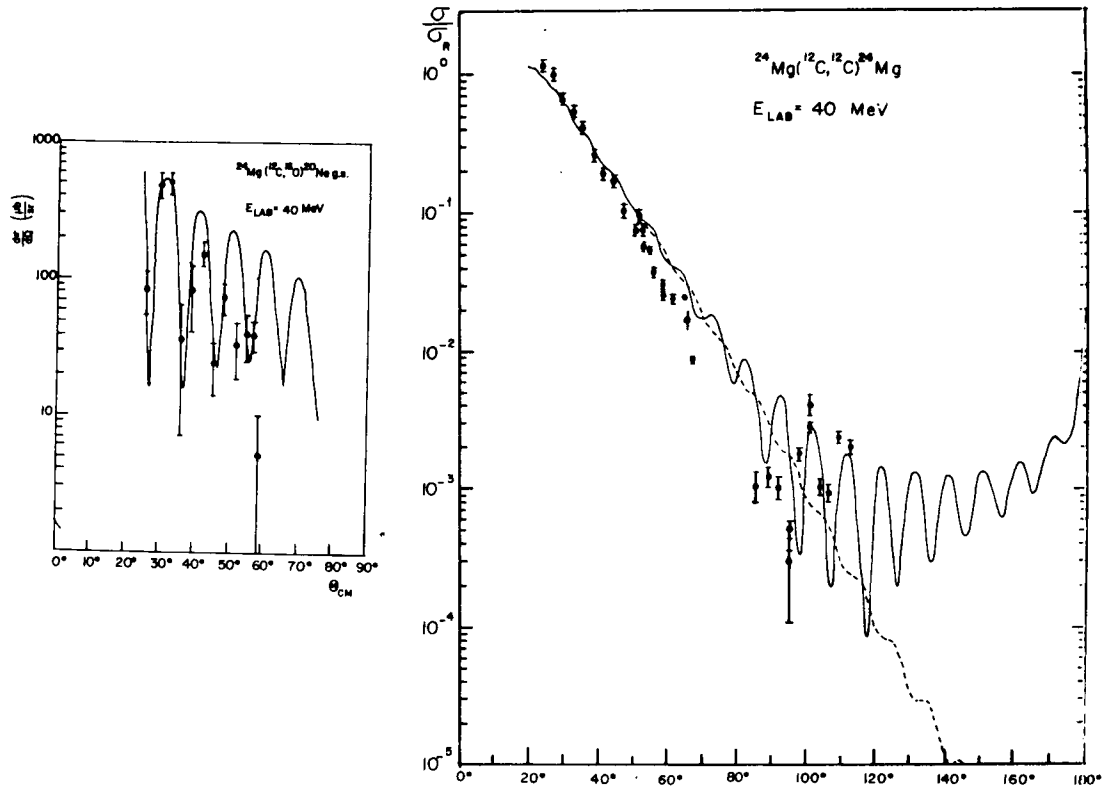


Figure 1

<sup>+</sup> Supported by FAPESP, CNPq

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# ELASTIC SCATTERING FOR HEAVY ION SYSTEMS NEAR THE COULOMB BARRIER

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The aim of this work is to obtain corrections to the optical interaction potential in order to reproduce the oscillations observed in the  $^{12}\text{C}+^{28}\text{Si}$  angular distributions.

The elastic scattering angular distributions have been measured at  $E_{\text{lab}} = 19, 21, 23$  and  $25$  MeV from  $\Theta_{\text{CM}} = 30^\circ$  to  $150^\circ$  using a  $^{12}\text{C}$  beam accelerated by the Pelletron Tandem at Sao Paulo. The scattered particles were detected by a PSD at forward angles and by a telescope system at back angles.

The analysis consists of an iterative method to relate variations in the S-matrix with corrections to the optical potential<sup>1)</sup>. The lowest order variation of  $S_l$  can be written expressed in terms of a perturbation  $V$  as:

$$\delta S_l = \frac{-i}{2k} \frac{2m}{\hbar^2} \int_0^\infty \delta V(r) u_l^2(r) dr \quad [1]$$

where  $\delta S_l = \Delta S_l = S_l - S_l^0$  and

$$\delta V(r) = \Delta V(r) = V(r) - V^0(r)$$

The values of  $S_l$  are given by an optical model analysis of the data using a Woods-Saxon potential  $V^0(r)$ .

Expanding  $\delta V(r)$  in terms of an arbitrary function  $f(r)$  and substituting it in the relation [1], one obtains:

$$\delta S_l = \frac{-i}{2k} \frac{2m}{\hbar^2} \sum_{i=0}^{i_{\text{max}}} F_i \int_{i\Delta R}^{(i+1)\Delta R} u_l^2(r) f(r) dr \quad [2]$$

where  $f(r)$  is chosen as a Woods-Saxon function. The fit obtained by the first iteration is presented in figure 1.

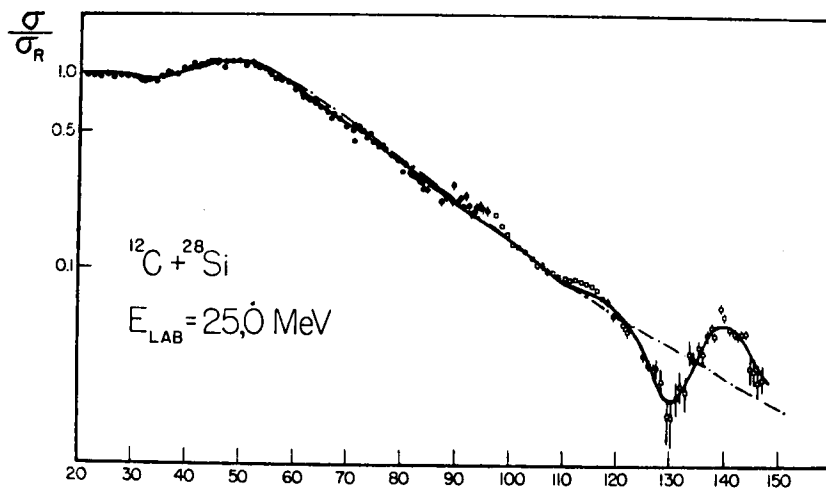


Fig. 1

Figure 1

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<sup>+</sup>  
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# Quantal description of the discontinuity in Mallmann's plot

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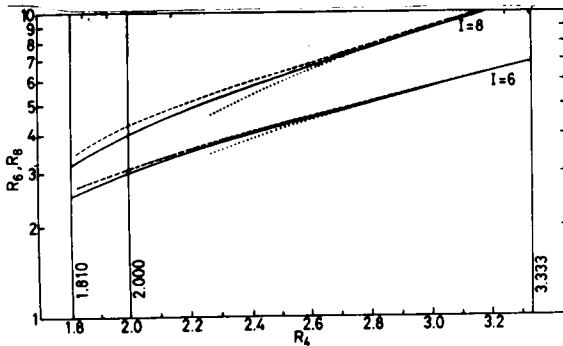
The first step in the direction of a universal description of ground state bands(gsb) of even-even nuclei was given by Mallmann<sup>1</sup> who showed that for a very large class of nuclei, their gbs-energies, when plotted in the form of energy ratios  $R_J = E_J/E_2$  ( $J=6,8$ ) vs  $R_4$ , arrange themselves on smooth curves suggesting the existence of a common mode of motion for all of them. Several semiclassical approaches were proposed to describe these curves, the most successful of which is known as the VMI model<sup>2,3</sup>. This model gave a surprisingly good description of gsb's of nuclei ranging from well deformed to near-harmonic and was subsequently extended<sup>4,5</sup> and shown to include some magic nuclei as well<sup>4</sup>. It was this last work which revealed the existence of a discontinuity in Mallmann's plot implying that two, instead of one, sets of curves were needed. Thus, almost all nuclei seem to lie approximately on two disjunct families of curves in Mallmann's plot separated by a sharp break which may reflect two fundamentally different types of motion of the nucleus. There certainly is at the present time a lack of deep understanding of this striking behavior. In the present work a quantal description is given for the right hand side of this discontinuity.

In spite of the success of the VMI in the near-harmonic to deformed region it was shown<sup>6</sup> that the centrifugal stretching model<sup>7</sup> (CSM) gives a somewhat improved description for so called spherical nuclei (defined through  $1.8 \leq R_4 \leq 2.0$ ). The idea here is to investigate the values of the ratios  $R_J$  but for the most simple quantized version of the CSM, i.e.:

$$\left[ -(\hbar^2/2B) \frac{d^2}{d\beta^2} + \hbar^2 I(I+1)/2B\beta^2 + \frac{1}{2}C(\beta-\beta_0)^2 \right] \chi(\beta) = E\chi(\beta) \quad (1)$$

The relevant limit  $\beta_0 \rightarrow -\infty$  corresponds to a situation in which the system is pushed against the origin with a force proportional to  $\beta_0$  thus giving maximum stability to the spherical shape. Eq. (1) gives a harmonic spectrum for zero equilibrium distance and follows the  $I(I+1)$  law for  $x_0 \rightarrow +\infty$ . The value of  $R_4$  for  $x_0 \rightarrow -\infty$  is 1.809. This number agrees remarkably with the smallest value found in nature namely  $R_4=1.807$  for <sup>94</sup>Mo, a nucleus which lies slightly beyond the range covered by the VMI and has 2 protons and 2 neutrons outside the closed core <sup>90</sup>Zr. The figure shows the curves obtained (solid) from eq. (1) in a Mallmann plot.

These curves merge into the VMI ones (dashed lines) for large values of



the deformation running otherwise below them, giving a somewhat better description of the data for  $1.81 \leq R_4 \leq 2.00$ .

Finally the relation between eq.(1) and the  $\beta$ -dependent part of the Bohr Hamiltonian is investigated. The results for an axially symmetric nucleus ( $\gamma$ -degree of freedom frozen) are also shown in the figure (dotted) and they give a description of transitional nuclei of much

less quality than eq.(1). This case is governed by a differential eq. of the same type as (1) but with a different "effective" angular momentum. On the other hand the  $\gamma=0$ ,  $\gamma=30^\circ$  and the  $\gamma$ -unstable vibrator cases almost coincide with the results of eq.(1) in the whole range  $-\infty < \beta \leq 0$ , except that they end at a somewhat larger value of  $R_4$ .

Thus, a common feature of all cases considered is that there is a definite endpoint for the  $R_4$  ratio (significantly larger than unity), resulting from the balance between the force which tries to keep the system spherical and the opposing centrifugal pressure, beyond which it is not possible to go with a single degree of freedom. A quantal treatment of a very simple system containing these two basic ingredients gives a fair description of the discontinuity in Mallmann's plot. Also the cases which are traditionally expected to describe best a spherical situation (namely  $\gamma=30^\circ$  and  $\gamma$ -unstable) come relatively close to the experimental break. The existence of this limiting value seems to reflect a basic property of the rotating-vibrating systems. Nuclei (as far as gsb's are concerned) seem to belong to two classes. The first one encompasses doubly and singly magic nuclei in the interval  $1 \leq R_4 \leq 1.8$  (and possibly some others) and no appropriate description seems possible in the framework of the stretching models considered here. The second class, apparently requires the opening of shells for both protons and neutrons and the underlying mode spans a wide range from spherical to well deformed nuclei. The "seed" for this mode (which may be associated with a collective degree of freedom) seems to be laid already very near to magic numbers and an understanding of the discontinuity in Mallmann's plot from a more fundamental point of view might provide clues for the understanding of the phenomenon of deformation itself. Attempts are

underway to reproduce microscopically this feature<sup>8)</sup>. Meanwhile the significance of the striking accuracy with which solutions of eq.(1) give the  $R_4$  value of the break-point in Mallmann's plot and the dynamics associated with this eq. remains to be assessed.

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Structure of  $^{186}\text{Ir}$  and general conditions for decoupling in doubly odd nuclei

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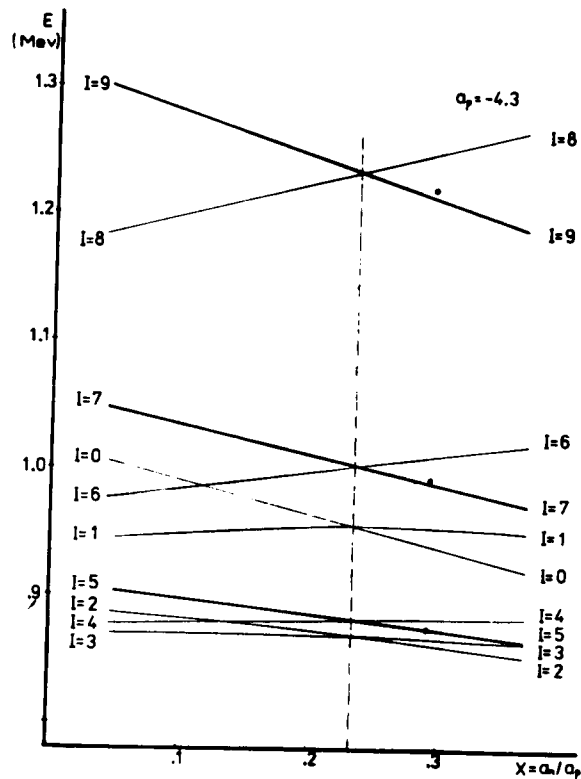
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Nothing was known until now about the structure of  $^{186}\text{Ir}$  except the existence of two long lived isomeric states of spin-parities ( $2^-$ ) and  $5(+)$ . In this contribution we discuss the first results obtained for this nucleus from in-beam  $\gamma$ -ray spectroscopy experiments performed in two different laboratories.  $^{186}\text{Ir}$  was populated using the  $^{181}\text{Ta}(^9\text{Be}(40-55\text{MeV}), 3n)$  and the  $^{187}\text{Re}(\alpha(55\text{MeV}), 5n)$  (carried out at the Brookhaven National Lab. Tandem and at the Buenos Aires Cyclotron respectively). The most interesting result is a cascade of five transitions (energies are 117.1, 241.3, 362.0, 473.6, and 574.0 keV) showing positive anisotropies. This result together with the lack of cross-over transitions indicate the quite uncommon existence of a decoupled band. Since the  $(\text{H.I.}, xn)$  reaction brings a sizable amount of angular momentum into the compound system the feeding is most likely predominantly into the  $5(+)$  state. Hence, the available evidence suggests the presence of a band with a spin-parity sequence  $5(+), 7(+), 9(+)$ , etc. It is known<sup>1</sup> that in doubly odd systems involving high-j excitations for both protons and neutrons, decoupling (i.e.  $\Delta I=2$  bands) exists only if both particles satisfy the conditions<sup>2</sup> for decoupling. In the present case, however, the existing information about intrinsic states in neighboring odd-mass nuclei only offers a decoupled band for protons, namely the  $\tilde{\pi}_{h_{9/2}}$  in odd Ir isotopes. In particular, in  $^{185}\text{Ir}$  the ground state band is based on this excitation<sup>3</sup>. On the other hand,  $K^\pi=1/2^-$  states of low spin parentage are known<sup>4</sup> for the neutrons. The coupling of these two types of states and the conditions for decoupling for such a system are studied. Also in this case it is concluded that to have  $\Delta I=2$  behavior in the odd-odd nucleus it is necessary that both participating particles decouple. The figure shows the level structure obtained from a two-quasiparticle plus rotor calculation, neglecting the p-n residual force, for the  $\tilde{\pi}_{h_{9/2}} \otimes \nu(K^\pi=1/2^-)$ -system as a function of

the ratio between the two decoupling parameters  $a_n/a_p$ . The dots correspond to the experimental points and it is seen that the present theoretical framework is able to provide a natural explanation for the situation encountered in  $^{186}\text{Ir}$ .



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# AN INTERACTING QUARTET-BOSON MODEL

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An alternative boson model is proposed for the description of positive-parity spectra in heavy even-even nuclei. The building blocks of this model are obtained by grouping the valence neutrons and protons into as many quartets (Q) as possible, and the remaining alike nucleons into pairs<sup>1)</sup> (P). Both quartets and pairs are then represented by s- and d-bosons, in analogy to the Interacting Boson Model (IBM)<sup>2)</sup>. The attractive features of such a model are: a) The quartets include the neutron-proton (n-p) interaction, leading to a model in which the strongest part of the nucleon-nucleon force is included automatically; b) Its relative simplicity, since the number of bosons is quite smaller than in the IBM.

The n-p interaction is responsible for nuclear deformation<sup>1,3)</sup>, it is expected that the Q-bosons will be responsible for the rotational features of the spectra, while the P-bosons will give rise to the vibrational patterns.

The model is applied as a first state to the very fashionable case of <sup>168</sup>Er, with nine Q-bosons ( $N_Q=9$ ). Experimentally its spectrum is known in great detail, including nine positive parity K-bands<sup>4)</sup>. Theoretically, it has been the subject of several recent studies<sup>4,5)</sup>. The most general Q-Q Hamiltonian contains in this case only six parameters and the small size of the energy matrices allows for a least-square search of them. Thirty-three experimental energy levels were included in the search, with a r.m.s. deviation of only 50 keV. The results are compared with experiment in Table 1.



Experimental and calculated positive-parity energy levels of  $^{168}\text{Er}$ .

TABLE I

| $K_n$   | J   | $E_{\text{exp}}$<br>keV | $E_{\text{cal}}$<br>keV | $K_n$   | J   | $E_{\text{exp}}$<br>keV | $E_{\text{cal}}$<br>keV |
|---------|-----|-------------------------|-------------------------|---------|-----|-------------------------|-------------------------|
| $0_1^+$ | 0   | 0                       | 0                       | $0_3^+$ | 0   | 1422                    | 1412                    |
|         | 2   | 79                      | 77                      |         | 2   | 1493                    | 1526                    |
|         | 4   | 264                     | 255                     |         | 4   | 1656                    | 1584                    |
|         | 6   | 548                     | 536                     |         | 6   | 1902                    | 1792                    |
|         | 8   | 928                     | 919                     | $3_1^+$ | 3   | 1653                    | 1722                    |
|         | 10  | 1396                    | 1404                    |         | 4   | 1736                    | 1650                    |
|         | 12  | 1942 <sup>a,b)</sup>    | 1994                    |         | 5   | 1839                    | 1715                    |
|         | 14  | (2564) <sup>b,d)</sup>  | 2654                    |         | 6   | 1961                    | 1904                    |
|         | 16  | -----                   | 3520                    |         | 7   | -----                   | 1988                    |
|         | 18  | -----                   | 4491                    |         |     |                         |                         |
| $2_1^+$ | 2   | 821                     | 868                     | $2_2^+$ | 2   | 1848                    | 1775                    |
|         | 3   | 895                     | 937                     |         | (3) | 1915                    | 1961                    |
|         | 4   | 994                     | 1033                    |         | (4) | 2002                    | 1927                    |
|         | 5   | 1117                    | 1150                    |         | 5   | 2108                    | 2035                    |
|         | 6   | 1263                    | 1288                    | $2_3^+$ | 2   | 1930                    | 1942                    |
|         | 7   | 1432                    | 1452                    |         | (3) | 1994 <sup>b)</sup>      | 2398                    |
|         | 8   | 1624                    | 1642                    |         | 4   | 2080                    | 2077                    |
|         | 9   | (1836) <sup>b,d)</sup>  | 1851                    | $4_1^+$ | 4   | 2030                    | 2087                    |
|         | 10  | (2071) <sup>b,d)</sup>  | 2081                    |         |     |                         |                         |
|         |     |                         |                         | $4_2^+$ | 4   | 2055 <sup>b)</sup>      | 2272                    |
| $0_2^+$ | 0   | 1217                    | 1185                    |         | 5   | 2169                    | 2172                    |
|         | 2   | 1276                    | 1262                    |         |     |                         |                         |
|         | 4   | 1411                    | 1426                    | c)      | 0   | -----                   | 1892                    |
|         | 6   | 1616                    | 1692                    | c)      | 0   | -----                   | 2216                    |
|         | (8) | (1890) <sup>b)</sup>    | 2034                    | c)      | 2   | 2137                    | 2119                    |

Also BE(2) relative transition rates were calculated and found in good agreement with experiment except for the transitions from the odd J members of the gamma-band. This may reflect the influence of a g-boson, or indicate that the calculations should be performed with  $N_Q=7$  and  $N_p=2$  rather than with  $N_Q=9$ , since in the later case the protons exceed the middle of the shell.

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"Cluster of nucleons as elementary modes of excitation in nuclei"

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Nuclear spectra have frequently been described in terms of elementary modes of excitation involving clusters of nucleons<sup>1)</sup>. An example of such modes is the use of pairs of nucleons coupled to  $J=0, T=1$  superfluidity and pairing vibrations<sup>2)3)</sup>. This cluster of two nucleons forms the building block which accounts for the collective aspects associated with two-particle correlations in nuclei. Pairs of nucleons coupled to higher angular momenta were also considered many years ago<sup>4)5)</sup> and revitalized more recently within the IBA<sup>6)</sup> in which the ground states of even nuclei are described as a condensate of pairs of nucleons carrying angular momenta 0(s-bosons) and 2(d-bosons).

A microscopic approach in the same direction is provided by the multi-step shell-model method<sup>7)</sup> (MSM), where clusters are successively added to an inert core. The size of the cluster can be left free within reasonable limits. The MSM provides an amenable procedure to deal with heavy systems where standard shell-model calculations are not feasible.

The existence of excitation modes with larger number of particles was considered within the quartet model<sup>8)</sup> some years ago. Within this picture the building blocks are  $\alpha$ -clusters. The reason for this choice was clearly suggested by the large stability of the  $\alpha$ -particle, and models using this cluster have been applied since the early times of nuclear physics<sup>9)</sup>. The quartet model had a reasonable success in the

description of low energy features of nuclear spectra of light nuclei ( $A \leq 50$ ).

Elementary modes of excitation must be constructed so that the effective interaction among them is as small as possible while the energy gained (correlation energy) in their construction is as big as possible. In order to study the correlation energy it is useful to define

$$S(A, \delta A) = B(A + \delta A) + B(A - \delta A) - 2 * B(A)$$

where  $B$  is minus the binding energy (i.e., they are negative numbers). If  $\delta A$  is taken to be one nucleon,  $S(A, 1)$  represents the two-quasiparticle energy. Although this eq. is generally used to support the interpretation of pairing modes as elementary degrees of freedom in nuclei<sup>10)</sup>, it does not give a direct measure of the interaction between the correlated clusters  $\delta A$ . The importance of this interaction is better seen by defining the quantity

$$C(A, \delta A_1, \delta A_2) = S(A, \delta A_1) + S(A, \delta A_2) - S(A, \delta A_1 + \delta A_2)$$

A cluster  $\delta A$  can be considered as a relevant mode of excitation when  $C(A, \delta A_1, \delta A_2)$  is positive for all possible partitions of  $\delta A$  into the fragments  $\delta A_1$  and  $\delta A_2$ . It is enough that one combination of  $\delta A_1$  and  $\delta A_2$  gives negative values of  $C$  to invalidate the cluster  $\delta A$  as a possible building block, because in this case it would be energetically favorable to use the uncorrelated configuration. The only cluster of four nucleons to fulfill this condition throughout the periodic table is the  $\alpha$ -like cluster, which includes two neutrons and two protons. One may think that other cases, like the four neutron cluster, should have

some relevance since the pairing interaction is very important in this case. However, since for identical nucleons

$$C(A,2,2) = 2 S(A,2) - S(A,4) = 2 S(A,2) (1 - R(A)/2)$$

and  $R$  is always larger than 2,  $C(A,2,2)$  turns out to be always negative. This is a manifestation of the blocking induced by the Pauli principle. In fact, of all possible four-nucleon clusters the  $\alpha$ -like cluster is the one which feels less that blocking.

In figs. 1(a) and 1(b) we show  $C(A, n^2, p^2)$  and  $C(A, d, d)$ , respectively, where  $n^2(p^2)$  indicates the two-neutron (proton) cluster and  $d$  indicates the deuteron cluster. In both cases  $C$  is always positive, showing that the  $\alpha$ -cluster gains energy over the uncorrelated  $T=1$  and  $T=0$  pairs. The quantities  $C(A, t, p)$  and  $C(A, {}^3\text{He}, n)$  are also always positive and of the same order of magnitude than  $C(A, d, d)$ . One can thus consider the  $\alpha$ -cluster as an elementary mode of excitation that includes simultaneously the  $T=0$  and  $T=1$  channels of the residual interaction. In fact, when an  $\alpha$ -cluster is built as two  $T=1$  pairs (fig. 1(a)) one may expect that the pairing correlations are already contained in these pairs giving a small value of  $C$ .

In this work we have studied different possible modes of excitation as clusters with more than one particle. The correlation aspects as well as the energy gained by these modes were analysed in terms of simple criteria associated with experimental binding energies. We have thus found for the two-particle cluster the known normal and superconductive behaviour of nuclei. However, we have also found that protons tend to behave as better superconductors than neutrons.

The application of the same criteria to more complex clusters, showed that only  $\alpha$ -like clusters can be considered as elementary modes. This suggests that the best way to study microscopically nuclei

away from magic numbers, would be first to maximize the number of  $\alpha$ -clusters. Afterwards one must bring in pairs of identical particles and, if necessary, consider, finally, the corresponding one, two or three particle cluster.

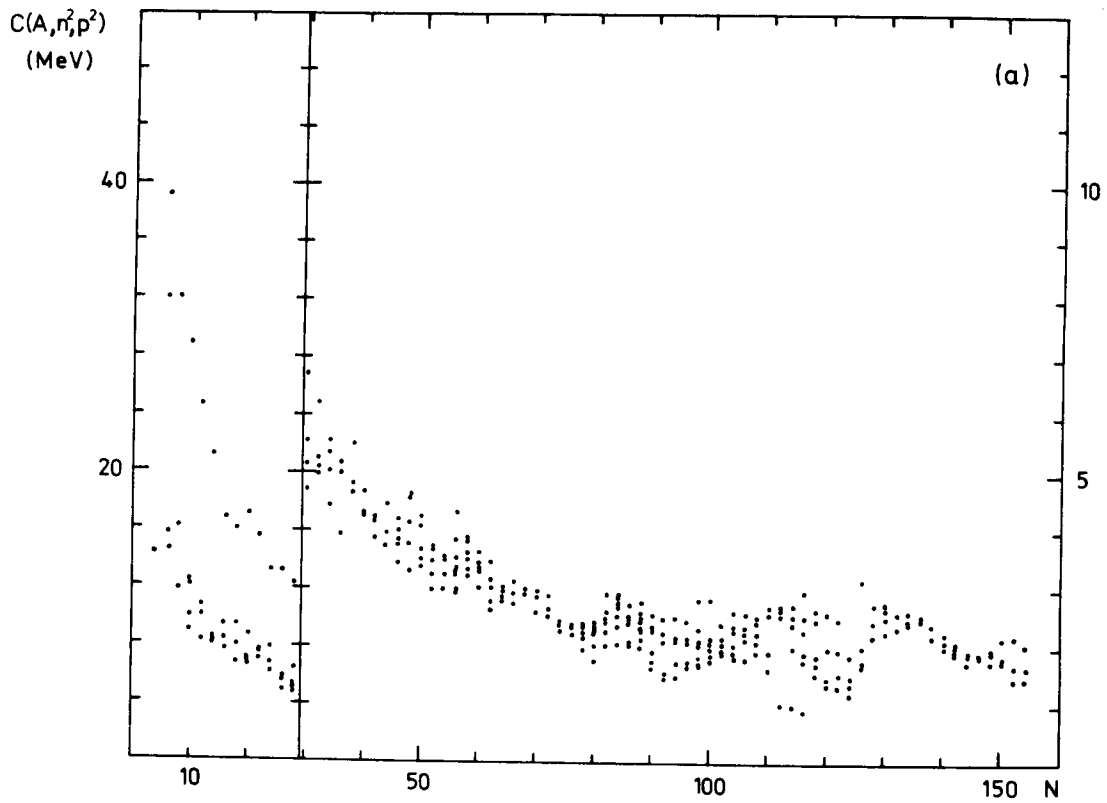


Figure 1a

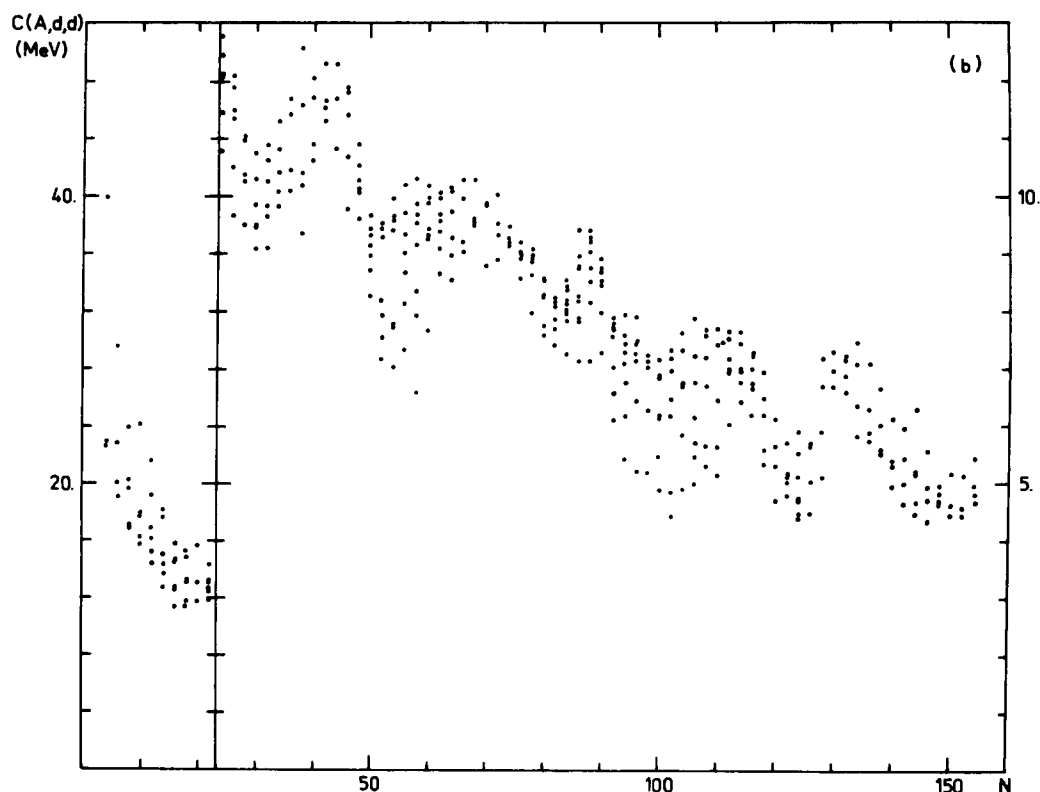


Figure 1b

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## LIFETIMES IN $^{91}\text{Zr}$

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Levels in  $^{91}\text{Zr}$  were populated upto an excitation energy of about 2.2 MeV by means of the  $^{88}\text{Sr}(\alpha, n)$  reaction. Angular distributions and excitation functions ( $E_{\alpha}=9.2-10.8$  MeV) of the gamma spectra were measured. The ground state transitions from the 1.205, 1.466, 1.882, 2.041 and 2.132 MeV levels show appreciable Doppler shifts and should yield measurable lifetimes for these levels. The known  $B(E2\uparrow)$  value for the 1.205 MeV transition leads to a lifetime of a factor of 4 larger than a previous DSAM measurement.

## ENERGY LOSS OF Ag IONS IN SOLIDS

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The measurements of the energy loss of heavy ions recoiling in foils of different atomic numbers (reported last year) have been completed. Now we have data for Ag recoiling in Al, Ti, V, Fe, Ni, Zn, Zr and Pd. In each case the energies of the recoiling Ag ions vary between  $\sim 90$  to  $\sim 160$  keV/AMU.

In the past year a program has been adapted to simulate the gamma line shapes resulting from the influence of the slowing down of recoiling ions. A Monte Carlo analysis takes into account the effects of particle-gamma angular correlations, finite detector size, and nuclear and electronic slowing down of the projectiles in the target and in the stopping medium.



## THE UPGRADE OF SADE

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### 1. Introduction

SADE (Stochastic Data Acquisition System for the São Paulo Pelletron Laboratory)<sup>1)</sup> was designed and built in the early 70's and has been operational for the last ten years. In the beginning a Honeywell DDP516 ran a stand alone program that supervised data storage and reduction in a multichannel basis. Afterwards an IBM/360-44 was linked to the system to supply software and mass storage. The front-end interface is fast and flexible allowing that 5 ADC's be scanned.

### 2. Today's Situation

After the development of a logical device that softened the task of multiparametric data collecting<sup>2)</sup>, this experiment mode became more frequent at the laboratory. This device allows the specification of a subset of the 5 ADC's as elements (parameters) that belong to an "event" where the reading (but not arriving) sequence and timing (time window) may be predefined in a simple way. Data acquired in this way are stored on /360 mass storage for afterwards reduction and exhibition in the form of bidimensional density plots<sup>3)</sup>. A primitive on line exhibition is still possible through the one-dimensional oriented CRT station.

### 3. Evolution

#### 3.1 Development in Course

The DDP516 was installed with 12 k words of 16 bits. This memory was extended to 16 kw and the next 16 kw (32 kw is maximum capacity) are

now being built. This extension aims at the real time exhibition of multidimensional spectra, but will also allow for improvements in man-machine interface.

A 24-bit word computer -PADE (Stochastic Data Acquisition Processor)<sup>4)</sup> built at the laboratory was attached to the /360-44 and to a graphic terminal (TEKTRONIKS 4014) and is being supplied with graphic support software.

### 3.2 Short Term

An intelligent graphic terminal (TELA) designed to display up to 32 K points (flicker free) will be linked to the lab's computers. It will be dedicated to on line processing, relieving the TEK 4014 from this task. A semi-interpretative language was designed to ease interaction with arriving data, providing also commands for file handling and graph manipulation. Terminal's architecture is adherent to 2-dimensional density plots exhibition but supports incremental plotting and graph mode.

A small subsystem designed to collect non-reaction information about the experiment will be included to permit fast logging of scalars, timers, etc. This device (DATAWAY) is under test (first module) and software for its support will be included in SADE's new release.

The old /360-44 is under change by a 2 Mb-4341 which is supposed to have enough processing power ( $\sim 1$  MIPS) for the increased system's load.

### 3.3 Medium Term

A 384 Kb memory will be added to PADE's to permit spectra manipulation for graphic purposes. A hardware event processor (around bit slices) with capability to fast select events pertaining to a subset of the n-dimensional space defined by parameter cluster will be

installed at the acquisition system's input port outperforming software based raw data processing.

### 3.4 Conclusion

The SADE, through the modifications exposed above seems still to exhibit certain longevity. The biggest menace the system faces is the aging of DDP516 for which maintenance needs may grow out of control. It is cogitated a complete reformulation of the acquisition facilities including substitution of DDP516 and its link to /360 by a self contained machine with a modern software design philosophy: high level languages and data base orientation.

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# CORE POLARIZATION EFFECTS ON THE MAGNETIC FORM FACTOR OF $^{17}\text{O}$

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The magnetic form factor of  $^{17}\text{O}$  (ground state) has been determined<sup>1)</sup> using elastic electron scattering. It is connected to the magnetic multipole operator  $T_{\lambda\mu}(q)$  by:

$$|F_M(q)|^2 = \frac{4\pi}{q^2} \frac{1}{2J+1} \sum_{\lambda} \langle J || T_{\lambda}(q) || J \rangle^2$$

For elastic scattering, the  $5/2^+$  ground state of  $^{17}\text{O}$  allows the transverse multipolarities  $M1, M3, M5$ . Using one body currents for the evaluation of  $T_{\lambda\mu}(q)$  and assuming the  $^{17}\text{O}$  to be a pure single particle state on top of an inert core, theoretical form factors were evaluated; the results show a considerable discrepancy with experimental data.

Two basic corrections are introduced<sup>2,3)</sup>; one is to include in the magnetic multipole operator the effects of two body meson exchange currents; the other consists in taking into account core polarization corrections. The pion exchange currents tend to produce an overall (but little) enhancement of single particle values. Calculations including up to second order core polarization effects are reported. This were evaluated using a G matrix obtained from a Reid soft core potential. Corrections thus induced, do not improve significantly the agreement with experimental points.

The idea of the present calculation is to evaluate up to second order effects, that do take into account explicitly the correlations already present in the core; as can be observed in the spectra of the  $^{16}\text{O}$ . This is done within the formalism of the Nuclear Field Theory (NFT)<sup>4)</sup>. The coupling between fermionic and bosonic degrees of freedom is given by

$$H_{FB} = - \sum_{\lambda\mu} K_{\lambda}(r) Y_{\lambda\mu}^*(\Omega) \alpha_{\lambda\mu}$$

where  $\alpha_{\lambda\mu}$  is related to the boson creation (annihilation) operator  $c_{\lambda\mu}^+$  ( $c_{\lambda\mu}$ ) by

$$\alpha_{\lambda\mu} = \frac{i^{-\lambda}}{2\lambda+1} (c_{\lambda\mu}^+ + c_{\lambda\mu})$$

The single particle form factor  $K_{\lambda}(r)$  is taken to be the derivative of the central Woods-Saxon potential. The fermion basis has an harmonic oscillator radial dependence, and the eigenenergies are the experimentally known from odd neighbour nuclei. The boson parameters are deduced from measured  $B(E\lambda)$ , electron or hadron scattering, or theoretically calculated for giant resonances.

Comparison with experimental curves show, the large effects of core polarization, which reproduce the correct trend although disagreement for large values of  $q$  (momentum transferred) continues. The fact that the agreement is considerable better than second order effects calculated with realistic forces; points to the conclusion that the renormalization procedure implied in the evaluation of the  $G$  matrix probably does not take into account the large collective surface oscillations already present in the core.

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