1.D.2

Nuclear Physics **A123** (1969) 81—98; © *North-Holland Publishing Co., Amsterdam*

ELECTRIC QUADRUPOLE TRANSITION PROBABILITIES IN EVEN NUCLEI

E. Y. DE AISENBERG, M. L. PÉREZ and J. F. SUÁREZ

Espectroscopía Nuclear, Comisión Nacional de Energía Atómica, Buenos Aires, Argentina

Received 2 June 1968

Abstract: The collective E2 transition rates between allowed positive-parity states of even nuclei are computed using the rotational component of the wave function as well as its β- and γ-part. In 65 % (90) of 163 experimental intensity ratios compared with predicted values, agreement is found within a factor 2 (10). Axial asymmetry plays an important role to predict transition probabilities in the lanthanide region.

1. Introduction

The Hamiltonian equation proposed by Bohr [1]) to describe the collective excitations of nuclei has been solved for the case of quadrupole deformation by Davydov [2]) and by one of the present authors [3]) in different approximations. In both papers the so-called displaced harmonic oscillator potential $V(\beta\gamma) = \frac{1}{2}C(\beta-\beta_0)^2+\frac{1}{2}C_\gamma(\beta_0^4/\beta^2)(\gamma-\gamma_0)^2$ has been adopted.

Numerical calculations of level schemes and their comparison with the positive-parity levels of even nuclei have been recently [3,4]) performed. The fitting of experimental data was satisfactory for β-deformed nuclei, and unexpectedly good in some cases, i.e. ^{112}Cd, two holes away from a magic number, for which the density of 0^+ excited states was correctly predicted.

The simple phenomenological model considers the nucleus as a rotating body with shape oscillations. The spatial orientation of the rotor is defined by the Euler angles θ_v and the nuclear shape by the intrinsic coordinates β and γ. In the approximations proposed, which consider approximately harmonic oscillations of small amplitude, the wave function can be written as a product of the rotational wave function and the β- and γ-wave functions. The transition probabilities can thus be evaluated separately for each of the variables.

As M1 transitions are to a high extent forbidden, one must attribute any substantial M1 admixture into the transition amplitude to degrees of freedom other than collective (in these cases the energy fitting is not good). We shall be concerned only with the most intense allowed transitions, i.e. the electric quadrupole ones.

The E2 transition probabilities of an asymmetric rotor have been calculated by Day *et al.* [5]). Davydov and Chaban [6]) have pointed out that for transitions between levels of the ground state rotational band, the contribution of the β-vibration to the transition probabilities is smaller than $\approx 20\,\%$ which is within experimental error and need not be considered. Since the completion of ref. [6]), Ge(Li) detectors are widely used, and relative gamma transition intensities have been measured much better and justify the calculation of this contribution.

Our present aim is to calculate with the complete wave function the E2 transition probabilities for transitions between levels of the lowest rotational bands and to compare the results with the available experimental data.

2. Notation, parameters and wave functions

The nth energy level of angular momentum J is labelled $i\lambda nJ$, where $i = 1, 2, 3, \dots$ and $\lambda = 0, 1, 2, \dots$ are quantum numbers corresponding, respectively, to the β- and γ-vibrational modes. Each pair $i\lambda$ labels a band whose levels are those allowed by the mirror symmetry imposed on the wave function, namely: ${}^{n}J = {}^{1}0;\ {}^{1,2}2;\ {}^{1}3;\ {}^{1,2,3}4;\ {}^{1,2}5; \dots$

In this paper we only deal with transitions between levels of the ground-state rotational band (g.s. band) ${}^{n}J \equiv 10nJ$; transitions from levels of the beta-vibration rotational band (β-excited band) ${}^{n}J \equiv 20nJ$ to levels of the g.s. band and transitions from levels of the gamma-vibration rotational band (γ-excited band) ${}^{n}J \equiv 11nJ$ to levels of the g.s. band; their gamma intensities and reduced transition probabilities we indicate respectively (e.g.) by $I({}^{1}2\beta \to {}^{2}4)$ and $B(\mathrm{E2};\ {}^{2}4_{\gamma} \to {}^{1}3)$.

The four parameters of the model are the energy of the first excited state $\varepsilon({}^{1}2)$, the axial deformation parameter γ_0 and the numbers $\mu[=(\hbar^2/BC\beta_0^4)^{\frac{1}{4}}]$ and $G[=(\hbar^2/BC_{\gamma}\beta_0^4)^{-\frac{1}{2}}]$ which are related to the mean square amplitudes of the β- and γ-vibration in the g.s. band by

$$\left\langle\left(\frac{\beta-p\beta_0}{\beta_0}\right)^2\right\rangle = \tfrac{1}{2}\mu^2\left(4-\frac{3}{p}\right)^{-\frac{1}{2}} \quad \text{and} \quad \langle(\gamma-\gamma_0)^2\rangle = \frac{1}{2G} \tag{1}$$

(for the meaning of p see eq. (5); G stands for $\sqrt{D}$ of ref. [2]) and Δ of ref. [3])). The values of the parameters used were those obtained in refs. [3,4]) for the fitting of energy levels.

As mentioned before, for a stationary state corresponding to a definite angular momentum J, the wave function may be written as a product of three functions

$$\Pi_{nJ}(\beta, \gamma, \theta_v) = F_i(\beta)g_{\lambda}(\gamma)\Psi_{nJ}(\theta_v). \tag{2}$$

The function Π_{nJ} is defined in a space volume element

$$\mathrm{d}\tau = \beta^4|\sin 3\gamma|\mathrm{d}\beta\mathrm{d}\gamma\mathrm{d}\theta_1 \sin\theta_2\mathrm{d}\theta_2\mathrm{d}\theta_3 .$$

The eigenfunction for the β-dependent part of the Hamiltonian (eq. (1.9) of ref. [2]); eq. (2.7) of ref. [3]))

$$\left[-\frac{\hbar^2}{2B}\frac{\mathrm{d}^2}{\mathrm{d}\beta^2}+\tfrac{1}{2}C(\beta-\beta_0)^2+\frac{\hbar^2(\Lambda+2)}{2B\beta^2}-E\right]\beta^2F_i(\beta) = 0, \tag{3}$$

is expressed in terms of the Hermite polynomials $H(\zeta)$,

$$F_i(\beta) = \beta^{-2} H_{i-1}(\zeta) \exp\left(-\tfrac{1}{2}\zeta^2\right),$$
$$\zeta = \frac{1}{\mu^4} \frac{\beta - p\beta_0}{\beta_0} \left(4 - \frac{3}{p}\right), \tag{4}$$

being p a solution of

$$p^3(p-1) = \mu^4(\Lambda+2), \tag{5}$$

where Λ is the separation parameter between the (γ, θ_v) and (β) parts of the Hamiltonian. The boundary condition $F(\beta) \to 0$ for $\beta \to \infty$ is fulfilled for $i = 1, 2, 3 \ldots$ while the boundary condition $F(\beta) \to 0$ at $\beta = 0$ is rather well satisfied if $p \geqq 0.95$ and $\mu \leqq 0.5$.

The expression depending on the γ-variable (eq. (3.10) of ref. [2]) is

$$\left\{\frac{\partial^2}{\partial\gamma^2} + \tfrac{9}{4}[1 + \sin^{-2} 3\gamma] - \tfrac{1}{4} \sum_{x=1}^{3} \frac{\hat{I}_x^2}{\sin^2(\gamma - \frac{2}{3}\pi x)} - G^2(\gamma-\gamma_0)^2 + \Lambda\right\} \varphi(\gamma, \theta_v) = 0. \tag{6}$$

The second and third terms are made γ-independent replacing them by their values at $\gamma = \gamma_0$ (ref. [2]) or by their mean values evaluated with the γ-dependent wave function (ref. [3]). Thus eq. (6) divides into two equations:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\gamma^2} - G^2(\gamma-\gamma_0)^2 + L\right] \sqrt{|\sin 3\gamma|}\, g_\lambda(\gamma) = 0, \tag{7}$$

whose eigenfunction is

$$g_\lambda(\gamma) = \frac{1}{\sqrt{|\sin \beta\gamma}} H_\lambda[G^{\frac{1}{2}}(\gamma-\gamma_0)] \exp\left[-\tfrac{1}{2}G(\gamma-\gamma_0)^2\right]$$

and

$$\left[\frac{1}{2} \sum_{x=1}^{3} \frac{\hat{I}_x^2}{\sin^2(\gamma_0 - \frac{2}{3}\pi x)} - \varepsilon_{nJ}\right] \Psi_{nJ}(\theta_v) = 0 \tag{8a}$$

or

$$\left[\frac{1}{2} \sum_{x=1}^{3} \frac{\hat{I}_x^2}{\langle\sin^2(\gamma - \frac{2}{3}\pi x)\rangle} - \varepsilon_{nJ}\right] \Psi_{nJ}(\theta_v) = 0, \tag{8b}$$

where Ψ_{nJ} results in

$$\Psi_{nJ, M} = \sum_K A_{nJ, K} \psi_{nJ, M}(\theta_v). \tag{9}$$

The coefficients $A_{nJ, K}$ are functions of the moments of inertia through the ratio $k = (2\mathscr{I}_2^{-1} - \mathscr{I}_3^{-1} - \mathscr{I}_1^{-1})/(\mathscr{I}_3^{-1} - \mathscr{I}_1^{-1})$; the $\psi_{nJ, M}(\theta_v)$ functions are

$$\psi_{nJ, M}(\theta_v) = \left(\frac{2J+1}{16\pi^2(1+\delta_{K, 0})}\right)^{\frac{1}{2}} [D^J_{M, K}(\theta_v) + (-)^J D^J_{M, -K}(\theta_v)].$$

3. Electric quadrupole transition probabilities

3.1. GENERAL

The probability per unit time T of the E2 radiative transition of energy E between the states $(i'\lambda'n'J')$ and $(i\lambda nJ)$ is

$$T(\mathrm{E2};\, i'\lambda'n'J' \to i\lambda nJ) = \frac{8\pi(3)}{2(5!!)^2}\,\frac{1}{\hbar}\left(\frac{E}{\hbar c}\right)^5 B(\mathrm{E2};\, i'\lambda'n'J' \to i\lambda nJ), \tag{10}$$

where

$$B(\mathrm{E2};\, i'\lambda'n'J' \to i\lambda nJ) = \sum_{M,\eta} |\langle i\lambda nJ|Q(2,\eta)|i'\lambda'n'J'\rangle|^2 \tag{11}$$

is the reduced transition probability. The electric quadrupole operator $Q(2, \eta)$ is referred to a space-fixed coordinate system. It is related to the operator referred to the body-fixed system by the transformation

$$Q(2,\eta) = \sum_{\eta'} D^2_{\eta,\eta'} Q'(2,\eta'), \tag{12}$$

being

$$Q'(2,\eta') = e\sum_{1}^{q} \alpha_j'^2 Y_{2,\eta'}(\theta'_{vj}, \varphi'_j),$$

where q is the total number of charges.

In the hydrodynamical model, the matrix elements are given by

$$\begin{aligned}\langle i\lambda nJ|Q(2,\eta)|i'\lambda'n'J'\rangle &= e\sum_{\eta'} \langle \psi_{nJ,M}(\theta_v)|D^2_{\eta,\eta'}|\psi_{n'J',M'}(\theta_v)\rangle\\ &\quad\times\langle F_i(\beta)g_\lambda(\gamma)\Gamma|\sum_{1}^{q} \alpha_j'^2 Y_{2,\eta'}(\theta'_{vj}, \varphi'_j)|F_{i'}(\beta)g_{\lambda'}(\gamma)\Gamma\rangle\\ &= \frac{3eR^2}{4\pi}\,[\langle \psi_{nJ,M}(\theta_v)|D^2_{\eta,0}|\psi_{n'J',M'}(\theta_v)\rangle\langle F_i(\beta)g_\lambda(\gamma)|\beta\cos\gamma|F_{i'}(\beta)g_{\lambda'}(\gamma)\rangle\\ &\quad+\langle \psi_{nJ,M}(\theta_v)|D^2_{\eta,2}+D^2_{\eta,-2}|\psi_{n'J',M'}(\theta_v)\rangle\langle F_i(\beta)g_\lambda(\gamma)\left|\frac{\beta\sin\gamma}{\sqrt{2}}\right|F_{i'}(\beta)g_{\lambda'}(\gamma)\rangle],\end{aligned} \tag{13}$$

and the reduced transition probability by

$$\begin{aligned}&\left(\frac{3eR^2}{4\pi}\right)^2 \sum_{M,\eta}\left[\langle \psi_{nJ,M}(\theta_v)|D^2_{\eta,0}+\frac{r}{\sqrt{2}}(D^2_{\eta,2}+D^2_{\eta,-2})|\psi_{n'J',M'}(\theta_v)\rangle\right]^2\\ &\qquad\times[\langle F_i(\beta)|\beta|F_{i'}(\beta)\rangle]^2[\langle g_\lambda(\gamma)|\cos\gamma|g_{\lambda'}(\gamma)\rangle]^2,\end{aligned} \tag{14}$$

where

$$r = \frac{\langle g_\lambda(\gamma)|\sin\gamma|g_{\lambda'}(\gamma)\rangle}{\langle g_\lambda(\gamma)|\cos\gamma|g_{\lambda'}(\gamma)\rangle}. \tag{15}$$

The matrix elements are products of three factors which can be evaluated separately.

3.2. THE ROTATIONAL REDUCED TRANSITION PROBABILITY

The rotational reduced transition probabilities $b(\mathrm{E2}; {}^{n}J \rightarrow {}^{n'}J')$ and $b(\mathrm{E2}; {}^{n}J_{\beta} \rightarrow {}^{n'}J')$ for $n \neq n'$ have been calculated by Day *et al.* [5]) as a function of the asymmetry parameters k and r. The rotational reduced transition probabilities $b(\mathrm{E2}; {}^{1}2_{\beta} \rightarrow {}^{1}2)$ and $b({}^{2}2_{\beta} \rightarrow {}^{2}2)$ can be calculated with the expression

$$b(\mathrm{E2}; {}^{1}2_{\beta} \rightarrow {}^{1}2) = b(\mathrm{E2}; {}^{2}2_{\beta} \rightarrow {}^{2}2) = \tfrac{2}{7}[-A_{{}^{1}2,0}^{2}+A_{{}^{1}2,2}^{2}+2rA_{{}^{1}2,0}A_{{}^{1}2,2}]^{2}, \tag{16}$$

where $A_{{}^{n}J,K}$ are the coefficients of the rotational wave functions (4) which are given in table 1 of ref. [5]) as a function of k.

Table 3 of ref. [5]) gives the values of $k(\gamma_0)$ for the rotor in the approximation used by Davydov [2]). For the approximation used in ref. [3]), the values of $k(\gamma_0, \Delta)$ are given for $\lambda = 0, 1$ in fig. 2 of that paper.

For transitions between members of the same rotational band ($i = i'$ and $\lambda = \lambda'$) or from levels of the β-excited band to levels of the g.s. band, $r = \mathrm{tg}\,\gamma_0$. For transitions from levels of the γ-excited band to levels of the g.s. band, $r = -1/\mathrm{tg}\,\gamma_0$.

Some rotational reduced transition probabilities are

$$\begin{aligned}
b(\mathrm{E2}; {}^{1}0_{\gamma} \rightarrow {}^{n}2) &= [A_{{}^{n}2,0}+rA_{{}^{n}2,2}]^{2},\\
b(\mathrm{E2}; {}^{n}2_{\gamma} \rightarrow {}^{1}0) &= \tfrac{1}{5}[A'_{{}^{n}2,0}+rA'_{{}^{n}2,2}]^{2},\\
b(\mathrm{E2}; {}^{1}2_{\gamma} \rightarrow {}^{1}2) &= \tfrac{2}{7}[(-A_{{}^{1}2,0}A'_{{}^{1}2,0}+A_{{}^{1}2,2}A'_{{}^{1}2,2})+r(A_{{}^{1}2,0}A'_{{}^{1}2,2}+A_{{}^{1}2,2}A'_{{}^{1}2,0})]^{2},\\
b(\mathrm{E2}; {}^{1}2_{\gamma} \rightarrow {}^{2}2) &= \tfrac{2}{7}[(A_{{}^{1}2,0}A'_{{}^{1}2,2}+A_{{}^{1}2,2}A'_{{}^{1}2,0})+r(A_{{}^{1}2,0}A'_{{}^{1}2,0}-A_{{}^{1}2,2}A'_{{}^{1}2,2})]^{2},\\
b(\mathrm{E2}; {}^{2}2_{\gamma} \rightarrow {}^{2}2) &= b(\mathrm{E2}; {}^{1}2_{\gamma} \rightarrow {}^{1}2).
\end{aligned} \tag{17}$$

The coefficients $A_{{}^{n}J,K}$ correspond to the band with $\lambda = 0$ and the $A'_{{}^{n}J,K}$ to that with $\lambda = 1$.

3.3. THE β-CONTRIBUTION TO THE REDUCED TRANSITION PROBABILITY

The factor of the transition probability corresponding to the β-variable is given by

$$\begin{aligned}
|\langle F_i(\beta)|\beta|F_{i'}(\beta)\rangle|^2 &= \left[\frac{\sqrt{2}\beta_0}{\sqrt{\mu_1\mu_1'}}\,\frac{a}{b}\sqrt{\frac{1}{b}}\exp\frac{a^2-bc}{2a}\right]^2 \qquad \text{for}\quad i = i' = 1\\
&= \left[\frac{2\beta_0}{\sqrt{\mu_1'^3\mu_1}}\,\frac{a}{b}\sqrt{\frac{1}{b}}\left(\frac{a}{b}-p'+\frac{1}{a}\right)\exp\frac{a^2-bc}{2a}\right]^2\\
&\qquad\qquad \text{for}\quad i' = 2;\quad i = 1\\
&= \left.\frac{2\sqrt{2}\beta_0}{\sqrt{\mu_1^3\mu_1'^3}}\,\frac{a}{b}\sqrt{\frac{1}{b}}\left\{pp'+\left(\frac{a}{b}\right)^2+\frac{3}{b}-(p+p')\left(\frac{a}{b}+\frac{1}{a}\right)\right\}\exp\frac{a^2-bc}{2a}\right]^2\\
&\qquad\qquad \text{for}\quad i = i' = 2,
\end{aligned} \tag{18}$$

TABLE 1

Comparison of experimental E2 probability ratios of transitions between levels of the g.s. band with theoretical values calculated with the parameters obtained in ref. [3]) (S) and in ref. [4]) (D)

		$\frac{I(^22\to{}^10)}{I(^22\to{}^12)}$	$\frac{I(^22\to{}^14)}{I(^22\to{}^12)}$	$\frac{I(^13\to{}^22)}{I(^13\to{}^12)}$	$\frac{I(^13\to{}^14)}{I(^13\to{}^12)}$	$\frac{I(^24\to{}^22)}{I(^24\to{}^12)}$	$\frac{I(^24\to{}^13)}{I(^24\to{}^12)}$	$\frac{I(^24\to{}^14)}{I(^24\to{}^12)}$
^{56}Fe	exp	9.0±3.0 (−1) a)		2.6±0.8 (−2)	2.9±0.9 (−1)			
	(D)	1.1 (0)		4.1 (−2)	1.6 (−1)			
	(S)	1.4 (0)		1.6 (−1)	1.4 (−1)			
^{76}Se	exp	7.1±0.7 (−1)						
	(D)	1.0 (0)						
	(S)	1.4 (0)						
^{106}Pd	exp	4.5±0.9 (−1)		5.6±1.0 (−1)	6.3±1.5 (−2)	3.0±0.6 (0)	3.2±1.0 (0)	4.4±0.8 (−1)
	(D)	5.9 (−1)		3.9 (−1)	5.4 (−2)	2.7 (0)	2.6 (−1)	1.2 (0)
	(S)	8.1 (−1)		3.9 (−1)	4.6 (−2)	2.2 (0)	3.1 (−1)	8.8 (−1)
^{110}Cd	exp	5.9±0.9 (−1)		5.0±1.2 (−1)	2.0±0.4 (−1)	3.5±0.7 (0)		8.6±1.7 (0)
	(D)	9.8 (−1)		4.2 (−1)	1.1 (−1)	7.8 (−1)		3.9 (−1)
	(S)	1.6 (0)		5.4 (−1)	8.7 (−2)	1.6 (0)		5.9 (−1)
^{112}Cd	exp	3.1±1.0 (−1)						
	(D)	1.1 (0)						
	(S)	2.0 (0)						
	exp	3.0±0.3 (−1)						

^{116}Cd	exp							
	(D)							
	(S)							
^{134}Ba	exp	2.2±0.4 (−1)		1.5±0.2 (0) d)	2.0±1.0 (−2) d)	2.8±0.5 (0)	6.1±3.0 (−3)	4.9±0.7 (0) b)
	(D)	1.7 (0)		4.8 (−1)	8.2 (−3)	2.0 (0)	1.7 (−2)	2.6 (−1)
	(S)	1.8 (−1)		2.9 (0)	5.6 (−2)	2.1 (0)	8.6 (−3)	2.5 (−1)
^{152}Sm	exp	9.1±1.8 (−1)			2.3±1.1 (−1)			1.4±0.8 (0)
	(D)	8.3 (−1)			2.4 (−1)			2.8 (0)
	(S)	1.2 (0)			1.1 (−1)			1.2 (0)
^{154}Gd	exp	2.7±0.7 (−1)	(1.5±1.0) (0)		5.2±2.6 (0)			3.5±1.0 (0)
	(D)	2.1 (−1)	6.5 (−1)		4.5 (0)			2.6 (0)
	(S)	2.1 (−1)	6.7 (−1)		4.8 (0)			2.4 (0)
^{156}Gd	exp	9.5±1.0 (−1)	5.1±1.5 (−2)		2.5±0.6(−1)	1.0±0.4 (0)	4.5±1.5 (−1)	2.5±1.0 (0)
	(D)	7.7 (−1)	3.3 (−2)		3.2 (−1)	6.9 (−2)	3.2 (−2)	3.0 (0)
	(S)	8.1 (−1)	4.9 (−2)		2.4 (−1)	1.2 (0)	5.3 (−1)	2.2 (0)
^{158}Dy	exp	7.6±0.3 (−1)			1.8±0.4 (−1)			1.7±0.3 (0)
	(D)	8.1 (−1)			2.6 (−1)			2.8 (0)
	(S)	8.2 (−1)			3.0 (−1)			1.4 (0)
^{160}Dy	exp	8.5±0.5 (−1)	2.1±0.6 (−2)	<8 (−3)	2.1±0.2 (−1)	<8 (−3)	<2.3 (−2)	2.3±0.2 (0)
	(D)	1.1 (0)	1.5 (−2)	→∞	1.3 (−1)	→∞	>4 (−2)	1.1 (0)
	(S)	7.4 (−1)	3.0 (−2)	2 (−4)	3.0 (−1)	1.4 (−2)	1.8 (−3)	3.3 (0)

TABLE 1 (continued)

		$\frac{I(^{2}4 \rightarrow {}^{1}6)}{I(^{2}4 \rightarrow {}^{1}2)}$	$\frac{I(^{1}5 \rightarrow {}^{1}3)}{I(^{1}5 \rightarrow {}^{1}4)}$	$\frac{I(^{1}5 \rightarrow {}^{2}4)}{I(^{1}5 \rightarrow {}^{1}4)}$	$\frac{I(^{1}5 \rightarrow {}^{1}6)}{I(^{1}5 \rightarrow {}^{1}4)}$	$\frac{B(E2; {}^{2}2 \rightarrow {}^{1}0)}{B(E2; {}^{1}2 \rightarrow {}^{1}0)}$	$\frac{B(E2; {}^{1}4 \rightarrow {}^{1}2)}{B(E2; {}^{1}2 \rightarrow {}^{1}0)}$	$\frac{B(E2; {}^{2}2 \rightarrow {}^{1}2)}{B(E2; {}^{1}2 \rightarrow {}^{1}0)}$
^{56}Fe	exp							
	(D)							
	(S)							
^{76}Se	exp						1.7±0.3 (0)	
	(D)						1.5 (0)	
	(S)						1.5 (0)	
^{106}Pd	exp		6.8±1.4 (−1)	9.8±0.4 (−1)				
	(D)		≈1 (4)	1.1 (1)				
	(S)		5.8 (2)	4.0 (0)				
^{110}Cd	exp		[f]	[f]	[f]	4.8±1.5 (−2)	1.5±0.2 (0)	1.3±0.3 (0)
	(D)					4.9 (−2)	1.8 (0)	1.0 (0)
	(S)					4.0 (−2)	1.8 (0)	5.0 (−1)
^{112}Cd	exp						1.8±0.2 (0)	1.5±0.3 (0)
	(D)						2.3 (0)	1.3 (0)
	(S)						1.6 (0)	4.7 (−1)
	exp						1.8±0.2 (0)	1.2±0.2 (0)

^{116}Cd	exp						1.6±0.4 (0)	7.4±1.7 (−1)
	(D)						2.1 (0)	8.7 (−1)
	(S)						1.9 (0)	3.2 (−1)
^{134}Ba	exp							
	(D)							
	(S)							
^{152}Sm	exp					2.0±0.4 (−2)		
	(D)					2.6 (−2)		
	(S)					2.2 (−3)		
^{154}Gd	exp					3.8±1.5 (−2)		
	(D)					3.2 (−2)		
	(S)					6.2 (−3)		
^{156}Gd	exp	2.8±1.2 (−1)	c)	1.5±0.6 (0) b)	8.0±3.0 (−1)	1.3±0.5 (−2)		
	(D)	2.0 (0)	5.1 (−2)	7.7 (−5)	8.6 (−1)	2.6 (−2)		
	(S)	1.7 (−1)	7.5 (−1)	3.4±2.0 (−3)	8.3 (−1)	1.1 (−3)		
^{158}Dy	exp				1.9±0.4 (−1)			
	(D)				2.6 (−1)			
	(S)				3.4 (−1)			
^{160}Dy	exp		3.3±0.6 (−2)	<1 (−2)	2.2±0.5 (−1)	1.6±0.5 (−2)		
	(D)		→∞	→∞	1.0 (−1)	1.6 (−4)		
	(S)		3.2 (−3)	2 (−3)	3.9 (−1)	5.5 (−2)		

TABLE 1 (continued)

		$\frac{I(^22 \to {}^10)}{I(^22 \to {}^12)}$	$\frac{I(^22 \to {}^14)}{I(^22 \to {}^12)}$	$\frac{I(^13 \to {}^22)}{I(^13 \to {}^12)}$	$\frac{I(^13 \to {}^14)}{I(^13 \to {}^12)}$	$\frac{I(^24 \to {}^22)}{I(^24 \to {}^12)}$	$\frac{I(^24 \to {}^13)}{I(^24 \to {}^12)}$	$\frac{I(^24 \to {}^14)}{I(^24 \to {}^12)}$
^{166}Er	exp	1.1±0.2 (0)	1.8±0.4 (−2)	6.8±3.0 (−4)	2.7±0.5 (−1)	1.2±0.3 (−2)	c)	1.8±0.4 (0)
	(D)	1.4 (0)	1.1 (−2)	→∞	1.0 (−1)	→∞	→∞	1.1 (0)
	(S)	8.8 (−1)	3.0 (−2)	2.6 (−4)	2.6 (−1)	2.0 (−2)	2.6 (−3)	3.0 (0)
^{182}W	exp	8.5±1.0 (−1)	1.6±0.2 (−3)		1.8±0.3 (−1)	7.1±3.0 (−1)		1.6±0.6 (−1)
	(D)	7.7 (−1)	2.9 (−2)		2.8 (−1)	2.2 (−3)		2.7 (0)
	(S)	1.0 (0)	2.9 (−2)		3.2 (−1)	1.2 (−2)		1.9 (0)
^{186}Os	exp	1.0±0.4 (0)	1.8±0.5 (−2)	4.8±1.0 (−3) a)	c)	1.4±0.5 (−1)	1.7±0.5 (−2) b)	1.3±0.4 (0)
	(D)	7.9 (−1)	6.3 (−3)	3.3 (−3)	2.6 (−2)	6.2 (−1)	5.9 (−2)	1.1 (1)
	(S)	8.9 (−1)	5.9 (−3)	5.5 (−3)	1.5 (−1)	4.7 (−1)	4.1 (−2)	4.6 (0)
^{188}Os	exp	1.3±0.3 (0)	c)	7.0±3.0 (−3)	5.9±1.0 (−2)	5.1±1.0 (−1) e)	e)	e)
	(D)	8.8 (−1)	5.9 (−4)	1.4 (−2)	8.8 (−2)	≈1 (2)		
	(S)	9.1 (−1)	6.6 (−4)	2.2 (−2)	8.4 (−2)	≈1 (2)		
^{190}Os	exp	1.4±0.2 (0)		<1.2 (−1)	≈2.1 (−2)	2.2±0.4 (0)	<5 (−1)	2.0±0.5 (0)
	(D)	1.4 (0)		7.6 (−2)	2.4 (−2)	6.4 (0)	3.6 (−1)	4.7 (0)
	(S)	1.2 (0)		1.1 (−1)	2.9 (−2)	6.7 (0)	4.1 (−1)	3.8 (0)
^{192}Pt	exp	1.8±0.4 (−1)		3.5±0.4 (0)	2.6±1.0 (−2)	1.6±0.2 (1)	1.0±0.5 (0)	2.5±0.3 (0)
	(D)	≈1 (−4)		≈1 (4)	≈1 (3)	≈3 (4)	2.9 (−1)	≈3 (3)
	(S)	2.0 (−1)		4.7 (0)	7.2 (−3)	3.7 (0)	1.4 (0)	3.4 (−1)
^{194}Pt	exp	9.1±0.2 (−2)						
	(D)	7.2 (−2)						
	(S)	8.5 (−2)						
^{196}Pt	exp	2 ±1 (−4)		6 ±3 (0)				
	(D)	≈1 (−4)		≈1 (4)				
	(S)	1.3 (−1)		5.6 (0)				
^{228}Th	exp	5.7±1.0 (−1)	4.4±1.0 (−2)		3.4±0.6 (−1)			3.8±0.7 (0)
	(D)	1.0 (0)	2.4 (−2)		2.0 (−1)			1.7 (0)
	(S)	7.6 (−1)	4.7 (−2)		4.1 (−1)			3.2 (0)
^{245}U	exp	(1.5) (0)			(4.4) (−1)			(1.7) (0)
	(D)	5.4 (0)			2.3 (−1)			1.5 (0)

TABLE 1 (continued)

		$\frac{I(^24 \to {}^16)}{I(^24 \to {}^12)}$	$\frac{I(^15 \to {}^13)}{I(^15 \to {}^14)}$	$\frac{I(^15 \to {}^24)}{I(^15 \to {}^14)}$	$\frac{I(^15 \to {}^16)}{I(^15 \to {}^14)}$	$\frac{B(\mathrm{E2}; {}^22 \to {}^10)}{B(\mathrm{E2}; {}^12 \to {}^10)}$	$\frac{B(\mathrm{E2}; {}^14 \to {}^12)}{B(\mathrm{E2}; {}^12 \to {}^10)}$	$\frac{B(\mathrm{E2}; {}^22 \to {}^12)}{B(\mathrm{E2}; {}^12 \to {}^10)}$
^{166}Er	exp	2 ±1 (−2)	4.5±1.5 (−2)		1.7±0.3 (−1)			
	(D)	6.8 (−3)	→∞		6.6 (−2)			
	(S)	5.7 (−2)	3.9 (−2)		2.9 (−1)			
^{182}W	exp							
	(D)							
	(S)							
^{186}Os	exp		1.6±0.6 (−1)		6.1±2.0 (−2)			
	(D)		3.5 (−1)		2.0 (−1)			
	(S)		4.8 (−1)		1.6 (−1)			
^{188}Os	exp					7.0±2.2 (−2)		1.9±0.6 (−1)
	(D)					7.2 (−2)		3.2 (−1)
	(S)					4.4 (−2)		2.0 (−1)
^{190}Os	exp		1.2±0.5 (0)			7.1±1.7 (−2)		4.1±0.9 (−1)
	(D)		3.7 (1)			7.2 (−2)	48 (1)	4.8 (−1)
	(S)		1.6 (1)			4.8 (−2)		3.1 (−1)
^{192}Pt	exp						4.5±1.0 (−1)	
	(D)						1.6 (0)	
	(S)						1.6 (0)	
^{194}Pt	exp							
	(D)							
	(S)							
^{196}Pt	exp							
	(D)							
	(S)							
^{228}Th	exp							
	(D)							
	(S)							
^{234}U	exp				3.0±0.6(−1)			
	(D)				2.5 (−1)			
	(S)				2.5 (−1)			

[a] M1 intensity deduced (97 % in ^{56}Fe; 86 % in the transition quoted in the numerator in ^{186}Os).

[b] The transition quoted in the numerator is M1+E2.

[c] The transition quoted in the numerator was observed, but no intensity assignment has been made.

[d] The transition quoted in the denominator is M1+E2.

[e] $\frac{I(^24 \to {}^22)}{I(^24 \to {}^14)}$ exp 4.6±1.0 (−1), (D) 2.6 (−1), (S) 4.0 (−1); $\frac{I(^24 \to {}^13)}{I(^24 \to {}^14)}$ exp [c], (D) 2.0 (−2), (S) 2.8 (−2)

[f] $\frac{I(^15 \to {}^13)}{I(^15 \to {}^24)}$ exp 1.4±0.3 (0), (D) 1.3 (0), (S) 1.3 (0); $\frac{I(^15 \to {}^24)}{I(^15 \to {}^16)}$ exp 4.9±1.0 (0), (D) 1.7 (1), (S) 2.0 (1)

TABLE 2

Comparison of experimental E2 probability ratios of transitions from a level of the β-excited band to levels of the g.s. band with theoretical values

	$\frac{I(^12_\beta \to {}^14)}{I(^12_\beta \to {}^12)}$	$\frac{I(^12_\beta \to {}^10)}{I(^12_\beta \to {}^14)}$		
^{152}Sm exp	7.5±2.0 (−1)	2.4±0.8 (−3)		
(D)	1.9 (−1)	1.6 (−3)		
(S)	2.0 (−1)	1.6 (−3)		
	$\frac{I(^12_\beta \to {}^10)}{I(^12_\beta \to {}^12)}$	$\frac{I(^12_\beta \to {}^14)}{I(^12_\beta \to {}^12)}$	$\frac{I(^14_\beta \to {}^12_\beta)}{I(^14_\beta \to {}^12)}$	$\frac{I(^14_\beta \to {}^16)}{I(^14_\beta \to {}^12)}$
^{154}Gd exp	5.9±4.5 (−1)	5.8±2.5 (−1)	6.7±1.6 (−1)	7.5±5.2 (−1)
(D)	6.9 (−1)	2.8 (−1)	1.2 (−1)	2.5 (−1)
(S)	6.9 (−1)	3.3 (−1)	1.2 (−1)	2.4 (−1)
	$\frac{I(^12_\beta \to {}^10)}{I(^12_\beta \to {}^12)}$	$\frac{I(^12_\beta \to {}^14)}{I(^12_\beta \to {}^12)}$	$\frac{B(\mathrm{E2};\, ^12_\beta \to {}^10)}{B(\mathrm{E2};\, ^22 \to {}^10)}$	
^{156}Gd exp	3.6±1.8 (−1)	6.0±3.0 (−1)	1.1±0.6 (0)	
(D)	8.0 (−1)	1.2 (0)	3.9 (−1)	
(S)	7.9 (−1)	1.2 (0)	9.2 (−1)	
	$\frac{I(^12_\beta \to {}^10)}{I(^12_\beta \to {}^12)}$			
^{158}Dy exp	7.5±1.5 (−1)			

		$\frac{I(^{1}2_\beta \to {}^{1}0)}{I(^{1}2_\beta \to {}^{1}2)}$	$\frac{I(^{1}2_\beta \to {}^{1}4)}{I(^{1}2_\beta \to {}^{1}2)}$	$\frac{I(^{1}4_\beta \to {}^{1}3)}{I(^{1}4_\beta \to {}^{2}2)}$	$\frac{I(^{1}4_\beta \to {}^{1}5)}{I(^{1}4_\beta \to {}^{2}2)}$	$\frac{I(^{1}4_\beta \to {}^{2}4)}{I(^{1}4_\beta \to {}^{2}2)}$	$\frac{I(^{1}6_\beta \to {}^{1}5)}{I(^{1}6_\beta \to {}^{1}4_\beta)}$
^{160}Dy	exp	≈6.4 (−1)	≈1.8 (1)	5.5±0.8 (−1)	1.4±0.3 (−2)	1.5±0.2 (−1)	2.5±1.3 (0)
	(D)	7.5 (−1)	1.3 (0)	8.4 (0)	3.6 (0)	6.6 (0)	0
	(S)	7.5 (−1)	1.3 (0)	6.4 (−1)	9.6 (−2)	6.0 (0)	1.1 (0)
		$\frac{I(^{1}0_\beta \to {}^{2}2)}{I(^{1}0_\beta \to {}^{1}2)}$					
^{166}Er	exp	3.2±1.6 (−2)					
	(D)						
	(S)	7.7 (−3)					
		$\frac{I(^{1}2_\beta \to {}^{2}2)}{I(^{1}2_\beta \to {}^{1}2)}$	$\frac{I(^{2}2_\beta \to {}^{1}0)}{I(^{2}2_\beta \to {}^{1}2)}$	$\frac{I(^{2}2_\beta \to {}^{2}2)}{I(^{2}2_\beta \to {}^{1}2)}$			
^{194}Pt	exp	7.0±1.5 (−1)	3.3±0.6 (−1)	2.1±0.4 (0)			
	(D)	4.3 (1)	≈2 (−4)	1.7 (−3)			
	(S)	4.7 (0)	4.9 (−3)	6.0 (0)			
		$\frac{I(^{1}3_\beta \to {}^{1}4)}{I(^{1}3_\beta \to {}^{1}2)}$	$\frac{I(^{2}4_\beta \to {}^{1}2)}{I(^{1}4_\beta \to {}^{1}4)}$				
^{228}Th	exp	2.5±0.5 (−1)	9.6±2.0 (−1)				
	(D)	1.4 (−1)	8.9 (−1)				
	(S)	2.8 (−1)	4.5 (−1)				
		$\frac{I(^{1}2_\beta \to {}^{1}0)}{I(^{1}2_\beta \to {}^{1}4)}$	$\frac{I(^{1}2_\beta \to {}^{1}4)}{I(^{1}2_\beta \to {}^{1}2)}$				
^{234}U	exp	1.0±0.3 (0)	4 ±2 (−1)				
	(D)	3.8 (−1)	9.4 (−1)				
	(S)	3.8 (−1)	9.4 (−1)				

Table 3

Comparison of experimental E2 probability ratios of transitions from a level of the γ-excited band to levels of the g.s. band with theoretical values

Nucleus		$\frac{I({}^12_\gamma \to {}^22)}{I({}^12_\gamma \to {}^12)}$	$\frac{I({}^12_\gamma \to {}^10)}{I({}^12_\gamma \to {}^12)}$	$\frac{I({}^10_\gamma \to {}^22)}{I({}^10_\gamma \to {}^12)}$	$\frac{I({}^22_\gamma \to {}^22)}{I({}^22_\gamma \to {}^12)}$	$\frac{I({}^22_\gamma \to {}^10)}{I({}^22_\gamma \to {}^12)}$	$\frac{I({}^12_\gamma \to 0_\gamma)}{I({}^12_\gamma \to {}^12)}$	$\frac{B(\mathrm{E2}; {}^12_\gamma \to {}^10)}{B(\mathrm{E2}; {}^12_\gamma \to {}^12)}$	$\frac{B(\mathrm{E2}; {}^10_\gamma \to {}^12}{B(\mathrm{E2}; {}^12 \to {}^10)}$
^{56}Fe	exp		>9.6 (−1)						
	(D)		6.5 (−1)						
	(S)		1.8 (−1)						
^{76}Se	exp	1.3±0.6 (−1)	2.0±1.0 (−1)						1.0±0.4 (0)
	(D)	2.4 (−2)	5.7 (−1)						3.7 (−2)
	(S)	2.1 (−2)	5.2 (−2)						3.0 (−2)
^{106}Pd	exp		1.1±0.4 (−1)						
	(D)		2.1 (−1)						
	(S)		4.7 (−2)						
^{112}Cd	exp							4.7±1.5 (−1)	
	(D)							5.8 (−2)	
	(S)							1.7 (−2)	
^{114}Cd	exp		1.2±0.4 (0)						8.5±1.7 (−1)
	(D)		5.8 (−1)						4.8 (−2)
	(S)		2.4 (−1)						3.8 (−2)
	exp								8.3±1.7 (−1)

^{152}Gd	exp		2.0±0.5 (−2)				1.4±0.5 (−1)
	(D)		1.9 (−2)				1.8 (−1)
	(S)		3.0 (−3)				2.4 (−1)
^{158}Dy	exp		6.2±1.2 (−1)				
	(D)		7.4 (−1)				
	(S)		8.8 (−1)				
^{166}Er	exp			3.2±1.5 (−2)			
	(D)			0			
	(S)						
^{188}Os	exp			1.1±0.2 (−1)			
	(D)			4.0 (−1)			
	(S)			6.5 (−1)			
^{190}Os	exp		6.5±1.5 (−1)	4.5±0.9 (−1)			
	(D)		3.7 (−1)	4.1 (−1)			
	(S)		1.4 (−1)	6.1 (−1)			
^{194}Pt	exp			2.6±0.3 (0)	1.4±0.2 (0)	1.3±0.2 (0)	
	(D)			7.6 (1)	9.8 (0)	7.9 (1)	
	(S)			5.2 (1)	7.6 (0)	6.2 (1)	
^{196}Pt	exp	2.4 (2)					
	(D)	0					
	(S)	3.6 (−3)					

where

$$a = \frac{p}{\mu_1^2} + \frac{p'}{\mu_1'^2}, \qquad b = \frac{1}{\mu_1^2} + \frac{1}{\mu_1'^2}, \qquad c = \left(\frac{p}{\mu_1}\right)^2 + \left(\frac{p'}{\mu_1'}\right)^2, \qquad \mu_1 = \mu\left(4 - \frac{3}{p}\right)^{\frac{1}{4}}.$$

3.4. THE γ-CONTRIBUTION TO THE REDUCED TRANSITION PROBABILITY

The factor of the transition probability corresponding to the γ-variable is given by

$$|\langle g_\lambda(\gamma)|\cos\gamma|g_{\lambda'}(\gamma)\rangle|^2 = \begin{cases} \cos^2\gamma_0 \exp(-1/2G) & \text{for } \lambda' = \lambda = 0 \\ \dfrac{\sin^2\gamma_0}{2G}\exp(-1/2G) & \text{for } \lambda' = 1, \quad \lambda = 0 \\ \left(\dfrac{1-2G}{2G}\right)^2 \cos^2\gamma_0 \exp(-1/2G) & \text{for } \lambda' = \lambda = 1. \end{cases} \qquad (19)$$

4. Results

The comparison of predicted E2 gamma-ray transition intensities (or reduced transition probabilities) with experimental data was made for 23 nuclei for which sufficient information was available.

The results for transition ratios between levels of the g.s. band are given in table 1, while results for transition ratios from a level of the β- or γ-excited band to levels of the g.s. band are given in tables 2 and 3, respectively.

5. Conclusions

Table 4 summarizes the degree of agreement of theory with experiment.

TABLE 4

Percentage of cases in agreement with experiment

Transitions		Within a factor		
		2 %	5 %	10 %
between levels of the g.s. band	D	62	77	83
	S	73	90	94
from levels of the β-band to g.s. band	D	61	70	70
	S	62	83	92
from levels of the γ-band to g.s. band	D	45	60	65
	S	32	63	63

The values of the reduced transition probability ratios between members of the g.s. band are mainly determined by the rotational part of the wave function. The corrections due to the β-part of the wave function depend on the parameters μ and on the energy of the involved levels; their values are smaller than 5 % for $\mu \lesssim 0.3$, about 10–25 % for $0.4 \leqq \mu \leqq 0.7$ and of the order of 35 % for $\mu \approx 0.8$. There are no corrections due to the γ-part of the wave function. The systematic deviation in the branching of gamma-ray transitions taking place between the $^{2}2$, $^{1}3$, $^{2}4, \ldots$ and $^{1}0$, $^{1}2$, $^{1}4$, $^{1}6, \ldots$ levels from the intensity rules governing quadrupole radiation of a symmetric rotor for nuclei of the lanthanide region have been removed by the small axial deformation ($0^\circ < \gamma_0 < 10^\circ$) which gives the best energy fit.

It should be emphasized that some b(E2) calculations are quite sensitive to the assumed axial-asymmetry k thus, the most striking discrepancies between predicted values in tables 1–3 are found generally for $k \approx \pm 1$. In cases like ^{160}Dy and ^{166}Er, the agreement between theoretical and experimental values is better using the parameters obtained in ref. ^{3}) than those found in ref. ^{4}).

The empirical material for transitions from levels of the β- or γ-excited bands is too scanty to allow definite conclusions to be drawn.

For transition ratios from a level of the β-excited band to levels of the g.s. band, the correction to the rotational transition probability ratios due to the β-part of the wave function may be of the order of 100 % even for $\mu < 0.3$. There is no correction due to the γ-part of the wave function.

The correction to the rotational transition probability ratios due to the β-part of the wave function for transitions from a level of the γ-excited band to levels of the g.s. band are of the same order as those for transitions between members of the g.s. band.

The 0^+ excited state of ^{166}Er is explained as a $^{1}0_\beta$ level in ref. ^{3}) and a $^{1}0_\gamma$ in ref. ^{4}); the transition probability ratio (see tables 2 and 3) is in agreement with the former prediction.

One of the authors (M.L.P.) acknowledges the moral and material support of the Bariloche and the J.S. Sauberan Foundations.

References

1) A. Bohr, Mat. Fys. Medd. Dan. Vid. Selsk. **26**, No. 14 (1952)
2) A. S. Davydov, Nucl. Phys. **24** (1961) 682
3) J. F. Suárez, to be published
4) E. Y. de Aisenberg and J. F. Suárez, Nucl. Phys. **A97** (1967) 529
5) P. P. Day and C. A. Mallmann, ANL 6184
6) A. S. Davydov and A. A. Chaban, Nucl. Phys. **20** (1960) 499

The references given below are those which did not appear in Nuclear Data up to and including vol. 2B, No 2, August 1967.

^{56}Fe C. Chasman and R. A. Ristinen, Phys. Rev. **159** (1967) 915;
P. F. Hinrichsen *et al.*, Nucl. Phys. **A101** (1967) 81
^{106}Pd K. D. Strutz, Z. Phys. **201** (1967) 20;
J. A. Moragues *et al.*, Nucl. Phys. **A106** (1968) 289
^{110}Cd J. A. Moragues *et al.*, Nucl. Phys. **A99** (1967) 652
^{110,112,114,116}Cd F. K. McGowan *et al.*, Nucl. Phys. **66** (1965) 97

^{134}Ba D. E. Raeside *et al.*, Nucl. Phys. **A98** (1967) 54;
R. A. Brown and G. T. Ewan, Nucl. Phys. **68** (1965) 325
^{154}Gd J. H. Hamilton *et al.*, Phys. Lett. **13** (1964) 43;
O. Lönsjö and G. B. Hagemann, Nucl. Phys. **88** (1966) 624
^{156}Gd N. F. Peek *et al.*, Phys. Rev. **136** (1964) B330;
P. F. Kenealy *et al.*, Nucl. Phys. **A105** (1967) 522
^{158}Dy A. A. Abdurazakov *et al.*, JINR P6-3464 (1967)
^{160}Dy N. A. Bonch-Osmolovskaya *et al.*, JINR P-2817 (1966);
J. Vrzal *et al.*, JINR P6-3312 (1967);
M. P. Avotina, Izv. Akad. Nauk SSSR (ser. fiz) **30** (1966) 530
^{166}Er S. B. Burson *et al.*, Phys. Rev. **158** (1967) 1161;
C. Gunther and D. R. Parsignault, Phys. Rev. **153** (1967) 1297;
G. Zylicz *et al.*, Nucl. Phys. **81** (1966) 88;
C. W. Reich and J. E. Cline, Phys. Rev. **137** (1965) Bl 424
^{190}Os B. Harmatz and T. H. Handley, Nucl. Phys. **56** (1964) 1;
M. A. Mariscotti *et al.*, BNL 11426 (1967)
^{192}Pt A. Schwarzschild, Phys. Rev. **141** (1966) 1206;
T. J. Palaska *et al.*, Nucl. Phys. **A95** (1967) 673;
L. Schellenberg and J. Kern, Helv. Phys. Acta **39** (1966) 420;
B. Lindström and J. Marklund, Nucl. Phys. **49** (1963) 609
^{194}Pt T. J. Palaska *et al.*, Nucl. Phys. **A95** (1967) 673
^{196}Pt J. F. W. Jansen and H. Pauw, Nucl. Phys. **A94** (1967) 235
^{234}U A. H. Wapstra, Nucl. Phys. **A97** (1967) 641;
S. Bjørnholm and O. B. Nielsen, Nucl. Phys. **30** (1962) 488